Edexcel (U.K.) Pre 2017

Questions By Topic

FP3 Chap04 Integration

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1.　The curve C has equation $y = 2x^3$, $0 \leqslant x \leqslant 2$.

　　The curve C is rotated through 2π radians about the x-axis.

　　Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.

(5)

2. Use calculus to find the exact value of $\int_{-2}^{1} \dfrac{1}{x^2 + 4x + 13}\, dx$.

(5)

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2. (a) Find

$$\int \frac{1}{\sqrt{(4x^2 + 9)}}\, dx$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2 + 9)}}\, dx$$

giving your answer in the form $k \ln(a + b \sqrt{5})$, where a and b are integers and k is a constant.

(3)

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2.
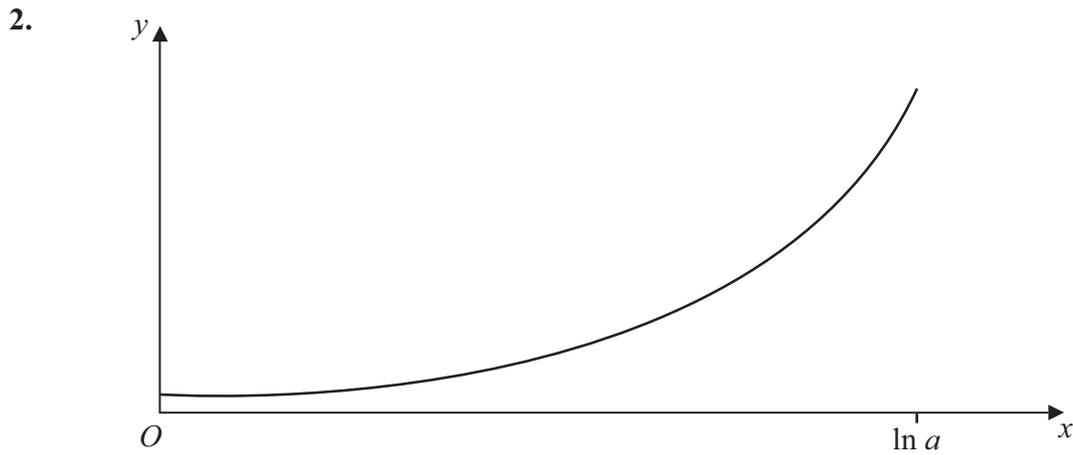
Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \leqslant x \leqslant \ln a$$

where a is a constant and $a > 1$

Using calculus, show that the length of curve C is

$$k(a^3 - \frac{1}{a^3})$$

and state the value of the constant k.

(6)

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3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4 \sinh \theta, \quad 0 \leqslant \theta \leqslant 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

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4. $I_n = \displaystyle\int_0^a (a-x)^n \cos x \, dx, \quad a > 0, \quad n \geqslant 0$

(a) Show that, for $n \geqslant 2$,
$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(b) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx.$

(3)

4.

$$I_n = \int_1^e x^2 (\ln x)^n \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$$

(4)

(b) Find the exact value of I_3.

(4)

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3. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2-10x+34}\,\mathrm{d}x = k\pi$, giving the value of the fraction k,

(5)

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2-10x+34)}}\,\mathrm{d}x = \ln(A+\sqrt{n})$, giving the values of the integers A and n.

(4)

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4. Given that $y = \text{arsinh}(\sqrt{x})$, $x > 0$,

(a) find $\dfrac{dy}{dx}$, giving your answer as a simplified fraction.

(3)

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\, dx,$$

giving your answer in the form $\ln\left(\dfrac{a + b\sqrt{5}}{2}\right)$, where a and b are integers.

(6)

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5. (a) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, dx$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

5.

$$I_n = \int_1^5 x^n (2x-1)^{-\frac{1}{2}}\, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$(2n+1)I_n = nI_{n-1} + 3 \times 5^n - 1$$

(5)

(b) Using the reduction formula given in part (a), find the exact value of I_2

(5)

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4.
$$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 2$,

$$I_n = \frac{1}{4} n \left(\frac{\pi}{4} \right)^{n-1} - \frac{1}{4} n(n-1) I_{n-2}$$

(5)

(b) Find the exact value of I_2

(4)

(c) Show that $I_4 = \dfrac{1}{64}(\pi^3 - 24\pi + 48)$

(2)

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5.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25 - x^2)}} \, dx, \qquad n \geqslant 0$$

(a) Find an expression for $\displaystyle\int \frac{x}{\sqrt{(25 - x^2)}} \, dx, \quad 0 \leqslant x \leqslant 5.$

(2)

(b) Using your answer to part (a), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2} \qquad n \geqslant 2$$

(5)

(c) Find I_4 in the form $k\pi$, where k is a fraction.

(4)

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8.

Figure 2

The curve C, shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \qquad 1 \leqslant x \leqslant 8$$

(a) Show that the length s of curve C is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)}\,dx$$

(2)

(b) Using the substitution $x = \sinh^2 u$, or otherwise, find an exact value for s.

Give your answer in the form $a\sqrt{2} + \ln(b + c\sqrt{2})$ where a, b and c are integers.

(9)

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6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,dx, \quad n \geqslant 0,$$

(a) prove that, for $n \geqslant 2$,

$$(n + 2)I_n = 16(n - 1)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of I_5

(5)

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7. $f(x) = 5\cosh x - 4\sinh x,$ $x \in \mathbb{R}$

(a) Show that $f(x) = \dfrac{1}{2}(e^x + 9e^{-x})$

(2)

Hence

(b) solve $f(x) = 5$

(4)

(c) show that $\displaystyle\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\, dx \ = \ \frac{\pi}{18}$

(5)

8. A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The curve is rotated through 2π radians about the x-axis.

(a) Show that the area of the surface generated is given by the integral

$$k\pi \int_0^\alpha \sqrt{(16c^2 + 9)}\, dc, \quad \text{where } c = \cos\theta,$$

and where k and α are constants to be found.

(6)

(b) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of

your working and giving the final answer to 3 significant figures.

(5)

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7.

Figure 1

The curves shown in Figure 1 have equations

$$y = 6 \cosh x \quad \text{and} \quad y = 9 - 2 \sinh x$$

(a) Using the definitions of sinh x and cosh x in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)