Edexcel (U.K.) Pre 2017

Questions By Topic

C2 Chap11 Integration

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1. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Q1

(Total 4 marks)

Leave blank

2.

$$y = \frac{x}{\sqrt{(1+x)}}$$

(a) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\,dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

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2.

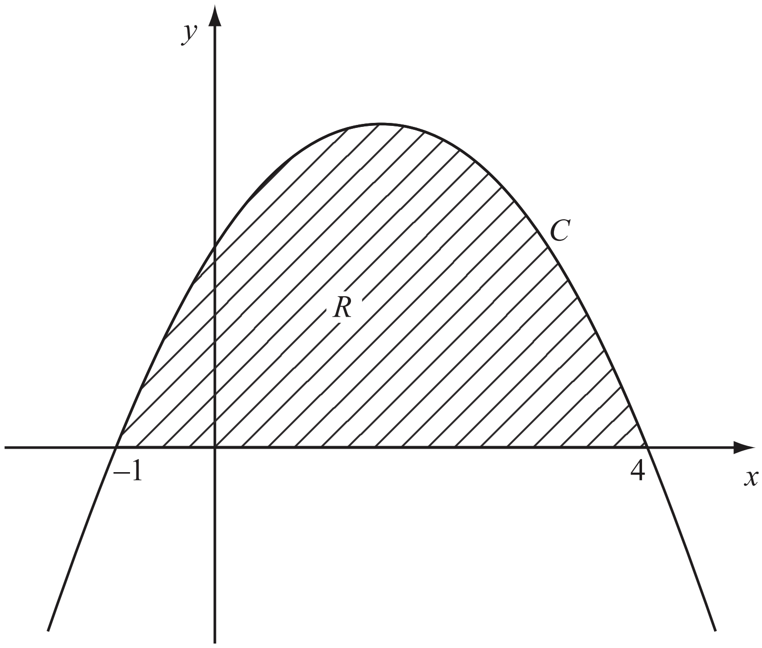

Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

Leave blank

2. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx.$

(5)

Q2

(Total 5 marks)

Leave blank

1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x})\, dx .$$

(5)

Q1

(Total 5 marks)

Leave blank

3. $$y = \sqrt{(10x - x^2)}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\, dx$.

(4)

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6. The speed, v m s^{-1}, of a train at time t seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \quad 0 \leqslant t \leqslant 30.$$

The following table shows the speed of the train at 5 second intervals.

t	0	5	10	15	20	25	30
v	0	1.22	2.28		6.11		

(a) Complete the table, giving the values of v to 2 decimal places.

(3)

The distance, s metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)}\,dt\ .$$

(b) Use the trapezium rule, with all the values from your table, to estimate the value of s.

(3)

Leave blank

2. $$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\,dx$.

(4)

Leave blank

1. $$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\,dx$.

(4)

Leave blank

7. $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of $\int_0^1 \sqrt{(3^x + x)}\, dx$

You must show clearly how you obtained your answer.

(4)

Leave blank

1. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

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4.

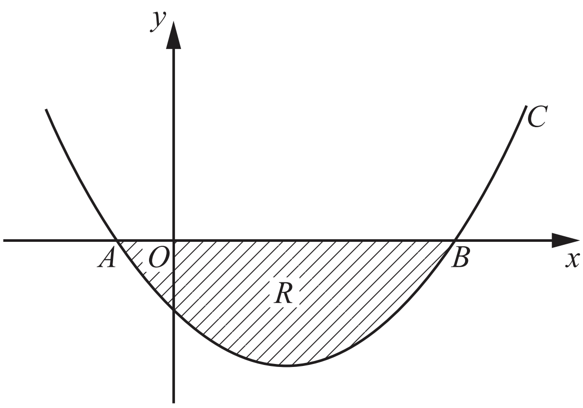

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

Leave blank

4. $$y = \frac{5}{(x^2 + 1)}$$

(a) Complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

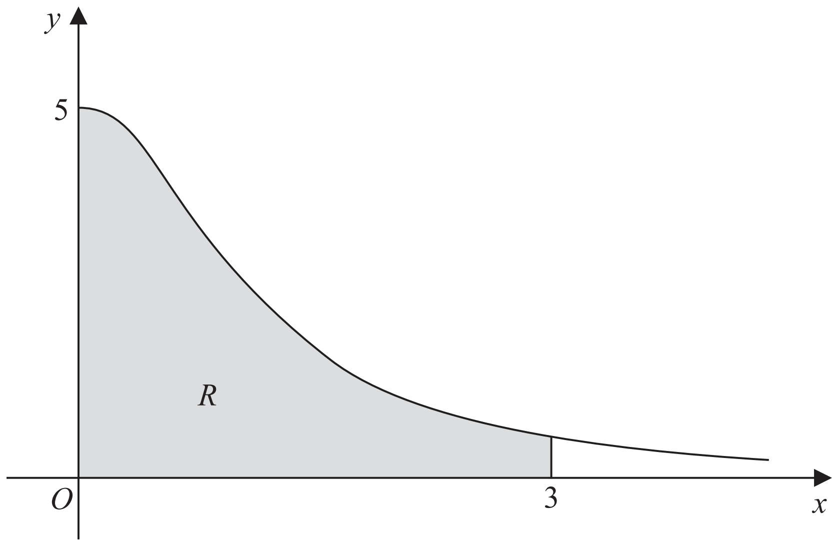

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2 + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^3 \left(4 + \frac{5}{(x^2 + 1)}\right) dx$$

giving your answer to 2 decimal places.

(2)

Leave blank

4. (a) Complete the table below, giving values of $\sqrt{(2^x+1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x+1)}$	1.414	1.554	1.732	1.957			3

(2)

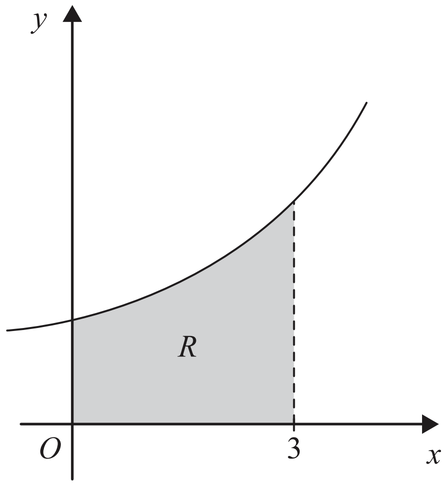

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y=\sqrt{(2^x+1)}$, the x-axis and the lines $x=0$ and $x=3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

Leave blank

5. (a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

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6.

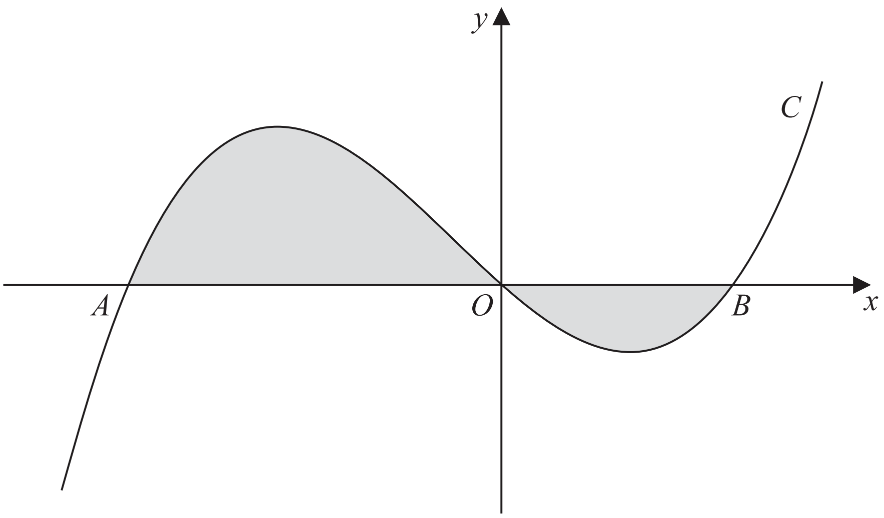

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

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6. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \quad 0 \leqslant x \leqslant 20.$$

(a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

(b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 ms^{-1},

(c) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

Leave blank

7.

Figure 1

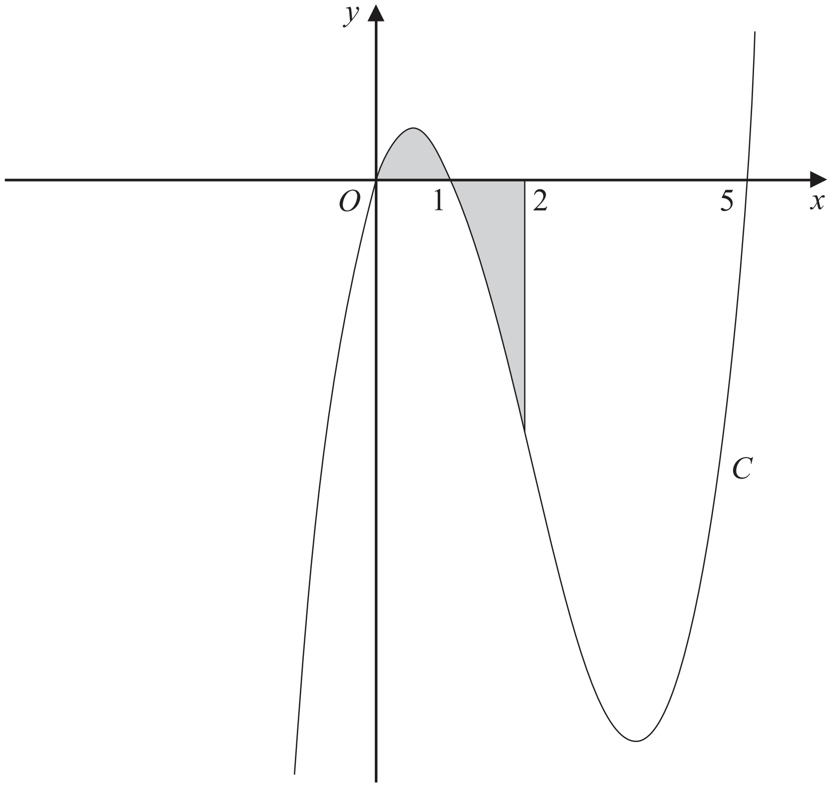

Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

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8.

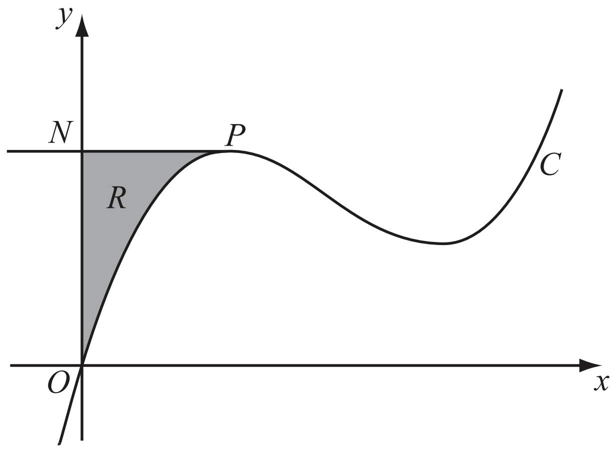

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

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7.

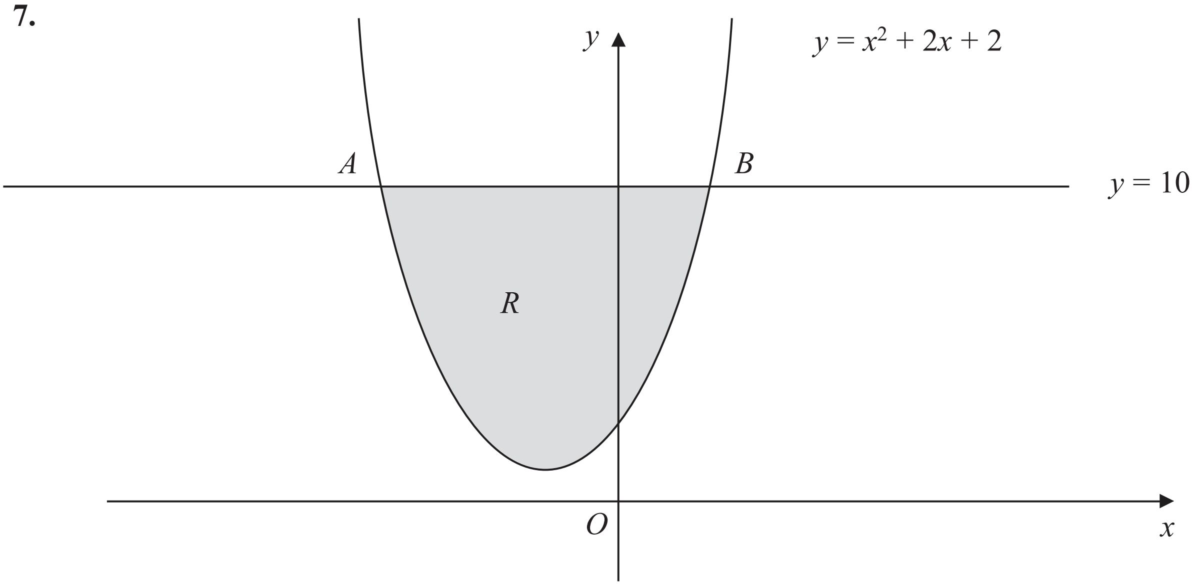

Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

Leave blank

6. $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

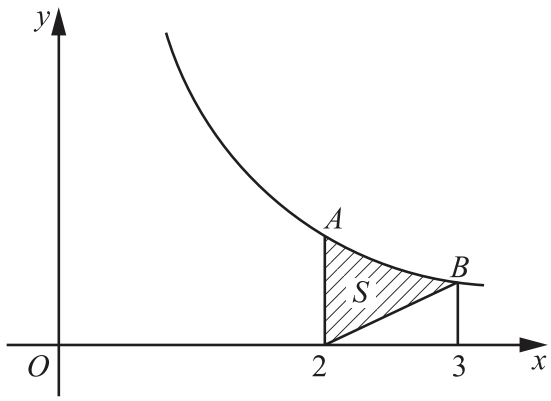

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

Leave blank

5. The curve C has equation

$$y = x\sqrt{(x^3+1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,dx$, giving your answer to 3 significant figures.

(4)

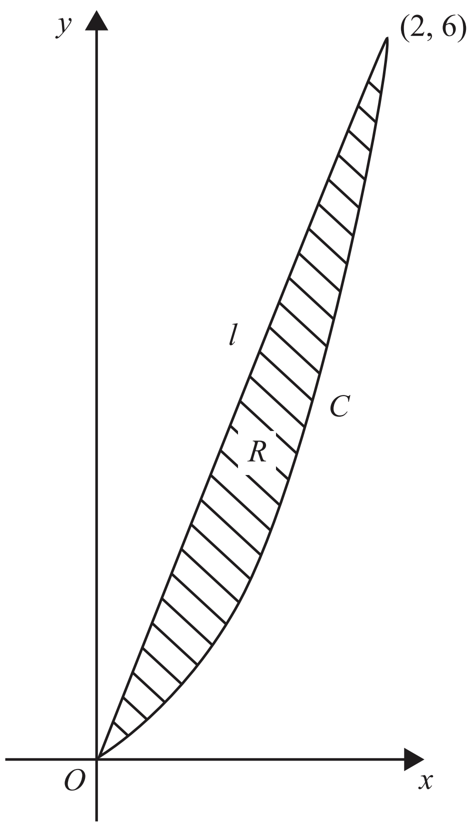

Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3+1)}$, $0 \leqslant x \leqslant 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

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7.

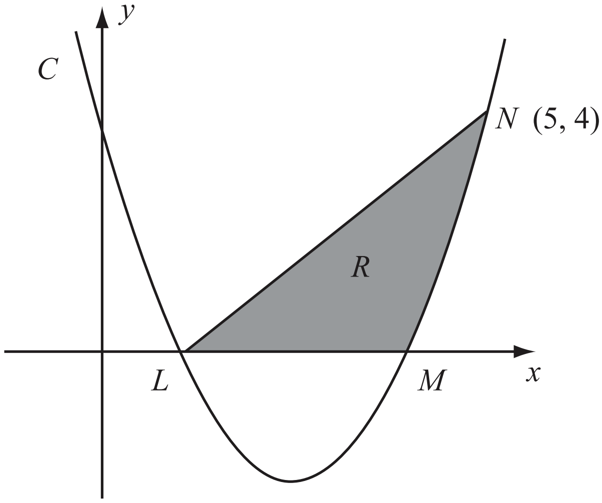

Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point N (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\, dx$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

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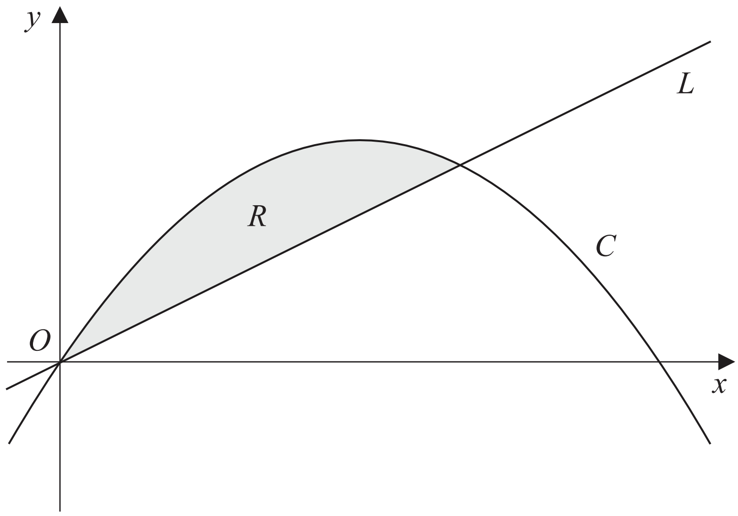
7. **Figure 2**



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

Leave blank

Question 8 continued

(b) Find all the values of θ, to 1 decimal place, in the interval $0° \leqslant \theta < 360°$ for which

$$\tan^2\theta = 4.$$

(5)

Q8

(Total 9 marks)

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9.

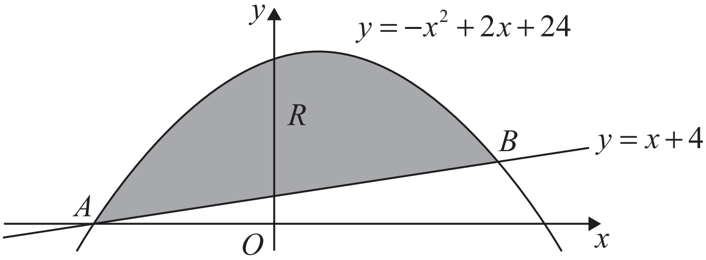

Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

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8.

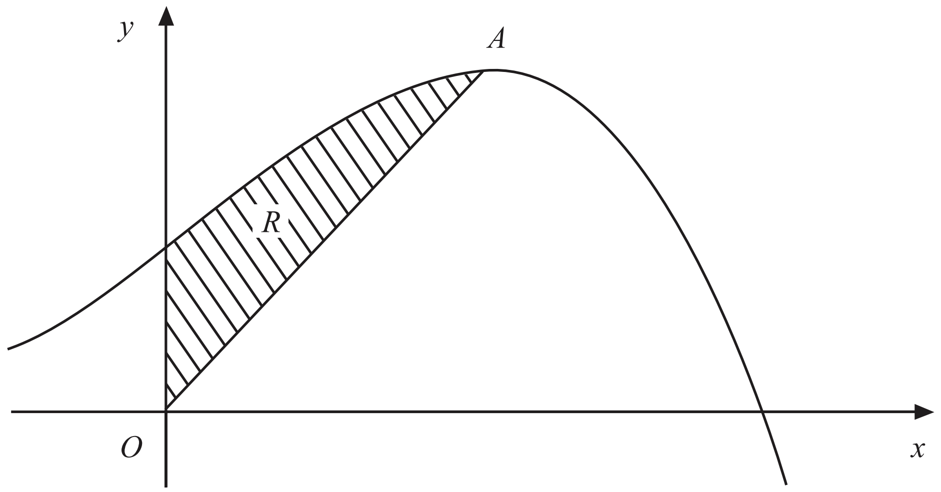

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

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6.

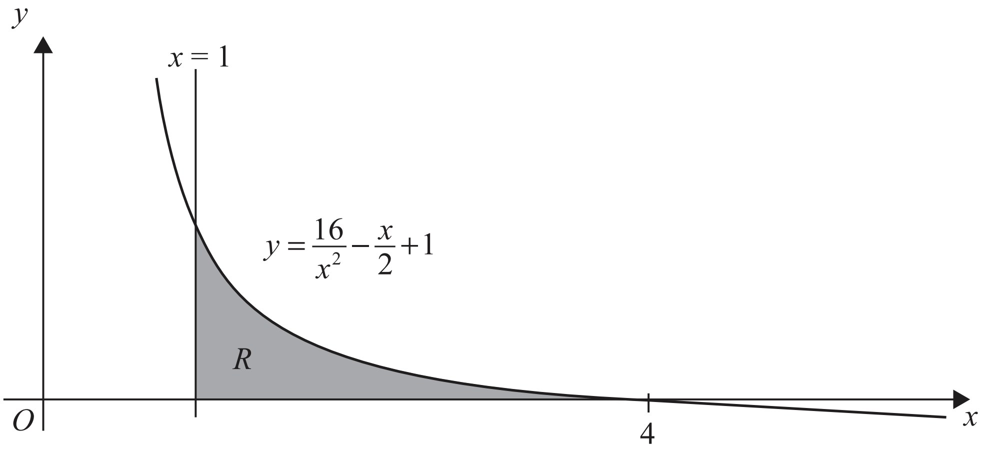

Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(5)

Leave blank

9.

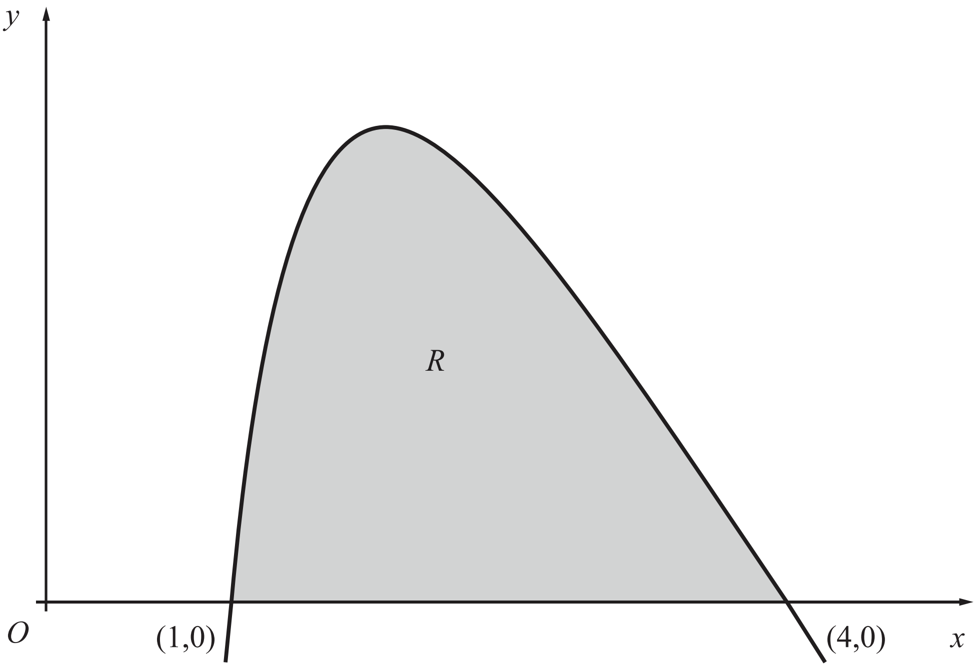

Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)

Leave blank

10.

Figure 1

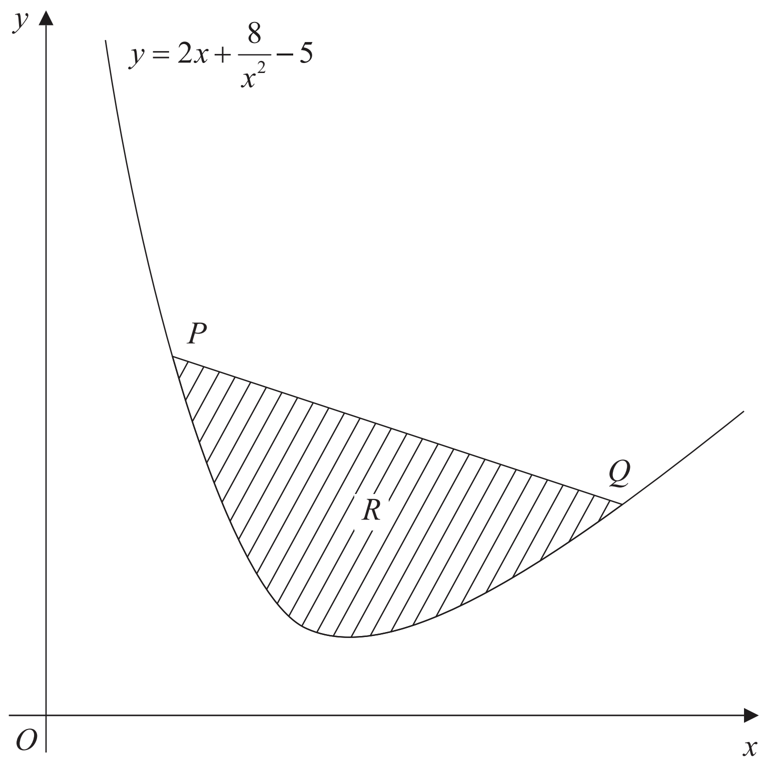

Figure 1 shows part of the curve C with equation $y = 2x + \frac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(a) Find the exact area of R.

(8)

(b) Use calculus to show that y is increasing for $x > 2$.

(4)

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5.

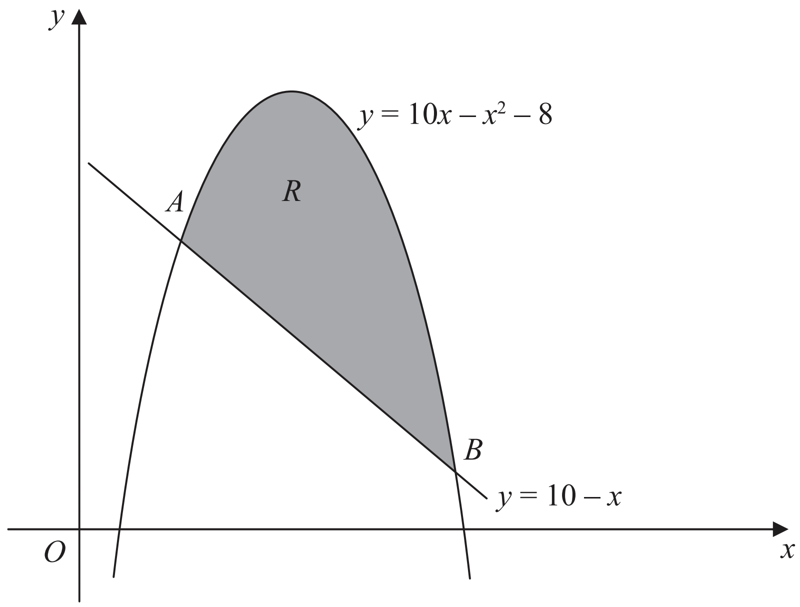

Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

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10.

Figure 3

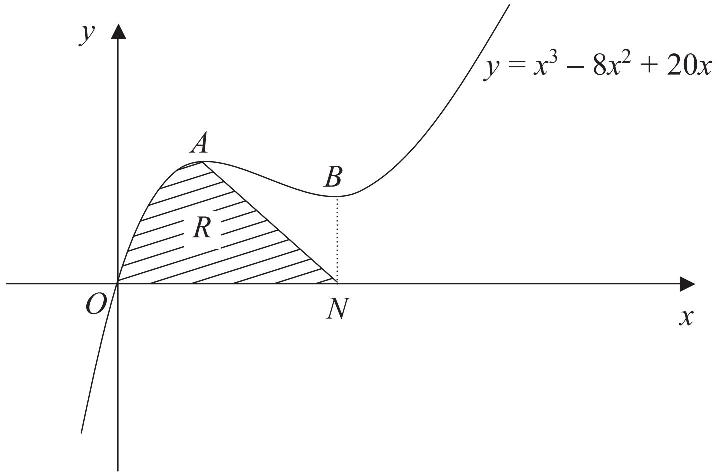

Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)