## Solomon Practice Paper

Core Mathematics 2K
Time allowed: 90 minutes

Centre: www.CasperYC.club
Name:
Teacher:

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 4 |  |
| 2 | 4 |  |
| 3 | 7 |  |
| 4 | 9 |  |
| 5 | 9 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| 8 | 11 |  |
| 9 | 11 |  |
| Total: | 75 |  |

## How I can achieve better:

1. Evaluate

$$
\int_{1}^{4} x^{2}-5 x+4 \mathrm{~d} x
$$ Last updated: May 5, 2023

2. Figure shows the curve with equation $y=\sqrt{4 x-1}$.


Use the trapezium rule with five equally-spaced ordinates to estimate the area of the shaded region bounded by the curve, the $x$-axis and the lines $x=1$ and $x=3$.
3. (a) Given that $y=\log _{2}(x)$, find expressions in terms of $y$ for
i. $\log _{2}\left(\frac{x}{2}\right)$,
ii. $\log _{2}(\sqrt{x})$.
(b) Hence, or otherwise, solve the equation

$$
2 \log _{2}\left(\frac{x}{2}\right)+\log _{2}(\sqrt{x})=8
$$

4. 

$$
\mathrm{f}(x)=2-x-x^{3}
$$

(a) Show that $\mathrm{f}(x)$ is decreasing for all values of $x$.
(b) Verify that the point $(1,0)$ lies on the curve $y=\mathrm{f}(x)$.
(c) Find the area of the region bounded by the curve $y=\mathrm{f}(x)$ and the coordinate axes.
5. Figure shows triangle $P Q R$ in which $P Q=7$ and $P R=3 \sqrt{5}$.


Given that $\sin (\angle Q P R)=\frac{2}{3}$ and that $\angle Q P R$ is acute,
(a) find the exact value of $\cos (\angle Q P R)$ in its simplest form,
(b) show that $Q R=2 \sqrt{6}$,
(c) find $\angle P Q R$ in degrees to 1 decimal place.
6. The polynomial $\mathrm{p}(x)$ is defined by

$$
\mathrm{p}(x)=2 x^{3}+x^{2}+a x+b,
$$

where $a$ and $b$ are constants.
Given that when $\mathrm{p}(x)$ is divided by $(x+2)$ there is a remainder of 20 ,
(a) find an expression for $b$ in terms of $a$.

Given also that $(x+3)$ is a factor of $\mathrm{p}(x)$,
(b) find the values of $a$ and $b$,
(c) fully factorise $\mathrm{p}(x)$.
7. (a) Find, to 2 decimal places, the values of $x$ in the interval $0 \leq x<2 \pi$ for which

$$
\tan \left(x+\frac{\pi}{4}\right)=3
$$

(b) Find, in terms of $\pi$, the values of $y$ in the interval $0 \leq y<2 \pi$ for which $2 \sin (y)=\tan (y)$.
8. The point $A$ has coordinates $(4,6)$.

Given that $O A$, where $O$ is the origin, is a diameter of circle $C$,
(a) find an equation for $C$.

Circle $C$ crosses the $x$-axis at $O$ and at the point $B$.
(b) Find the coordinates of $B$.
(c) Find an equation for the tangent to $C$ at $B$, giving your answer in the form $a x+b y=c$, where $a, b$ and $c$ are integers.
9. Figure shows part of a design being produced by a computer program.


The program draws a series of circles with each one touching the previous one and such that their centres lie on a horizontal straight line.

The radii of the circles form a geometric sequence with first term 1 mm and second term 1.5 mm . The width of the design is $w$ as shown.
(a) Find the radius of the fourth circle to be drawn.
(b) Show that when eight circles have been drawn, $w=98.5 \mathrm{~mm}$ to 3 significant figures.
(c) Find the total area of the design in square centimetres when ten circles have been drawn.

