Edexcel (U.K.) Pre 2017

Questions By Topic

C3 Chap02 Functions

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3. The function f is defined by

$$f : x \to \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \frac{2}{x-1}, \quad x > 1.$ **(4)**

(b) Find $f^{-1}(x)$. **(3)**

The function g is defined by

$$g : x \to x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$. **(3)**

Leave blank

8. The functions f and g are defined by

$$\mathrm{f}: x \to 2x + \ln 2, \quad x \in \mathbb{R},$$

$$\mathrm{g}: x \to \mathrm{e}^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function gf is

$$\mathrm{gf}: x \to 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}.$$

(4)

(b) In the space provided on page 19, sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

Leave blank

7. For the constant k, where $k > 1$, the functions f and g are defined by

$$\begin{aligned} \mathrm{f}:\ & x \mapsto \ln(x + k), \quad x > -k, \\ \mathrm{g}:\ & x \mapsto |2x - k|, \quad x \in \mathbb{R}. \end{aligned}$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f.

(1)

(c) Find $\mathrm{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \mathrm{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k.

(4)

Leave blank

6. The function f is defined by

$$f : x \mapsto \ln(4-2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \; x_0 = -0.3$$

is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

Leave blank

5. The functions f and g are defined by

$$f : x \mapsto \ln(2x-1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(a) Find the exact value of fg(4).

(2)

(b) Find the inverse function $f^{-1}(x)$, stating its domain.

(4)

(c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

(3)

(d) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.

(3)

Leave blank

8. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 1-2x^3, \ \ x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \frac{3}{x}-4, \ \ x>0, \ \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$\mathrm{gf}: x \mapsto \frac{8x^3-1}{1-2x^3}.$$

(4)

(c) Solve $\mathrm{gf}(x)=0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

(5)

Leave blank

4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(b) Find the range of f.

(2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \dfrac{1}{8}$.

(3)

Leave blank

5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}\ .$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\dfrac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

Leave blank

5.

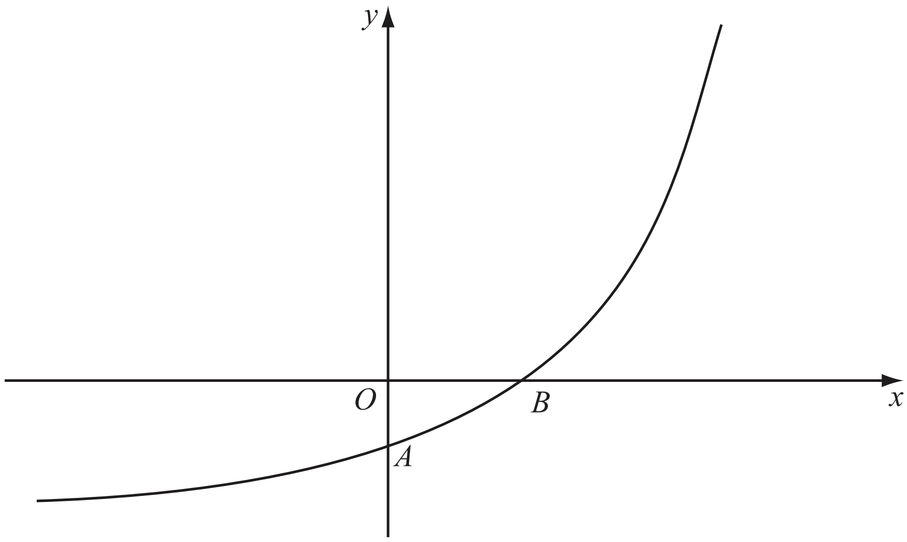

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$,

where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,

(1)

(d) find $\mathrm{f}^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

(1)

Leave blank

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

Leave blank

6. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R},\ x \neq 5$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

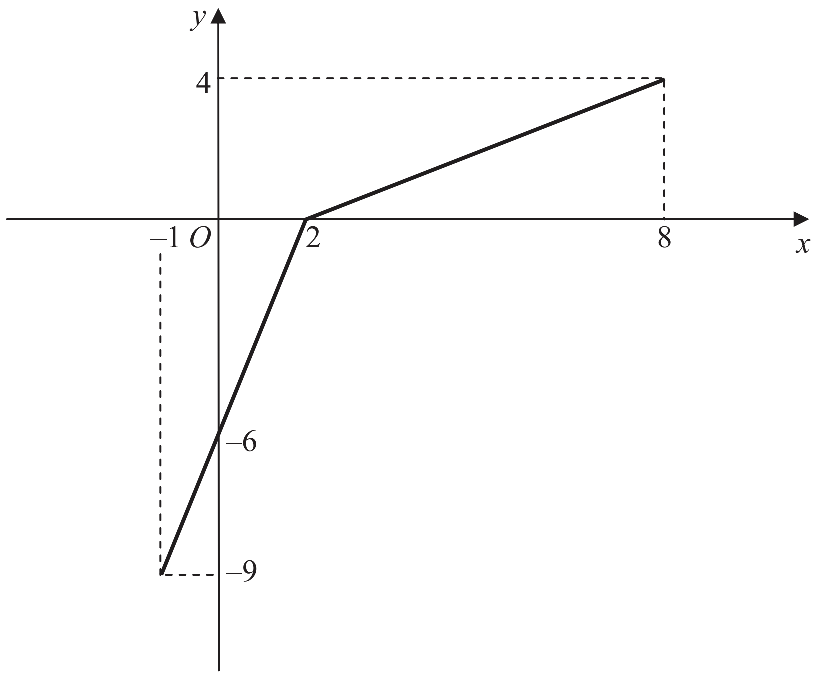

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = \mathrm{g}(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |\mathrm{g}(x)|$,

(ii) $y = \mathrm{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

Leave blank

4. The function f is defined by

$$\mathrm{f}: x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R},\ x \geqslant -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$\mathrm{g}: x \mapsto \mathrm{e}^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

Leave blank

7. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{2x-1}$

(4)

(b) Find $\mathrm{f}^{-1}(x)$

(3)

(c) Find the domain of f^{-1}

(1)

$$\mathrm{g}(x) = \ln(x+1)$$

(d) Find the solution of $\mathrm{fg}(x) = \dfrac{1}{7}$, giving your answer in terms of e.

(4)

Leave blank

6. The functions f and g are defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^{x} + 2, \qquad x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $\mathrm{f}(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

Leave blank

7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

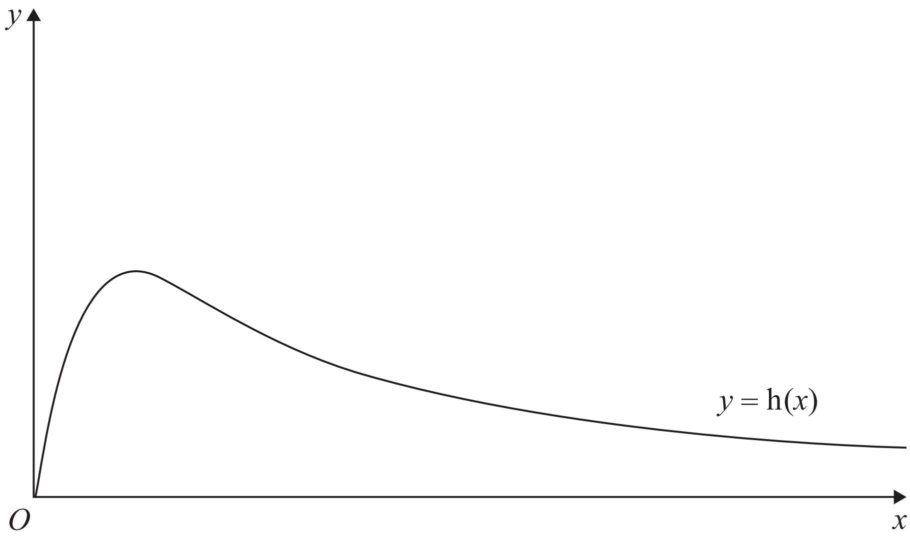

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$. **(5)**

Leave blank

8.

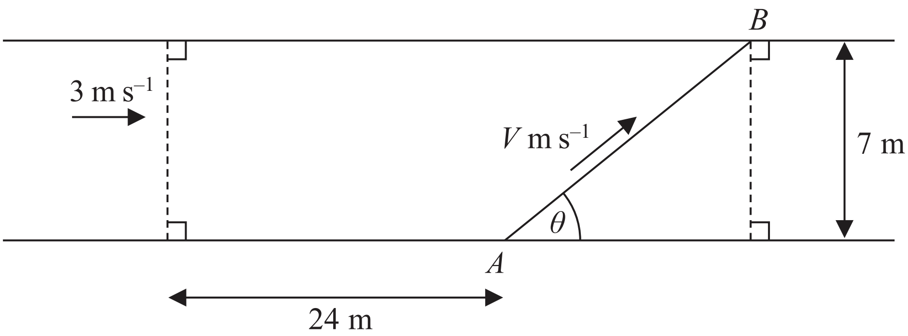

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.
Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.
(3)

Given that θ varies,

(b) find the minimum value of V.
(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance AB.
(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.
(6)

Leave blank

4. The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.
(2)

(b) Find fg(1).
(2)

(c) Find g^{-1}, the inverse function of g.
(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)