Question Number	Scheme	Marks
1(a)	$27(3-5x)^{-2} = 27\times\frac{1}{9}\left(1-\frac{5}{3}x\right)^{-2}$ o.e. $3\times\left(1-\frac{5}{3}x\right)^{-2}$	B1
	$=(3)\left(1+(-2)\left(-\frac{5}{3}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5}{3}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{5}{3}x\right)^3+...\right)$	M1, A1
	$=3+10x+25x^2+\frac{500}{9}x^3+...$	A1
		(4)
(b)	$27(3+5x)^{-2} = 27(3+5x)^{-2} = 3-10x+25x^2-\frac{500}{9}x^3+...$	B1ft
		(1)
(c)	$27(3-x)^{-2} = 3+\frac{10}{5}x+\frac{25}{5^2}x^2+\frac{500}{9\times5^3}x^3 = 3+2x+x^2+\frac{4}{9}x^3+...$	M1A1
		(2)
		(7 marks)
Alt(a)	$27(3-5x)^{-2} = 27\left(3^{-2}+(-2)\times3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times3^{-4}\times(-5x)^2+\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3\right)$	B1 M1
	$=27\left(\frac{1}{9}+\frac{10x}{27}+\frac{25x^2}{27}+\frac{500x^3}{243}+..\right)$	A1 A1
	$=3+10x+25x^2+\frac{500}{9}x^3+...$	

(a)

B1: Correctly takes out a factor and proceeds to $3\times\left(1-\frac{5}{3}x\right)^{-2}$ which may be unsimplified. May be implied

M1: Expands $(1+kx)^{-2}$ $k\neq\pm1$ to give the correct structure for 3rd or 4th term

$\frac{(-2)(-3)}{2}(kx)^2$ or $\frac{(-2)(-3)(-4)}{3!}(kx)^3$ with or without the brackets around kx

A1: A correct simplified or unsimplified form of the expansion of $\left(1-\frac{5}{3}x\right)^{-2}$ May be implied by their final answer. Allow terms to be listed, which may be on different lines for this mark.

A1: Fully correct simplified expansion $=3+10x+25x^2+\frac{500}{9}x^3$ with all terms written on one line. Do not isw if they e.g. multiply through by 9

Alt(a)

B1: For the first term $27(3^{-2}+....$

M1: For correct structure for 3rd or 4th terms $\frac{(-2)(-3)}{2}\times 3^{-4}\times(-5x)^2$ or $\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3$ with or without the brackets around $-5x$

A1A1: See main scheme notes

In both parts ignore earlier workings if it is a restart and just mark their final answer

(b)

B1ft: $3-10x+25x^2-\frac{500}{9}x^3$ or if (a) was incorrect either score for a correct answer or follow through on their part (a) i.e. $A+Bx+Cx^2+Dx^3 \to A-Bx+Cx^2-Dx^3$

(c)

M1: Comparing their final answer line in (c) to their answer to part (a) of the form $A+Bx+Cx^2+Dx^3$ look for at least two of $B\to\frac{B}{5}$ or $C\to\frac{C}{25}$ or $D\to\frac{D}{125}$

A1: $3+2x+x^2+\frac{4}{9}x^3$ **which can only be scored following full marks in (a)**

Question Number	Scheme	Marks
2.	$\cancel{\dfrac{dy}{dx}} = \underline{8x - 2y\dfrac{dy}{dx}} + \underline{\underline{2x\dfrac{dy}{dx} + 2y}} = 0$	M1 B1 A1
	Sets $\dfrac{dy}{dx} = 2, \Rightarrow 8x - 4y + 4x + 2y = 0 \Rightarrow y - 6x = 0$	M1
	Sub "$y = 6x$" into $4x^2 - y^2 + 2xy + 5 = 0$ $\Rightarrow 4x^2 - 36x^2 + 12x^2 + 5 = 0 \Rightarrow x^2 = \dfrac{1}{4} \Rightarrow x = \pm\dfrac{1}{2}$	dM1, A1
	$\left(\dfrac{1}{2}, 3\right) \left(-\dfrac{1}{2}, -3\right)$	A1
		(7)
		(7 marks)

M1: Differentiates $4x^2 - y^2 + 5$ wrt x to obtain $Ax + By\dfrac{dy}{dx}$ (inc. $5 \to 0$) Condone $\dfrac{dy}{dx} = ...$ at start

B1: Sight of $\dfrac{d}{dx}(2xy) = 2x\dfrac{dy}{dx} + 2y$ oe e.g. $2\left(y + x\dfrac{dy}{dx}\right)$

A1: $8x - 2y\dfrac{dy}{dx} + 2x\dfrac{dy}{dx} + 2y = 0$ oe e.g. $\dfrac{dy}{dx} = \dfrac{-8x - 2y}{2x - 2y}$ oe The =0 may be implied by further work

M1: Sets $\dfrac{dy}{dx} = 2$ in their differentiated function in x and y and proceeds to a linear equation in x and y.
Brackets do not need to be multiplied out. Condone arithmetical slips in their rearrangement.
There must have been some attempt at implicit differentiation so look for at least a $...y\dfrac{dy}{dx}$ or $...x\dfrac{dy}{dx}$ term in their differentiated function – not just e.g. $\dfrac{dy}{dx} = 8x - 2y + 2$

Note: $\boldsymbol{\dfrac{dy}{dx} = 0 \Rightarrow y = -4x \Rightarrow 20x^2 = 5 \Rightarrow x = \pm\dfrac{1}{2}}$ **scores M0dM0A0A0**

dM1: Dependent on the previous method mark only. For this mark:

- substitutes their $y = 6x$ or $x = \dfrac{y}{6}$ into the equation of curve C to form an equation in one variable
- reaches a **two-term quadratic** $Ax^2 = B$ oe or $Ay^2 = B$ oe for any values of A or B where $A \times B > 0$:

  e.g. $(4x^2 - 36x^2 + 12x^2 + 5 = 0 \Rightarrow)$ $20x^2 = 5$

  e.g. $\left(4\left(\dfrac{y}{6}\right)^2 - y^2 + \dfrac{y^2}{3} + 5 = 0 \Rightarrow\right)$ $\dfrac{5}{9}y^2 = 5$

  but condone slips in the working. Also allow if both terms are on one side of an equation. The = 0 may be implied by further work.
- solves their **two term quadratic** to find a value for x or y (you do not need to be check this)

  It cannot be implied by a correct value for *x* or *y*.

A1: Either **one correct pair** of coordinates **or both *x* or both *y* values**. It must follow evidence of a two term quadratic. If it is a pair of coordinates, condone the omission of brackets.

A1: **Both** $\left(\frac{1}{2},3\right)$ and $\left(-\frac{1}{2},-3\right)$ (two answers correct following a correct quadratic in *x* or *y* only can score dM1A1A1) Any extra solutions (obtained by substituting the variable found first into the quadratic, instead of the linear equation for example) result in A0.

Unsimplified answers lose the final A mark e.g. $\left(\frac{\sqrt{5}}{\sqrt{20}},3\right)$ and $\left(-\frac{\sqrt{5}}{\sqrt{20}},-3\right)$

Allow (0.5, 3) and $(-0.5,\ -3)$ and condone without brackets.

Coordinates may be written separately if pairing is clear but a wrong pairing or just stating the *x* coordinates together and the *y* coordinates together is A0.

i.e. $x=\frac{1}{2}, y=3$ and $x=-\frac{1}{2}, y=-3$ is acceptable but $x=\pm\frac{1}{2}, y=\pm 3$ is not sufficient

Question Number	Scheme	Marks
3	$\frac{dh}{dt}=\frac{2h^{\frac{3}{2}}}{5t^2}$	
(a)	$\int \frac{1}{2h^{\frac{3}{2}}}\,dh=\int \frac{1}{5t^2}\,dt$	B1
	$-h^{-\frac{1}{2}}=-\frac{1}{5}t^{-1}(+c)$ oe	M1, A1
	$-(1)^{-\frac{1}{2}}=-\frac{1}{5}(1)^{-1}+c\Rightarrow c=...\left(\text{e.g.}=-\frac{4}{5}\right)$	M1
	-- e.g. $-h^{-\frac{1}{2}}=-\frac{1}{5}t^{-1}-\frac{4}{5}\Rightarrow\frac{1}{\sqrt{h}}=\frac{1}{5t}+\frac{4}{5}=\frac{1+4t}{5t}\Rightarrow\sqrt{h}=\frac{5t}{1+4t}\Rightarrow h=...$ or $\frac{1}{\sqrt{h}}=\frac{1}{5t}+\frac{4}{5}\Rightarrow 5t=\sqrt{h}(1+4t)\Rightarrow h=...$ --	M1
	$h=\frac{25t^2}{(1+4t)^2}\quad(a=25,\ b=4)$	A1
		(6)
(b)	Max $h=\frac{25}{16}$ or 1.5625 (m)	M1, A1ft
		(2)
		Total 8

(a)

B1: Correct separation of variables (allow equivalent forms) There is no need to see the integral signs but the dh and dt must be present and in the correct positions or may be implied by further work.

M1: Reaches $\alpha h^{-\frac{1}{2}}=\beta t^{-1}(+c)$ oe

A1: Correct integration with or without +c Accept other correct equivalent forms of $-h^{-\frac{1}{2}}=-\frac{1}{5}t^{-1}\ (+c)$

e.g. $-10h^{-\frac{1}{2}}=-2t^{-1}(+c)$

Note that if they do not have a constant of integration then no further marks can be scored

M1: Uses given conditions at an appropriate point in their integrated form (however poor) to find a constant of integration for their equation in h and t. Alternatively uses $\left[-h^{-\frac{1}{2}}\right]_1^h=\left[-\frac{1}{5}t^{-1}+c\right]_1^t$

Allow stating $h=1,\ t=1\Rightarrow c=...$ or equivalent but if a value for c is stated without any method seen then you will need to check this on your calculator.

M1: Uses correct algebra starting from their integrated expression of the form $\alpha h^{-\frac{1}{2}} = \beta t^{-1} + c$ oe to make h the subject . Look for an attempt that has

- a single fraction being formed before "inverting" oe. Do not allow attempts where the candidate inverts each term. Alternatively they may multiply throughout and factorise. Condone sign slips only in their manipulation
- a correct attempt to square i.e. each side is squared rather than each term. Condone the slip e.g. $5t^2$ instead of $25t^2$

A1: cao $h = \dfrac{25t^2}{(1+4t)^2}$ $\quad (a = 25,\ b = 4)$

(b)

M1: Max $h = \dfrac{\text{Their } a}{\text{Their } b^2}$ from an answer to (a) in the form $h = \dfrac{at^2}{(c+bt)^2}$ $\quad a > 0$ or allow from an equation of the required form stated in this part.

Implied by 1.56 (i.e. answer to 3sf)

A1ft: $\dfrac{25}{16}$ (m) or exact equivalent e.g. or 1.5625 (m) following through on their $h = \dfrac{"25"t^2}{(1+"4"t)^2}$ oe $\quad a > 0$

Condone e.g. $h \leqslant \dfrac{25}{16}$ or $0 < h < \dfrac{25}{16}$ or any other correct notation including interval notation

e.g. $\left(0,\ \dfrac{25}{16}\right)$, $\left(0,\ \dfrac{25}{16}\right]$ Units not required but if given they must be correct.

Question number	Scheme	Marks
4	For fractions $\frac{A}{x}+\frac{B}{x-4}=-\frac{2}{x}+\frac{14}{x-4}$	M1 A1
	$\frac{3x^2+8}{x^2-4x}=3-\frac{2}{x}+\frac{14}{x-4}$	B1ft
	$\int_1^3 \frac{3x^2+8}{x^2-4x}\,dx=\int_1^3 3-\frac{2}{x}+\frac{14}{x-4}\,dx=\left[3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert\right]_{(x=1)}^{(x=3)}$	M1 A1ft
	$=9-2\ln 3-3-14\ln 3$	dM1
	$=6-16\ln 3$	A1
		(7)
		[7 marks]

M1: Attempts to write in partial fractions form and finds values A **and** B for $\frac{A}{x}+\frac{B}{x-4}$ $A,B\neq 0$ Note this may be scored without the "3". They cannot just state values for A and B for this mark – it must be written in partial fraction form.

A1: $-\frac{2}{x}+\frac{14}{x-4}$ oe

B1ft: Correct partial fraction form $\frac{3x^2+8}{x^2-4x}=3-\frac{2}{x}+\frac{14}{x-4}$ oe such as $3-\frac{2}{x}-\frac{14}{4-x}$ **ft on their non-zero values for A and B**. Condone e.g. $\frac{3x^2+8}{x^2-4x}=3+-\frac{2}{x}+\frac{14}{x-4}$

Allow use of the letters "A" and "B" instead of numerical values i.e. $3+\frac{A}{x}+\frac{B}{4-x}$

Also allow this mark to be scored for $3+\frac{12x+8}{x^2-4x}$ or ft on their remainder $Cx+D$ both non-zero.

M1: For $\int \frac{1}{x}\,dx\to \ln x$ **and** $\int \frac{1}{x-4}\,dx\to \ln\lvert x-4\rvert$ but allowing $\ln(x-4)$ and $-\ln(4-x)$. Condone invisible brackets for this mark.

A1ft For $\int 3-\frac{2}{x}+\frac{14}{x-4}\,dx\to 3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert$ oe following through on their non-zero coefficients. $x-4$ requires modulus signs and/or brackets unless implied by later work (condone not around x)

E.g. allow $\int 3-\frac{2}{x}-\frac{14}{4-x}\,dx\to 3x-2\ln x+14\ln(4-x)$

dM1: Uses both limits 3 and 1, subtracts either way round and proceeds to the expression $P+Q\ln 3$ where P and Q are rational and non-zero. Must have simplified to this form.

It cannot be implied by $6-16\ln 3$ on its own. We must see some evidence of limits being used which as a minimum is the limits substituted in the integrand.

It is dependent on the previous method mark.

A1: $6-16\ln 3$ **following all previous marks scored**

Special Case:

Some candidates know to use PF but fail to see it is an improper fraction.

Note that use of $3x^2+8\equiv A(x-4)+Bx$ would result in $A=-2, B=14$ via substitution. The solution may appear as:

$$\frac{3x^2+8}{x^2-4x}=\frac{14}{x-4}-\frac{2}{x}$$

$$\int_1^3 \frac{3x^2+8}{x^2-4x}\,\mathrm{d}x=\int_1^3 \frac{14}{x-4}-\frac{2}{x}\,\mathrm{d}x=\left[14\ln|x-4|-2\ln|x|\right]_{(x=1)}^{(x=3)}$$

$$=14\ln 1-2\ln 3-(14\ln 3-2\ln 1)=-16\ln 3$$

This potentially will score M1A1B0M1A0dM0A0 for 3 out of 7

However, a candidate who correctly adds 3 to their partial fraction can potentially score full marks using the main scheme – send to review if unsure.

Question Number	Scheme	Marks
5	Point of intersection is $(6,-1,-5)$ or the value of $\mu = 4$	B1
	Uses their μ and **i** components $10+\mu a = "6"$ to find a	M1
	$a=-1$	A1
	Uses perpendicular vectors $\Rightarrow \begin{pmatrix}-2\\1\\4\end{pmatrix}.\begin{pmatrix}a\\b\\-2\end{pmatrix}=0 \Rightarrow -2a+b-8=0 \Rightarrow b=(6)$	dM1
	Uses **j** component $"c+\mu b"="-1" \Rightarrow c+24=-1 \Rightarrow c=(-25)$	ddM1
	$b=6, c=-25$	A1
		(6)
		[6 marks]

B1: States or uses the point of intersection as $(6,-1,-5)$ or the value of $\mu = 4$ Allow values or vectors seen or used or implied. These values are sometimes embedded within the work e.g. $2-2(-2)$, $1-2$ and $3+4(-2)$

M1: A full method to find "a". This involves

- using the **k** component of l_2 to find μ
- followed by the **i** component to find a

A1: $a=-1$ May be seen or implied by further work.

dM1: A full method of finding "b".

This involves using the fact that $\begin{pmatrix}-2\\1\\4\end{pmatrix}.\begin{pmatrix}a\\b\\-2\end{pmatrix}=0$ and using their value of a. Look for at least 2 correct products or implied by a correct value for b.

Condone arithmetical slips. It is dependent on the previous method mark.

ddM1: A full method of finding "c" using the **j** component. For using their values for a, b and μ and proceeding from $"c+\mu b"="-1"$ to a value for c. If just a value for c is stated you may have to check using their values for a, b and μ. It is dependent on the previous two method marks.

A1: $b=6, c=-25$

(Note a common misread is $\lambda = 2$ and will lead to a point of intersection $(-2, 3, 11)$ of values of $\mu=-4,\ a=3,\ b=14,\ c=59$ apply the misread rule in situations like this so max score will be B0M1A0dM1ddM1A1 – send to review if unsure)

If they attempt to find *c* before *b* then send to review (unless fully correct and scores full marks)

Question Number	Scheme	Marks
6(a)	$\frac{\mathrm{d}u}{\mathrm{d}x}=2$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{2}$	B1
	$\int \frac{(3x+2)^2}{2x-1}\mathrm{d}x=\int \frac{\left(3\frac{u+1}{2}+2\right)^2}{u}\frac{1}{2}\mathrm{d}u$	M1
	$=\frac{1}{8}\int\left(9u+42+\frac{49}{u}\right)\mathrm{d}u$	dM1
	$=\frac{1}{8}\left[\frac{9u^2}{2}+42u+49\ln u\right]$	A1
	$=\frac{1}{8}\left[\frac{9(9)^2}{2}+42(9)+49\ln(9)-\left(\frac{9(3)^2}{2}+42(3)+49\ln(3)\right)\right]$	M1
	$72+\frac{49}{8}\ln 3$ *	A1*
		(6)
(b)	$V=\pi\int\left(\frac{3x+2}{2\sqrt{2x-1}}\right)^2\mathrm{d}x=\frac{\pi}{4}\int\frac{(3x+2)^2}{2x-1}\mathrm{d}x$ $=\pi\left(18+\frac{49}{32}\ln 3\right)$	M1 A1
		(2)
		Total 8

(a) Note if no algebraic integration is seen then max score B1M1dM1A0M0A0*
Correct answer with no working scores 0 marks

You may see other credit-worth methods e.g. partial fractions followed by substitution – send to review

B1: $\frac{\mathrm{d}u}{\mathrm{d}x}=2$ o.e such as $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{2}$ or $\mathrm{d}u=2\mathrm{d}x$

M1: An attempt at a complete substitution in terms of u Look for $A\int\frac{(\text{linear expression in }u)^2}{u}(\mathrm{d}u)$ $A\neq 0$

Condone slips. The du may be implied by further work.

dM1: Proceeds to the form $\int\left(\alpha u+\beta+\frac{\gamma}{u}\right)(\mathrm{d}u)$ oe which may be unsimplified $\alpha,\ \beta,\ \gamma\neq 0$. It is dependent on the previous method mark.

A1: $\frac{1}{8}\left[\frac{9u^2}{2}+42u+49\ln u\right]$ oe (note may see e.g. $\frac{49}{8}\ln 8u$ for $\frac{49}{8}\ln u$) May be unsimplified

M1: Correct use of changed limits (3 and 9) within their integrand in 'u' of the form $pu^2+qu+r\ln ku$ or $x=2$ and $x=5$ within the integral of this form where 'u' has been changed back to $2x-1$

It cannot be for proceeding directly to the given answer which is M0 – must see some evidence of the use of limits.

A1*: Obtains the given answer with no errors seen and all previous marks scored. Condone invisible brackets to be recovered provided it is before the final answer. Do not penalise incorrect limits on the integral in their working, provided the correct values were substituted in to their integrand.

Note $=\frac{1}{8}\left[\frac{9u^2}{2}+42u+49\ln u\right]_3^9$ = given answer then M0A0*

(b)

M1: $V = K\pi \times$ answer to (a). Condone $K=1$ Implied by $K\pi\times\left(72+\frac{49}{8}\ln 3\right)$

A1: Correct exact answer (allow equivalent exact forms) e.g. $\frac{\pi}{4}\left(72+\frac{49}{8}\ln 3\right)$ isw

Question Number	Scheme	Marks
7 (a)	$(1,4.5)$	B1B1
		(2)
(b)	Attempts $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{12\sin\theta\cos\theta}{-24\cos^2\theta\sin\theta}=\left(-\dfrac{1}{2\cos\theta}\right)$	M1A1
	Subs $\theta=\dfrac{\pi}{3}$ into $\dfrac{dy}{dx}\Rightarrow\dfrac{dy}{dx}=...(=-1)$	dM1
	Uses gradient of normal with $(1,4.5)\Rightarrow(y-4.5)=1(x-1)$	ddM1
	$y=x+3.5$	A1
		(5)
(c)	Attempts $\displaystyle\int y\frac{dx}{d\theta}\,d\theta=\int 6\sin^2\theta\times-24\cos^2\theta\sin\theta\,d\theta$	M1A1
	Uses $\sin^2\theta=1-\cos^2\theta\Rightarrow\displaystyle\int y\frac{dx}{d\theta}\,d\theta=\int A\left(\cos^2\theta\sin\theta-\cos^4\theta\sin\theta\right)d\theta$	dM1
	Area of trapezium= $\dfrac{1}{2}(3.5+4.5)=4$	B1ft
	Attempts trapezium + area under curve $=\dfrac{1}{2}(3.5+4.5)-144\displaystyle\int_{\frac{\pi}{3}}^{0}\sin^3\theta\cos^2\theta\,d\theta$	dM1
	Area $=4+144\displaystyle\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)d\theta$ *	A1*
		(6)
(d)	Area $=4+144\left[-\dfrac{\cos^3\theta}{3}+\dfrac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{3}}=4+144\left(\left(-\dfrac{1}{24}+\dfrac{1}{160}\right)-\left(-\dfrac{1}{3}+\dfrac{1}{5}\right)\right)=\dfrac{181}{10}$	M1A1A1
		(3)
		(16 marks)

(a)

B1: Either of $(1,4.5)$ Accept any exact equivalent for 4.5 (May be seen on a diagram) May be written separately e.g. $x=1,\ y=4.5$ Condone coordinates without brackets

B1: Both $(1,4.5)$ Accept any exact equivalent for 4.5 (May be seen on a diagram) May be written separately e.g. $x=1,\ y=4.5$ Condone coordinates without brackets. isw once a correct pair is seen

Note for parts (b), (c) and (d) do not penalise incorrect notation provided the intention is clear or notation is recovered. E.g. $\sin\theta^2$ **for** $\sin^2\theta$**.**

(b)

M1: Attempts $\left(\dfrac{dy}{dx}=\right)\dfrac{dy/d\theta}{dx/d\theta}$ Allow e.g. "$12\sin\theta\cos\theta$"$\times\dfrac{1}{-24\cos^2\theta\sin\theta}$ or e.g. "$6\sin2\theta$"$\times\dfrac{1}{-12\sin2\theta\cos\theta}$ Condone poor differentiation on y and/or x provided the functions are both "changed". If the attempts at $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$ are not seen separately then look for the formula stated or a numerator of the form $p\sin\theta\cos\theta$ or a denominator of the form $q\cos^2\theta\sin\theta$. May be seen as an attempt to directly find $-\frac{\mathrm{d}x}{\mathrm{d}y}$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{12\sin\theta\cos\theta}{24\cos^2\theta\sin\theta}=\left(-\frac{1}{2\cos\theta}\right)$ oe May be unsimplified and may be implied by further work

dM1: Substitutes $\theta=\frac{\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to achieve a value. If just a value is stated then you may need to check this on your calculator. Condone slips provided the intention is clear. Dep. on 1st M. May be implied by their gradient in the equation of a normal.

ddM1: Attempts to find the normal by using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $(1,4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5)=1(x-1)$ If using $y=mx+c$ then $\Rightarrow c=...$ Dep on both previous Ms

A1: $y=x+3.5$ o.e. in the required form

Alt(b) Finding the cartesian equation and differentiating

M1: Finds the cartesian equation $y=A\pm Bx^{\frac{2}{3}}$ and attempts to differentiate proceeding to the form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)Cx^{-\frac{1}{3}}$ May be unsimplified and index does not need to be processed

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-x^{-\frac{1}{3}}$ May be unsimplified but index processed

dM1: Substitutes $x=1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}=(-1)$ to achieve a value. If just a value is stated then you may need to check this on your calculator. Condone slips provided the intention is clear. Dep. on 1st M. May be implied by their gradient in the equation of a normal.

ddM1: Attempts to find the normal by using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $(1,4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5)=1(x-1)$ If using $y=mx+c$ then $\Rightarrow c=...$ Dep on both previous Ms

A1: $y=x+3.5$ o.e. in the required form

(c) Note: Do not be concerned with the order of the limits on the integral and being consistent with 144 until the final mark.

M1: Attempts $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$ (Allow omission of $\mathrm{d}\theta$ and allow unsimplified). Condone poor attempts at $\frac{\mathrm{d}x}{\mathrm{d}\theta}$

A1: $\int 6\sin^2\theta\times-24\cos^2\theta\sin\theta\,(\mathrm{d}\theta)$

dM1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ to proceed from $\int A\sin^3\theta\cos^2\theta\,\mathrm{d}\theta$ to produce an expression in an 'integrable form'. Dep. on 1st M

$\int A\sin^3\theta\cos^2\theta\,\mathrm{d}\theta = \int A\sin\theta\left(1-\cos^2\theta\right)\cos^2\theta\,\mathrm{d}\theta = A\int\left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right)\mathrm{d}\theta$ Condone sign slips only for this mark.

B1ft: Area of trapezium$=\frac{1}{2}$("3.5"+"4.5") or alternatively $\int_0^1 ("x+3.5")\mathrm{d}x = \left[\frac{x^2}{2}+3.5x\right]_0^1 \left(=0.5+3.5\right)=4$

Where integration is used you must see the integrated expression but they can proceed directly to the answer from this point. You may need to check this on your calculator.

Must see any correct calculation to find the area. Just stating area of trapezium = “4” is B0ft

dM1: Attempts trapezium + area under curve. Look for $="\frac{1}{2}(3.5+4.5)"\pm A\int_0^{\frac{\pi}{3}} \sin^3\theta\cos^2\theta\left(\mathrm{d}\theta\right)$

OR alternatively $="\frac{1}{2}(3.5+4.5)"\pm\ A\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)\left(\mathrm{d}\theta\right)$ There must be an attempt to use the correct limits either way around and add a trapezium. Dep on 1st M

A1*: Achieves the given answer including $\mathrm{d}\theta$ provided all previous marks have been awarded. **Must be seen in (c) so do not condone at the start of (d).** Withhold this mark if the limits written on the integral are not consistent with ± 144 oe on any line of their solution but do not be concerned with the positions of 8 and 1 on the integral before they are replaced. Do not condone sign slips but allow invisible brackets to be recovered before the final answer.

Alt(c) – finding the cartesian equation first

M1: Finds the cartesian equation and proceeds to the form $\int A \pm Bx^{\frac{2}{3}}\,\mathrm{d}x$ oe (ignore limits)

A1: $\int 6-\frac{3}{2}x^{\frac{2}{3}}\ (\mathrm{d}x)\ \left(=\frac{3}{2}\int 4-x^{\frac{2}{3}}\,\mathrm{d}x\right)$ oe

dM1: $\frac{3}{2}\int\left(4-\left(8\cos^3\theta\right)^{\frac{2}{3}}\right)(-24\cos^2\theta\sin\theta) \Rightarrow$ integrable form as in main scheme notes

B1ftdM1A1*: Main scheme notes

(d)

M1: $\int \sin\theta\cos^n\theta = \pm\frac{\cos^{n+1}\theta}{n+1}$ in either term. This cannot be implied by a correct answer. May use substitution e.g. $u=\cos\theta$

A1: Any correct (unsimplified) answer following a correct integrated expression. Trigonometric expressions should be evaluated.

A1: 18.1 or exact equivalent cso

Question Number	Scheme	Marks
8 (a)	$4p^2 - q^2 = (2p+q)(2p-q)$	B1
		(1)
(b)	Makes one valid statement :Either $\begin{matrix} 2p+q=46 \\ 2p-q=1 \end{matrix}$ **or** $\begin{matrix} 2p+q=23 \\ 2p-q=2 \end{matrix}$	M1
	Solves correctly one of the pairs (see notes): e.g. $2p+q=46,\ 2p-q=1 \Rightarrow\ p=11.75, (q=22.5)$ oe **or** e.g. $2p+q=23,\ 2p-q=2 \Rightarrow p=6.25, (q=10.5)$ oe	A1
	States and attempts to solve **both equations (see above)**	dM1
	Fully correct proof by contradiction requiring (1) correct set up, (2) correct solution of both equations and (3) correct conclusion *	A1*
		(4)
		Total 5

(a)

B1: Correct factorisation

Note on epen this is B1M1A1A1* but we are marking this M1A1dM1A1*

Mark the attempts at valid pairs of equations in the order which scores the most marks

(b) If you see any other proofs which appear to be credit worthy then send to review

M1: Deduces either of the valid pairs of equations. There is no requirement to justify that $2p+q$ must be greater than $2p-q$ and therefore solve the other possible equations. Ignore these attempts at these pairs of equations.

A1: Correctly solves one of the two valid pairs of equations. No working is required, just the correct solution. Note that it is acceptable to only go as far as finding either one of p or q as that is a non-integer. However, if they find the other one of p or q then they must both be correct. Beware that the correct solving may be their second valid pair.

Note if they only proceed to e.g. $4p = 25$ **they must explain the contradiction** that e.g. p cannot be an integer **because** 25 is not divisible by 4 or LHS=even, RHS=odd or LHS=multiple of 4, RHS=not a multiple of 4

dM1: States and attempts to solve **both** valid pairs of equations. Condone proceeding as far as e.g. $4p = 47$ for this mark. It is dependent on the previous method mark.

A1*: Full proof by contradiction. This must follow M1A1dM1

Look for

- a correct set up with words such as "assume there is" or "let there be" with "integers p and q"
- correct solution of **both** valid pairs of equations (and explanations where required e.g. if only proceeding as far as $4p = 25$) If p and q are both found for both valid pairs then they must be correct.
- a (minimal) conclusion

Minimal proof for 4 marks:

Assume that there are integers p and q such that $4p^2 - q^2 = 46$

(So) $2p + q = 46$, $2p - q = 1 \Rightarrow p = 11.75$ not an integer

(and) $2p + q = 23$, $2p - q = 2 \Rightarrow p = 6.25$ not an integer

Hence, (we have a contradiction), so statement is proven

Alt(b) – using odd and even factor argument example

M1: Identifies that $46 = 1 \times 46,\ 2 \times 23$ **and** considers either the case when q is odd OR when q is even

- when q is odd then $(2p + q)$ is odd and $(2p - q)$ is odd OR
- when q is even then $(2p + q)$ is even and $(2p - q)$ is even

A1: Deduces correctly for the case when q is odd that 46 cannot be the product of two odd numbers OR deduces correctly for the case when q is even that 46 cannot be the product of two even numbers

dM1: Considers both cases (see first M1). It is dependent on the previous method mark

A1*: Full proof by contradiction. This must follow M1A1dM1

Look for

- a correct set up with words such as "assume there is" or "let there be" with "integers p and q"
- correct deductions for both odd and even cases
- a (minimal) conclusion

Alt(b) – condone using even property example

M1: e.g. $4p^2 - q^2 = 46 \left(\Rightarrow q^2 = 4p^2 - 46\right)$ and deduces q^2 (or q) is even.

A1: **States** q^2 is even (then q is even) and writes in the form $q = 2m$

dM1: Attempts $(2m)^2 = 4p^2 - 46$ and rearranges to a form where a contradiction can be deduced It is dependent on the previous method mark. e.g. $4m^2 = 4p^2 - 46 \Rightarrow m^2 = p^2 - \frac{46}{4}$ (which means m^2 and hence m cannot be an integer so there is a contradiction)

A1*: Full proof by contradiction. This must follow M1A1dM1

Look for

- a correct set up with words such as "assume there is" or "let there be" with "integers p and q"
- correct deductions
- a (minimal) conclusion

Question Number	Scheme	Marks
9 (a)	$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ $\quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + 6\mathbf{k}$ $\quad \mathbf{c} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$	
	e.g. $\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ (so $\overrightarrow{CD} = -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$)	M1
	$\mathbf{d} = \mathbf{c} + \overrightarrow{CD} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} + -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k} = -3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$	dM1 A1
		(3)
(b)	$\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -4\mathbf{i} + 4\mathbf{j}$	M1
	$\cos\theta = \dfrac{\begin{pmatrix}-2\\2\\-8\end{pmatrix}.\begin{pmatrix}-4\\4\\0\end{pmatrix}}{\sqrt{(-2)^2+2^2+(-8)^2}\sqrt{(-4)^2+4^2}} = \dfrac{16}{\sqrt{72}\sqrt{32}} = \dfrac{1}{3}$	dM1 A1
	So angle is awrt 1.23 radians or awrt 70.5 degrees	A1
		(4)
(c)	Area = $\sqrt{72}\sqrt{32} \times \sin"70.5" = 45.3$ **or** $32\sqrt{2}$	M1A1
		(2)
(d)	Area = $\dfrac{3}{2} \times "45.3" = 67.9$ or $48\sqrt{2}$	M1 A1
		(2)
		(11 marks)

(a) Note that one correct component of $-3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ scores M1dM1A0 if no incorrect working seen

M1: For attempting one of $\mathbf{b} - \mathbf{a}$ or $\mathbf{a} - \mathbf{b}$ or $\mathbf{c} - \mathbf{b}$ or $\mathbf{b} - \mathbf{c}$. It must be correct for at least one of the components $\pm(-2\mathbf{i} + 2\mathbf{j} - 8\mathbf{k})$ or $\pm(-4\mathbf{i} + 4\mathbf{j})$. Condone coordinate notation. May be implied by at least one correct component of $-3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$

dM1: For attempting $\mathbf{d} = \mathbf{a} - \mathbf{b} + \mathbf{c}$ and proceeding to a position vector for *D*. It must be correct for at least one of the components. Condone coordinate notation. It is dependent on the previous method mark.

A1: $-3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ oe $\begin{pmatrix}-3\\5\\-2\end{pmatrix}$ Condone without brackets. Condone $(-3\mathbf{i},\ 5\mathbf{j}, -2\mathbf{k})$

Correct answer no working scores all 3 marks. Condone coordinates.

Alt(a) Finding the mid point *E* of *AC* and then using $\mathbf{d} = \mathbf{b} + 2\overrightarrow{BE}$ Must be a full method:

M1: Attempts $mp_{AC} = (0,2,2)$ and uses $\mathbf{d} = \mathbf{b} + 2\overrightarrow{BE}$

M1: Attempts $(3,-1,6) + 2\times(-3,3,-4) \Rightarrow ....$

A1: $-3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ oe (see main scheme notes)

(b)

M1: Uses a correct pair of vectors , so $\pm p\overrightarrow{BA}$ and $\pm p\overrightarrow{BC}$. Must be seen or used in (b). May be implied by further work e.g. in the dot product formula or an attempt to find the magnitude of each of these vectors. Each must be correct for at least two of the components.

dM1: Attempts to use the dot product formula to find $\cos\theta = k, -1 < k < 1$. Dep. on the previous M. Condone arithmetical slips in both their dot product calculation and the moduli, but the process must be correct.

A1: For $\frac{1}{3}$ or $-\frac{1}{3}$ oe from correct working - may be implied by 70.5 or 109.5 or 1.23 radians or 1.91 radians.

A1: cso for awrt 70.5 degrees or awrt 1.23 radians. (Note that $\cos^{-1} = -\frac{1}{3} \Rightarrow$ 109.5 followed by 70.5 is A0 unless accompanied by a convincing argument that the angle 109.5 is the exterior angle, and therefore the interior angle is 70.5. It is not awarded for simply finding the acute angle. A diagram with correct angles would be ok)

Alt(b)

M1: See main scheme notes

dM1: Attempts $\pm p\overrightarrow{AC}$, finds all three magnitudes and uses the cosine rule with the values in the correct positions to find $\cos\theta = k, -1 < k < 1$. e.g. $\cos\theta = \frac{"72"+"32"-"72"}{2"\sqrt{72}""\sqrt{32}"} \Rightarrow \theta = ...$ Dep on previous M

Condone slips in finding the magnitudes but the process must be correct.

A1A1: See main scheme notes

(c)

M1: Uses correct area formula for parallelogram with their angle from (b). The values embedded is sufficient. They must be using their $\pm p\overrightarrow{BA}$ and $\pm p\overrightarrow{BC}$, each must be correct for at least two of the components. Condone slips. May see the area of the triangle *ABC* doubled. May be implied by awrt 45.2/45.3

A1: Obtains awrt 45.3. Allow this from an angle of 109.5

Alt(c) – vector cross product example

M1: Attempts e.g. $\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -8 \\ -4 & 4 & 0 \end{vmatrix} = 32\mathbf{i} - 32\mathbf{j} \Rightarrow \text{Area} = \left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \sqrt{32^2 + 32^2}$ using $\pm p\overrightarrow{BA}$ and $\pm p\overrightarrow{BC}$, each must be correct for at least two of the components. Condone sign slips for their cross product

A1: Obtains awrt 45.3. Allow this from an angle of 109.5

(d) Must use their answer to part (c) to score

M1: Uses 1.5 times answer to the area of their *ABCD* (It can be implied by awrt 67.9). Alternatively, uses their answer to part (c) and adds the area of the triangle

e.g. $h = 4\sqrt{2} \times \sqrt{1 - \frac{16}{9\sqrt{2} \times 6\sqrt{2}}} = \frac{16}{3} \Rightarrow \text{Area} = \frac{1}{2} \times \frac{16}{3} \times 6\sqrt{2} + "32\sqrt{2}"$

A1: awrt 67.9 or $48\sqrt{2}$