

Question Number	Scheme		Marks
1 (a)	Po(isson) with ($\lambda =$)4		B1 (1)
(b)	Pairs of shoes (are sold) singly/randomly/independently/at a constant (average) rate		B1 (1)
(c) (i)	$X =$ number of sales per hour $\Rightarrow X \sim \text{Po}(4)$		
	$P(X > 4) = 1 - P(X \leq 4)$		M1
	$= 0.3712$ awrt 0.371		A1
(ii)	$(\text{'0.371...'})^3$		M1
	$= 0.051147...$ 0.05115 or awrt 0.0511		A1
			(4)
(d)	$H_0 : \lambda = '4'$ $H_1 : \lambda > '4'$		B1ft
	$P(X \geq 7) = 1 - P(X \leq 6)$ or $P(X \geq 9) = 1 - P(X \leq 8) = 0.0214$		M1
	$= 0.1107$ or CR $X \geq 9$ awrt 0.111		A1
	Not significant/Do not reject H_0 /Not in the critical region		M1
	There is insufficient evidence of an <u>increase in sales</u> following the appearance of the advert/ <u>manager's belief</u> is not supported.		dA1
			(5)
Notes			Total 11
(a)	B1	For Po or Poisson and 4 must be seen in part (a). Do not allow P(4)	
(b)	B1	For one of the given assumptions in context (must have context of shoes or sales). Ignore extraneous non-contradictory comments.	
(c) (i)	M1	For writing or using $P(X > 4) = 1 - P(X \leq 4)$	
	A1	awrt 0.371	
(ii)	M1	'part (i)' ³	
	A1	0.05115 or awrt 0.0511 (Calculator gives 0.051132...)	
(d)	B1ft	Both hypotheses correct. Must be in terms of λ or μ ft their λ from part (a) Must be attached to H_0 and H_1	
	M1	For writing or using $P(X \geq 7) = 1 - P(X \leq 6)$ If a CR approach is taken then award M1 for $P(X \geq 9) = 1 - P(X \leq 8)$ written or used This mark may be implied by a correct p -value or CR	
	A1	awrt 0.111 or CR $X \geq 9$	
	M1	Any correct ft statement consistent with their p -value and 0.05 or their CR and 7 – no context needed but do not allow contradicting non contextual comments. The comparison of their p -value and the significance level is not counted as a non contextual statement. May be implied by a correct ft conclusion in context.	
	dA1	Dependent on 1 st M1 - Correct conclusion in context which must be not rejecting H_0 . Must include the underlined words (oe).	

Question Number	Scheme			Marks	
2 (a)	20, 20, 20	20, 20, 50 (×3)	20, 50, 50 (×3)	50, 50, 50	B2 (2)
(b)	$a = 30$ and $b = 40$			B1 (1)	
(c)	$p^3 = \frac{4913}{8000}$ or $q^3 = \frac{27}{8000}$			M1	
	$p = \frac{17}{20}$ (0.85) and $q = \frac{3}{20}$ (0.15)			A1 (2)	
(d)	$[P(30)] = 3 \times p^2 \times q$ $[P(40)] = 3 \times p \times q^2$			M1 M1	
	$c = \frac{2601}{8000}$ $d = \frac{459}{8000}$			A1 (3)	
(e)	M	20	50	B1 M1 A1ft (3)	
	$P(M = m)$	$\frac{3757}{4000}$	$\frac{243}{4000}$		
Notes				Total 11	
(a)	B2	For all 4 correct combinations (B1 for 3 correct combinations) Ignore extraneous repetitions of any of the given combinations			
(b)	B1	For $a = 30$ and $b = 40$			
(c)	M1	Either $p^3 = \frac{4913}{8000}$ or $q^3 = \frac{27}{8000}$			
	A1	$p = 0.85$ oe and $q = 0.15$ oe			
(d)	M1	$[P(30)] = 3 \times (\text{their } p)^2 \times (\text{their } q)$ or $[P(40)] = 3 \times (\text{their } p) \times (\text{their } q)^2$ must see values substituted and must be using their values from part (c)			
	M1	$[P(30)] = 3 \times (\text{their } p)^2 \times (\text{their } q)$ and $[P(40)] = 3 \times (\text{their } p) \times (\text{their } q)^2$ or use of sum of probabilities = 1 i.e. $c + d = \frac{153}{400}$			
	A1	For $c = \frac{2601}{8000}$ (= 0.325125) and $d = \frac{459}{8000}$ (= 0.057375)			
(e)	B1	For 20 and 50 only (ignore notation used for M)			
	M1	Either $\frac{4913}{8000} + \text{their } c$ or $\frac{27}{8000} + \text{their } d$ for ft answers only values will need to be checked			
	A1ft	For $\frac{3757}{4000}$ oe and $\frac{243}{4000}$ oe Follow through their values for c and d but $P(M = 20) + P(M = 50)$ must sum to 1 (A table is not required).			
	NB	If a and b are reversed then allow $a = 40$ and $b = 30$ – this will mean $p = 0.15$ and $q = 0.85$, $c = \frac{459}{8000}$ $d = \frac{2601}{8000}$			

Question Number	Scheme		Marks
3 (a) (i)	$X \sim B(10, 0.1)$		
	$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9872$		M1
	$= 0.0128$		awrt 0.0128
			A1
(ii)	$P(1 < X < 5) = P(X \leq 4) - P(X \leq 1) = 0.9984 - 0.7361$		M1
	or $P(X=2) + P(X=3) + P(X=4) = 0.1937 + 0.0574 + 0.0112$		
	$= 0.2623$		awrt 0.262
			(4)
(b)	$H_0 : p = 0.1 \quad H_1 : p < 0.1$		B1
	$X \sim B(50, 0.1)$		
	$P(X \leq 2) = 0.1117$ or CR $X \leq 1$		B1
	Do not reject H_0 /Not in the critical region		M1
	There is insufficient evidence to suggest that this result supports the managing <u>director's claim</u> /not enough evidence to suggest a <u>reduction</u> in the probability of a tennis ball <u>failing</u> the bounce test		A1
			(4)
(c)	$X \sim B(n, 0.1)$ and we reject H_0 if $P(X=0) < 0.01$		
	$P(X=0) = [{}^n C_0 \times 0.1^0] \times 0.9^n [< 0.01]$		M1
	$0.9^{44} = 0.00969... [< 0.01]$	$n > \frac{\ln 0.01}{\ln 0.9} \Rightarrow n > 43.7$	M1
	$n = 44$		A1
			(3)
Notes			Total 11
(a) (i)	M1	for writing or using $P(X \geq 4) = 1 - P(X \leq 3)$	
	A1	awrt 0.0128	
(ii)	M1	for writing or using $P(X \leq 4) - P(X \leq 1)$	
	A1	or for writing or using $P(X=2) + P(X=3) + P(X=4)$	
(b)	B1	Both hypotheses correct. Must be in terms of p or π Must be attached to H_0 and H_1	
	B1	awrt 0.112 or CR ≤ 1	
	M1	A correct fit statement consistent with their p -value and 0.05 or their CR and 2- no context needed but do not allow contradicting non contextual comments. The comparison of their p -value and the significance level is not counted as a non contextual statement. May be implied by a correct fit conclusion in context. Must have a p -value or CR to access this mark.	
	A1	Correct conclusion in context which must be not rejecting H_0 . Must use underlined words (oe). No hypotheses then A0	
(c)	M1	For recognising $P(X=0) = 0.9^n$	
	M1	For $0.9^{44} (= 0.00969...)$ or $0.9^{43} (= 0.01077...)$ or rearranging to $n > \frac{\ln 0.01}{\ln 0.9} \dots$ (Allow =)	
	A1	$n >$ awrt 43.7 implies M1M1 (Allow $n =$ awrt 43.7 for M1M1)	
	A1	Cao	
	SC	Use of tables only, $n = 40, p = 0.0148$ and $n = 50, p = 0.0052$ scores M1M0A0	

Question Number	Scheme		Marks
4 (a)	$\frac{9}{20}$		B1 (1)
(b)	$(21k - k) \times \frac{\pi}{20} = 1$		M1
	$k = \frac{1}{\pi} *$		A1*
			(2)
(c) (i)	$\left[E(X) = \frac{1}{2}(k + 21k) \right] = \frac{11}{\pi}$		B1
(ii)	$\text{Var}(X) = \frac{1}{12}(21k - k)^2$ or $\text{Var}(X) = \int_{\frac{1}{\pi}}^{\frac{21}{\pi}} \frac{\pi}{20} x^2 dx - \left(\frac{11}{\pi} \right)^2$		M1
	$= \frac{100}{3\pi^2}$		A1
			(3)
(d)	$E(A) = \pi E(X^2) + 4E(X) + \frac{4}{\pi}$	$E(A) = \int_k^{21k} f(x)(A) dx = \int_k^{21k} \frac{\pi}{20} (\pi) \left(x^2 + \frac{4}{\pi} x + \frac{4}{\pi^2} \right) dx$	M1
	$E(X^2) = \frac{100}{3\pi^2} + \left(\frac{11}{\pi} \right)^2 = \frac{463}{3\pi^2}$	$E(A) = \frac{\pi}{20} (\pi) \left(\frac{x^3}{3} + \left(\frac{4}{\pi} \right) \frac{x^2}{2} + \frac{4}{\pi^2} x \right)$	M1
	$E(A) = \frac{463}{3\pi} + \frac{44}{\pi} + \frac{4}{\pi}$	sub limits $\frac{21}{\pi}$ and $\frac{1}{\pi}$	M1
	$= \frac{607}{3\pi}$	$= \text{awrt } 64.4$	A1
			(4)
Notes			Total 10
(a)	B1	0.45oe cao	
(b)	M1	use of the area of the rectangle = 1 Any equivalent rearrangement, allow $20k$ instead of $(21k - k)$	
	A1*	answer is given so a fully correct solution must be seen	
(c)(i)	B1	oe must be in terms of π (isw after correct answer seen)	
(ii)	M1	use of $\frac{(b-a)^2}{12}$ or $\text{Var}(X) = \int_{\frac{1}{\pi}}^{\frac{21}{\pi}} \frac{\pi}{20} x^2 dx - \left(\frac{11}{\pi} \right)^2$	
	A1	for $\frac{100}{3\pi^2}$ oe must be in terms of π (isw after correct answer seen)	
	SC	If both final answers are given in terms of k , score B1M1A0 for (c)(i) $11k$ and (c)(ii) $\frac{100}{3}k^2$	
(d)	M1	for expanding $E(A) = E\left(\pi X^2 + 4X + \frac{4}{\pi} \right)$ or for setting up correct integral (ignore limits)	
	M1	Valid method for finding $E(X^2)$ i.e. use of $\text{Var}(X) + E(X)^2$ or integration of $x^2 f(x)$ or for integration of their $f(x)A$ with at least one $x^n \rightarrow x^{n+1}$	
	M1	substitution of their $E(X)$ and their $E(X^2)$ into their $E(A)$ or for use of correct limits	
	A1	for $\frac{607}{3\pi}$ or awrt 64.4	

Question Number	Scheme		Marks
5 (a)	$X \sim \text{Po}(5)$		
	$P(X \leq 5) = 0.6160$	awrt 0.616	M1 A1 (2)
(b)	$X \sim \text{B}(4, "0.616")$		B1ft
	$P(X < 2) = P(X \leq 1)$		M1
	$= 0.384^4 + 4 \times 0.616 \times 0.384^3$		M1
	$= 0.16126\dots$		awrt 0.161 A1 (4)
(c)	$X = \text{The number of defects per } x \text{ meters}$		
	$X \sim \text{N}\left(\frac{x}{16}, \frac{x}{16}\right)$		B1
	$P(X < 26) = P\left(Z < \frac{25.5 - \frac{x}{16}}{\sqrt{\frac{x}{16}}}\right) = 0.5398$		M1
	$\frac{25.5 - \frac{x}{16}}{\frac{1}{4}\sqrt{x}} = 0.1$		B1 M1 A1ft
	$\frac{1}{16}x + \frac{1}{40}\sqrt{x} - 25.5 = 0 \rightarrow \sqrt{x} = 20 \text{ (or } \sqrt{x} = -20.4)$		M1
	$(\sqrt{x})^2 = 20^2$		M1
	$x = 400$		A1 (8)
	Notes		
(a)	M1	For writing or using $P(X \leq 5)$	
	A1	awrt 0.616	
(b)	B1ft	For $X \sim \text{B}(4, 0.616)$ Follow through their part (a). May be implied by a correct ft expression for the 2 nd M1	
	M1	For writing or using $P(X \leq 1)$ (May be implied by 2 nd M1)	
	M1	For $= [{}^4C_0](1-p)^4 + {}^4C_1 \times p \times (1-p)^3 \quad 0 < p < 1$	
	A1	awrt 0.161 correct answer on its own scores 4 out of 4	
(c)	B1	For $X \sim \text{N}\left(\frac{x}{16}, \frac{x}{16}\right)$ May be implied by values in standardisation.	
	M1	For use of a continuity correction either 25.5 or 26.5 (Allow 24.5)	
	B1	$z = \pm 0.1$ Allow calculator value if seen $\pm 0.0999(2986\dots)$	
	M1	Standardising using either 24.5 or 25 or 25.5 or 26 or 26.5 and equate to a z value. Follow through their mean and variance	
	A1ft	A correct equation with compatible signs ft their mean and variance provided mean = variance	
	M1	For solving their 3 term equation by factorising, completing the square or use of formula. May be implied by -20.4 , otherwise if answer is incorrect working must be shown.	
	M1	For correct squaring of both sides. May be implied by 416[.16] from correct equation This mark may be scored prior to solving a 3TQ, e.g. $\left(25.5 - \frac{x}{16}\right)^2 = \left(\frac{1}{40}\sqrt{x}\right)^2$. Do not award if squaring each individual term	
	A1	$x = 400$ only. This is dependent upon all previous marks in (c).	
	SC	Use of $X \sim \text{N}\left(\frac{x}{16}, \frac{15x}{256}\right)$ leading to $x = 400$ scores max B0M1B1M1A0M1M1A0	

Question Number	Scheme	Marks
6 (a)	$[F(k) = 1 \Rightarrow] ak + bk^2 = 1 \Rightarrow ak = 1 - bk^2 *$	B1*
		(1)
(b)	$f(x) = a + 2bx$	B1
	$E(X) = \int_0^k (ax + 2bx^2) dx \left[= \frac{6}{5} \right] \Rightarrow \left[\frac{ax^2}{2} + \frac{2bx^3}{3} \right]_0^k \left[= \frac{6}{5} \right]$	M1
	$\frac{ak^2}{2} + \frac{2bk^3}{3} = \frac{6}{5}$	dM1, A1
	$15ak^2 + 20bk^3 = 36$	
	$15k(1 - bk^2) + 20bk^3 = 36$	M1
	$5bk^3 = 36 - 15k *$	A1*
		(6)
(c)	$E(X^2) = \int_0^k (ax^2 + 2bx^3) dx \Rightarrow \left[\frac{ax^3}{3} + \frac{bx^4}{2} \right]_0^k$	M1
	$\text{Var}(X) = \frac{ak^3}{3} + \frac{bk^4}{2} - \frac{36}{25} = \frac{22}{75}$	dM1 A1
	$10ak^3 + 15bk^4 = 52$	
	$10k^2(1 - bk^2) + 15bk^4 = 52$	M1
	$5bk^4 = 52 - 10k^2 *$	A1*
		(5)
(d)	$\frac{1}{k} = \frac{36 - 15k}{52 - 10k^2}$	M1
	$5k^2 - 36k + 52 = 0$	A1
	$(k - 2)(5k - 26) = 0$	M1
	$k = 2$	A1
		(4)
(e)	'40'b = 36 - '30' $\Rightarrow b = \frac{3}{20}$ or '80'b = 52 - '40' $\Rightarrow b = \frac{3}{20}$	B1ft
	$2a + \frac{3}{5} = 1 \Rightarrow a = \frac{1}{5}$	B1ft
		(2)
Notes		Total 18
(a)	B1* Answer is given so no incorrect working can be seen	
(b)	B1 For a correct expression for f(x) (may be implied by a correct expression for E(X))	
	M1 For an attempt to integrate x f(x) (Ignore limits) at least one ($x^n \rightarrow x^{n+1}$). F.t. their f(x) f(x) must be a changed expression from F(x) so integrating xF(x) is M0	
	dM1 Dependent on the previous M mark. For equating to $\frac{6}{5}$ and substitution of k (no need to see substitution of lower limit 0).	
	A1 For a correct equation any form	
	M1 For substitution of $ak = 1 - bk^2$ oe into their equation	
	A1* Answer is given so no incorrect working can be seen	

(c)	M1	For an attempt to integrate $x^2 f(x)$ (Ignore limits) at least one $(x^n \rightarrow x^{n+1})$ F.t. their $f(x)$ $x^2 F(x)$ is M0
	dM1	Dependent on previous M mark. For substitution of correct limits and subtraction of $\frac{36}{25} = \frac{22}{75}$
	A1	For a correct equation any form
	M1	For substitution of $ak = 1 - bk^2$ oe into their equation
	A1*	Answer is given so no incorrect working can be seen
(d)	M1	For solving simultaneously to set up an equation in k only
	A1	For a correct 3 term quadratic
	M1	For solving their 3 term quadratic by factorising, completing the square or using formula. $k = 5.2$ implies M1A1M1
	A1	2 only cao. Correct answer on its own scores 4 out of 4
(e)	B1ft	For $b = \frac{3}{20}$ ft their k $b = \frac{36-15k}{5k^3}$ Common ft answer is $b = \frac{-525}{8788} = \text{awrt } -0.0597$ coming from choosing $k = 5.2$
	B1ft	For $a = \frac{1}{5}$ ft their k and their b $a = \frac{1-bk^2}{k}$ Common ft answer is $a = \frac{85}{169} = \text{awrt } 0.503$ coming from choosing $k = 5.2$