

Edexcel (U.K.) Pre 2017

Questions By Topic

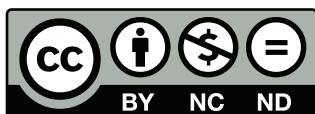
C4 Chap06 Integration

Compiled By: Dr Yu

Editors: Betül, Signal, Vivian

www.CasperYC.club

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DrYuFromShanghai@QQ.com

3. (a) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions. (3)

(5)

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4.

(a) Find the values of the constants A , B and C .

(4)

(6)

3.

(a) Find the values of the constants A , B and C .

(4)

(3)

(3)

1.
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(4)

(6)

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6. The curve C has parametric equations

Find

- (a) an equation of the normal to C at the point where $t = 3$,

(6)

- (b) a cartesian equation of C .

(3)

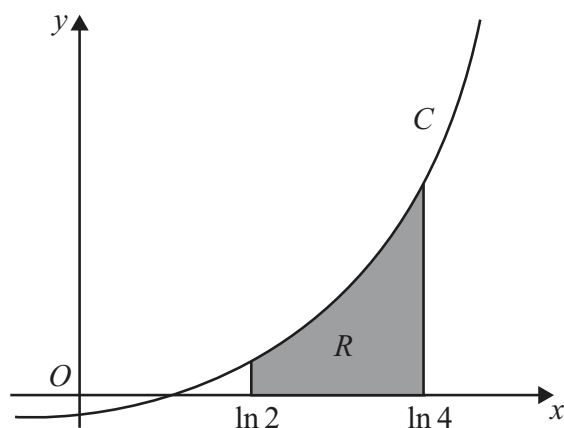


Figure 1

The finite area R , shown in Figure 1, is bounded by C , the x -axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x -axis.

- (c) Use calculus to find the exact volume of the solid generated.

(6)

7.

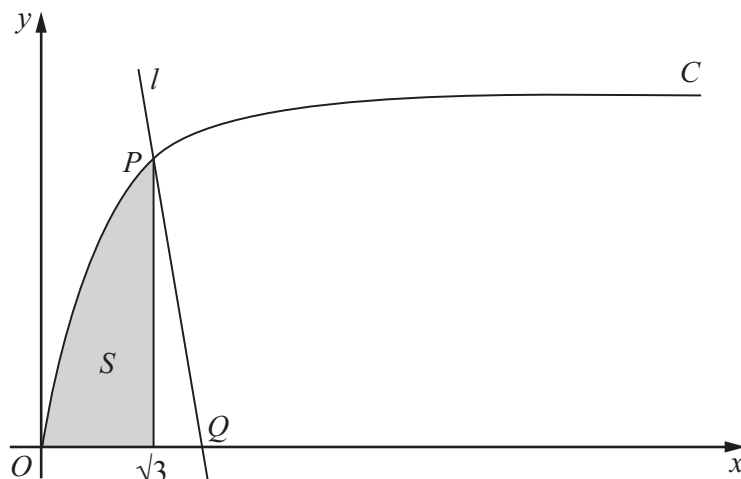
**Figure 3**

Figure 3 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P .

(2)

The line l is a normal to C at P . The normal cuts the x -axis at the point Q .

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k .

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C , the line $x = \sqrt{3}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

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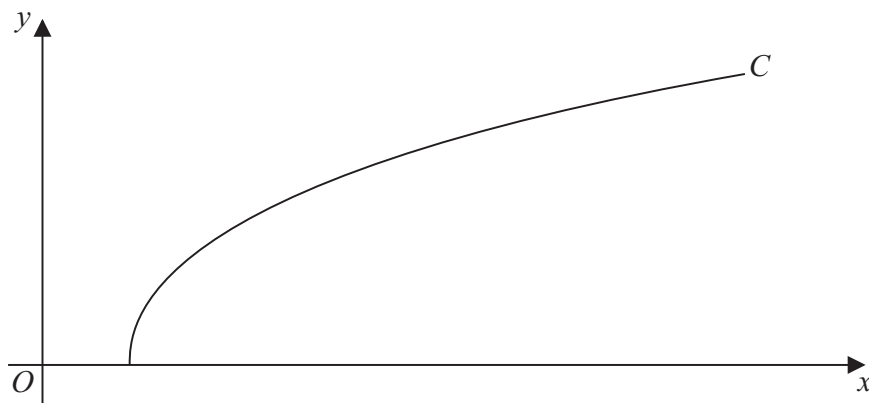
**Figure 2**

Figure 2 shows a sketch of the curve C with parametric equations

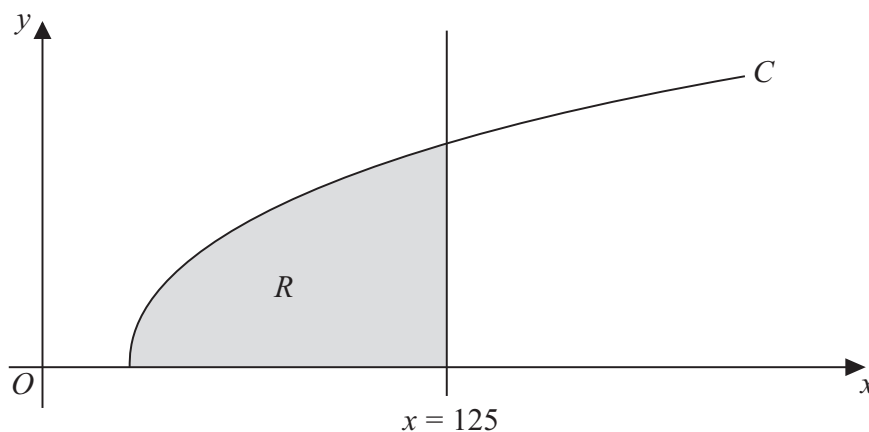
$$x = 27 \sec^3 t, \quad y = 3 \tan t, \quad 0 \leq t \leq \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \frac{\pi}{6}$ (4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \quad a \leq x \leq b$$

stating the values of a and b .

(3)**Figure 3**

The finite region R which is bounded by the curve C , the x -axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution. (5)

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4. Use the substitution $x = \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx.$$

(7)

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Figure 1

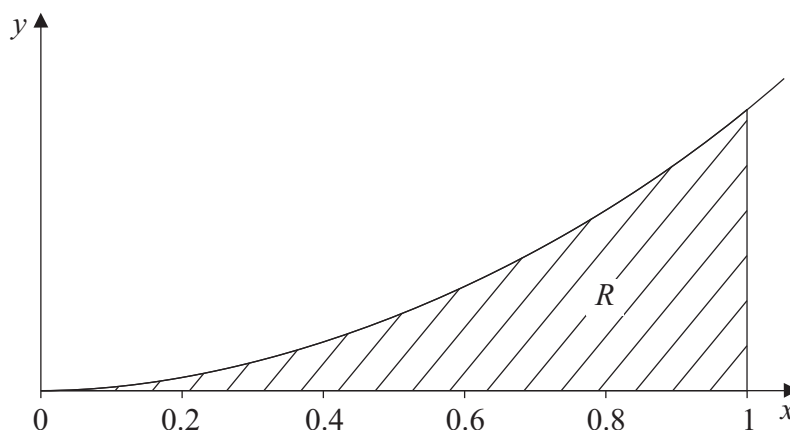


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R .

(5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

2. (a) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(b) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, dx$. Show all the steps of your working, and give your answer to 4 decimal places.

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, dx$ is $\ln(1 + \sqrt{2})$.

(2)

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3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}} dx.$$

(8)

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Figure 1

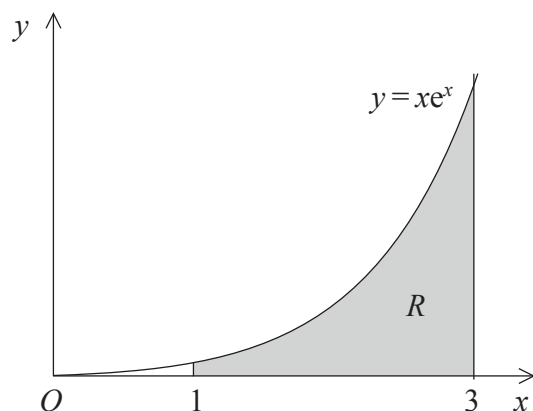


Figure 1 shows the finite shaded region, R , which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x -axis.

The region R is rotated through 360 degrees about the x -axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

3.

A graph of the function $y = \sin(x)$ on the interval $[0, 2\pi]$. The x-axis is labeled with 0 and 2π , and the y-axis is labeled with y . The origin is marked with O . The curve starts at $(0, 0)$, reaches a maximum at $(\pi, 1)$, and ends at $(2\pi, 0)$. The area under the curve is shaded gray.

(a) Find, by integration, the area of the shaded region.

(b) Find the volume of the solid generated.

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Figure 3

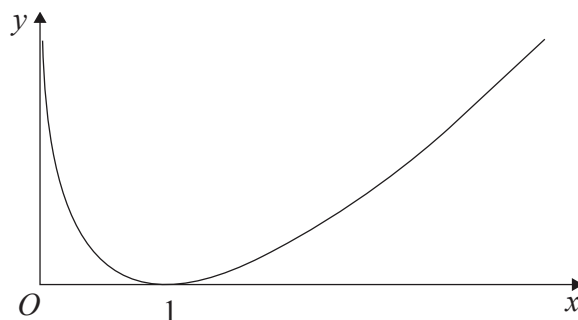


Figure 3 shows a sketch of the curve with equation $y = (x-1) \ln x$, $x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2 \ln 3$

Given that $I = \int_1^3 (x-1) \ln x \, dx$, (1)

(b) use the trapezium rule

(i) with values of y at $x = 1, 2$ and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1, 1.5, 2, 2.5$ and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

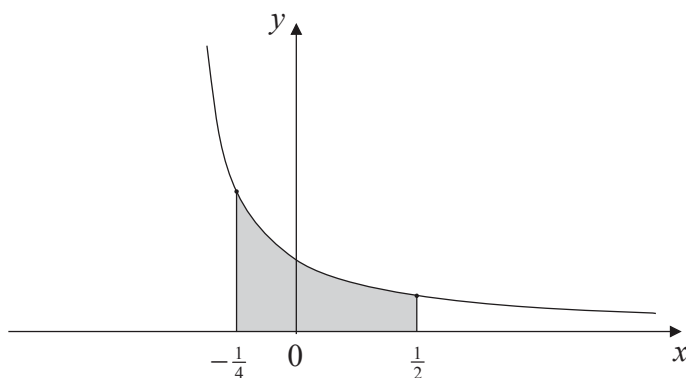
(d) Show, by integration, that the exact value of $\int_1^3 (x-1) \ln x \, dx$ is $\frac{3}{2} \ln 3$.

(6)

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2.

Figure 1



The curve with equation $y = \frac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x -axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x -axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

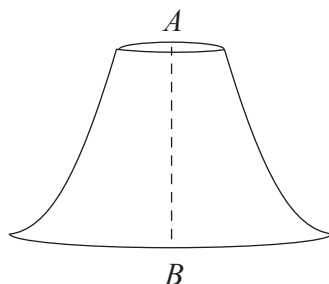


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)

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8.

$$I = \int_0^5 e^{\sqrt{3x+1}} dx.$$

- (a) Given that $y = e^{\sqrt{3x+1}}$, complete the table with the values of y corresponding to $x = 2, 3$ and 4 .

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I , giving your answer to 4 significant figures.

(3)

- (c) Use the substitution $t = \sqrt{3x+1}$ to show that I may be expressed as $\int_a^b kte^t dt$, giving the values of a, b and k .

(5)

- (d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)

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2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2} dx.$$

(6)

3. (a) Find $\int x \cos 2x \, dx$.

(b) Hence, using the identity $\cos 2x = 2 \cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$. (3)

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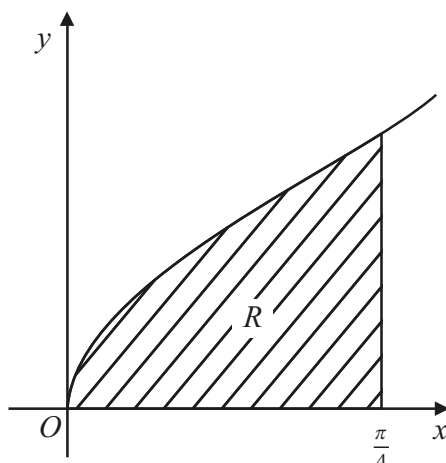
**Figure 1**

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R , which is bounded by the curve, the x -axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

- (a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

- (b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R , giving your answer to 4 decimal places.

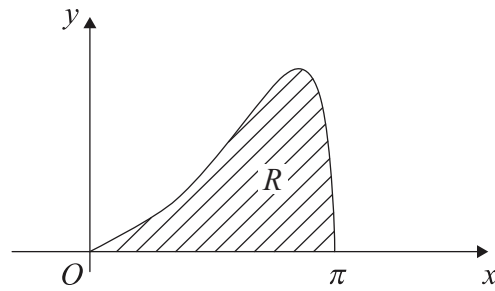
(4)

The region R is rotated through 2π radians around the x -axis to generate a solid of revolution.

- (c) Use integration to find an exact value for the volume of the solid generated.

(4)

1.



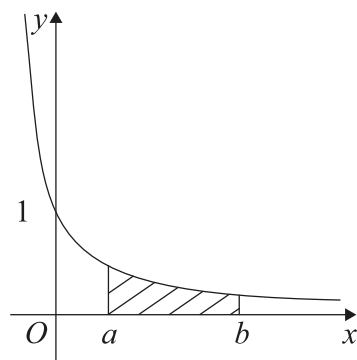
The curve shown in Figure 1 has equation $y = e^x \sqrt{(\sin x)}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x -axis is shown shaded in Figure 1.

- | | | | | | |
|-----|---|-----------------|-----------------|------------------|-------|
| x | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | π |
| y | 0 | | | 8.87207 | 0 |

(4)

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3.

**Figure 2**

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x -axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x -axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b .

(5)

Leave
blank4. (i) Find $\int \ln\left(\frac{x}{2}\right) dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$.

(5)

1.



x	0	0.4	0.8	1.2	1.6	2
y	\mathbf{e}^0	$\mathbf{e}^{0.08}$		$\mathbf{e}^{0.72}$		\mathbf{e}^2

(3)

2. (a) Use integration by parts to find $\int x e^x dx$.

(b) Hence find $\int x^2 e^x \, dx$.

(3)

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2.

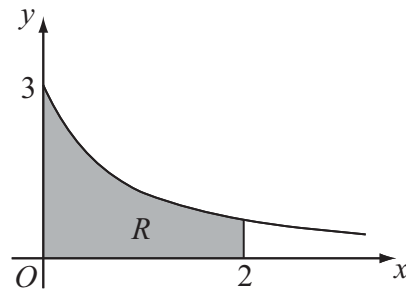


Figure 1 shows part of the curve $y = \frac{3}{\sqrt{1+4x}}$. The region R is bounded by the curve, the x -axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

- The region R is rotated 360° about the x -axis.

- (b) Use integration to find the exact value of the volume of the solid formed. (5)

6. (a) Find $\int \tan^2 x \, dx$. (2)

(c) Use the substitution $u = 1 + e^x$ to show that

where k is a constant. (7)

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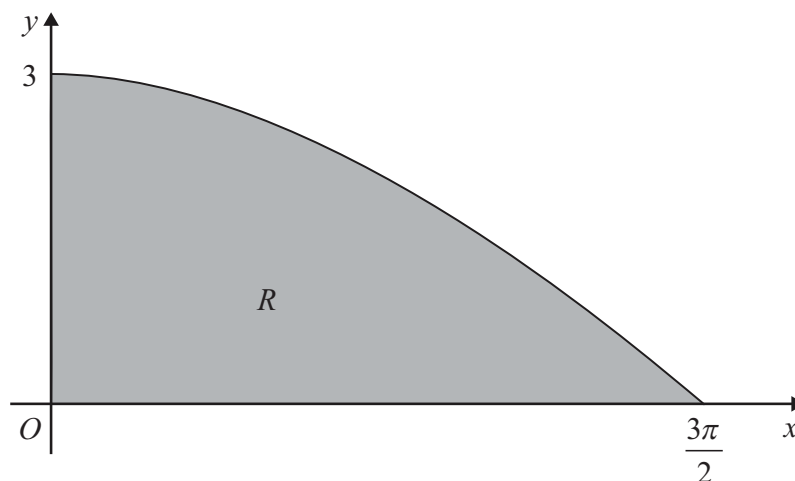
**Figure 1**

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. (4)
- (c) Use integration to find the exact area of R . (3)

6. (a) Find $\int \sqrt[3]{(5-x)} \, dx$.

$$y = (x - 1) \sqrt[3]{5 - x}, \quad 1 \leq x \leq 5$$
$$\int (x-1) \sqrt[4]{(5-x)} \, dx$$

(ii) Hence find $\int_1^5 (x-1)\sqrt{5-x} \, dx$.

8. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2 \theta$, find $\int \sin^2 \theta \, d\theta$.

(2)

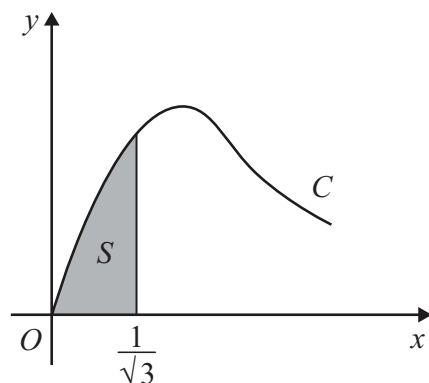


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = 2 \sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C , the line $x = \frac{1}{\sqrt{3}}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Show that the volume of the solid of revolution formed is given by the integral

$$k \int_0^{\frac{\pi}{6}} \sin^2 \theta \, d\theta$$

where k is a constant.

(5)

- (c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

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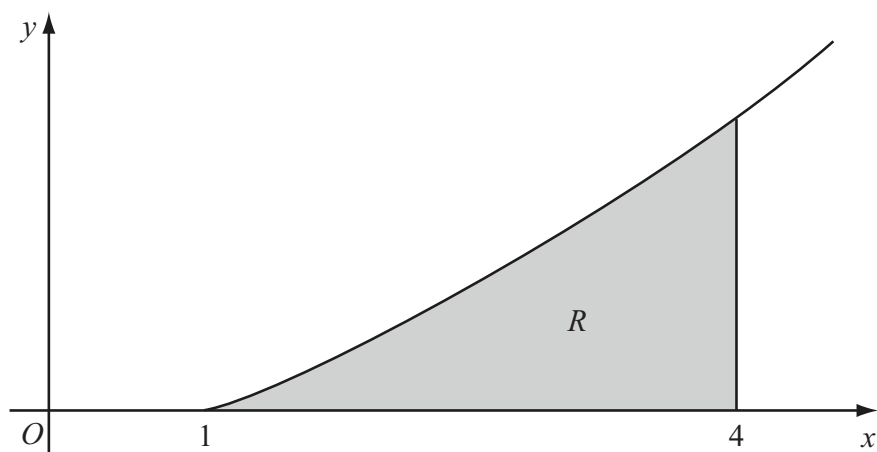
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places.

(4)

(c) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

8. (a) Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \quad (7)$$

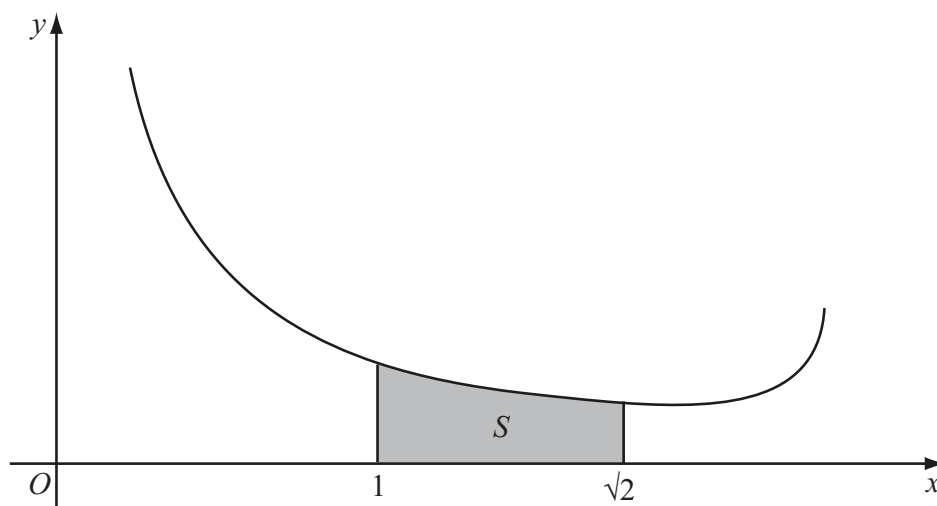


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S , shown in Figure 3, is bounded by the curve, the x -axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)

1.

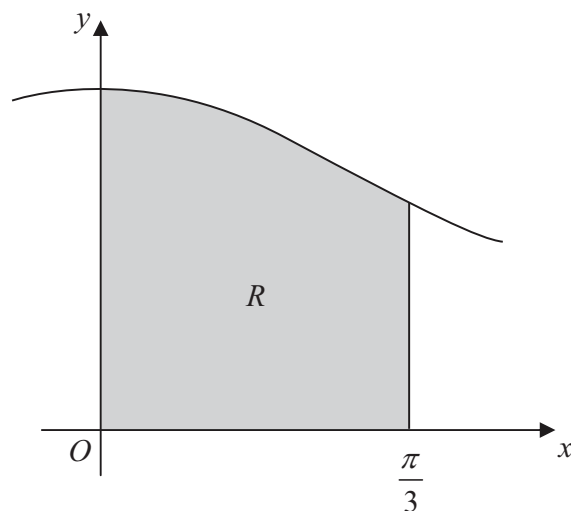
**Figure 1**

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the y -axis, the x -axis and the line with equation $x = \frac{\pi}{3}$.

- (a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

- (b) Use the trapezium rule

- (i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R .

Give your answer to 3 decimal places.

- (ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a

further estimate of the area of R . Give your answer to 3 decimal places.

(6)

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2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1)$$

(6)

1. Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

7.

$$I = \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx$$

- | | | | | |
|-----|-----|---|--------|---|
| x | 2 | 3 | 4 | 5 |
| y | 0.2 | | 0.1745 | |

(8)

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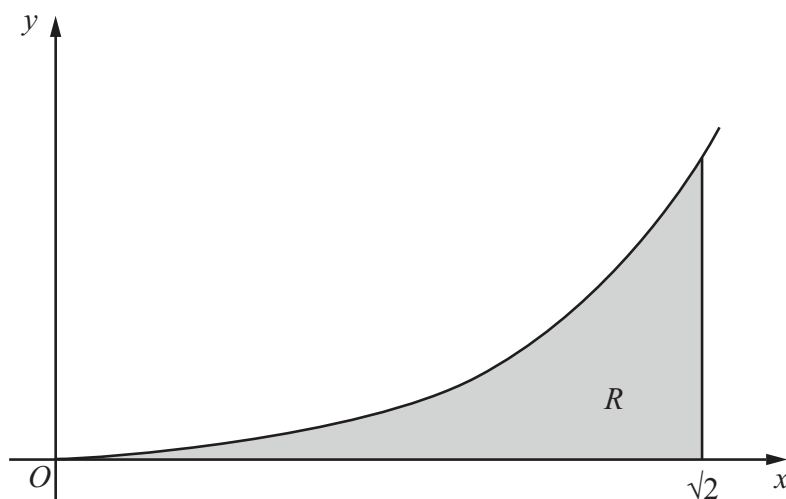
**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$. The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

(d) Hence, or otherwise, find the exact area of R . (6)

2. (a) Use integration by parts to find $\int x \sin 3x \, dx$. (3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$. (3)

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4.

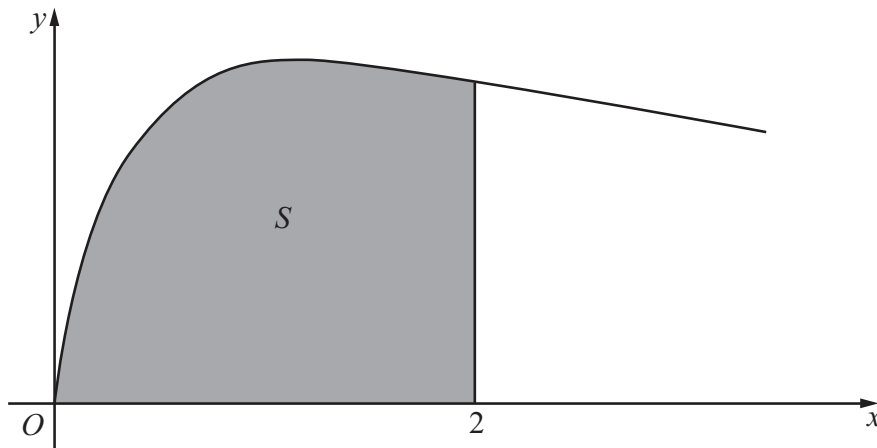


Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0$$

The finite region S , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 2$

The region S is rotated 360° about the x -axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

6.

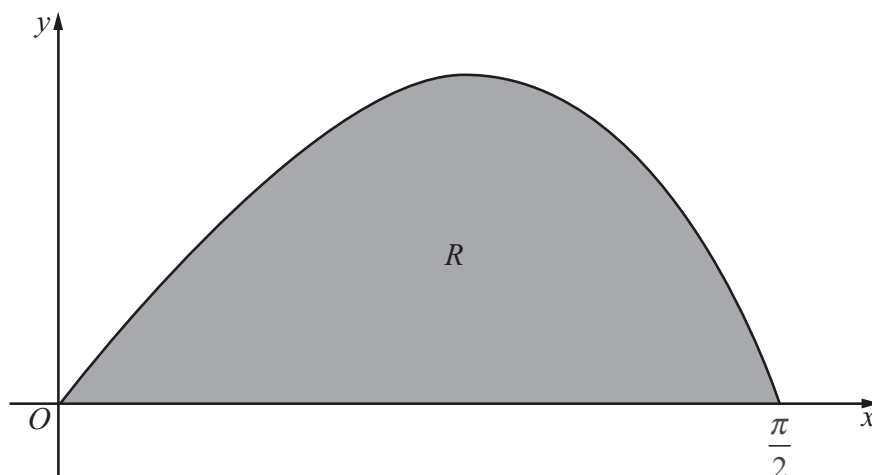
**Figure 3**

Figure 3 shows a sketch of the curve with equation $y = \frac{2 \sin 2x}{(1 + \cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

The table below shows corresponding values of x and y for $y = \frac{2 \sin 2x}{(1 + \cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 4 decimal places. (3)
- (c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2 \sin 2x}{(1 + \cos x)} dx = 4 \ln(1 + \cos x) - 4 \cos x + k$$

where k is a constant.

(5)

- (d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)

7.



(4)

(4)

(3)

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2. (a) Use integration to find

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx \quad (2)$$

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4.

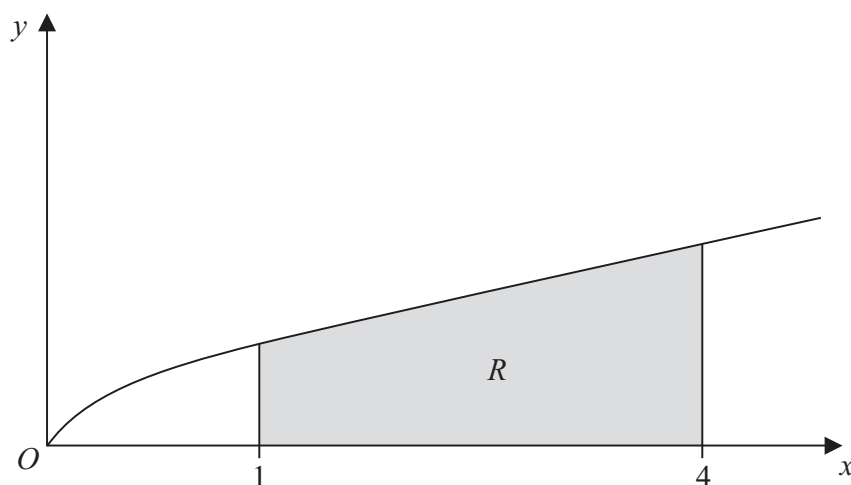


Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1 + \sqrt{x}}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis, the line with equation $x = 1$ and the line with equation $x = 4$.

- (1)

x	1	2	3	4
y	0.5	0.8284		1.3333

- (3)

- (8)

6.



(b) Find, by integration, the exact value of the volume of the solid generated. (6)

1. (a) Find $\int x^2 e^x dx$.

(5)

(b) Hence find the exact value of $\int_0^1 x^2 e^x dx$.

(2)

Leave
blank

3.

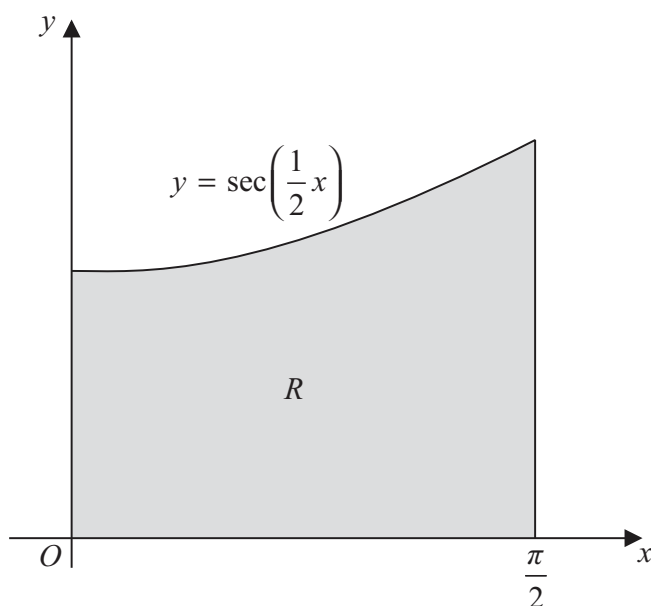
**Figure 1**

Figure 1 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places. (1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

(c) Use calculus to find the exact volume of the solid formed. (4)

Leave
blank

5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)} dx = \int \frac{2}{u(2u - 1)} du \quad (3)$$

- (b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)} dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

5.

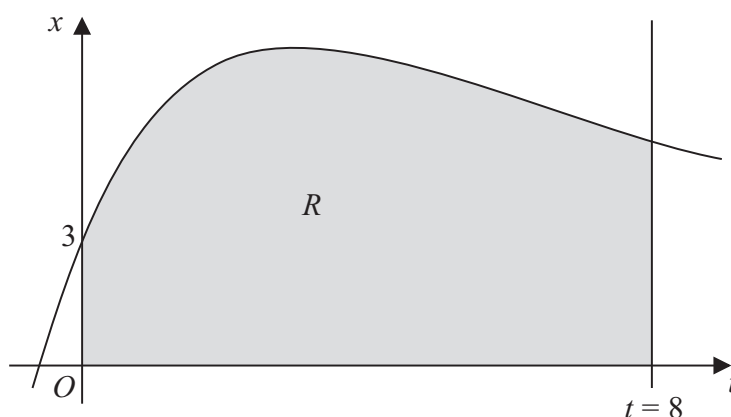


Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x -axis, the t -axis and the line $t = 8$.

- | | | | | | |
|-----|---|-------|-------|---|-------|
| t | 0 | 2 | 4 | 6 | 8 |
| x | 3 | 7.107 | 7.218 | | 5.223 |

(3)

- (6)

- (1)