

Question Number	Scheme		Marks
2(a)	$\bar{x} = \frac{4530}{60} = 75.5$		B1
	$s^2 = \frac{350761 - 60 \times 75.5^2}{59}$		M1
	$= \frac{8746}{59} = 148.237...$	awrt 148	A1
			(3)
(b)	$H_0 : \mu_B - \mu_A = 2, \quad H_1 : \mu_B - \mu_A > 2$		B1
	$\pm \frac{82.3 - 75.5 - 2}{\sqrt{\frac{17.4^2}{80} + \frac{148}{60}}} = 1.919...$	awrt 1.92	M1 dM1 A1
	Critical value $z = 1.6449$ (p value awrt $0.0274 < 0.05$)		B1
	Reject H_0 / Significant result		ddM1
	Sufficient evidence to suggest that the mean length of salmon in region B is more than 2 cm greater than the mean length of salmon in region A / Sufficient evidence to support the biologist claim.		A1ft
			(7)
(c)	Assumed that the sample variance (s^2) is equal to the population variance (σ^2) for both regions		B1
			(1)
Notes			Total 11
(a)	B1	75.5 cao	
	M1	A correct expression for either s or s^2 (ft their mean)	
	A1	$\frac{8746}{59}$ or awrt 148	
(b)	B1	Both hypotheses stated correctly, must be in terms of μ . Allow equivalent hypotheses. Condone the use of X and Y for A and B	
	M1	Attempt at the SE using their part (a). e.g. $\sqrt{\frac{17.4^2}{80} + \frac{148}{60}}$. May be in their standardisation. (Implied by awrt ± 1.92 seen or awrt 0.0274)	
	dM1	Dependent on 1st M1 for standardising correctly. Allow $\pm$. Must be in the form $\frac{82.3 - 75.5 - 2}{\sqrt{\frac{p}{q} + \frac{r}{s}}}$ or equivalent. (Implied by awrt ± 1.92 seen or awrt 0.0274)	
	A1	awrt 1.92 or awrt -1.92 or awrt 0.0274	
	B1	Critical value of ± 1.6449 or better (seen). Allow for a p value of awrt $0.0274 < 0.05$ using tables)	
	ddM1	Dependent on both previous M marks. A correct non-contextual statement based on their normal CV and their test statistic. – Do not allow contradicting non contextual comments.	
	A1ft	Correct comment in context rejecting the null hypothesis. Must mention mean, length, region Allow sufficient evidence to support the biologists claim (o.e)	

(c)	B1	Suitable comment assuming $s^2 = \sigma^2$ for both regions (o.e) (Allow $s_x^2 = \sigma_x^2$ and $s_y^2 = \sigma_y^2$ (o.e))
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Question Number	Scheme		Marks
3(a)	It involves an unknown (population) parameter σ		B1
			(1)
(b)	$\left[E(S) = E\left(\frac{2}{5}X_1 + X_2 - \frac{1}{3}X_3\right) \right] = \frac{2}{5}E(X_1) + E(X_2) - \frac{1}{3}E(X_3) = \frac{2}{5}\mu + \mu - \frac{1}{3}\mu$		M1
	$\frac{2}{5}\mu + \mu - \frac{1}{3}\mu = \frac{16}{15}\mu \neq \mu$ (so S is a biased estimator of μ)		A1
			(2)
(c)	Bias $E(S) - \mu = \frac{16}{15}\mu - \mu = \frac{1}{15}\mu$		B1ft
			(1)
(d)	$E(Y) = aE(X_1) + bE(X_2) - 3E(X_3) = \mu$ $\Rightarrow (a + b - 3)\mu = \mu$		M1
	$a + b = 4$		A1
			(2)
(e)	$\text{Var}(Y) = a^2\text{Var}(X_1) + b^2\text{Var}(X_2) + (-3)^2\text{Var}(X_3) = (a^2 + b^2 + 9)\sigma^2$		M1
	$\text{Var}(Y) = (a^2 + (4 - a)^2 + 9)\sigma^2$		M1
	$\text{Var}(Y) = (2a^2 - 8a + 25)\sigma^2$		A1
			(3)
(f)	$\text{Var}(Y) = (2(a - 2)^2 + 17)\sigma^2$		M1
	Smallest value of $\text{Var}(Y) = 17\sigma^2$		A1ft
			(2)
	Notes		Total 11
(a)	B1	Suitable comment referring to the bold words and including σ Ignore any references to μ	
(b)	M1	Writing or using $E(S) = \frac{2}{5}E(X_1) + E(X_2) - \frac{1}{3}E(X_3)$. Must be in terms of $E(X)$ or μ .	
	A1	Implied by correct expression $\frac{2}{5}\mu + \mu - \frac{1}{3}\mu$ (o.e). Condone missing subscripts.	
(c)	B1ft	$\frac{1}{15}\mu$ (awrt 0.07μ) or follow through their part (b) $-\mu$	
(d)	M1	For writing or using $E(Y) = aE(X_1) + bE(X_2) - 3E(X_3) = \mu$. Implied by $a + b = 4$. Condone missing subscripts.	
	A1	cao	
	SC	If they state, $a = b = 2$ then score M1A1	
(e)	M1	Writing or using $\text{Var}(Y) = a^2\text{Var}(X_1) + b^2\text{Var}(X_2) + (-3)^2\text{Var}(X_3)$. Condone missing subscripts.	
	M1	For substituting of $b = 4 - a$ ft their part (d) into their expression for $\text{Var}(Y)$	
	A1	$\text{Var}(Y) = (2a^2 - 8a + 25)\sigma^2$ (o.e)	
	SC	Using $-9\text{Var}(X_3)$ instead of $(-3)^2\text{Var}(X_3)$ leads to $(2a^2 - 8a + 7)\sigma^2$ M1M1A0	

(f)	M1	For a valid method to find minimum e.g. differentiation, complete the square. Implied by 17 seen if their values of p , q and r are correct.
	A1ft	$17\sigma^2$ or ft their a and using the correct expression $(2a^2 - 8a + 25)\sigma^2$. $\text{Var}(Y) < 0$ A0

Question Number	Scheme		Marks
4(a)	Use of $\frac{335.6}{28} \pm 2.3263 \times \frac{0.9}{\sqrt{28}}$		M1B1
	$= (11.590..., 12.381...)$ awrt (11.6, 12.4)		A1A1
			(4)
(b)	The weights are normally distributed		B1
			(1)
(c)	As 12 is in the CI there is insufficient evidence against the manufacturer's claim		B1ft
			(1)
(d)	$\frac{5.07 - 5}{\frac{0.3}{\sqrt{n}}} = 1.19$		M1B1
	$\sqrt{n} = \frac{"1.19" \times 0.3}{0.07}$ (o.e)		dM1
	$[n =]26$		A1
			(4)
	Notes		Total 10
(a)	M1	For use of correct expression with 0.9, 28 and $1 < z < 3$ Ignore their value for $\bar{x}$	
	B1	For $z = 2.3263$ or better (calc: 2.326347931)	
	1 st A1	For awrt 11.6 or awrt 12.4 (must come from awrt 2.33)	
	2 nd A1	For awrt 11.6 and awrt 12.4 (must come from awrt 2.33)	
	Note	M1B0A1A1 is possible	
(b)	B1	For a suitable comment referring weights being normally distributed. It is a normal distribution – B0 Any comments implying not being normally distributed – B0	
(c)	B1ft	Comment suggesting claim is accepted / true (o.e) and that 12 is in the CI ft on their confidence interval. 12 needs to be seen. It is in the confidence interval and claim is accepted / rejected (o.e) B0. “awrt 11.6 < 12 < awrt 12.4” is equivalent to 12 is in the confidence interval.	
(d)	M1	For correct standardisation using 5.07, 5 and $\frac{0.3}{\sqrt{n}}$ (o.e) set equal to $1 < z < 3$. Allow $\pm$ provided the signs are compatible.	
	B1	awrt 1.19	
	dM1	Dependent on 1 st method mark. For rearranging to and equation in which n can be found e.g. $\sqrt{n} = \frac{"1.19" \times 0.3}{0.07}$, $\frac{0.3}{n} = \frac{1}{289}$ or equivalents. Implied by $n = 26$	
	A1	cao (Correct answer with no incorrect work seen 4/4)	

Question Number	Scheme		Marks
5(a)(i) (ii)	Quota (sampling)		B1
	e.g. May not reach the quota of customers.		dB1
			(2)
(b)	$\frac{65 \times 60}{200}$ or $\frac{57 \times 60}{200}$		M1
	19.5 and 17.1		A1
			(2)
(c)	H ₀ : The type of products bought, and customer age are independent (not associated) H ₁ : The type of products bought, and customer age are not independent (are associated)		B1
	Observed	Expected	$\frac{(O - E)^2}{E}$
	29	"19.5"	$\frac{(29 - "19.5")^2}{"19.5"} = 4.628...$
	8	"17.1"	$\frac{(8 - "17.1")^2}{"17.1"} = 4.84...$
	$\sum \frac{(O - E)^2}{E} = 6.456 + "4.628..." + "4.84..."$		M1
	= 15.926... awrt 15.9		A1
	$\nu = (3 - 1)(3 - 1) = 4$ $\chi^2_4(0.01) = 13.277 \Rightarrow CR : X^2 \geq 13.277$		B1 B1ft
	[In the CR / Significant / Reject H ₀] Sufficient evidence to suggest there is evidence of an association between the range of products bought and customer age		dA1
			(7)
	Notes		Total 11
(a)(i)	B1	Correct sampling method	
(a)(ii)	dB1	Dependent on B mark awarded in a(i). For a suitable contextual comment e.g., customers divided into groups which may not be appropriate or customers who do not want to participate are not recorded. Allow e.g. not all people may want to say what their age is or may give a different age. No reason that this would give a sample representative of the customers, (Condone introduces bias, it is not random so can introduce bias (o.e)) Random on its own is not enough . eg. It is not random\ non-random sampling)– B0,	
(b)	M1	A correct method for finding one expected value. Implied by one correct value.	
	A1	Both answers correct	
(c)	B1	For both hypotheses and attached to H ₀ and H ₁ Must mention product and age at least once. Use of "relationship" or "correlation" or "connection" is B0.	
	M1	A correct method for finding both missing values for the χ^2 value or awrt 4.63 or awrt 4.84 Allow truncated answers. May be implied by awrt 15.9. May be seen by the table.	
	M1	For adding their two values to 6.456 (may be implied by a full χ^2 calculation, with at least 3 correct expressions or values. May be seen by the table.	
	A1	awrt 15.9 (Correct answer only with no incorrect working seen M1 M1 A1)	
	B1	$\nu = 4$ This mark can be implied by a correct critical value of 13.27[7] or better	
	B1ft	13.277 or better, ft their degrees of freedom and using a 1% significance level. e.g. $\nu = 2$ 9.21[0] or better, $\nu = 3$ 11.34[5] or better, $\nu = 9$ 21.66[6].or better, $\nu = 8$ 20.09[0] or better	
	dA1	Independent of the hypotheses mark but dependent on all other previous marks.	

		A correct contextual conclusion compatible with their values, which has the words product and age . Contradictory statements score A0. Condone 'relationship' or 'connection' here but do not allow correlation. Ignore their hypotheses
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Question Number	Scheme		Marks
6(a)	$D = F - M$		M1
	$D \sim N(5, 23)$		A1A1
	$[P(D > 2)] = P\left(Z > \frac{2 - '5'}{\sqrt{'23'}}\right) [= 1 - P(Z < -0.6255..)]$ or		M1
	$[P(D < -2)] = P\left(Z > \frac{-2 + '5'}{\sqrt{'23'}}\right) [= 1 - P(Z < -0.6255..)]$		
	$= 0.7357$	awrt 0.736	A1
			(5)
(b)	$X = M_1 - M_2$		M1
	$X \sim N(0, 18)$		A1A1
	$[P(X > 1)] = P\left(Z > \frac{1 - '0'}{\sqrt{'18'}}\right) [= P(Z > 0.2357...)]$ $\left(\text{or } [P(X < -1)] = P\left(Z < \frac{-1 - '0'}{\sqrt{'18'}}\right) \right)$		dM1
	$P(X > 1) = 2 \times P(X > 1) = 2 \times '0.4052'$		M1
	$= 0.8104$	awrt 0.810	A1
			(6)
(c)	$T = F_1 + F_2 - 2M_1$		M1
	$T \sim N(\pm 10, 64)$		A1A1
	$P(T > 0) = P\left(Z > \frac{0 - '10'}{\sqrt{'64'}}\right) [= P(Z > -1.25)]$		dM1
	$= 0.8944$	awrt 0.894	A1
			(5)
	Notes		Total 16
	Parts a), b) and c) If they have not written down the correct model ($D \sim N(5, 23)$, $X \sim N(0, 18)$ or $T \sim N(\pm 10, 64)$) but you see the correct values (simplified or un-simplified) for the E(X) and Var (X) in their standardisation, then award first 4 marks (In b) award the M for seeing 2 x) and then the final A mark for a correct answer.		
(a)	M1	States or uses a correct model $D = F - M$ or $D = M - F$. May be implied by a correct E(D) of 5 or -5. Condone the use of different letters.	
	A1	Correct E(D) = 5 or 29-24 If they use $M - F$ you should see E(D) = -5 or 24-29	
	A1	Correct Var(D) = 23 or 9 + 14	
	M1	Attempts standardising ($\pm$), with their mean and variance. (Implied by $\pm$ awrt 0.626)	
	A1	awrt 0.736 or 0.734 (calc: 0.73419...)	
(b)	M1	States or uses a correct model. $X = M_1 - M_2$ or $X = M_2 - M_1$ Implied by E(X) = 0. Condone the use of different letters.	
	A1	Correct E(X) = 0 or 24-24	
	A1	Correct Var(X) = 18 or 9 + 9	
	dM1	Dependent on 1st M1 for standardising ($\pm$) with their mean and variance (Implied by $\pm$ awrt 0.236). Condone a missing 0 in their standardisation.	

	M1	For 2 x seen or implied (calc: 0.4068....)
	A1	awrt 0.810 or 0.814 (calc: 0.81366...)
(c)	M1	States or uses a correct model $T = F_1 + F_2 - 2M_1$ or $T = 2M_1 - F_1 - F_2$ Implied a correct $\text{Var}(T)$. Condone the use of different letters and missing subscripts
	A1	Correct $E(T)$ of 10 ($29+29-2 \times 24$) or -10 ($2 \times 24 - 29 - 29$)
	A1	Correct $\text{Var}(T) = 64$ or $14 + 14 + 2^2 \times 9$
	dM1	Dependent on 1st M1 for standardising ($\pm$) with their mean and variance (Implied by ± 1.25). Condone a missing 0 in their standardisation.
	A1	awrt 0.894 (calc: 0.89435...)

Question Number	Scheme		Marks
7 (a)	$[E(X) = \frac{k-6+k+10}{2} \quad [=k+2]$		M1
	$[Var(X) = \frac{(k+10-k+6)^2}{12} \quad \left[= \frac{64}{3} \right]$		M1
	$\bar{X} \sim N\left(k+2, \frac{4}{15}\right)$		A1ft
			(3)
(b)	$17.5 - 2.5758\sqrt{\frac{4}{15}} < \mu < 17.5 + 2.5758\sqrt{\frac{4}{15}}$		M1 B1
	$17.5 - 2.5758\sqrt{\frac{4}{15}} - 4 < k - 2 < 17.5 + 2.5758\sqrt{\frac{4}{15}} - 4$		M1
	$12.2 < Q_1 < 14.8$	awrt (12.2, 14.8)	A1
			(4)
	Notes		Total 7
(a)	M1	For attempting to find the mean of X	
	M1	For attempting to find the variance of X	
	A1ft	For a correct distribution. Condone awrt 0.267, $\frac{64/3}{80}$ for $\frac{4}{15}$. They must have divided their $Var(X)$ by 80.	
(b)	M1	For attempting at least one tail of confidence interval $17.5 \pm "z"(\text{their } SE_{\bar{X}})$. Follow their z value, $1 < z < 3$ Alternatively, may estimate the lower quartile as 13.5 and attempt at least one tail of confidence interval for the lower quartile . Dividing by 80 again is M0 Follow through the $\sqrt{Var(\bar{X})}$ Implied by the 2 nd M mark	
	B1	For either awrt +2.5758 or -2.5758	
	M1	For attempting both tails of $17.5 \pm "z"(\text{their } SE_{\bar{X}}) - 4$. Follow their z value, $1 < z < 3$	
	A1	$12.2 < Q_1 < 14.8$ or awrt (12.2, 14.8)	