Edexcel (U.K.) Pre 2017

Questions By Topic

FP1 Chap01 Complex Numbers

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1. The complex numbers z and w are given by

$$z = 8 + 3i, \quad w = -2i$$

Express in the form $a + bi$, where a and b are real constants,

(a) $z - w$,

(1)

(b) zw.

(2)

Leave blank

4. Given that 2 – 4i is a root of the equation

$$z^2 + p\,z + q = 0\,,$$

where p and q are real constants,

(a) write down the other root of the equation,

(1)

(b) find the value of p and the value of q.

(3)

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1.
$$f(x) = 2x^3 - 8x^2 + 7x - 3$$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

$$f(x) = 2x^3 - 8x^2 + 7x - 3$$

Leave blank

1.
$$z = 5 - 3i, \quad w = 2 + 2i$$

Express in the form $a + bi$, where a and b are real constants,

(a) z^2,

(2)

(b) $\dfrac{z}{w}$.

(3)

Q1

(Total 5 marks)

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1.

$$f(x) = 2x^3 - 6x^2 - 7x - 4$$

(a) Show that $f(4) = 0$

(1)

(b) Use algebra to solve $f(x) = 0$ completely.

(4)

Leave blank

4.
$$f(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of f(x) = 0

(4)

(b) Show the four roots of f(x) = 0 on a single Argand diagram.

(2)

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5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3.

(a) Given that $z_1 = 3 + i$, find z_2 and z_3.

(4)

(b) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

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6.   Given that $z = x + \mathrm{i}y$, find the value of x and the value of y such that

$$z + 3\mathrm{i}\,z^* = -1 + 13\mathrm{i}$$

where z^* is the complex conjugate of z.

(7)

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4.
$$f(x) = x^3 + x^2 + 44x + 150$$

Given that $f(x) = (x+3)(x^2 + ax + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Find the three roots of $f(x) = 0$.

(4)

(c) Find the sum of the three roots of $f(x) = 0$.

(1)

Leave blank

1. $$z = 2 - 3i$$

    (a) Show that $z^2 = -5 - 12i$.

                                              (2)

    Find, showing your working,

    (b) the value of $\left| z^2 \right|$,

                                              (2)

    (c) the value of $\arg(z^2)$, giving your answer in radians to 2 decimal places.

                                              (2)

    (d) Show z and z^2 on a single Argand diagram.

                                            (1)

Leave blank

5.
$$f(x) = (4x^2 + 9)(x^2 - 6x + 34)$$

(a) Find the four roots of $f(x) = 0$

Give your answers in the form $x = p + iq$, where p and q are real.

(5)

(b) Show these four roots on a single Argand diagram.

(2)

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3. $$f(x)=(x^2+4)(x^2+8x+25)$$

(a) Find the four roots of $f(x)=0$.

(5)

(b) Find the sum of these four roots.

(2)

Leave blank

3. Given that $x = \dfrac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(b) the other 2 roots of the equation.

(4)

Leave blank

1. Given that $z_1 = 1 - i$,

(a) find arg(z_1).

(2)

Given also that $z_2 = 3 + 4i$, find, in the form $a + ib$, $a, b \in \mathbb{R}$,

(b) $z_1 z_2$,

(2)

(c) $\dfrac{z_2}{z_1}$.

(3)

In part (b) and part (c) you must show all your working clearly.

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8i \quad \text{and} \quad z_2 = 1 - i$$

Find, showing your working,

(a) $\dfrac{z_1}{z_2}$ in the form $a + bi$, where a and b are real,

(3)

(b) the value of $\left| \dfrac{z_1}{z_2} \right|$,

(2)

(c) the value of $\arg \dfrac{z_1}{z_2}$, giving your answer in radians to 2 decimal places.

(2)

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2.
$$z_1 = -2 + i$$

(a) Find the modulus of z_1.

(1)

(b) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(c) Find z_2 and z_3, giving your answers in the form $p \pm i\sqrt{q}$, where p and q are integers.

(3)

(d) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2)

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6. Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \qquad c, d \in \mathbb{R},$$

(a) write down the other complex root of the equation.

(1)

(b) Find the value of c and the value of d.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - i \quad \text{and} \quad z_2 = -8 + 9i$$

(a) Show z_1 and z_2 on a single Argand diagram.

(1)

Find, showing your working,

(b) the value of $|z_1|$,

(2)

(c) the value of $\arg z_1$, giving your answer in radians to 2 decimal places,

(2)

(d) $\dfrac{z_2}{z_1}$ in the form $a + bi$, where a and b are real.

(3)

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2.
$$z = \frac{50}{3+4i}$$

Find, in the form $a + i\,b$ where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Find

(c) $|z|$,

(2)

(d) $\arg z^2$, giving your answer in degrees to 1 decimal place.

(2)

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9. The complex number w is given by

$$w = 10 - 5i$$

(a) Find $|w|$.

(1)

(b) Find arg w, giving your answer in radians to 2 decimal places.

(2)

The complex numbers z and w satisfy the equation

$$(2 + i)(z + 3i) = w$$

(c) Use algebra to find z, giving your answer in the form $a + bi$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9i + w) = \frac{\pi}{4}$$

where λ is a real constant,

(d) find the value of λ.

(2)

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7.
$$z = -24 - 7i$$

(a) Show z on an Argand diagram.

(1)

(b) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

It is given that

$$w = a + bi, \quad a \in \mathbb{R}, \, b \in \mathbb{R}$$

Given also that $|w| = 4$ and $\arg w = \dfrac{5\pi}{6}$,

(c) find the values of a and b,

(3)

(d) find the value of $|zw|$.

(3)

Leave blank

9. Given that $z_1 = 3 + 2i$ and $z_2 = \dfrac{12 - 5i}{z_1}$,

(a) find z_2 in the form $a + ib$, where a and b are real.

(2)

(b) Show on an Argand diagram the point P representing z_1 and the point Q representing z_2.

(2)

(c) Given that O is the origin, show that $\angle POQ = \dfrac{\pi}{2}$.

(2)

The circle passing through the points O, P and Q has centre C. Find

(d) the complex number represented by C,

(2)

(e) the exact value of the radius of the circle.

(2)

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7. $\qquad z_1 = 2 + 3i, \quad z_2 = 3 + 2i, \quad z_3 = a + bi, \quad a, b \in \mathbb{R}$

(a) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(b) find w in terms of a and b, giving your answer in the form $x + iy, \quad x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}i$,

(c) find the value of a and the value of b,

(3)

(d) find arg w, giving your answer in radians to 3 decimal places.

(2)

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blank

7.
$$z = 2 - \mathrm{i}\sqrt{3}$$

(a) Calculate arg z , giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(b) $z + z^2$ in the form $a + b\,\mathrm{i}\sqrt{3}$, where a and b are integers,

(3)

(c) $\dfrac{z + 7}{z - 1}$ in the form $c + d\,\mathrm{i}\sqrt{3}$, where c and d are integers.

(4)

Given that

$$w = \lambda - 3\mathrm{i}$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(d) find the value of λ.

(2)