## S3\_2023\_06\_MS

Question Number		Scheme	Marks	
1 (a)	When th	e data is ordinal e.g. Judges' ranks	B1	
	When a	non-linear relationship might be expected	B1	
			(2)	
(b)	$H_0: \rho =$	$0, H_1: \rho \neq 0$	B1	
	Critical	value $r_s = -0.6485$ or CR: $r_s \leq -0.6485$ (and $r_s \geq 0.6485$ )	B1	
	Reject H	$I_0$ or significant or lies in the critical region	M1	
	-	arman's rank correlation coefficient shows there is sufficient evidence of a on [between the length and maximum diameter of the melons]	A1	
			(4)	
(c)	$H_0: \rho =$	$0, H_1: \rho < 0$	B1	
	Critical	value $r = -0.5494$ or CR: $r \leq -0.5494$	B1	
	-	luct moment correlation coefficient shows there is insufficient evidence of a correlation [between the length and maximum diameter of the melons]	B1	
			(3)	
		Notes	Total 9	
(a)	B1	For one correct condition		
	B1	For a second correct condition. Condone not underlying normal		
(b)	B1For both hypotheses correct. Must be in terms of $\rho$ . Must be attached to H <sub>0</sub> and HB1For critical value of -0.6485 (Allow -0.5636 if a one tailed test is stated for H <sub>1</sub> ) Condone 0.6485 if compared with 0.673		d H <sub>1</sub>	
			)	
	M1	A correct statement – no context needed but do not allow contradicting non contextual comments. ft their CV provided the CV is negative (May be implied by a correct conclusion) Condone a positive CV if a comparison with 0.673 seen		
	A1	For a correct conclusion which is rejecting $H_0$ Allow negative correlation This independent of the hypotheses		
(c)	<b>B1</b>	For both hypotheses correct. Must be in terms of $\rho$ . Must be attached to H <sub>0</sub> and		
	<b>B1</b> For critical value of -0.5494 (Allow -0.6319 if a two tailed test is stated for H <sub>1</sub> ) Condone 0.5494 if compared with 0.525		ı)	
	<b>B1</b>	For a correct conclusion which is not rejecting $H_0$		

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Question Number	Scheme				Marks	
2 (a)	$\frac{60 \times 60}{240}$ or $\frac{60 \times 84}{240}$ or $\frac{60 \times 96}{240}$				M1	
	15 and 2	21 and 24				A2
					(3)	
(b)	0			n the payment amount and n the payment amount and		B1
	Obse	erved	Expected	$\frac{\frac{(O-E)^2}{E}}{\frac{(23-'15')}{'15'}} = 4.2667$		
	2	23	15	$\frac{(23-'15')}{'15'} = 4.2667$		M1
	2	21	21	$\frac{(21 - '21')}{'21'} = 0$ $\frac{(16 - '24')}{'24'} = 2.6667$		
	1	16	24	$\frac{(16-'24')}{'24'} = 2.6667$		
	$\chi^2 = 2.4$	4048 + '4.2	667'+'0'+'2.666	7'		M1
	= 9.3381 awrt 9.34					A1
	$v = (3-1)(3-1) = 4$ $\chi_4^2(0.05) = 9.488 \implies CR: X^2 \ge 9.488$				B1 B1ft	
	[Not in the CR/Not significant/Do not reject H <sub>0</sub> ] There is no evidence of an association between <b>the payment amount and payment method used</b>			dA1		
	octween the payment amount and payment method used			(7)		
				Notes		Total 10
(a)	M1	For a co	rrect method for	finding one expected value	2	•
	A2		answers correct		that sum to $60$	
(b)	<b>R2</b> (A1 for 2 correct answers or 1 correct and 3 values that sum to 60) <b>B1</b> Both hypotheses correct. Must mention method <b>and</b> amount with payment at least on (may be written in terms of independence)			least once.		
	M1 For a correct method for finding all three contributions to the $\chi^2$ value ft their p be implied by 3 correct values If expected values are incorrect then working m shown					part a May
	M1 For adding their values to 2.4048 (If all 9 values are calculated the 6 values no part (a) must have working shown or the correct values seen or awrt 9.34)				ot found in	
	A1	awrt 9.34				
	B1	v = 4 This mark can be implied by a correct critical value of 9.488				
	B1ft	9.488 or	better ft their Do	oF		
	$\begin{array}{c} \text{Dependent on both M marks. A correct contextualised conclusion which is not H_0} \\ Must mention method and amount. If no hypotheses on they are the years we have the set of the$					
	<b>dA1</b> Must mention <b>method</b> and <b>amount</b> . If no hypotheses or they are the wrong wa then A0 here. Contradictory statements score A0. e.g. "Significant, do not rejo". Condone "relationship" or "connection" here but <b>not</b> "correlation".					
		".Condo	one "relationship"	or "connection" here but	not "correlation".	

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Question Number		Scheme	Marks	
3 (a)	It is not a	a statistic as it involves unknown [population] parameter	B1	
			(1)	
(b)	$\mathrm{E}(S) = \mathrm{H}$	$E\left(\frac{3}{5}X_{1} + \frac{5}{7}X_{2}\right) = \frac{3}{5}E(X_{1}) + \frac{5}{7}E(X_{2})$	M1	
	$=\frac{3}{5}\mu + \frac{3}{5}\mu$	$\frac{5}{7}\mu = \frac{46}{35}\mu \neq \mu \qquad \text{So } S \text{ is a biased estimator for } \mu$	A1	
			(2)	
(c)	$\frac{46}{35}\mu' - \mu'$	$\mu = \frac{11}{35}\mu$	B1ft	
			(1)	
(d)	$\mathbf{E}(Y) = a\mathbf{E}(X_1) + b\mathbf{E}(X_2) = \mu$			
(u)	$\Rightarrow (a+b)\mu = \mu$			
	a+b=1		A1	
			(2)	
(e)	$\operatorname{Var}(Y)$	$= a^{2} \operatorname{Var}(X_{1}) + b^{2} \operatorname{Var}(X_{2}) = (a^{2} + b^{2}) \sigma^{2}$	M1	
	$\operatorname{Var}(Y)$	$=(a^2+(1-a)^{\prime 2})\sigma^2$	M1	
	$\operatorname{Var}(Y)$	$= \left(2a^2 - 2a + 1\right)\sigma^2 *$	A1*	
			(3)	
		Notes	Total 9	
(a)	B1	For a correct explanation Allow $\sigma$ is unknown (Do not allow $\sigma$ is unknown variable)	riance)	
(b)	M1	For writing or using $E(S) = aE(X_1) + bE(X_2)$ Condone missing subscripts		
	A1	cao (Allow $1.31 \mu \neq \mu$ )		
(c)	B1ft	Follow through their part (a) – $\mu$		
(d)	M1	For writing or using $E(Y) = aE(X_1) + bE(X_2) = \mu$ (May be implied by $a + b =$ Condone missing subscripts	1)	
	A1	Cao		
(e)	M1	For writing or using $\operatorname{Var}(Y) = a^2 \operatorname{Var}(X_1) + b^2 \operatorname{Var}(X_2)$ Condone missing subs	cripts	
	M1	For substitution of $b = 1 - a$ ft their part (d) into their expression for Var(Y)		
	A1*	Answer is given so no incorrect working must be seen		

Question Number		Scheme	Mark
		$\frac{2}{25}t  dt = \frac{2}{25} \left[ \frac{t^2}{2} \right]_a^{a+1} \text{ or } F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{25}t^2 & 0 \le t < 5 \text{ or} \\ 1 & t > 5 \end{cases}$ $\frac{5}{5}(a+1) + \frac{2}{25}a \left( a+1-a \right)$	M1
		$(a+1)^2 - a^2$ or $\frac{1}{25}(a+1)^2 - \frac{1}{25}a^2$ or $\left(\frac{1}{25}a + \frac{1}{25}a + \frac{1}{25}a\right)$	M1
	$\frac{1}{25}(a$	$a^{2} + 2a + 1 - a^{2}$ ) oe $\left[ = \frac{1}{25} (2a + 1) \right]^{*}$	A1*
	н.,	The data could be modelled by the p.d.f	(3)
(b)	0	The data could not be modelled by the p.d.f	B1
	Expec	eted frequencies: 6, 18, 30, 42, 54	M1 A1
		$\frac{(2-E)^2}{E} = \frac{(10-6')^2}{6'} + \dots + \frac{(68-54')^2}{54'}$ $\frac{(2-E)^2}{E} - N = \frac{10^2}{6'} + \dots + \frac{68^2}{54'} - 150 \text{ or } 2.666\dots + 1.388\dots + 1.2 + 1.166\dots + 3.629$	M1
	= 10.0		A1
	v = 4		B1
	$\chi_{4}^{2}(0$	$(0.05) = 9.488 \implies CR \ge 9.488$	B1ft
		e CR so there is sufficient evidence to reject H <sub>0</sub> ]	
	-	ient evidence to say that data does not fit the given p.d.f	dA1 (8)
		Notes	Total 11
(a)	M1	For correct integration, ignore limits or finding the area of a trapezium	
	M1	For substitution of the limits. May be implied by $\frac{1}{25}(a^2+2a+1-a^2)$ or simplifyin expression for the area of the trapezium	g the
	A1*	Answer is given so no incorrect working should be seen. At least one correct line of from the method mark to the final answer should be seen	working
(b)	B1	Both hypotheses correct. Allow $H_0$ : The p.d.f/f(t) is a suitable model $H_1$ : The p.d.f/f(t) is not a suitable model $H_1$ .	nodel
	M1	For a correct method to find at least one expected frequency e.g. $\frac{1}{25} \times 150$ Ignore and	ıy
		reference to limits	
	A1	For all 5 expected frequencies correct	ad
	M1	For an attempt at the test statistic, at least 2 correct expressions/values ft their expect frequencies	ea
	A1	awrt 10.1	
	B1	v = 4 This mark can be implied by a correct critical value of 9.488	
	B1ft	9.488 or better ft their DoF	
	dA1	Dependent on 2 <sup>nd</sup> M1. A correct conclusion based on their $\chi^2$ critical value	
	UAI	If no hypotheses or they are the wrong way round, then A0 here.	

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Question Number		Scheme	Marks
5 (a)	$\overline{x} \pm 1.64$	$49 \times \frac{5}{\sqrt{10}}$	M1 B1
	$\overline{x} \pm 2.60$	$0 \Rightarrow (\overline{x} - 2.60, \overline{x} + 2.60) *$	A1*
			(3)
(b)	$\overline{y} \pm 1.96$	$ \Rightarrow \frac{3}{\sqrt{20}}  \Rightarrow (\overline{y} - 1.31, \overline{y} + 1.31) $	M1 B1
	$\overline{y} \pm 1.31$	$\Rightarrow (\overline{y} - 1.31, \overline{y} + 1.31)$	A1
			(3)
(c)(i)	$\overline{X} - \overline{Y} \sim$	$ \cdot \operatorname{N}\left(\mu - \mu, \ \frac{5^2}{10} + \frac{3^2}{20}\right) \Longrightarrow \overline{X} - \overline{Y} \sim \operatorname{N}(0, \ 2.95) $	M1 A1
(ii)	Do not o	overlap when either	
		$y > \overline{y} + 1.31'$ or $\overline{x} + 2.60 < \overline{y} - 1.31'$	M1
		3.91 or $\bar{x} - \bar{y} < -3.91$	Alft
	$2 \times P(\overline{X})$	$-\overline{Y} > 3.91) = 2 \times P\left(Z > \frac{'3.91' - '0'}{'\sqrt{2.95'}}\right) = \left[2 \times P(Z > 2.276)\right]$	M1 M1
		$[13] = 0.0226$ (calculator gives $[2 \times 0.0114] = 0.0228$ )	A1
1			111
			(7)
		Notes	
(a)	M1		(7)
(a)		Notes	(7)
(a)	 M1	For use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$	(7) Total 3
(a) (b)	M1 B1	NotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or better	(7) Total 3
	M1 B1 A1*	Notes         For use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or better         Answer is given so no incorrect working should be seen (condone use of 1.645)	(7) Total 3
	M1 B1 A1* M1	Notes         For use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or better         Answer is given so no incorrect working should be seen (condone use of 1.645)         For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$	(7) Total 3
	M1 B1 A1* M1 B1	NotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645)For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or better	(7) <b>Total 3</b>
(b)	M1 B1 A1* M1 B1 A1	NotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645)For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or betterFor use of $z = 1.96$ or betterFor $(\overline{y} - awrt1.31, \overline{y} + awrt1.31)$ Allow 1.315	(7) Total 3
(b)	M1 B1 A1* M1 B1 A1 M1	NotesNotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or betterFor $(\overline{y} - \operatorname{awrt1.31}, \overline{y} + \operatorname{awrt1.31})$ Allow 1.315For a correct method to find the variance (May be seen in a standardisation expression)For N(0, 2.95) (May be seen in a standardisation expression)Allow N $\left(0, \frac{5}{16}\right)$ For $\overline{x} - 2.60 > \overline{y} + 1.31$ oe or $\overline{x} + 2.60 > \overline{y} - 1.31$ oe ft part (b)	(7) Total 3
(b) (c)(i)	M1 B1 A1* M1 B1 A1 M1 A1	NotesNotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645)For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or betterFor $(\overline{y} - \operatorname{awrt1.31}, \overline{y} + \operatorname{awrt1.31})$ Allow 1.315For a correct method to find the variance (May be seen in a standardisation expression) Allow N $\left(0, \frac{5}{10}\right)$ For N $(0, 2.95)$ (May be seen in a standardisation expression) Allow N $\left(0, \frac{5}{10}\right)$	(7) Total 3
(b) (c)(i)	M1 B1 A1* M1 B1 A1 M1 A1 M1 M1	NotesNotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or betterFor $(\overline{y} - awrt1.31, \overline{y} + awrt1.31)$ Allow 1.315For a correct method to find the variance (May be seen in a standardisation expression)For N(0, 2.95) (May be seen in a standardisation expression) Allow N $\left(0, \frac{5}{10}, \frac{1}{10}\right)$ For $\overline{x} - 2.60 > \overline{y} + 1.31$ oe or $\overline{x} + 2.60 > \overline{y} - 1.31$ oe ft part (b)For $\overline{x} - \overline{y} > '3.91'$ or $\overline{x} - \overline{y} < -'3.91'$ ft part (b)For multiplying by 2 (may be seen at any stage of their working)	(7) <b>Total 3</b> $(5)$ $(7)$
(b) (c)(i)	M1 B1 A1* M1 B1 A1 M1 A1 M1 A1 ft	NotesNotesFor use of $\overline{x} \pm z$ value $\times \frac{5}{\sqrt{10}}$ For use of $z = 1.6449$ or betterAnswer is given so no incorrect working should be seen (condone use of 1.645For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $\overline{y} \pm z$ value $\times \frac{3}{\sqrt{20}}$ For use of $z = 1.96$ or betterFor use of $z = 1.96$ or betterFor $(\overline{y} - \operatorname{awrt1.31}, \overline{y} + \operatorname{awrt1.31})$ Allow 1.315For a correct method to find the variance (May be seen in a standardisation expression)For N(0, 2.95) (May be seen in a standardisation expression) Allow N $\left(0, \frac{5}{10}, \frac{5}{10}, \frac{1}{10}, \frac{1}{10}$	(7) <b>Total 3</b> $(5)$ $(7)$

Question Number		Scheme	Marks		
6 (a)	$\alpha = 5.1$		B1		
	$\beta = \sqrt{\frac{16}{3}}$	$\frac{694.65 - 65 \times ('5.1')^2}{64}$	M1		
	= 0.25		A1		
			(3)		
(b)	$ \begin{aligned} \mathbf{H}_{0} &: \boldsymbol{\mu}_{A} \\ \mathbf{H}_{1} &: \boldsymbol{\mu}_{A} < \end{aligned} $	$<\mu_{\scriptscriptstyle R}$	B1		
		5.0-'5.1'			
	$z = \pm -$	$\frac{5.0 - 5.1'}{0.24^2} + \frac{0.25'^2}{65}$	M1 M1		
	√⁻	70 + 65			
	= -2.3		A1		
	One tail	ed c.v. $z = -1.6449$ or CR: $z \le -1.6449$	B1		
	In CR/S	ignificant/Reject H <sub>0</sub>	M1		
	Sufficier	nt evidence to support Roxane's claim	A1		
			(7)		
(c)	Since th	e sample is <b>large</b> the <b>CLT</b> applies.	M1		
	No [nee	d to assume that the fat content is normally distributed]	A1		
			(2)		
(d)	Assumed that $s^2 = \sigma^2$ in <b>both</b> groups				
			(1)		
		Notes	Total 13		
(a)	<b>B</b> 1	cao			
	M1	For a correct method to find $\beta$ using their $\alpha$			
	A1	Cao			
(b)	<b>B</b> 1				
	M1	For correct standard error ft their <i>s</i> in part a			
	M1	For an attempt to find the test statistic, ft their SE and their $\alpha$			
	A1	awrt –2.37 (Allow 2.37)			
	<b>B</b> 1	-1.6449 or better (seen) (Allow 1.6449 or better if comparing to their 2.37)			
	<b>M1</b>	A correct statement – need not be contextual but do not allow contradicting n contextual comments. ft their CV and test statistic	on		
	A1	A correct contextual statement e.g sufficient evidence to support that crisps from brand			
(c)	M1	A suitable comment that mentions large and CLT			
	A1	A correct answer, context not required.			
(d)	<b>B</b> 1	For the assumption that sample variance = population variance for <b>both</b> grou	ps		

Question Number		Scheme	Marks	
7 (a)	$E(X) = 4 \times 15 - 3 \times 10[=30]$			
	Var(X)	$= 4^2 \times 5^2 + 3^2 \times 4^2 [= 544]$	M1	
	So <i>X</i> ~ 1	N(30, 544)		
	P(X < 4	$0) = P\left(Z < \frac{40 - '30'}{'\sqrt{544'}}\right) \left[ = P\left(Z < 0.428\right) \right]$	M1	
		= 0.6664 (Calculator gives 0.6659) awrt 0.666	A1	
			(4)	
(b)	E(A+B)	$(2+D) = 15+10+3\times 20 = [85]$	M1	
	Var(A +	$B + D) = 5^{2} + 4^{2} + 3 \times \sigma^{2} = [41 + 3\sigma^{2}]$	M1	
	So $A + B$	$B + D \sim N(85, 41 + 3\sigma^2)$		
	$P(A+B+D<76) = P\left(Z < \frac{76-85}{\sqrt{41+3\sigma^2}}\right) = 0.242$			
	So $\frac{-}{\sqrt{41}}$	$\frac{-9}{+3\sigma^2} = -0.7$ or $\frac{9}{\sqrt{41+3\sigma^2}} = 0.7$ (Calculator gives -0.69988)	M1 A1	
	$3\sigma^2 = \left(\frac{1}{2}\right)$	$\left(\frac{-9}{-0.7}\right)^2 - 41$	dM1	
	$\sigma = 6.43$	awrt 6.44	A1	
			(6)	
		Notes	Total 10	
(a)	M1	For a correct method to find $E(X)$ . May be implied by a correct standardisati expression.	on	
	M1	Ear a correct method to find $Var(X)$ Allow $\sqrt{544}$ or or 23.3 <sup>2</sup> or better. May be implied		
	M1	For standardising $(\pm)$ using their mean and their variance		
	A1	awrt 0.666		
(b)	M1	For a correct method to find $E(A + B + D)$		
	M1	For a correct method to find $Var(A + B + D)$		
	1.61	For standardising $(\pm)$ using their mean and their standard deviation which is	in terms of	
	M1	$\sigma^2$ and setting equal to -0.7 or better. Allow +0.7		
	A1	For the correct equation		
	dM1	Dependent on the previous M mark. For squaring and rearranging leading to a in $\sigma^2$	an equation	
	A1	awrt 6.44 (Do not award if previous A mark was not awarded)		