Question Number	Scheme	Marks
1	$(8+32x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+4x)^{\frac{1}{3}}$	B1
	$(1+4x)^{\frac{1}{3}} = 1+\left(\tfrac{1}{3}\right)(4x)+\dfrac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(4x)^2+\dfrac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(4x)^3+\ldots$	M1A1
	$(8+32x)^{\frac{1}{3}} = 2+\dfrac{8}{3}x-\dfrac{32}{9}x^2+\dfrac{640}{81}x^3+\ldots$	A1
		(4)
		Total 4

Notes

B1: For taking out a factor of $8^{\frac{1}{3}}$ or e.g. 2 to obtain e.g. $8^{\frac{1}{3}}(1+\ldots)$. Seen anywhere in their work

M1: Correct attempt at the 3rd or 4th term of a binomial expansion of $(1\pm ax)^{\frac{1}{3}}$

Look for $\dfrac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}(\pm ax)^2$ or $\dfrac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(\pm ax)^3$ where a could be 1

For this mark we are looking for the correct binomial coefficient paired with the correct power of x

A1: Correct unsimplified (or simplified) expansion of the **bracket** e.g.:

$$(1+4x)^{\frac{1}{3}} = 1+\left(\tfrac{1}{3}\right)(4x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(4x)^2+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(4x)^3+\ldots$$

NB simplified is: $1+\dfrac{4}{3}x-\dfrac{16}{9}x^2+\dfrac{320}{81}x^3+\ldots$

or for 2 correct simplified terms for the final expansion from $\dfrac{8}{3}x-\dfrac{32}{9}x^2+\dfrac{640}{81}x^3$

A1cao: $2+\dfrac{8}{3}x-\dfrac{32}{9}x^2+\dfrac{640}{81}x^3+\ldots$

All coefficients must be simplified, accept as a list and ignore any extra terms

Alternative by direct expansion:

$$(8+32x)^{\frac{1}{3}} = 8^{\frac{1}{3}}+\frac{1}{3}\left(8^{-\frac{2}{3}}\right)(32x)+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(8^{-\frac{5}{3}}\right)(32x)^2+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6}\left(8^{-\frac{8}{3}}\right)(32x)^3+\ldots$$

B1: For $8^{\frac{1}{3}}$ or 2 seen as the constant term.

M1: For the correct term 3 or term 4 of the expansion.

A1: For two correct and simplified terms.

A1: Correct expansion from correct work. Accept as a list and ignore any extra terms.

NOTE: Isw any further 'simplification' once the correct response has been seen.

Question Number	Scheme	Marks
2(a)	$\dfrac{2x^2-x+11}{(x+2)(2x-3)}=1+\dfrac{-2x+17}{(x+2)(2x-3)}$	M1A1
	$\dfrac{-2x+17}{(x+2)(2x-3)}=\dfrac{B}{x+2}+\dfrac{C}{2x-3}\Rightarrow B=\ldots,\ C=\ldots$	M1
	$\dfrac{2x^2-x+11}{(x+2)(2x-3)}=1-\dfrac{3}{x+2}+\dfrac{4}{2x-3}$	A1
	ALT 1	
	$2x^2-x+11=A(x+2)(2x-3)+B(2x-3)+C(x+2)$ Let e.g., $x=-2$ or $x=\dfrac{3}{2}$ $\Rightarrow$ eliminates either B or $C\Rightarrow B=-3$ or $C=4$	M1A1
	$2x^2-x+11=A(x+2)(2x-3)+B(2x-3)+C(x+2)$ Let e.g., $x=-2$ and $x=\dfrac{3}{2}\Rightarrow$ finds B,C and A	M1
	$\dfrac{2x^2-x+11}{(x+2)(2x-3)}=1-\dfrac{3}{x+2}+\dfrac{4}{2x-3}$	A1
	ALT 2	
	$2x^2-x+11=2Ax^2+x(A+2B+C)+(-6A-3B+2C)$ $\Rightarrow$ equates coefficients to find $A\Rightarrow 2=2A\Rightarrow A=1$	M1A1
	$A+2B+C=-1\Rightarrow 1+2B+C=-1$ $-6A-3B+2C=11\Rightarrow -6-3B+2C=11$ $\Rightarrow B=\ldots\ldots,\quad C=\ldots\ldots.$	M1
	$\dfrac{2x^2-x+11}{(x+2)(2x-3)}=1-\dfrac{3}{x+2}+\dfrac{4}{2x-3}$	A1
		(4)
(b)	$\displaystyle\int\left(1-\frac{3}{x+2}+\frac{4}{2x-3}\right)\mathrm{d}x=x-3\ln\lvert x+2\rvert+2\ln\lvert 2x-3\rvert(+c)$	M1A1ftA1ft
		(3)
		Total 7

Notes
(a)

M1: Uses e.g. long division or inspection to express $\mathrm{f}(x)$ as a constant with a linear remainder of the

form $1+\dfrac{\pm\alpha x\pm\beta}{(x+2)(2x-3)}$ where α, β are non zero constants.

A1: Correct expression. i.e.,$1+\dfrac{-2x+17}{(x+2)(2x-3)}\Rightarrow A=1$

M1: Uses an appropriate method to both B and C from an expression of the form $\dfrac{\alpha x+\beta}{(x+2)(2x-3)}$. E.g.
substituting values or comparing coefficients to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1$, $B = -3$, $C = 4$

ALT 1

M1: Multiplies through by the denominator to obtain the shown equation, substitutes an appropriate value of x to find a value for B or C Allow sign errors for this mark.
Some candidates are deducing that $A = 1$ in this method. Award this mark for the equation and for B or C.

A1: Either B or C correct. **following fully correct processing**

M1: For **complete** processing to find both A and either B or C to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1$, $B = -3$, $C = 4$ o.e

ALT 2

M1: Multiplies through by the denominator to obtain the shown equation [can be deduced], multiplies out the RHS,
equates coefficients and finds a value for A. Allow sign errors for this mark.

A1: A correct **following fully correct processing**.

M1: For **complete** processing to find both B and C to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1$, $B = -3$, $C = 4$ o.e

isw further simplification

(b)

M1: $\displaystyle\int\frac{\ldots}{x+2}\,\mathrm{d}x=\ldots\ln(x+2)$ **or** $\displaystyle\int\frac{\ldots}{2x-3}\,\mathrm{d}x=\ldots\ln(2x-3)$ Allow modulus or round brackets.

Do not allow errors on $(x+2)$ or $(2x-3)$

A1ft: For any 2 terms correctly integrated. Follow through their stated A, B and C.
The brackets must be present but allow e.g. $-3\ln(x+2)$ or e.g. $2\ln(2x-3)$

A1ft: All **3** of their terms fully simplified. Follow through their stated A, B and C. The "+ c" is not required. If their $A = 0$ this mark is not awarded.
Allow $x-3\ln(x+2)+2\ln(2x-3)(+c)$
isw further simplification

Question Number	Scheme	Marks
3	$V = x^3 \Rightarrow \left(\dfrac{\mathrm{d}V}{\mathrm{d}x}\right) = 3x^2$	B1
	$V = 100 \Rightarrow x^3 = 100 \Rightarrow x = \sqrt[3]{100}$	B1
	$k = \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{1}{3\times 100^{\frac{2}{3}}} \times 0.085$	M1
	$= 0.00132$	A1
		(4)
		Total 4

Notes

$\left[\text{Note: } 0.085 = \dfrac{17}{200}\right]$

Allow other variables in place of x but not t.

B1: $\left(\dfrac{\mathrm{d}V}{\mathrm{d}x}\right) = 3x^2$ seen anywhere in the working

B1: M1 in Epen Finds x when $V = 100 \Rightarrow x = \sqrt[3]{100}$ or awrt 4.64

M1: Applies $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t}$ with their $\dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ provided it is of the form $\dfrac{\mathrm{d}V}{\mathrm{d}x} = \mu x^2$ and their numerical value for x substituted correctly

This must lead to $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \ldots\ldots$

Do not award this mark for $x = 100$ substituted here.

A1: Awrt 0.00132 (oe e.g. awrt 1.32×10^{-3}) Ignore units given.

Question Number	Scheme	Marks
4(a)	$x=\dfrac{3}{5t-2}\Rightarrow 5tx-2x=3\Rightarrow t=\dfrac{2x+3}{5x}\Rightarrow y=\dfrac{\frac{2x+3}{5x}}{4-\frac{2x+3}{5x}}$ o.e.	M1A1
	$y=\dfrac{2x+3}{18x-3}$ oe	A1
		(3)
(b)	Assume that C and l do intersect	B1
	At intersection $\dfrac{2x+3}{18x-3}=-x+1\Rightarrow 2x+3=-18x^2+21x-3\Rightarrow 18x^2-19x+6\ (=0)$ or $y=-x+1\Rightarrow \dfrac{t}{4-t}=\dfrac{3}{2-5t}+1\Rightarrow 2t-5t^2=3(4-t)+(2-5t)(4-t)$ $\Rightarrow 10t^2-27t+20\,(=0)$	M1
	$b^2-4ac=19^2-4\times18\times6(=-71)$ or $b^2-4ac=27^2-4\times10\times20(=-71)$	M1
	$b^2-4ac<0$ so there are **no real (roots)** **So C and l do not intersect**	A1
		(4)
		Total 7

Notes

(a)

M1: Finds t in terms of x **and** substitutes into y

A1: Any correct equation connecting y and x which need not be simplified for this mark.

A1: Correct equation in the required form with no errors. Accept equivalent correct expressions.

(b)

B1: Sets up a correct assumption statement. This step **must** be seen.

M1: Sets their $\dfrac{2x+3}{18x-3}=-x+1$ and multiplies up to obtain a 3 term quadratic in x

or

Uses the parametric form and sets $\dfrac{t}{4-t}=-\left(\dfrac{3}{5t-2}\right)+1$ oe and multiplies up to obtain a 3 term quadratic in t

M1: Any correct strategy for establishing the contradiction e.g. attempts the discriminant on their 3TQ. This can be part of the quadratic formula.

Complex roots are: $\dfrac{19\pm\sqrt{71}i}{36}$

A1: Fully correct work including -71 seen evaluated, **and** conclusion stated that **roots are not (real), so *C* and *l* do not intersect.** Mark the intention to write these statements and mark positively. **This is a formal proof. Only award the final A mark if B1M1M1 has been scored.**

Question Number	Scheme	Marks
5	$u=\sqrt{2x-1}\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(2x-1)^{-\frac{1}{2}}\times 2\Rightarrow\left(\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2u}\times 2\right)\Rightarrow(\mathrm{d}x=u\mathrm{d}u)$ **or** e.g. $u^2=2x-1\Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}x}=2\Rightarrow(\mathrm{d}x=u\mathrm{d}u)$ **or** e.g. $u=\sqrt{2x-1}\Rightarrow x=\frac{u^2+1}{2}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=u\Rightarrow(\mathrm{d}x=u\mathrm{d}u)$	B1
	$\int\frac{x}{\sqrt{2x-1}}\mathrm{d}x=\int\frac{1}{2}(u^2+1)\frac{1}{u}\times u\ \mathrm{d}u$	M1
	$\int\frac{1}{2}(u^2+1)\frac{1}{u}\times u\ \mathrm{d}u=\int\frac{1}{2}(u^2+1)\ \mathrm{d}u=\frac{u^3}{6}+\frac{u}{2}(+k)$	M1
	$=\frac{(2x-1)^{\frac{3}{2}}}{6}+\frac{(2x-1)^{\frac{1}{2}}}{2}(+k)$	ddM1
	$=\frac{1}{3}(2x-1)^{\frac{1}{2}}(x+1)+k$	A1
		(5)
		Total 5

Notes

B1: Any correct equation involving $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$ See the alternatives above.

M1: Makes a complete substitution to obtain an integral in terms of u including their dx in terms of u..du

M1: Integrates to the form $...u^3+...u(+k)$

ddM1: Reverses the substitution **correctly** into their integrated expression in the form au^3+bu+k where a and b are rational numbers. The minimum form required is $=\alpha(2x-1)^{\frac{3}{2}}+\beta(2x-1)^{\frac{1}{2}}(+k)$

Note: This mark is dependent on **both** previous M marks.

A1: Correct answer in the required form including the "+ k" allow + C for this mark.

Question Number	Scheme	Marks
6(a)	$x^2 - xy + y^2 + 4x = 12$ $\Rightarrow 2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} + 4 = 0$	M1A1
	$\frac{dy}{dx}(2y - x) = y - 2x - 4 \Rightarrow \frac{dy}{dx} = \ldots$	M1
	$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{y - 2x - 4}{2y - x}$ oe	A1
		(4)
(b)	$(-4, -6) \rightarrow \frac{dy}{dx} = \frac{-6 + 8 - 4}{-12 + 4} = \frac{1}{4} \Rightarrow m_N = -4$	M1
	$y + 6 = -4(x + 4)$	M1
	$y = -4x - 22$	A1
		(3)
(c)	$y = -4x - 22 \Rightarrow 21x^2 + 202x + 472 = 0$ or $x = \frac{y + 22}{-4} \Rightarrow 21y^2 + 116y - 60 = 0$	M1
	$21x^2 + 202x + 472 = 0 \Rightarrow x = -\frac{118}{21}, (-4)$ or $21y^2 + 116y - 60 = 0 \Rightarrow y = \frac{10}{21}, (-6)$	dM1
	$x = -\frac{118}{21}$, o.e.e $\quad y = \frac{10}{21}$ o.e.e.	A1
		(3)
		Total 10

(a)

M1: For $xy \rightarrow \alpha y + \beta x\frac{dy}{dx}$ **or** $y^2 \rightarrow \alpha y\frac{dy}{dx}$

A1: Fully correct differentiation on the whole equation unsimplified or simplified.

M1: Makes $\frac{dy}{dx}$ the subject from **2** different terms in $\frac{dy}{dx}$ coming from the correct terms only

A1: Any correct expression

(b)

M1: Correct method for the gradient of the normal using their numerical value for $\frac{dy}{dx}$

M1 Correct method for finding the equation of the normal. Ft their normal gradient which must not be their tangent gradient.

A1: Correct equation in the required form

(c)

M1:	Substitutes for x or y into the given equation to obtain a 3TQ in x or y.
dM1:	Solves 3TQ using any method [including using a calculator] to obtain either the x or y coordinate of Q. This mark is dependent on the first M mark awarded.
A1:	Both correct **exact** coordinates. Accept values given separately. Ignore any reference to $(-4,-6)$

Question Number	Scheme	Marks
7(a)	$\pm\left(5\mathbf{i}-3\mathbf{j}+\mathbf{k}-\left(3\mathbf{i}-2\mathbf{j}+3\mathbf{k}\right)\right)$	M1
	e.g. $\mathbf{r}=5\mathbf{i}-3\mathbf{j}+\mathbf{k}+\lambda\left(2\mathbf{i}-\mathbf{j}-2\mathbf{k}\right)$ $\mathbf{r}=5\mathbf{i}-3\mathbf{j}+\mathbf{k}+\lambda\left(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$ $\mathbf{r}=3\mathbf{i}-2\mathbf{j}+3\mathbf{k}+\lambda\left(2\mathbf{i}-\mathbf{j}-2\mathbf{k}\right)$ $\mathbf{r}=3\mathbf{i}-2\mathbf{j}+3\mathbf{k}+\lambda\left(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$	A1
		(2)
(b)	e.g. $1-2\lambda=-5\Rightarrow\lambda=3$ $\lambda=3\Rightarrow\mathbf{r}=5\mathbf{i}-3\mathbf{j}+\mathbf{k}+3\left(2\mathbf{i}-\mathbf{j}-2\mathbf{k}\right)$	M1
	$\Rightarrow\alpha=11,\ \beta=-6$	A1
		(2)
(c)	$\overrightarrow{OC}\bullet\overrightarrow{AB}=\left(11\mathbf{i}-6\mathbf{j}-5\mathbf{k}\right)\bullet\left(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)=-22-6-10=(-38)$	M1
	$-38=\sqrt{11^2+6^2+5^2}\sqrt{2^2+1^2+2^2}\cos\theta\Rightarrow\theta=...$	dM1
	$\theta=20.1°$	A1
		(3)
(d)	e.g., $\overrightarrow{OD}\bullet\begin{pmatrix}2\\-1\\-2\end{pmatrix}=0\Rightarrow\begin{pmatrix}2\lambda+5\\-\lambda-3\\-2\lambda+1\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\-2\end{pmatrix}\Rightarrow\lambda=...\left(-\frac{11}{9}\right)$ or e.g. $\overrightarrow{OD}\bullet\begin{pmatrix}-2\\1\\2\end{pmatrix}=0\Rightarrow\begin{pmatrix}3-2\lambda\\-2+\lambda\\3+2\lambda\end{pmatrix}\bullet\begin{pmatrix}-2\\1\\2\end{pmatrix}\Rightarrow\lambda=...\left(\frac{2}{9}\right)$	M1

	e.g. $\lambda = -\frac{11}{9} \Rightarrow \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} - \frac{11}{9}\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ or e.g. $\lambda = \frac{2}{9} \Rightarrow \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + \frac{2}{9}\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	M1
	$\left(\frac{23}{9}, -\frac{16}{9}, \frac{31}{9}\right)$	A1
		(3)
		Total 10

Notes

(a)

M1: Attempts the direction of l

Two out of three components correct in the direction implies the M mark.

A1: Any correct equation including '**r** =' Do not accept **l** = …..

Accept column vectors throughout, e.g., $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ but do **NOT** accept $\mathbf{r} = \begin{pmatrix} 3\mathbf{i} \\ -2\mathbf{j} \\ 3\mathbf{k} \end{pmatrix} + \lambda\begin{pmatrix} -2\mathbf{i} \\ 1\mathbf{j} \\ 2\mathbf{k} \end{pmatrix}$

(b)

M1: Uses the z component to find the value of the parameter for their equation and uses their parameter value to find the value of α and the value of β.

A1: Correct values or vector.

(c)

M1: Attempts the scalar product $\pm\overrightarrow{OC} \bullet \overrightarrow{AB}$ Ft their $\overrightarrow{OC}$ and $\overrightarrow{AB}$

Look for at least two correct products can be implied by the correct answer.

dM1: A complete method to find the angle (allow acute or obtuse, 20.13° or 159.87° respectively allow working in radians for the M marks)

A1: Awrt 20.1°

(d)

M1: Attempts the scalar product between their $\overrightarrow{OD}$ and the direction of l, sets = 0 and solves for λ for their equation.

M1: Uses their λ correctly to find D

A1: Correct exact coordinates (condone vector)

Do **NOT** accept the final answer in the form $\begin{pmatrix} \frac{23}{9}\mathbf{i} \\ -\frac{16}{9}\mathbf{j} \\ \frac{31}{9}\mathbf{k} \end{pmatrix}$

Question Number	Scheme	Marks
8(a)	$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$	M1A1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	A1
		(3)
(b)	$V = (\pi)\int (4x+3\cos 2x)^2\ dx$ $V = (\pi)\int (16x^2 + 24x\cos 2x + 9\cos^2 2x)\,dx$	M1
	$\int 24x\cos 2x\,dx = 12x\sin 2x + 6\cos 2x(+c)$	M1
	$\int 9\cos^2 2x\,dx = \frac{9}{2}\int(1+\cos 4x)\,dx$	M1
	$\left[(\pi)\int(16x^2 + 24x\cos 2x + 9\cos^2 2x)\,dx\right]$ $= (\pi)\left(\frac{16x^3}{3} + 12x\sin 2x + 6\cos 2x + \frac{9x}{2} + \frac{9}{8}\sin 4x(+c)\right)$	A1
	$V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(4x+3\cos 2x)^2\,dx = \pi\left(\frac{16\pi^3}{24} + 0 - 6 + \frac{9\pi}{4} + 0 - \left(\frac{16\pi^3}{192} + 3\pi + 0 + \frac{9\pi}{8} + 0\right)\right)$	M1
	$\pi\left(\frac{7\pi^3}{12} - \frac{15\pi}{8} - 6\right)$	A1
		(6)
		Total 9

Notes
(a)

M1: Applies integration by parts to obtain the form $...x\sin 2x \pm ...\int \sin 2x\,\mathrm{d}x$

A1: Correct expression

A1: Completes the integration to obtain the correct answer including + c

If the "DI" method is used:

x	$\cos 2x$
1	$\frac{1}{2}\sin 2x$
0	$-\frac{1}{4}\cos 2x$

Award M1 for reaching the correct form of the answer e.g. $...x\sin 2x + ...\cos 2x$ with A1 implied by their table and A1 as above.

(b)

Condone the missing π from $\pi \int ...$ for the 4 marks.

M1: Attempts $\int (4x + 3\cos 2x)^2\,\mathrm{d}x$ and multiplies out the bracket with 2 out of 3 terms correct.

M1: Uses part (a) to attempt their $\int 24x\cos 2x\,\mathrm{d}x$ provided part (a) is in the form $...x\sin 2x \pm ...\cos 2x$

An attempt is defined as using their part (a) correctly here with their '24'

M1: Attempts $\int \cos^2 2x\,\mathrm{d}x$ using the correct identity $\cos 4x = 2\cos^2 2x - 1$ and reaches

$$\int \cos^2 2x\,\mathrm{d}x = ...x + ...\sin 4x\,(+c)$$

A1: Fully correct integration in any form.

M1: Complete method to find the volume with π and the given limits [accept substituted either way around] using their integral which must include at least two trigonometrical terms.

A1: Correct volume in the required form.

Question Number	Scheme	Marks
9(a)	$\frac{d\theta}{dt} = -k(\theta+15) \Rightarrow \int \frac{d\theta}{\theta+15} = \int -k\,dt$ **OR** $\frac{dt}{d\theta} = \frac{1}{-k(\theta+15)}$ $\Rightarrow \ln(\theta+15) = -kt(+c)$ o.e. $\Rightarrow t = -\frac{1}{k}\ln(\theta+15)+c$	M1A1
	$t=0, \theta=21 \Rightarrow c = ...(\text{e.g.}, c=\ln 36)$	M1
	$\ln(\theta+15) = -kt+\ln 36$ $\Rightarrow \theta+15 = e^{-kt+\ln 36}$ **OR** $\ln(\theta+15) = -kt+\ln 36$ $\Rightarrow [\ln(\theta+15)-\ln 36 = -kt]$ $\Rightarrow \ln\left(\frac{\theta+15}{36}\right) = -kt \Rightarrow \theta = 36e^{-kt}-15$	M1
	$\theta = 36e^{-kt}-15$	A1
		(5)
(b)	$12.9 = 36e^{-20k}-15 \Rightarrow e^{-20k} = \frac{31}{40} \Rightarrow -20k = \ln\frac{31}{40} \Rightarrow k = ...$	M1
	$k = 0.0127$	A1
		(2)
(c)	$t = 60 \Rightarrow \theta = '36'e^{-60\times'0.0127'}-'15'$	M1
	$= 1.8\,(^\circ C)$	A1
		(2)
		Total 9

Notes

(a)

M1: Separates the variables **and** integrates to obtain $...\ln(\theta+15) = ...kt(+c)$ (or equivalent) with or without a constant of integration.

A1: Correct equation with or without a constant of integration.

If they do not have a constant of integration the following 3 marks are not available

M1: States or uses $t=0$ and $\theta=21$ to find a value for "c". Do not be concerned by the rearrangement as long as a value of c is found.

M1: Rearranges using correct application of log processing to make θ the subject

A1: Correct equation

(b)

M1: Sets their $\theta = 12.9$, substitutes $t = 20$ into their expression which is minimally of the form $\theta = \alpha e^{-kt} \pm \beta$ where α can equal 1 and rearranges **using correct explicit log work** to find k.

For the award of this mark their $'\frac{31}{40}'$ must be a positive number.

A1: Awrt 0.0127

(c)

M1: Substitutes $t = 60$ and their k into their $\theta = 36e^{-kt}-15$ which must be minimally of the form $\theta = \alpha e^{-kt} - \beta$

	Unless the final answer is correct coming from a correct equation, substitution of 60 must be explicit.
A1:	Awrt $1.8\,(^\circ\mathrm{C})$ A correct answer coming a correct expression implies A1
	Units are not required, but if the units are incorrect score A0.

Question Number	Scheme	Marks
10(a)	$\theta=\dfrac{\pi}{3}\Rightarrow x=8\cos\dfrac{\pi}{3}=4,\ y=5\sin\dfrac{2\pi}{3}=\dfrac{5\sqrt{3}}{2}$	B1
	$x=8\cos\theta\Rightarrow\dfrac{dx}{d\theta}=-8\sin\theta \qquad y=5\sin 2\theta\Rightarrow\dfrac{dy}{d\theta}=10\cos 2\theta$ $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}=\dfrac{10\cos 2\theta}{-8\sin\theta}=\dfrac{10\cos\frac{2\pi}{3}}{-8\sin\frac{\pi}{3}}=\ldots\left(\dfrac{5\sqrt{3}}{12}\right)$	M1
	$y-\dfrac{5\sqrt{3}}{2}=-\dfrac{4\sqrt{3}}{5}(x-4)$	M1
	$y=-\dfrac{4\sqrt{3}}{5}x+\dfrac{57\sqrt{3}}{10}$	A1
		(4)
(b)	$\displaystyle\int y\,dx=\int y\frac{dx}{d\theta}(d\theta)=\int 5\sin 2\theta\times-8\sin\theta(d\theta)$	M1
	$\displaystyle\int y\,dx=\int y\frac{dx}{d\theta}(d\theta)=\int 5(2\sin\theta\cos\theta)\times-8\sin\theta\,(\,d\theta)=\pm 80\int\sin^2\theta\cos\theta\,(d\theta)$	M1
	$=\left(-\dfrac{80}{3}\right)\sin^3\theta+c$	M1
	$0=-\dfrac{4\sqrt{3}}{5}x+\dfrac{57\sqrt{3}}{10}\Rightarrow x=\ldots\dfrac{57}{8}$ and $A_\Delta=\dfrac{1}{2}\left(\dfrac{57}{8}-4\right)\times\dfrac{5\sqrt{3}}{2}=\left(\dfrac{125\sqrt{3}}{32}\right)$	M1
	$\pm\dfrac{80}{3}\left[\sin^3\theta\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$ or $\pm\dfrac{80}{3}\left[\sin^3\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}=\dfrac{80}{3}\left(1-\dfrac{3\sqrt{3}}{8}\right)$	M1
	Total area $=\dfrac{80}{3}-\dfrac{195\sqrt{3}}{32}$	A1
		(6)
		Total 10

Notes
NB: If you see attempts to answer this question using a Cartesian equation please send to Review.
(a)

B1: Correct coordinates for P stated or implied by working.

M1: Attempts $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$ at $\theta=\frac{\pi}{3}$

Minimally required differentiation is: $\cos\theta\Rightarrow\beta\sin\theta \quad \sin2\theta\Rightarrow\alpha\cos2\theta \quad \frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$

M1: Attempts to find the equation of the normal at $\theta=\frac{\pi}{3}$

A1: Correct equation in the required form.

(b)

Allow sign slips in this part for the M marks.

M1: Attempts $\int y\frac{dx}{d\theta}d\theta$ with the given y and their

and obtains $\frac{dx}{d\theta}$ which must be as a minimum $\pm8\sin\theta$ $\quad\Rightarrow\int5\sin2\theta\times'\pm8\sin\theta'(d\theta)$

M1: **This is an A mark in Epen** Uses $\sin2\theta=2\sin\theta\cos\theta$ to obtain $\pm80\int\sin^2\theta\cos\theta\,(d\theta)$ oe

Ignore limits for this mark.

M1: Integrates to the form $\alpha\sin^3\theta(+c)$

M1: Attempts the x intercept of l and finds the area of the triangle using their equation of the line. You must check this. Ft their line provided the intercept on the x-axis > 4
This may be completed by integration

M1: Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{3}$ to evaluate the integral. Accept substitution of limits either way around for this mark, but must include subtraction..

A1: Correct area of the required region R in the required form.
[Allow recovery from a negative integral].
Score this mark ONLY if all previous marks in part (b) have been awarded.