Edexcel (U.K.) Pre 2017

Questions By Topic

S2 Chap03 Continuous Random Variables

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6. A continuous random variable X has probability density function f(x) where

$$f(x) = \begin{cases} k(4x - x^3), & 0 \leqslant x \leqslant 2, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive integer.

(a) Show that $k = \frac{1}{4}$.
(4)

Find

(b) E(X),
(3)

(c) the mode of X,
(3)

(d) the median of X.
(4)

(e) Comment on the skewness of the distribution.
(2)

(f) Sketch f(x).
(2)

Leave blank

6. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{1+x}{k}, & 1 \leqslant x \leqslant 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \dfrac{21}{2}$.

(3)

(b) Specify fully the cumulative distribution function of X.

(5)

(c) Calculate E(X).

(3)

(d) Find the value of the median.

(3)

(e) Write down the mode.

(1)

(f) Explain why the distribution is negatively skewed.

(1)

Leave blank

5. The continuous random variable X is uniformly distributed over the interval $\alpha < x < \beta$.

(a) Write down the probability density function of X, for all x.

(2)

(b) Given that $E(X) = 2$ and $P(X < 3) = \frac{5}{8}$ find the value of α and the value of β.

(4)

A gardener has wire cutters and a piece of wire 150 cm long which has a ring attached at one end. The gardener cuts the wire, at a randomly chosen point, into 2 pieces. The length, in cm, of the piece of wire with the ring on it is represented by the random variable X. Find

(c) $E(X)$,

(1)

(d) the standard deviation of X,

(2)

(e) the probability that the shorter piece of wire is at most 30 cm long.

(3)

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7. The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 2x^2 - x^3, & 0 \leqslant x \leqslant 1, \\ 1, & x > 1. \end{cases}$$

(a) Find $P(X > 0.3)$.

(2)

(b) Verify that the median value of X lies between $x = 0.59$ and $x = 0.60$.

(3)

(c) Find the probability density function $f(x)$.

(2)

(d) Evaluate $E(X)$.

(3)

(e) Find the mode of X.

(2)

(f) Comment on the skewness of X. Justify your answer.

(2)

Leave blank

8. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 0 < x \leqslant 3 \\ 2 - \frac{1}{2}x & 3 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the probability density function of X.

(3)

(b) Find the mode of X.

(1)

(c) Specify fully the cumulative distribution function of X.

(7)

(d) Using your answer to part (c), find the median of X.

(3)

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4. The continuous random variable Y has cumulative distribution function F(y) given by

$$\mathrm{F}(y)=\begin{cases}0 & y<1\\ k(y^4+y^2-2) & 1\leqslant y\leqslant 2\\ 1 & y>2\end{cases}$$

(a) Show that $k=\frac{1}{18}$.

(2)

(b) Find P($Y > 1.5$).

(2)

(c) Specify fully the probability density function f(y).

(3)

Leave blank

8. The continuous random variable X has probability density function f(x) given by

$$\mathrm{f}(x)=\begin{cases}2(x-2) & 2\leqslant x\leqslant 3\\ 0 & \text{otherwise}\end{cases}$$

(a) Sketch f(x) for all values of x. **(3)**

(b) Write down the mode of X. **(1)**

Find

(c) E(X), **(3)**

(d) the median of X. **(4)**

(e) Comment on the skewness of this distribution. Give a reason for your answer. **(2)**

Leave blank

7. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leqslant x < 1 \\ kx^3 & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{5}$ **(4)**

(b) Calculate the mean of X. **(4)**

(c) Specify fully the cumulative distribution function F(x). **(7)**

(d) Find the median of X. **(3)**

(e) Comment on the skewness of the distribution of X. **(2)**

Leave blank

4. The length of a telephone call made to a company is denoted by the continuous random variable T. It is modelled by the probability density function

$$f(t) = \begin{cases} kt & 0 \leqslant t \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is $\frac{1}{50}$.
(3)

(b) Find $P(T > 6)$.
(2)

(c) Calculate an exact value for $E(T)$ and for $Var(T)$.
(5)

(d) Write down the mode of the distribution of T.
(1)

It is suggested that the probability density function, $f(t)$, is not a good model for T.

(e) Sketch the graph of a more suitable probability density function for T.
(1)

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7. A random variable X has probability density function given by

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function F(x) can be written in the form $ax^2 + bx + c$, for $1 \leqslant x \leqslant 4$ where a, b and c are constants.

(3)

(b) Define fully the cumulative distribution function F(x).

(2)

(c) Show that the upper quartile of X is 2.5 and find the lower quartile.

(6)

Given that the median of X is 1.88

(d) describe the skewness of the distribution. Give a reason for your answer.

(2)

Leave blank

6. The three independent random variables A, B and C each has a continuous uniform distribution over the interval [0, 5].

(a) Find $P(A > 3)$.
(1)

(b) Find the probability that A, B and C are all greater than 3.
(2)

The random variable Y represents the maximum value of A, B and C.

The cumulative distribution function of Y is

$$F(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^3}{125} & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(c) Find the probability density function of Y.
(2)

(d) Sketch the probability density function of Y.
(2)

(e) Write down the mode of Y.
(1)

(f) Find $E(Y)$.
(3)

(g) Find $P(Y > 3)$.
(2)

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7.

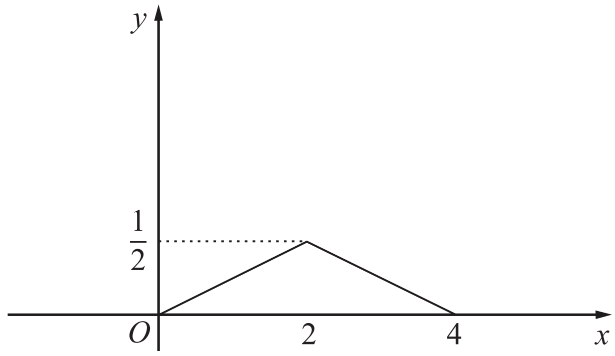

Figure 1

Figure 1 shows a sketch of the probability density function f(x) of the random variable X. The part of the sketch from $x = 0$ to $x = 4$ consists of an isosceles triangle with maximum at (2, 0.5).

(a) Write down E(X).

(1)

The probability density function f(x) can be written in the following form.

$$f(x) = \begin{cases} ax & 0 \leqslant x < 2 \\ b - ax & 2 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Find the values of the constants a and b.

(2)

(c) Show that σ, the standard deviation of X, is 0.816 to 3 decimal places.

(7)

(d) Find the lower quartile of X.

(3)

(e) State, giving a reason, whether $P(2 - \sigma < X < 2 + \sigma)$ is more or less than 0.5

(2)

Leave blank

2. A continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0, & x < -2 \\ \dfrac{x+2}{6}, & -2 \leqslant x \leqslant 4 \\ 1, & x > 4 \end{cases}$$

(a) Find $\mathrm{P}(X < 0)$.
(2)

(b) Find the probability density function $\mathrm{f}(x)$ of X.
(3)

(c) Write down the name of the distribution of X.
(1)

(d) Find the mean and the variance of X.
(3)

(e) Write down the value of $\mathrm{P}(X = 1)$.
(1)

Leave blank

4. The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 0 < x \leqslant 3 \\ 3k & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$

(4)

(b) Find the cumulative distribution function F(x).

(6)

(c) Find the mean of X.

(3)

(d) Show that the median of X lies between $x = 2.6$ and $x = 2.7$

(4)

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4. The lifetime, X, in tens of hours, of a battery has a cumulative distribution function F(x) given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3) & 1 \leqslant x \leqslant 1.5 \\ 1 & x > 1.5 \end{cases}$$

(a) Find the median of X, giving your answer to 3 significant figures.
(3)

(b) Find, in full, the probability density function of the random variable X.
(3)

(c) Find $P(X \geqslant 1.2)$
(2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

(d) Find the probability that the lantern will still be working after 12 hours.
(2)

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7. The random variable Y has probability density function f(y) given by

$$\mathrm{f}(y)=\begin{cases} ky(a-y) & 0\leqslant y\leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are positive constants.

(a) (i) Explain why $a \geqslant 3$

(ii) Show that $k=\dfrac{2}{9(a-2)}$

(6)

Given that E(Y) = 1.75

(b) show that a = 4 and write down the value of k.

(6)

For these values of a and k,

(c) sketch the probability density function,

(2)

(d) write down the mode of Y.

(1)

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5. A continuous random variable X has the probability density function f(x) shown in Figure 1.

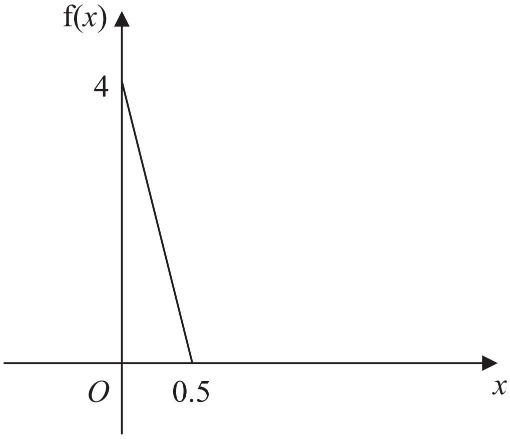

Figure 1

(a) Show that $f(x) = 4 - 8x$ for $0 \leqslant x \leqslant 0.5$ and specify f(x) for all real values of x.
(4)

(b) Find the cumulative distribution function F(x).
(4)

(c) Find the median of X.
(3)

(d) Write down the mode of X.
(1)

(e) State, with a reason, the skewness of X.
(1)

Leave blank

7. The queuing time in minutes, X, of a customer at a post office is modelled by the probability density function

$$f(x)=\begin{cases} kx(81-x^2) & 0\leqslant x\leqslant 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k=\dfrac{4}{6561}$.

(3)

Using integration, find

(b) the mean queuing time of a customer,

(4)

(c) the probability that a customer will queue for more than 5 minutes.

(3)

Three independent customers shop at the post office.

(d) Find the probability that at least 2 of the customers queue for more than 5 minutes.

(3)

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3.

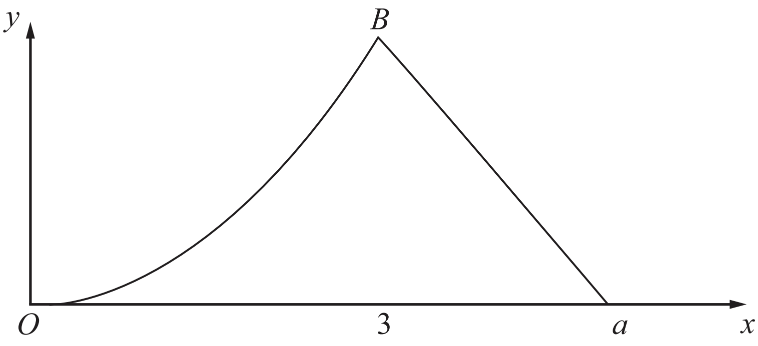

Figure 1

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X.

For $0 \leqslant x \leqslant 3$, $f(x)$ is represented by a curve OB with equation $f(x) = kx^2$, where k is a constant.

For $3 \leqslant x \leqslant a$, where a is a constant, $f(x)$ is represented by a straight line passing through B and the point $(a, 0)$.

For all other values of x, $f(x) = 0$.

Given that the mode of X = the median of X, find

(a) the mode,

(1)

(b) the value of k,

(4)

(c) the value of a.

(3)

Without calculating $E(X)$ and with reference to the skewness of the distribution

(d) state, giving your reason, whether $E(X) < 3$, $E(X) = 3$ or $E(X) > 3$.

(2)

Leave blank

7. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{32}(x-1)(5-x) & 1 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ showing clearly the points where it meets the x-axis. **(2)**

(b) Write down the value of the mean, μ, of X. **(1)**

(c) Show that $E(X^2) = 9.8$ **(4)**

(d) Find the standard deviation, σ, of X. **(2)**

The cumulative distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{32}(a - 15x + 9x^2 - x^3) & 1 \leqslant x \leqslant 5 \\ 1 & x > 5 \end{cases}$$

where a is a constant.

(e) Find the value of a. **(2)**

(f) Show that the lower quartile of X, q_1, lies between 2.29 and 2.31 **(3)**

(g) Hence find the upper quartile of X, giving your answer to 1 decimal place. **(1)**

(h) Find, to 2 decimal places, the value of k so that

$$P(\mu - k\sigma < X < \mu + k\sigma) = 0.5$$

(2)

Leave blank

6. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leqslant x < 1 \\ x - \frac{1}{2} & 1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$.
(2)

(b) Show that $k = \frac{1}{2}(1+\sqrt{5})$.
(4)

(c) Define fully the cumulative distribution function $F(x)$.
(6)

(d) Find $P(0.5 < X < 1.5)$.
(2)

(e) Write down the median of X and the mode of X.
(2)

(f) Describe the skewness of the distribution of X. Give a reason for your answer.
(2)

Leave blank

5. The queueing time, X minutes, of a customer at a till of a supermarket has probability density function

$$f(x) = \begin{cases} \frac{3}{32}x(k-x) & 0 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is 4
(4)

(b) Write down the value of E(X).
(1)

(c) Calculate Var(X).
(4)

(d) Find the probability that a randomly chosen customer's queueing time will differ from the mean by at least half a minute.
(3)

Leave blank

7. The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} \dfrac{x^2}{45} & 0 \leqslant x \leqslant 3 \\[2ex] \dfrac{1}{5} & 3 < x < 4 \\[2ex] \dfrac{1}{3} - \dfrac{x}{30} & 4 \leqslant x \leqslant 10 \\[2ex] 0 & \text{otherwise} \end{cases}.$$

(a) Sketch f(x) for $0 \leqslant x \leqslant 10$

(4)

(b) Find the cumulative distribution function F(x) for all values of x.

(8)

(c) Find $P(X \leqslant 8)$.

(2)

Leave blank

4. The continuous random variable X is uniformly distributed over the interval $[-4, 6]$.

(a) Write down the mean of X.

(1)

(b) Find $P(X \leqslant 2.4)$

(2)

(c) Find $P(-3 < X - 5 < 3)$

(2)

The continuous random variable Y is uniformly distributed over the interval $[a, 4a]$.

(d) Use integration to show that $E(Y^2) = 7a^2$

(4)

(e) Find $Var(Y)$.

(2)

(f) Given that $P(X < \frac{8}{3}) = P(Y < \frac{8}{3})$, find the value of a.

(3)

Leave blank

5. The continuous random variable T is used to model the number of days, t, a mosquito survives after hatching.

The probability that the mosquito survives for more than t days is

$$\frac{225}{(t+15)^2}, \quad t \geqslant 0$$

(a) Show that the cumulative distribution function of T is given by

$$\mathrm{F}(t) = \begin{cases} 1 - \dfrac{225}{(t+15)^2} & t \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(1)

(b) Find the probability that a randomly selected mosquito will die within 3 days of hatching.

(2)

(c) Given that a mosquito survives for 3 days, find the probability that it will survive for at least 5 more days.

(3)

A large number of mosquitoes hatch on the same day.

(d) Find the number of days after which only 10% of these mosquitoes are expected to survive.

(4)

Leave blank

7. The continuous random variable X has the following probability density function

$$f(x) = \begin{cases} a + bx & 0 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.

(a) Show that $10a + 25b = 2$

(4)

Given that $E(X) = \frac{35}{12}$

(b) find a second equation in a and b,

(3)

(c) hence find the value of a and the value of b.

(3)

(d) Find, to 3 significant figures, the median of X.

(3)

(e) Comment on the skewness. Give a reason for your answer.

(2)

Leave blank

5. The continuous random variable X has a cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x^3}{10} + \dfrac{3x^2}{10} + ax + b & 1 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where a and b are constants.

(a) Find the value of a and the value of b.

(4)

(b) Show that $f(x) = \dfrac{3}{10}(x^2 + 2x - 2), \quad 1 \leqslant x \leqslant 2$

(1)

(c) Use integration to find E(X).

(4)

(d) Show that the lower quartile of X lies between 1.425 and 1.435

(3)

Leave blank

2. The continuous random variable Y has cumulative distribution function

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{4}(y^3 - 4y^2 + ky) & 0 \leqslant y \leqslant 2 \\ 1 & y > 2 \end{cases}$$

where k is a constant.

(a) Find the value of k.

(2)

(b) Find the probability density function of Y, specifying it for all values of y.

(3)

(c) Find $P(Y > 1)$.

(2)

Leave blank

4. The random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} k(3 + 2x - x^2) & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$ **(3)**

(b) Find the mode of X. **(2)**

(c) Use algebraic integration to find E(X). **(4)**

By comparing your answers to parts (b) and (c),

(d) describe the skewness of X, giving a reason for your answer. **(2)**