Solomon Practice Paper

Core Mathematics 3E

Time allowed: 90 **minutes**

Question	Points	Score
1	5	
2	10	
3	11	
4	11	
5	12	
6	13	
7	13	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: July 14, 2025

1. Express [5]

$$\frac{2x^3 + x^2}{x^2 - 4} \times \frac{x - 2}{2x^2 - 5x - 3}$$

as a single fraction in its simplest form.

2. (a) Prove that, for $\cos(x) \neq 0$, [5]

$$\sin(2x) - \tan(x) \equiv \tan(x)\cos(2x).$$

(b) Hence, or otherwise, solve the equation [5]

$$\sin(2x) - \tan(x) = 2\cos(2x).$$

for x in the interval $0 \leq x \leq 180°$.

Total: 10

3.

$$f(x) = x^2 + 5x - 2\sec(x), \quad x \in \mathbb{R}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

(a) Show that the equation $f(x) = 0$ has a root in the interval $[1, 1.5]$. [2]

A more accurate estimate of this root is to be found using iterations of the form

$$x_{n+1} = \cos^{-1}(g(x_n)).$$

(b) Find a suitable form for $g(x)$ and use this formula with $x_0 = 1.25$ to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 3 decimal places. [6]

The curve $y = f(x)$ has a stationary point at P.

(c) Show that the x-coordinate of P is 1.0535 correct to 5 significant figures. [3]

Total: 11

4. (a) Differentiate each of the following with respect to x and simplify your answers. [6]

   i. $\sqrt{1-\cos(x)}$

   ii. $x^3\ln(x)$

   (b) Given that [5]

$$x = \frac{y+1}{3-2y},$$

find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

Total: 11

5. (a) Express $\sqrt{3}\sin(\theta) + \cos(\theta)$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

   (b) State the maximum value of $\sqrt{3}\sin(\theta) + \cos(\theta)$ and the smallest positive value of θ for which this maximum value occurs. [3]

   (c) Solve the equation [5]

$$\sqrt{3}\sin(\theta) + \cos(\theta) + \sqrt{3} = 0,$$

   for θ in the interval $-\pi \leq \theta \leq \pi$, giving your answers in terms of π.

Total: 12

6. The function f is defined by

$$\mathrm{f}(x) \equiv 3 - x^2, \qquad x \in \mathbb{R}, \qquad x \geq 0.$$

(a) State the range of f. [1]

(b) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}$ on the same diagram. [3]

(c) Find an expression for f^{-1} and state its domain. [4]

The function g is defined by

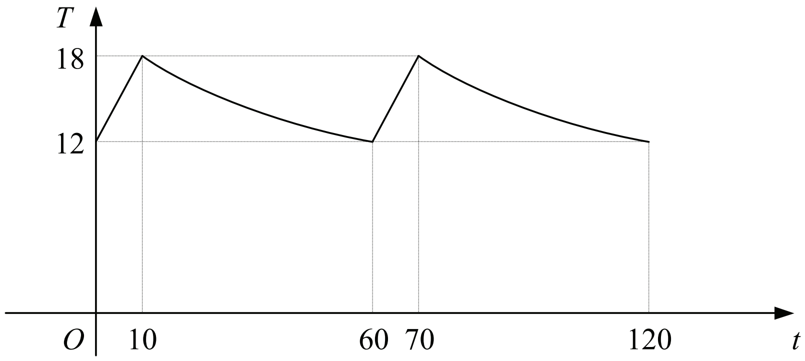
$$\mathrm{g}(x) \equiv \frac{8}{3 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 3.$$

(d) Evaluate $\mathrm{fg}(-3)$. [2]

(e) Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$. [3]

Total: 13

7. Figure shows a graph of the temperature of a room, T°C, at time t minutes.



The temperature is controlled by a thermostat such that when the temperature falls to 12°C, a heater is turned on until the temperature reaches 18°C. The room then cools until the temperature again falls to 12°C.

For t in the interval $10 \leq t \leq 60$, T is given by

$$T = 5 + Ae^{-kt},$$

where A and k are constants.

Given that $T = 18$ when $t = 10$ and that $T = 12$ when $t = 60$,

(a) show that $k = 0.0124$ to 3 significant figures and find the value of A, [6]

(b) find the rate at which the temperature of the room is decreasing when $t = 20$. [4]

The temperature again reaches 18°C when $t = 70$ and the graph for $70 \leq t \leq 120$ is a translation of the graph for $10 \leq t \leq 60$.

(c) Find the value of the constant B such that for $70 \leq t \leq 120$ [3]

$$T = 5 + Be^{-kt}.$$

Total: 13