Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Answer	Mark
1 (a)	$\frac{2+11x}{(1+x)(2+5x)} \equiv \frac{A}{1+x}+\frac{B}{2+5x} \Rightarrow$ Values for A and B	M1
	One correct value, either $A=3$ or $B=-4$	A1
	Correct PF form $\frac{3}{1+x}-\frac{4}{2+5x}$	A1
		(3)
(b)	$\frac{'3'}{1+x}='3'(1+x)^{-1}='3'\left(1-x+x^2+...\right)$	B1
	$\frac{'4'}{2+5x}=\frac{'4'}{2}\left(1+\frac{5x}{2}\right)^{-1}=\frac{'4'}{2}\left(1+(-1)\frac{5x}{2}+\frac{(-1)(-2)}{2}\left(\frac{5x}{2}\right)^2+\right)$	M1, A1
	$\mathrm{f}(x)='3'\times\left(1-x+x^2+...\right)-\frac{'4'}{2}\left(1-\frac{5x}{2}+\frac{25x^2}{4}+...\right)$	dM1
	$=1+2x-\frac{19}{2}x^2+...$	A1
		(5)
(c)	Valid $\lvert x\rvert<\frac{2}{5}$ o.e	B1
		(1)
		(9 marks)

(a)

M1: Attempts correct PF. What is seen in the scheme is acceptable. There is no need to see any working. Just look for the correct PF form followed by values for A and B

A1: One correct value for A or B or one correct term

A1: Correct PF form $\frac{3}{1+x}-\frac{4}{2+5x}$ oe. such as $\frac{3}{1+x}+\frac{-4}{2+5x}$. The correct values for A and B are insufficient but you can award if the correct expression is seen in (b)

(b)

B1: $\frac{'3'}{1+x}='3'(1+x)^{-1}='3'\left(1-x+x^2+...\right)$ correct and simplified which may be implied by a multiplied-out bracket. Ignore any additional terms in x^3 etc, follow through on their '3'

M1: Attempts $\frac{1}{2+5x}=(2+5x)^{-1}=....$

Look for $\left(...(1+kx)^{-1}\right)=...\left(1+(-1)kx+\frac{(-1)(-2)}{2}(kx)^2+\right)$ with or without the bracket on the second term. Look for correct powers of x with correct binomial coefficients and the value of k ($k\neq 1$) used consistently.

Also allow $(2+5x)^{-1}=2^{-1}+(-1)2^{-2}(5x)^1+\frac{(-1)(-2)}{2!}2^{-3}(5x)^2+...$ condoning missing brackets

A1: $\frac{'4'}{2+5x}=\pm\frac{'4'}{2}\left(1+\frac{5x}{2}\right)^{-1}=\pm\frac{'4'}{2}\left(1+(-1)\frac{5x}{2}+\frac{(-1)(-2)}{2}\left(\frac{5x}{2}\right)^2+\right)$ which is OK left unsimplified.

Follow through on their '4'. FYI the fully simplified form is $\frac{'4'}{2}\left(1-\frac{5x}{2}+\frac{25}{4}x^2+\right)$

dM1: Uses their coefficients and attempts to correctly combine the two series.

Look for $'3'\left(1\pm x\pm x^2+...\right)\pm\frac{'4'}{2}\left(1\pm\alpha x\pm\beta x^2+\right)=a+bx+cx^2+$

but condone $'3'\left(1\pm x\pm x^2+...\right)\pm 2\times'4'\left(1\pm\alpha x\pm\beta x^2+\right)=a+bx+cx^2$

To score this mark

- the previous method mark must have been awarded
- there must be some evidence of an attempt to expand both brackets and collect like terms, seen or implied in at least two out of the three cases. Condone the series being added instead of subtracted

A1: $1+2x-\frac{19}{2}x^2+...$ or equivalent including $1+(2x)+\left(-\frac{19}{2}x^2\right)+...$

Condone additional terms even if incorrect. Condone terms being given as a list $1, 2x, -9.5x^2$

If they reach the correct answer and then multiply by 2 it is A0. We do not isw this error.

(c)

B1: $|x|<\frac{2}{5}$ ONLY or exact equivalent such as $-\frac{2}{5}<x<\frac{2}{5}$ or $x\in\left(-\frac{2}{5},\frac{2}{5}\right)$

Condone $|x| \leqslant \frac{2}{5}$ o.e. If they give $|x| \leqslant 1$ as well (without discounting it) it will be B0

Question Number	Scheme	Marks
2 (a)	States that $\begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$ is not a scalar multiple of $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$	B1
		(1)
(b)	Assume that there exists a λ and a μ such that $\begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$	B1
	Attempts to solve two of the equations and find either λ or μ .	M1
	Correct values for either λ or μ for the appropriate equation E.g. (1) and (2) $-2+2\lambda = 6+4\mu$ $5-1\lambda = 12-3\mu$ $\Rightarrow \lambda = 26$ or $\mu = 11$	A1
	E.g Uses $\lambda = 26,\ \mu = 11$ in equation (3) LHS = $3+5\lambda = 133$ RHS = $-5+10\mu = 105$	dM1
	$133 \neq 105$, so lines don't intersect	A1
		(5) **(6 marks)**

(a)

B1: Explains that $\begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$ is not a (scalar) multiple of $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. E.g States that $\begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix} \neq k \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

Allow attempts via use of ratios such as $4 = \underline{\underline{2}} \times 2$, $10 = \underline{\underline{2}} \times 5$ but $-3 = \underline{\underline{3}} \times -1$ so cannot be parallel.

Condone for this mark a statement that 'the direction vectors are not multiples of each other'

(b)

B1: Sets up the contradiction

There must be some words and a reference to what it would mean for the equations

E.g.I Assume that l_1 and l_2 intersect so

$-2+2\lambda = 6+4\mu$, $5-1\lambda = 12-3\mu$ and $3+5\lambda = -5+10\mu$

E.g.II Assume the lines meet so

$-2+2\lambda = 6+4\mu$, $5-1\lambda = 12-3\mu$ and $3+5\lambda = -5+10\mu$

The equations may appear some way after the assumption

Note that B0, M1, A1, dM1, A1 is possible

Standard/usual method:

M1: Attempts to solve two of the equations to find the value of λ OR μ

E.g. (1) and (2) $-2+2\lambda=6+4\mu \quad 5-1\lambda=12-3\mu \quad \Rightarrow \lambda=...\text{or} \quad \mu=...$

E.g. (2) and (3) $3+5\lambda=-5+10\mu \quad 5-\lambda=12-3\mu \quad \Rightarrow \lambda=...,\text{or} \quad \mu=...$

Condone slips in the coefficients of the initial equations but it must lead to a value for either λ or μ

A1: Find the correct value of λ or μ for the equations they choose.

E.g. Equations (1) and (2) $\lambda=26$ or $\mu=11$ or Equations (2) and (3) $\lambda=9.2$ or $\mu=5.4$

dM1: Dependent upon the previous M. This mark is scored for substituting their values of λ and μ into both sides of the third equation and comparing values/equating values. Alternatively, they may substitute their value of λ or μ into their third equation and find a contradictory value for μ or λ. Again, condone slips, but the intention must be clear.

A1: Requires all of the following

- accurate calculations and values for the approach they use
- a reason e.g. '133 $\neq$105'
- a minimal conclusion such as 'lines don't intersect' 'contradiction', 'hence proven' or ✓.

So, using equations **(2) and (3)** expect to see:

$3+5\lambda=-5+10\mu \quad 5-\lambda=12-3\mu \quad \Rightarrow \lambda=9.2,\ \mu=5.4$

which when substituted into (1) gives LHS = $-2+2\lambda=16.4$ RHS = $6+4\mu=27.6$

Other correct methods exist. You should look very carefully at what the candidates are doing.
Three alternative methods that have been seen are:

Alt I: Uses two of the equations to find one unknown.

Uses two different equations to find the same unknown and compares

E.g. (1) and (2) $-2+2\lambda=6+4\mu \quad 5-1\lambda=12-3\mu \quad \Rightarrow \mu=11$

(2) and (3) $3+5\lambda=-5+10\mu \quad 5-\lambda=12-3\mu \quad \Rightarrow \mu=5.4$

As these are different, lines don't meet and are skew

Score M1: For attempting to solve two equations and find one unknown

A1: For correctly finding one correct value for their equations

dM1: For attempting to solve two different equations and find the same unknown

A1: For correct values, valid statement and suitable conclusion

Alt II: Uses two equations to find one unknown

Uses two different equations to find the other unknown.

Substitutes in one of the equations and compares

E.g. (1) and (2) $-2+2\lambda=6+4\mu \quad 5-1\lambda=12-3\mu \quad \Rightarrow \mu=11$

(2) and (3) $3+5\lambda=-5+10\mu \quad 5-\lambda=12-3\mu \quad \Rightarrow \lambda=9.2$

Then substitute into (1) $-2+2\lambda=16.4, \quad 6+4\mu=50$

As these are different, lines don't meet and are skew

Score M1: For attempting to solve two equations and find one unknown

A1: For finding one correct value for their equations

dM1: For attempting to solve two different equations and find the other unknown AND

substitute both values into one of the equations

A1: For correct values, valid statement and suitable conclusion

Alt III

Note that if equations **(1) and (3) are attempted they cannot be solved but the method is valid**:

Expect to see something like $-2+2\lambda=6+4\mu \quad 3+5\lambda=-5+10\mu \quad \Rightarrow 5\lambda-10\mu=20$ and $5\lambda-10\mu=-8$ and $20\neq-8$.

Score M1, A1 for considering both equations and finding two equations that can be compared, e.g $\lambda-2\mu=4$ and $\lambda-2\mu=-1.6$

Then dM1: For an attempt at the comparison with A1 scored for accurate calculations, reason and conclusion

Question Number	Scheme	Marks
3 (a)	Coordinates of $P=(1,2)$	B1
	Attempts $\frac{dx}{dt}=\frac{-4}{(t+2)^2}$ and $\frac{dy}{dt}=\frac{(t+4)\times6-6t}{(t+4)^2}$	M1
	Attempts gradient of the tangent using $m=\frac{dy/dt}{dx/dt}=\left(\frac{-6(t+2)^2}{(t+4)^2}\right)$ at $t=2$	M1
	Equation of tangent $y-2=-\frac{8}{3}(x-1)\Rightarrow y=-\frac{8}{3}x+\frac{14}{3}$	ddM1, A1
		(5)
(b)	Attempts to get t in terms of x or y. E.g. $x=\frac{4}{t+2}\Rightarrow t=\frac{4}{x}-2$	M1

Question Number	Scheme	Marks
	Achieves a Cartesian Equation E.g. $y=\dfrac{6t}{t+4} \Rightarrow y=\dfrac{6\left(\dfrac{4}{x}-2\right)}{\dfrac{4}{x}-2+4}$	dM1
	$\Rightarrow y=\dfrac{-6x+12}{x+2}$, $0<x<4$	A1, B1
		(4)
(c)	Range $-2<\mathrm{f}<6$	M1, A1
		(2)
		(11 marks)

(a)

B1: Coordinates of $P=(1,\,2)$ but this may be implied within the tangent equation via use of $x=1,\ y=2$

M1: Attempts both $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ and reaching a correct form for each.

Look for $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\pm\dfrac{\beta}{(t+2)^2}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{\lambda(t+4)-\delta t}{(t+4)^2}$ $\lambda,\delta>0$ o.e. condoning missing brackets

For the differentiation of y with respect to t. You may see either chain rule or product rule used.

E.g. Chain rule; $y=\dfrac{6t}{t+4}=6-\dfrac{24}{t+4}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{24}{(t+4)^2}$. Look for $\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{\pm p}{(t+4)^2}$ following division.

E.g. Product rule $y=6t(t+4)^{-1}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}t}=6(t+4)^{-1}-6t(t+4)^{-2}$. Look for $e(t+4)^{-1}\pm ft(t+4)^{-2}$

M1: Attempts gradient of the tangent at $t=2$ using $m=\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$.

It is dependent upon getting one of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ or $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ in the correct form

ddM1: For an attempt at the equation of the tangent at $t=2$ using their gradient and their $(1,\,2)$

If the form $y=mx+c$ is used they must proceed as far as $c=\ldots$
The previous two M's must have been awarded.

A1: $y=-\frac{8}{3}x+\frac{14}{3}$ or exact equivalent in the form $y = mx + c$ following **fully correct work** including division if attempted. ISW after a correct answer.

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Note that the demand is to use parametric differentiation so candidates who use their answer to (b) cannot achieve all marks. If they have an equation of the form $y=\frac{ax+b}{cx+d}$ they can score a S.C. maximum of

B1 for $(1, 2)$, M0, M1 for correct attempt at quotient rule (see main scheme), ddM1, A0 for 3 out of 5.

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(b)

M1: Attempts to get t in terms of x or y.

E.g. $x=\frac{4}{t+2}\Rightarrow t=\frac{4}{x}-2$ **or** $y=\frac{6t}{t+4}\Rightarrow t=\frac{4y}{6-y}$ condoning slips. The order of operations must be correct. Expect the form of the answer to be correct. E.g.

$t=\pm\frac{py}{q\pm ry}$, $t=\pm\frac{r}{x}\pm s$ or equivalents

dM1: And then fully substitutes this into the other equation to form an expression for $\mathrm{f}(x)$ or the Cartesian equation

E.g. $(\mathrm{f}(x))=\frac{6\left(\frac{4}{x}-2\right)}{\frac{4}{x}-2+4}$ **or** $x=\frac{4}{t+2}\Rightarrow x=\frac{4}{\frac{4y}{6-y}+2}$

It is dependent upon the previous M. Again, expect the form to be correct allowing for slips.

Condone the omission of the lhs

A1: Correct equation in required form. The left-hand side must be present (somewhere) linked to their final answer.

Either $y=\frac{-6x+12}{x+2}$ or $\mathrm{f}(x)=\frac{-6x+12}{x+2}$. The order doesn't matter so $y=\frac{12-6x}{2+x}$ is fine

B1: Either $k = 4$ or $0<x<4$

(c)

M1: Either limit seen condoning strict or non-strict inequalities.

E.g. $\mathrm{f}<6$ or $y\leqslant 6$ allowable for the upper limit. Do not allow $\mathrm{f}>6$

Condone the use of x for f or y for this mark only

A1: $-2<\mathrm{f}<6$ o.e. such as $-2<\mathrm{f}(x)<6$, $-2<y<6$, $y\in(-2, 6)$ or just $(-2, 6)$

The notation must be correct so $-2 < x < 6$ is A0

Question Number	Scheme	Marks
4. (a)	Attempts to find the time = $\left(\frac{1}{3}\times 21^2\sqrt{(21+4)}\right)\div 30$	M1
	Time = $\frac{735}{30} = 24.5$ seconds	A1
		(2)
(b)	$V = \frac{1}{3}h^2\sqrt{(h+4)} \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2h}{3}\times\sqrt{(h+4)} + \frac{1}{6}h^2(h+4)^{-\frac{1}{2}}$	M1, A1
	States or uses $\frac{\mathrm{d}V}{\mathrm{d}t} = 30$	B1
	Uses $30 = \frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{dt}$ with $h = 5 \Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{108}{41}$ cms^{-1}	M1, A1
		(5)
		(7 marks)

(a)

M1: Attempts to find the time = $\left(\frac{1}{3}\times 21^2\sqrt{(21+4)}\right)\div 30$ condoning slips when substituting $h = 21$ into the formula. E.g. condone slips where the 21^2 appears as 21

A1: Reaches a time of 24.5 seconds (including units). Allow variations like $\frac{49}{2}$ s or 24.5 sec.

(b)

M1: Attempts to differentiate using the product rule achieving a correct form.

Look for $\frac{\mathrm{d}V}{\mathrm{d}h} = Ah\sqrt{(h+4)} + Bh^2(h+4)^{-\frac{1}{2}} \quad A, B > 0$

A1: Correct $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be left unsimplified

B1: States or uses $\frac{\mathrm{d}V}{\mathrm{d}t} = 30$ which may be awarded from anywhere within the solution and even in the body of the question. Note that using 30 in part (a) does not score this unless you see the lhs

M1: Uses $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{dt}$ with $\frac{\mathrm{d}V}{\mathrm{d}t} = 30$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ with $h = 5$ to find $\frac{\mathrm{d}h}{\mathrm{d}t}$. A correct $\left.\frac{\mathrm{d}V}{\mathrm{d}h}\right|_{h=5}$ is $\frac{205}{18}$

There must have been some attempt at differentiation to find $\frac{dV}{dh}$ but the form is not important and the M1 does not need to have been scored. If the attempt to find $\frac{dh}{dt}$ is via $\frac{dV}{dt} \times \frac{dh}{dV}$ DON'T allow attempts at the reciprocal like the following

$$\frac{dh}{dV} = \frac{1}{Ah\sqrt{(h+4)}} + \frac{1}{Bh^2(h+4)^{-\frac{1}{2}}}$$

A1: $\frac{108}{41}$ o.e. such as $\frac{540}{205}$ but isw after a correct answer seen.

Note that the exact answer is required so 2.63 scores A0 unless $\frac{108}{41}$ is seen first

Question Number	Scheme	Marks
5 (a)	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix} \bullet \begin{pmatrix}6\\-2\\-8\end{pmatrix} = 2\times6+3\times-2+-2\times-8$	M1
	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix} \bullet \begin{pmatrix}6\\-2\\-8\end{pmatrix} = \begin{vmatrix}2\\3\\-2\end{vmatrix}\begin{vmatrix}6\\-2\\-8\end{vmatrix}\cos(BAC) \Rightarrow 22 = \sqrt{17}\times\sqrt{104}\times\cos(BAC) \Rightarrow \cos(BAC) = \frac{22}{\sqrt{17}\sqrt{104}}$	dM1
	angle $BAC = 58.5°$	A1
		(3)
(b)	Area triangle $ABC = \frac{1}{2}\times\sqrt{17}\times\sqrt{104}\sin 58.5° = 17.9$	M1, A1
		(2)
(c)	Attempts $\pm\overrightarrow{DC} = \begin{pmatrix}6-2\lambda\\-2-3\lambda\\-8+2\lambda\end{pmatrix}$ with any constant λ	M1, A1
	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix} \bullet \begin{pmatrix}6-2\lambda\\-2-3\lambda\\-8+2\lambda\end{pmatrix} = 0 \Rightarrow \lambda = \frac{22}{17}$	dM1
	$\overrightarrow{AD} = \frac{44}{17}\mathbf{i} + \frac{66}{17}\mathbf{j} - \frac{44}{17}\mathbf{k}$ o.e. $\frac{22}{17}(2\mathbf{i}+3\mathbf{j}-2\mathbf{k})$	A1
		(4)
		(9 marks)

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}\bullet\pm\overrightarrow{AC}$.

Condone slips, e.g. in sign, but the intention must be clear.

If the calculation is not seen in full, expect to see 2 out of 3 of $(\pm12)+(\pm6)+(\pm16)$

If no intermediary calculation is seen it must be correct, so look for ±22

dM1: Full method to find $\cos(BAC)$ or BAC. It is dependent upon the previous M.

If the calculations for the modulus are not written out, at least one must be correct

A1: Angle $BAC =$ awrt $58.5°$. Note that $BAC =$ awrt $121.5°$ will be A0

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Alternatives exist, for example using the cosine rule

M1: Attempts the length of all 3 sides including an attempt at $\pm\overrightarrow{BC}$ and $\left|\pm\overrightarrow{BC}\right|$

$\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=(6\mathbf{i}-2\mathbf{j}-8\mathbf{k})-(2\mathbf{i}+3\mathbf{j}-2\mathbf{k})=4\mathbf{i}-5\mathbf{j}-6\mathbf{k}$ leading to

$\left|\overrightarrow{BC}\right|=\sqrt{4^2+5^2+6^2}$

If no calculations are seen then score if 2 out of 3 are correct.

dM1: Full attempt at the cosine rule to find $\cos(BAC)$, BAC or $180-BAC$. E.g

$\cos(BAC)=\dfrac{17+104-77}{2\times\sqrt{17}\times\sqrt{104}}$

A1: Angle $BAC =$ awrt $58.5°$

(b)

M1: Attempts the area using a correct combination of lengths and angles. E.g.

$\dfrac{1}{2}\times\sqrt{17}\times\sqrt{104}\sin 58.5°$

Follow through on their lengths and angle from part (a)

A1: AWRT 17.9. Can be scored when angle $BAC =$ awrt $121.5°$

Alt (b)

You may see vector product used. $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 3 & -2\\ 6 & -2 & -8\end{vmatrix}=-28\mathbf{i}+4\mathbf{j}-22\mathbf{k}$ then

$\dfrac{1}{2}\sqrt{28^2+4^2+22^2}=17.9$

M1: Full attempt condoning slips

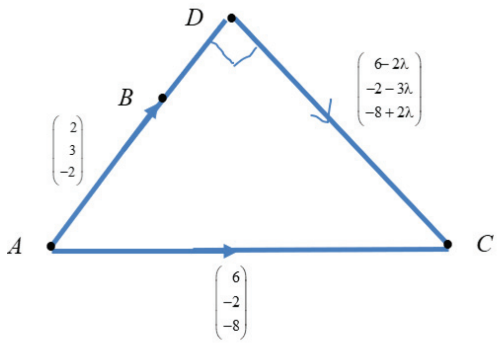
A1: $\sqrt{321}$ or AWRT 17.9

(c)

M1: Attempts $\pm\overrightarrow{DC}$ Look for a calculation that involves

- $\overrightarrow{AD}=\lambda\overrightarrow{AB}$ allowing with any constant
- use of $\overrightarrow{DC}=\overrightarrow{DA}+\overrightarrow{AC}$ or equivalent either way around

Condone slips but the intention must be clear



A1: Correct $\pm\overrightarrow{DC} = \pm\begin{pmatrix} 6-2\lambda \\ -2-3\lambda \\ -8+2\lambda \end{pmatrix}$

dM1: Uses a correct attempt at the scalar product of $\pm\overrightarrow{AD} \bullet \pm\overrightarrow{DC}$, sets equal to 0 and reaches a value for their constant 'λ'. Dependent upon previous M

A1: Correct $\overrightarrow{AD}$ using correct notation. Allow either version but not a mixture or coordinates

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Alternative methods exist and can be scored in a similar way with the first three marks for a correct method of finding the **exact** value for $\lambda\left(=\frac{22}{17}\right)$.

For example, using the trigonometry of a right-angled triangle (and possibly Pythagoras' theorem too).

M1: For attempting the length of AD using $\left|\overrightarrow{AC}\right| \cos BAC = \sqrt{104}\cos BAC$ allowing for use of degrees here

A value must be found, exact or approximate. An alternative is to use $\left|\overrightarrow{AC}\right| \sin\left(90 - BAC\right) = '\sqrt{104}'\sin'31.5'$

A1: Correct and exact length $\left|\overrightarrow{AD}\right| = \left|\overrightarrow{AC}\right| \times \cos BAC = \sqrt{104} \times \frac{22}{\sqrt{17}\sqrt{104}} = \frac{22}{\sqrt{17}}$

dM1: Uses ratio $\overrightarrow{AD} : \overrightarrow{AB} = \frac{22}{\sqrt{17}} : \sqrt{17} = \frac{22}{17} : 1$ to find the exact value of λ.

Other alternatives exist such as $\left(2\lambda\right)^2 + \left(3\lambda\right)^2 + \left(-2\lambda\right)^2 = \left(\frac{22}{\sqrt{17}}\right)^2 \Rightarrow \lambda = \ldots$

A1: Hence $\overrightarrow{AD}=\frac{22}{17}\times\begin{pmatrix}2\\3\\-2\end{pmatrix}=\begin{pmatrix}\frac{44}{17}\\ \frac{66}{17}\\ -\frac{44}{17}\end{pmatrix}$ allow alternatives such as $\overrightarrow{AD}=\frac{22}{17}\times(2\mathbf{i}+3\mathbf{j}-2\mathbf{k})$

Question Number	Scheme	Marks
6 (a)	$\frac{d}{dx}(12y)=12\frac{dy}{dx}$	B1
	$4y^2+8xy\frac{dy}{dx}+12\frac{dy}{dx}+6=0$	M1, A1
	$\Rightarrow\frac{dy}{dx}=\frac{-4y^2-6}{8xy+12}=\frac{-2y^2-3}{4xy+6}$	M1, A1
		(5)
(b)	Deduces that $4xy+6=0$	B1ft
	Substitutes $y=-\frac{3}{2x}$ into $4xy^2+12y+6x=53$	M1
	$\Rightarrow\frac{9}{x}-\frac{18}{x}+6x=53\Rightarrow 6x^2-53x-9=0$	A1
	$\Rightarrow(6x+1)(x-9)=0\Rightarrow x=\ldots$	dM1
	$\Rightarrow a=9, b=-\frac{1}{6}$ but allow $P=\left(9,-\frac{1}{6}\right)$	A1
		(5) **(10 marks)**

(a)

B1: Correct differentiation $\frac{d}{dx}(12y)=12\frac{dy}{dx}$ but allow equivalents such as $12y'$

M1: Correct attempt at the product rule on $4xy^2\to pxy\frac{dy}{dx}+qy^2$

A1: Correct differentiation.

If they write $\frac{dy}{dx}=4y^2+8xy\frac{dy}{dx}+12\frac{dy}{dx}+6$ it is A0 unless further work suggests that this is not what they meant. Allow y' or $\dot{y}$ for $\frac{dy}{dx}$. You may also see $4y^2dx+8xy\,dy+12\,dy+6dx=0$ which is fine.

M1: Collects exactly two $\frac{dy}{dx}$ terms together, one from each from the differentiation of $4xy^2$ and $12y$, and makes $\frac{dy}{dx}$ the subject. Note that the differentiation of the product rule could be incorrect e.g. $4xy^2 \to 8xy\frac{dy}{dx}$

A1: $\frac{dy}{dx} = \frac{-2y^2-3}{4xy+6}$ o.e such as $\frac{dy}{dx} = -\frac{2y^2+3}{4xy+6}$ but must be in simplest form

(b) Via HENCE

B1ft: Correctly deduces that $4xy+6=0$. Condone if they set $2y^2+3=0$ as well as $4xy+6=0$

Follow through on their denominator of $\frac{dy}{dx}$ as long as it is of the form $axy+b \quad a,b \neq 0$

M1: Substitutes their $y=-\frac{3}{2x}$ into $4xy^2+12y+6x=53$ to set up a 3TQ equation in x.

Alternatively substitutes their $x=-\frac{3}{2y}$ into $4xy^2+12y+6x=53$ to set up a 3TQ equation in y.

The denominator in (a) needs to be of the form $axy+b$ but the rearrangement may be incorrect.

A1: Correct 3TQ equation in x or y. FYI the correct simplified equation in y is $6y^2-53y-9=0$

dM1: It is dependent upon the previous M. Solves their 3TQ equation in x or y by a correct method.

Condone the use of a calculator here but the solution(s) must be correct for their equation

A1: CSO a or $x=9$, b or $y=-\frac{1}{6}$ but allow $P=\left(9,-\frac{1}{6}\right)$. Note that if $P=\left(-\frac{1}{6},9\right)$ is also found it must be rejected. All previous marks must have been awarded in (b). Note also that this mark can only be awarded following a correct simplified or unsimplified $\frac{dy}{dx}$ under this method.

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Some candidates will use their equation solvers to make short cuts to solve equations that are not in the 3TQ form, e.g. $-\frac{9}{x}+6x=53$. If they proceed to $x=9, y=-\frac{1}{6}$ o.e. they can score

SC: B1, M0, A0, dM1, A1

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(b) Alt via OTHERWISE. Works on the premise that y will have real roots

$$4xy^2+12y+(6x-53)=0$$
$$(12)^2-4\times4x\times(6x-53)\geqslant0$$
$$6x^2-53x-9\leqslant0$$
$$(x-9)(6x+1)\leqslant0$$
$$-\frac{1}{6}\leqslant x\leqslant9$$

The following is a suggested way of applying the MS but if you see a solution that you feel deserves more credit then please send to review.

B1: Sets up as a quadratic in y with $a=4x, b=12, c=6x-53$ and uses within $b^2-4ac...0$

M1: Attempts $b^2-4ac\geqslant0$ or $b^2-4ac>0$ with $a=4x, b=12, c=6x-53$ condoning slips

A1: $6x^2-53x-9\geqslant0$ or $6x^2-53x-9>0$

dM1: Solves their $6x^2-53x-9...0\Rightarrow x=...,...$ and finds the inside region.

A1: CSO $a=9, b=-\frac{1}{6}$ but allow $P=\left(9,-\frac{1}{6}\right)$.

Note that, via this method, this mark can be awarded following an incorrect part (a)

Question Number	Scheme	Marks
7 (i)	Way One: $\int e^{2x}\cos4x\,dx=\frac{1}{4}e^{2x}\sin4x-\int\frac{1}{2}e^{2x}\sin4x\,dx$	M1, A1
	$=\frac{1}{4}e^{2x}\sin4x-\left(-\frac{1}{8}e^{2x}\cos4x+\int\frac{1}{4}e^{2x}\cos4x\,dx\right)$	dM1, A1
	$\frac{5}{4}\int e^{2x}\cos4x\,dx=\frac{1}{4}e^{2x}\sin4x+\frac{1}{8}e^{2x}\cos4x$	ddM1
	$\Rightarrow\int e^{2x}\cos4x\,dx=\frac{1}{5}e^{2x}\sin4x+\frac{1}{10}e^{2x}\cos4x+c$	A1
		(6)
(ii)	$2x=\sin u\Rightarrow2dx=\cos u\,du$	
	$\int\frac{2x^3}{\sqrt{1-4x^2}}dx=\int\frac{1}{8}\frac{\sin^3u}{\sqrt{1-\sin^2u}}\times\cos u\,du$	M1

Question Number	Scheme	Marks
	$=\int \frac{1}{8}\sin^3 u\ \mathrm{d}u$	A1
	$=\int \frac{1}{8}\sin u\left(1-\cos^2 u\right)\mathrm{d}u=\int \frac{1}{8}\sin u-\frac{1}{8}\sin u\cos^2 u\ \mathrm{d}u$	dM1
	$=\left[-\frac{1}{8}\cos u+\frac{1}{24}\cos^3 u\right]$	ddM1 A1
	States or uses limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ E.g. $\left(-\frac{1}{8}\cos\frac{\pi}{3}+\frac{1}{24}\cos^3\frac{\pi}{3}\right)-\left(-\frac{1}{8}\cos\frac{\pi}{6}+\frac{1}{24}\cos^3\frac{\pi}{6}\right)$	M1
	$=\frac{3}{64}\sqrt{3}-\frac{11}{192}$	A1
		(7)
		(13 marks)

(i) Condone missing dx's throughout

M1: Integrates by parts to a correct form. E.g.

$\int \mathrm{e}^{2x}\cos 4x\ \mathrm{d}x=\pm A\mathrm{e}^{2x}\sin 4x\pm\int B\mathrm{e}^{2x}\sin 4x\ \mathrm{d}x$

A1: Correct integration $\frac{1}{4}\mathrm{e}^{2x}\sin 4x-\int\frac{1}{2}\mathrm{e}^{2x}\sin 4x\ \mathrm{d}x$ which may be left unsimplified

dM1: Integrates again in the same direction to

$\pm A\mathrm{e}^{2x}\sin 4x\pm B\mathrm{e}^{2x}\cos 4x\pm C\int \mathrm{e}^{2x}\cos 4x\ \mathrm{d}x$

A1: Fully correct $\frac{1}{4}\mathrm{e}^{2x}\sin 4x-\left(-\frac{1}{8}\mathrm{e}^{2x}\cos 4x+\int\frac{1}{4}\mathrm{e}^{2x}\cos 4x\ \mathrm{d}x\right)$ which may be left unsimplified

ddM1: Attempts to collect terms in $\int \mathrm{e}^{2x}\cos 4x\ \mathrm{d}x$ following award of both previous M's

Condone arithmetical slips when combining the terms in $\int \mathrm{e}^{2x}\cos 4x\ \mathrm{d}x$

A1: $\frac{1}{5}\mathrm{e}^{2x}\sin 4x+\frac{1}{10}\mathrm{e}^{2x}\cos 4x+c$ with or without the $+c$

Alternative scheme which can be applied in exactly the same way

Way Two:	$\int \mathrm{e}^{2x}\cos 4x\ \mathrm{d}x=\frac{1}{2}\mathrm{e}^{2x}\cos 4x+\int 2\mathrm{e}^{2x}\sin 4x\ \mathrm{d}x$	M1, A1

$= \frac{1}{2}e^{2x}\cos 4x + \left(e^{2x}\sin 4x - \int 4e^{2x}\cos 4x\,dx\right)$	dM1, A1
$5\int e^{2x}\cos 4x\,dx = e^{2x}\sin 4x + \frac{1}{2}e^{2x}\cos 4x \Rightarrow \int e^{2x}\cos 4x\,dx = \frac{1}{5}e^{2x}\sin 4x + \frac{1}{10}e^{2x}\cos 4x + c$	ddM1, A1

Note that attempts using the D and I method are acceptable. For example, under Way One, the first 4 marks are allocated as follows;

M2: Integrates to a form $\int e^{2x}\cos 4x\,dx = \pm Ae^{2x}\sin 4x \pm Be^{2x}\cos 4x \pm C\int e^{2x}\cos 4x\,dx$

A2: Integrates correctly $\int e^{2x}\cos 4x\,dx = \frac{1}{4}e^{2x}\sin 4x + \frac{1}{8}e^{2x}\cos 4x - \frac{1}{4}\int e^{2x}\cos 4x\,dx$

(ii)

M1: Fully applies the substitution WITH $k\mathrm{d}x = \cos u\,(\mathrm{d}u)$.

Look for $\int \pm k\frac{\sin^3 u}{\sqrt{1-\sin^2 u}} \times \cos u\,\mathrm{d}u$ You may condone a missing du only if the intention is clear

A1: $\int \frac{1}{8}\sin^3 u\,(\mathrm{d}u)$ which may be unsimplified but the $\frac{\cos u}{\sqrt{1-\sin^2 u}}$ must be fully cancelled.

Condone a missing du here and for all remainder of the question

dM1: Puts into a form that can be integrated. It is dependent upon having achieved

$\pm\int k\sin^3 u\,\mathrm{d}u$

$$\int \sin^3 u\,\mathrm{d}u = \int \sin u\left(1-\cos^2 u\right)\mathrm{d}u = \int \sin u - \sin u\cos^2 u\,\mathrm{d}u$$

You may see the substitution $\sin^3 u = \pm\frac{3}{4}\sin u \pm \frac{1}{4}\sin 3u$

ddM1: And then integrates to a correct form. Look for $\pm\alpha\cos u \pm \beta\cos^3 u$ or $\pm\alpha\cos u \pm \beta\cos 3u$

A1: $-\frac{1}{8}\cos u + \frac{1}{24}\cos^3 u$ or $\frac{3}{32}\cos u + \frac{1}{96}\cos 3u$

M1: States the correct limits or uses the correct limits (either way around) to an integrated form

If the limits are in degrees, this mark can only be awarded when the value of sin30, sin60, cos30 and cos60 are correctly applied to a function that must involve sin and/or cos only

A1: $\frac{3}{64}\sqrt{3}-\frac{11}{192}$ or $\frac{9\sqrt{3}-11}{192}$ but isw after a correct answer

Question Number	Scheme	Marks
8 (a) (i)	$18=140-\frac{k}{0.3\times0+8}\Rightarrow\frac{k}{8}=122\Rightarrow k=976$	B1
(ii)	$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{0.3\times976}{(0.3\times10+8)^2}=\text{awrt }2.42\ \left(\mathrm{m}^2/\text{day}\right)$	M1, A1
		(3)
(b)	$\int\frac{\mathrm{d}S}{128-S}=\int\frac{1}{50}\,\mathrm{d}t$	B1
	$-\ln(128-S)=\frac{1}{50}t+c$	M1, A1
	Uses $t=0,\ S=18\Rightarrow-\ln110=c$	M1
	$\ln\left(\frac{110}{128-S}\right)=\frac{1}{50}t\Rightarrow\frac{128-S}{110}=\mathrm{e}^{-\frac{1}{50}t}\Rightarrow S=128-110\mathrm{e}^{-\frac{1}{50}t}$	ddM1, A1
		(6)
(c)	128 (m^2)	B1
		(1)
		(10 marks)

(a)(i)

B1: Substitutes $t=0$ $S=18$ to reach $k=976$

(a)(ii)

M1: Substitutes $t=10$ into $\frac{\mathrm{d}S}{\mathrm{d}t}=\pm\frac{\delta}{(0.3t+8)^2}$ or if S is initially written as a single fraction, substitutes $t=10$ into a $\frac{\mathrm{d}S}{\mathrm{d}t}$ of the form $\frac{\alpha(0.3t+8)-\beta(at+b)}{(0.3t+8)^2}$

A1: $\frac{1464}{605}$ or awrt 2.42 following M1. No requirement for units. If two answers are given then it is A0

(b)

B1: Correctly separates the variables. The dS and the dt must be correctly placed but there is no requirement for the integral signs

M1: Integrates to $P\ln(128-S)=Qt\left(+c\right)$ There is no need for the + c

A1: Correct integration including the + c

M1: Uses t =0 S =18 to find a non-zero value for c. There must have been some attempt to integrate

Note that M1, A1, M1 can be achieved via

$$\left[-\ln(128-S)\right]_{18}^{S}=\left[\frac{1}{50}t\right]_{0}^{t}\Rightarrow \ln 110-\ln(128-S)=\frac{1}{50}t$$

ddM1: Dependent upon having achieved $P\ln(128-S)=Qt+c$. It is for the correct **order** of operations leading to S in terms of $e^{\pm nt}$. The ln/exp work must be sound. A common error scoring M0 is $\ln 110-\ln(128-S)=\frac{1}{50}t\Rightarrow 110-(128-S)=e^{\frac{1}{50}t}$

A1: $S=128-110e^{-\frac{1}{50}t}$ or exact equivalent such as $S=128-\frac{110}{e^{0.02t}}$ Condone $S=128-e^{-0.02t+\ln 110}$

(c)

B1: 128 (m^2). Units are not important here. Allow $S\leqslant 128$