Question Number	Scheme	Marks
1 (a)	$nax = \pm 4x$ OR $\frac{n(n-1)(ax)^2}{2} = \frac{24}{5}x^2$	M1
	$na = -4$ AND $\frac{n(n-1)a^2}{2} = \frac{24}{5}$	A1
	Sub $a = \frac{-4}{n} \Rightarrow \frac{\not{n}(n-1)16}{2n^{\not{2}}} = \frac{24}{5} \Rightarrow 80(n-1) = 48n \Rightarrow n = ...$	
	$a = -\frac{8}{5}, n = \frac{5}{2}$	dM1, A1, A1
		(5)
(b)	$p = \frac{"\frac{5}{2}"\left("\frac{5}{2}"-1\right)\left("\frac{5}{2}"-2\right) \times \left("-\frac{8}{5}"\right)^3}{6} = -\frac{32}{25}$	M1, A1
		(2)
(c)	Allow $\lvert x \rvert < \frac{5}{8}$ OR $\lvert x \rvert \leqslant \frac{5}{8}$ or equivalents such as $-\frac{5}{8} < x < \frac{5}{8}$	B1ft
		(1)
		(8 marks)

(a)

M1: For $nax = \pm 4x$ OR $\frac{n(n-1)(ax)^2}{2} = \pm\frac{24}{5}x^2$ which may be implied. Condone missing brackets.

Allow the method mark to be scored for

- non-simplified versions such as $\binom{n}{1}ax = \pm 4x$ or ${}^nC_2(ax)^2 == \frac{24}{5}x^2$
- simplified versions (with x's cancelled) such as $na = -4$ or $\frac{n(n-1)a^2}{2} = \frac{24}{5}$

A1: Correct equations not involving x's (and not in combination notation form)

Scored for both $na = -4$ AND $\frac{n(n-1)a^2}{2} = \frac{24}{5}$ o.e.

dM1: Solves two equations of the correct form and proceeds to find a value for either a or n.
Expect to see the following

- 'simplified' initial equations of the form $na = \pm 4$ and $\frac{n(n-1)a^2}{2} = \pm\frac{24}{5}$
- a simplified intermediate equation in a single variable. Look, for example, for equations of the form $80(n-1) = 48n$ or $-2(-4-a) = \frac{24}{5}$ that are easily solvable. Condone slips
- followed by a value for either a or n.

A1: Either $a = -\frac{8}{5}$ or $n = \frac{5}{2}$ following two correct equations which may be in an un-simplified form

Allow equivalent fractions or decimals

A1: Both $a=-\frac{8}{5}$ and $n=\frac{5}{2}$ following correct equations **AND** following the award of all previous marks. Allow equivalent fractions or decimals

(b) The M1 can be awarded from a made up $n \notin \mathbb{N}$ and *a*. For example, they may say let $n=\frac{1}{2}, a=6$

M1: Attempts $\frac{n(n-1)(n-2)}{3!}\times a^3\left(x^3\right)$ with their values for *a* and *n* with or without the x^3

Condone ${}^nC_3(ax)^3 = ...x^3$ provided the value produced implies correct processing

A1: $-\frac{32}{25}$ o.e which must be seen without the x^3

(c)

B1ft: $|x|<\frac{5}{8}$, $|x|\leqslant\frac{5}{8}$ or equivalents such as $-\frac{5}{8}<x<\frac{5}{8}$ but follow through on their *a*. E.g. $|x|<\frac{1}{a}$

If they haven't got a value for *a* they can score this for $|x|<\frac{1}{a}$. If they have a value for *a* it must be used.

....

Note the demand is to show working and not rely on calculator technology

Solutions that don't show an intermediate equation in a single variable score as follows. If unsure please use review

Scenario I: Writes down two correct simplified initial equations and then correct values for *a* and *n*, but no simplified intermediate equation

E.g.I $\quad na=-4 \quad$ AND $\frac{n(n-1)a^2}{2}=\frac{24}{5} \Rightarrow a=-\frac{8}{5}, n=\frac{5}{2}$

Score 1,1,0,1,0. All marks in (b) and (c) are available

E.g.II $\quad na=-4 \quad$ AND $\frac{n(n-1)a^2}{2}=\frac{24}{5} \Rightarrow \frac{-\frac{4}{a}\left(-\frac{4}{a}-1\right)a^2}{2}=\frac{24}{5} \Rightarrow a=-\frac{8}{5}, n=\frac{5}{2}$

Score 1,1,0,1,0. All marks in (b) and (c) are available

Scenario II: Writes down one or two correct un-simplified initial equations and then correct values for *a* and *n*

E.g. $\quad na=-4 \quad$ AND ${}^nC_2(ax)^2 = \frac{24}{5}x^2 \Rightarrow a=-\frac{8}{5}, n=\frac{5}{2}$

Score 1,0,0,1,0. All marks in (b) and (c) are available

Scenario III: Writes down one or two correct un-simplified initial equations and then a correct value for just *n*

E.g. $\quad na=4 \quad$ AND ${}^nC_2(ax)^2 = \frac{24}{5}x^2 \Rightarrow a=\frac{8}{5}, n=\frac{5}{2}$

Score 1,0,0,1,0. As the value of *a* is incorrect, then only the M mark in (b) is available

Scenario IV: Only writes down one correct equation and then uses trial and improvement methods to find/show correct values for *a* and *n*

E.g. $\quad na=-4 \quad$ and then follows with trials including $a=-\frac{8}{5}, n=\frac{5}{2}$ and shows $\frac{\frac{5}{2}\times\frac{3}{2}}{2}\times\left(-\frac{8}{5}\right)^2=\frac{24}{5}$

Score 1,0,0,1,0. All marks in (b) and (c) are available

Question Number	Scheme	Marks
2	$\int x^3 \ln\left(\frac{1}{2}x\right) dx = \frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \int \frac{1}{4}x^4 \times \frac{1}{x} dx$	M1
	$= \frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4 (+c)$	dM1 A1
	$\left[\frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4\right]_2^{2e^2} = \left(\frac{16e^8}{4}\ln e^2 - \frac{16e^8}{16}\right) - \left(\frac{16}{4}\ln 1 - \frac{16}{16}\right)$	ddM1
	$= 7e^8 + 1$	A1
		(5 marks)

Condone missing dx's throughout

M1: Attempts to integrate by parts the correct way around.

Look for $\int x^3 \ln\left(\frac{1}{2}x\right) dx = ax^4 \ln\left(\frac{1}{2}x\right) \pm \int bx^4 \times \frac{1}{x} dx$ o.e. such as $ax^4 \ln\left(\frac{1}{2}x\right) \pm \int bx^3 dx$ with $a > 0$

dM1: And then integrates again to a form $px^4 \ln\left(\frac{1}{2}x\right) - qx^4$ where p and q are **positive constants**.

A1: Correct simplified or un-simplified integration. $\frac{1}{4}x^4 \ln\left(\frac{1}{2}x\right) - \frac{1}{16}x^4$ with or without the bracket

ddM1: Attempts to substitute in both limits and subtract either way around, condoning slips. It is dependent upon having an integral of the correct form. To score this mark

- Look for integration that leads to the form $px^4 \ln\left(\frac{1}{2}x\right) \pm qx^4$ where p and q are **positive constants**
- Substitution of both limits and an attempt to subtract either way around
- Correct calculation of both $\ln\left(\frac{1}{2} \times 2e^2\right) = 2$ and $\ln\left(\frac{1}{2} \times 2\right) = 0$ which may be implied

A1: $7e^8 + 1$ which must be in simplest form

………

Note: You may see D and I methods attempted. For the first three marks score M1, dM1 for integration to the form $px^4 \ln\left(\frac{1}{2}x\right) - qx^4$ where p and q are **positive constants.**

You may see something like this;

D	I
$\ln\left(\frac{1}{2}x\right)$	x^3
$\frac{1}{x}$	$\frac{x^4}{4}$

You could also see methods involving substitution in which the marks are scored as in the main scheme.

E.g. Let $u = \frac{x}{2}$ then $\int_2^{2e^2} x^3 \ln\left(\frac{1}{2}x\right) dx = \int_1^{e^2} 16u^3 \ln u \, du = \left[4u^4 \ln u - u^4\right]_1^{e^2} = 7e^8 + 1$

………

Question Number	Scheme	Marks
3 (a)	$(x+2y)(x-2y)$	B1
		(1)
(b)	Makes one valid statement. Either $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$	M1
	Solves correctly one of the pair; $(x+2y)=27,(x-2y)=1 \Rightarrow x=14, y=6.5$ $(x+2y)=9,(x-2y)=3 \Rightarrow x=6, y=1.5$	A1
	States and attempts to solve both valid equations	dM1
	Fully correct proof by contradiction (1) including correct set up, (2) correct solution of both equations and (3) correct conclusion*	A1*
		(4)
		(5 marks)

(a)

B1: $x^2-4y^2 \equiv (x+2y)(x-2y)$

(b)

M1: Deduces either of the valid pairs of equations, $(x+2y)=27,(x-2y)=1$ or $(x+2y)=9,(x-2y)=3$.

There is no requirement to justify that $x+2y$ must be greater than $x-2y$ and therefore find the other equations $(x+2y)=1,(x-2y)=27$ and $(x+2y)=3,(x-2y)=9$

If they have part (a) incorrect, e.g $x^2-4y^2 \equiv (x+4y)(x-4y)$ award for $(x+4y)=27,(x-4y)=1$ or $(x+4y)=9,(x-4y)=3$

A1: Correctly solves one of the two valid pairs of equations. No working is required, just the correct solution.

Look for either of $(x+2y)=27,(x-2y)=1 \Rightarrow y=6.5,(x=14)$

Or $(x+2y)=9,(x-2y)=3 \Rightarrow y=1.5,(x=6)$

Note that it is acceptable to only go as far as finding y as it is the non-integer solution

dM1: States and attempts to solve both valid pairs of equations. It is dependent upon the previous M mark

Note that it is acceptable to only go as far as finding y as that is the non-integer

You can **IGNORE** working from non-relevant equations like $(x+2y)=1,(x-2y)=27 \Rightarrow x=..., y=...$ or $(x+2y)=-3,(x-2y)=-9 \Rightarrow x=..., y=...$ and incorrect equations like $(x+2y)=18,(x-2y)=1.5 \Rightarrow x=..., y=...$

A1*: Full proof by contradiction. This must follow the award of M1, A1, dM1

Look for

- a correct set up with words such as 'assume there is' or 'let there be' with 'positive integers *x* and *y*' or 'positive integers'
- correct solution of both valid pair of equations
- acceptable (minimal) conclusion

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Minimal proof for 4 marks

Assume that there are positive integers x and y such that $x^2 - 4y^2 = 27$

So either $(x+2y) = 27, (x-2y) = 1 \Rightarrow y = 6.5$ ✗

Or $(x+2y) = 9, (x-2y) = 3 \Rightarrow y = 1.5$ ✗

Hence, we have a contradiction, so statement proven.

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Other methods exist.

For example:

There exist positive integers x and y such that $x^2 - 4y^2 = 27 \Rightarrow x^2 = 27 + 4y^2$

Then deduces that x^2 is odd which means that x is odd.

So $x = 2m+1 \Rightarrow (2m+1)^2 - 4y^2 = 27$

$$\Rightarrow 4m^2 + 4m - 4y^2 = 26$$

$$\Rightarrow y^2 = m^2 + m - \frac{13}{2}$$

Hence y^2 and so y cannot be an integer, so we have a contradiction,

Score M1: For correct work in deducing that x^2 or x is odd

A1: For stating that both x^2 and x are odd and x is of the form $2m+1$ o.e.

dM1: For squaring $2m+1$ and attempting to make y^2 the subject

A1: Full proof with correct set up, equations and deductions

Candidates who attempt odd and even combinations of x and y will only score marks when they reach an equivalent stage to the above. Please send these to review if you are unsure.

………………………………………………………………………………………………

Other more elaborate methods will be seen. If you feel that they deserve credit and you cannot see how to score the response, then please send to review.

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Question Number	Scheme	Marks
4 (a)	Limits $a=5, b=10$	B1
	$\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2\times(u-4)+3}{u}\times\frac{du}{(u-4)} = \int \frac{2u-5}{u(u-4)}\,du$	M1 A1
		(3)
(b)	Sets their $\frac{2u-5}{u(u-4)}$ as $\frac{A}{u}+\frac{B}{u-4}$	B1
	$"2u-5" = A(u-4)+Bu \Rightarrow A=\frac{5}{4}, B=\frac{3}{4}$	M1, A1
	$\int_0^{\ln 6} \frac{2e^x+3}{e^x+4}\,dx = \int_5^{10} \frac{5/4}{u}+\frac{3/4}{(u-4)}\,du = \left[\frac{5}{4}\ln u+\frac{3}{4}\ln(u-4)\right]_5^{10}$	M1
	$=\left(\frac{5}{4}\ln 10+\frac{3}{4}\ln 6\right)-\left(\frac{5}{4}\ln 5+\frac{3}{4}\ln 1\right)$	
	$=\frac{5}{4}\ln\frac{10}{5}+\frac{3}{4}\ln 6 = \frac{5}{4}\ln 2+\frac{3}{4}\ln 2+\frac{3}{4}\ln 3$	dM1
	$=2\ln 2+\frac{3}{4}\ln 3$	A1
		(6) **(9 marks)**

(a)

B1: States either $a=5, b=10$ or correctly embeds the values as limits in the correct place within the integral. May be awarded from the limits seen or used in part (b)

M1: Attempts to write $\frac{2e^x+3}{e^x+4}\,dx$ in terms of u.

Look for both of

- A correct $\frac{dx}{du}$ or $\frac{du}{dx}$ in terms of u. E.g. $u=e^x+4 \Rightarrow \frac{du}{dx}=e^x=u-4$ or $x=\ln(u-4) \Rightarrow \frac{dx}{du}=\frac{1}{u-4}$
- $\frac{2e^x+3}{e^x+4}\,dx$ going to $\frac{cu+d}{u(u-4)}(du)$

A1: Shows that $\int \frac{2e^x+3}{e^x+4}\,dx = \int \frac{2u-5}{u(u-4)}\,du$ with the dx's and du's appropriately placed throughout the solution. This mark does not depend on correct limits but the integral sign and du must present on the final line

(b)

B1: Scored for realising that this integral needs to be attempted via PF. So sets their $\frac{2u-5}{u(u-4)} \equiv \frac{A}{u}+\frac{B}{u-4}$

This may be implied by writing their expression in a form with numerical values for A and B

M1: Valid attempt to find the values for A and B.

Score for a correct identity followed by methods of substitution or equating terms to find values for A and B. For example, sets '$cu+d = A(u-4)+Bu$ followed by $u=4 \Rightarrow B=...$ and $u=0 \Rightarrow A=...$

This can be implied by a correct value for A or B

A1: Correct values for A and B. This is not a follow through mark

Note that other methods of splitting the fraction are possible but the scheme can be applied.

FYI: $\frac{2u-5}{u(u-4)} \equiv \frac{2}{(u-4)}-\frac{5}{u(u-4)} \equiv \frac{2}{(u-4)}-\left\{-\frac{5}{4u}+\frac{5}{4(u-4)}\right\}$

M1: Integrates the form $\frac{A}{u}+\frac{B}{u-4}$ to $...\ln u+...\ln(u-4)$. Don't be concerned by the coefficients

Note that $...\ln pu+...\ln q(u-4)$ is also correct so you may see $\frac{5}{4u} \to \frac{5}{4}\ln 4u$

Condone missing brackets only if subsequent work involving limits implies this intention

dM1: Dependent upon the previous M. Scored for;

- Substituting 5 and 10 into $\alpha \ln u+\beta \ln(u-4)$ and subtracting either way around
- correctly applying at least one log law

A1: CAO $2\ln 2+\frac{3}{4}\ln 3$

Question Number	Scheme	Marks
5 (a)	$(1,10)$	B1, B1
		(2)
(b)	$\frac{dy}{dt}=4t-4, \quad \frac{dx}{dt}=\frac{4(2t+1)-2(4t-5)}{(2t+1)^2}$	B1, M1
	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{(4t-4)}{14/(2t+1)^2}=\frac{2(t-1)(2t+1)^2}{7}$	dM1 A1
		(4)
(c)	Substitutes $t=3$ into their $\frac{dy}{dx}=(28)$	
	$y-10=28(x-1)\Rightarrow y=28x-18$	M1, A1
		(2)
(d) (i)	$-5\leqslant x<2$	M1 A1
(ii)	$f(x)\geqslant 2$	B1
		(3)
		(11 marks)

(a)

B1: One correct coordinate. E.g. $(1,...)$ or $(...,10)$. Allow as $x=1$ or $y=10$

B1: $(1,10)$. Allow as $x=1,\ \ y=10$

(b)

B1: $\frac{dy}{dt}=4t-4$. You may see this written as $y'=4t-4$ or $\dot{y}=2\times 2t-4$ which is fine.

M1: Attempts to use the quotient rule (or equivalent) to differentiate $x=\frac{4t-5}{2t+1}$.

Look for the form $\frac{dx}{dt}=\frac{p(2t+1)-q(4t-5)}{(2t+1)^2}$ with $p, q>0$

You may see $x=\frac{4t-5}{2t+1}\equiv 2-\frac{7}{2t+1}\Rightarrow\frac{dx}{dt}=\frac{14}{(2t+1)^2}$ with M1 scored for the form

$x=a+\frac{b}{2t+1}\Rightarrow\frac{dx}{dt}=\frac{c}{(2t+1)^2}$

Or the product rule $x=(4t-5)(2t+1)^{-1}\Rightarrow\frac{dx}{dt}=4(2t+1)^{-1}-2(4t-5)(2t+1)^{-2}$ but condone sign and coefficient slips. Look for the form $\alpha(2t+1)^{-1}\pm\beta(4t-5)(2t+1)^{-2}$

dM1: Attempts to apply the rule $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Dependent upon the previous M and $\frac{dy}{dt}$ being linear

A1: A correct **simplified** expression for $\frac{dy}{dx}$ ISW after a correct answer

Allow $\frac{2(t-1)(2t+1)^2}{7}$, $\frac{(2t-2)(2t+1)^2}{7}$, $\frac{2(4t^3-3t-1)}{7}$ or $\frac{8t^3-6t-2}{7}$

Note that if division was used to find $\frac{dx}{dt}$, all calculations must have been correct.

(c)

M1: Uses the value of their gradient at $t = 3$ and their point from (a) to form the equation of the tangent.

Look for $y - '10' = m(x - '1')$ with m being the value of their $\frac{2(t-1)(2t+1)^2}{7}$ at $t = 3$

If the form $y = mx + c$ is used the method must proceed as far as $c = \ldots$

A1: CSO $y = 28x - 18$

(d) (i)

M1: Achieves the correct value for the upper or lower bound. Condone $x > -5$ or $x \leqslant 2$ or, with incorrect variables e.g. $y > -5$. Solutions such as $x < -5$ OR $x = 2$ will be M0

A1: $-5 \leqslant x < 2$ or equivalent form such as $[-5, 2)$. The notation must be correct

(d)(ii)

B1: $f(x) \geqslant 2$ but allow $f \geqslant 2$, $y \geqslant 2$ or $[2, \infty)$. Do not allow with x

Question Number	Scheme	Marks
6 (a)	Position vector of a general point on $l_1 = \begin{pmatrix} -3+2\lambda \\ 0+1\lambda \\ 5-3\lambda \end{pmatrix}$	
	$\begin{pmatrix} -3+2\lambda \\ 0+1\lambda \\ 5-3\lambda \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = 0 \Rightarrow -6+4\lambda+\lambda-15+9\lambda=0 \Rightarrow \lambda = \frac{3}{2}$	M1, A1
	Substitute $\lambda = \frac{3}{2}$ into $\begin{pmatrix} -3+2\lambda \\ 0+1\lambda \\ 5-3\lambda \end{pmatrix} \Rightarrow P = \left(0, \frac{3}{2}, \frac{1}{2}\right)$	dM1, A1
		(4)
(b)	Lines intersect when $\begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ a \\ -2 \end{pmatrix}$	
	Solves $-3+2\lambda = 4\mu$ and $5-3\lambda = -2\mu \Rightarrow \lambda = \frac{7}{4}, \mu = \frac{1}{8}$	M1, A1
	Uses $\mu a = \lambda \Rightarrow \frac{1}{8}a = \frac{7}{4} \Rightarrow a = 14$	dM1, A1
		(4)
(c)	For Example: Attempts $\mu \begin{pmatrix} 4 \\ a \\ -2 \end{pmatrix}$ o.e. with their a and $\mu \Rightarrow Q = \left(\frac{1}{2}, \frac{7}{4}, -\frac{1}{4}\right)$	M1, A1
		(2) **(10 marks)**
6 (a) ALT way	$OP^2 = (-3+2\lambda)^2 + (\lambda)^2 + (5-3\lambda)^2 = 14\lambda^2 - 42\lambda + 34$	
	Find the value of λ at the minimum, e.g. $\frac{\mathrm{d}(OP^2)}{\mathrm{d}\lambda} = 28\lambda - 42 = 0 \Rightarrow \lambda = \frac{3}{2}$	M1, A1
	Substitute $\lambda = \frac{3}{2}$ into $\begin{pmatrix} -3+2\lambda \\ 0+1\lambda \\ 5-3\lambda \end{pmatrix} \Rightarrow P = \left(0, \frac{3}{2}, \frac{1}{2}\right)$	dM1, A1
		(4)

(a)

M1: Sets up an equation and solves to find λ.

For example, attempts $\begin{pmatrix}-3+2\lambda\\0+1\lambda\\5-3\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\1\\-3\end{pmatrix}=0\Rightarrow -6+4\lambda+\lambda-15+9\lambda=0\Rightarrow \lambda=...$

Alternatively attempts to find the distance OP^2 and finds the value of λ at the minimum. For example, via differentiation $\frac{\mathrm{d}\left(OP^2\right)}{\mathrm{d}\lambda}=28\lambda-42=0\Rightarrow \lambda=...$ or completing the square $OP^2=14\lambda^2-42\lambda+34=14(\lambda-1.5)^2-14\times1.5^2+34$ but other methods may exist.

Condone slips, e.g. signs, but the overall method must be sound

A1: Achieves $\lambda=\frac{3}{2}$ following M1. **Achieving this by setting the *x* coordinate = 0 scores 0 marks**

dM1: Uses their value of λ (found using a correct method) to find the coordinates or position vector of *P*. If the calculation is not written out it is implied by two correct coordinates for their λ

A1: Correct coordinates or position vector of *P*.

Withold this mark if the only answer given is written with incorrect notation, e.g $\left(0\mathbf{i}, \frac{3}{2}\mathbf{j}, \frac{1}{2}\mathbf{k}\right)$

(b)

M1: Set up and solve equations using the *x* and *z* components. Condone slips, e.g signs and coefficients as long as the intention is clear. Scored for $-3+2\lambda=4\mu$ and $5-3\lambda=-2\mu\Rightarrow\lambda=...,\ \mu=...$. The method of solving the simultaneous equations is not important, just look for two equations leading to values for both λ and μ

A1: $\lambda=\frac{7}{4}, \mu=\frac{1}{8}$ o.e

dM1: Dependent upon previous M. Scored for a **correct** method of using the *y* components to find the value for *a*.

Look for $'\lambda'='\mu'\times a\Rightarrow a=\frac{'\lambda'}{'\mu'}$

A1: $a=14$. Condone an equivalent value such as $\frac{56}{4}$

(c)

M1: Uses the equation of either line to find the coordinates or position vector of *Q*.
It is dependent upon the first M mark in part (b)

Look for an attempt at $\mu\begin{pmatrix}4\\a\\-2\end{pmatrix}$ with their *a* and μ OR $\begin{pmatrix}-3+2\lambda\\0+1\lambda\\5-3\lambda\end{pmatrix}$ with their λ

If the calculation is not explicitly seen it can be implied by two correct coordinates for their values

A1: $Q=\left(\frac{1}{2}, \frac{7}{4}, -\frac{1}{4}\right)$ or correct position vector.

Withold this mark if the only answer given is written with incorrect notation, e.g $\left(\frac{1}{2}\mathbf{i}, \frac{7}{4}\mathbf{j}, -\frac{1}{4}\mathbf{k}\right)$ but only penalise this the first time it occurs….so not in both (a) and (c).

Question Number	Scheme	Marks
7(a)	$x^2\tan\frac{\pi}{4}+\frac{32}{\pi^2}\times\left(\frac{\pi}{4}\right)^2=11\Rightarrow x^2=(9)$	M1
	$x=\pm 3$	A1
		(2)
(b)	Attempt at chain rule $y^2\to ...y\frac{dy}{dx}$	M1
	Attempt at product rule $x^2\tan y\to x^2\sec^2 y\frac{dy}{dx}+...x\tan y$	M1
	Correct differentiation $2x\tan y+x^2\sec^2 y\frac{dy}{dx}+\frac{64y}{\pi^2}\frac{dy}{dx}=0$	A1
	Substitutes $x=3, y=\frac{\pi}{4}\Rightarrow 18\frac{dy}{dx}+6+\frac{16}{\pi}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{3\pi}{8+9\pi}$	dM1, A1
		(5)
(c)	Calculates the gradient of the tangent at Q is "$\frac{3\pi}{8+9\pi}$" o.e.	M1
	$(\text{grad of tan to } C \text{ at } P)\times(\text{grad of norm to } C \text{ at } Q)=-\frac{3\pi}{8+9\pi}\times-\frac{8+9\pi}{3\pi}=1$	A1
		(2)
		(9 marks)

(a)

M1: Substitutes $y=\frac{\pi}{4}$ in equation of curve and proceeds to $x^2=...$ using $\tan\frac{\pi}{4}=1$

This may be implied by $x^2+...=...\Rightarrow x=...$

A1: $x=\pm 3$ following the award of M1

(b)

M1: Attempts at chain rule $y^2\to ...y\frac{dy}{dx}$

M1: Attempt at product rule $x^2\tan y\to x^2\sec^2 y\frac{dy}{dx}+...x\tan y$

A1: Correct differentiation $x^2\sec^2 y\frac{dy}{dx}+2x\tan y+\frac{64y}{\pi^2}\frac{dy}{dx}=0$

You may see different forms such as $\pi^2x^2\sec^2 y\frac{dy}{dx}+2\pi^2x\tan y+64y\frac{dy}{dx}=0$ or

$x^2\sec^2 y\,dy+2x\tan y\,dx+\frac{64y}{\pi^2}dy=0$

Do not allow $\frac{dy}{dx}=x^2\sec^2 y\frac{dy}{dx}+2x\tan y+\frac{64y}{\pi^2}\frac{dy}{dx}$ unless subsequent work is correct

M1: Substitutes $x = "3", y = \frac{\pi}{4}$ into a differentiated function and finds a value for $\frac{dy}{dx}$.

There must be exactly two $\frac{dy}{dx}$ terms, one from differentiating each of y^2 and $x^2 \tan y$

A1: $\frac{dy}{dx} = -\frac{3\pi}{8+9\pi}$ or exact equivalent such as $\frac{-6\pi}{16+18\pi}$ or $-\frac{6}{18+\frac{16}{\pi}}$

(c) Via calculation

M1: Substitutes $x = "-3", y = \frac{\pi}{4}$ into a differentiated function and finds a value for $\frac{dy}{dx}$ at Q.

It is a method that must be seen, via embedded values of $x = "-3", y = \frac{\pi}{4}$ or else implied by a correct value for their $x = "-3", y = \frac{\pi}{4}$ following the calculation of their $\frac{dy}{dx}$ at P.

You may see at attempt at $-\frac{dx}{dy}$ at Q using $x = "-3", y = \frac{\pi}{4}$

A1: Achieves the value 1 following correct and exact values for $\frac{dy}{dx}$ at P and $\frac{dy}{dx}$ or $-\frac{dx}{dy}$ at Q

………………………………………………………………………………………………………..

Alt (c) Via reasoning/deduction. To score via this method

- their $\frac{dy}{dx}$ must be of the correct form e.g. $\frac{dy}{dx} = \frac{Ax \tan y}{Bx^2 \sec^2 y + Cy}$
- and at P and Q, the y values are the same and the x values $\pm k$

M1: Subject to the above conditions, deduces that

'Gradient of tangent at P is m, gradient of tangent at Q is $-m$'

A1: Hence gradient of tangent at $P \times$ gradient of normal at Q is $m \times -\frac{1}{-m} = 1$ o.e , following M1

This can only be scored if $\frac{dy}{dx}$ is correct. If unsure send to review

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Question Number	Scheme	Marks
8 (a)	States/ uses $V=\lambda r^3 \Rightarrow \frac{dV}{dr}=\mu r^2$ OR $\frac{dV}{dt}=-C$	M1
	Attempts to use $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt} \Rightarrow -C=\mu r^2\times\frac{dr}{dt}$	dM1
	$\Rightarrow \frac{dr}{dt}=-\frac{C}{4\pi r^2}=-\frac{k}{r^2}$	A1*
		(3)
(b)	$\frac{dr}{dt}=-\frac{k}{r^2} \Rightarrow \int r^2 dr=\int -k\, dt$	
	$\frac{1}{3}r^3=-kt+c$	M1 A1
	Uses $t=0,\ r=30$ $\quad 9000=c$	M1
	Substitutes $r=12, t=24$ $\quad \frac{1}{3}\times 12^3=-k\times 24+9000 \Rightarrow k=(351)$	ddM1
	$r=\sqrt[3]{27000-1053t}$	A1
		(5)
(c)	$27000-1053T=0 \Rightarrow T=...$	M1
	awrt 25.6	A1
		(2)
		(10 marks)

(a)

M1: States or uses either $\frac{dV}{dr}=\mu r^2$ with any constant value for μ or any constant e.g. μ (allow any letter)

or $\frac{dV}{dt}=-C$ (Allow with k) but **must be a negative** and not given a value such as 1

If the formula for V is used (attempted) condone slips but it must obey $V=\lambda r^3 \Rightarrow \frac{dV}{dr}=\mu r^2$

dM1: Attempts to use $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ o.e. with $\frac{dV}{dt}=-C$ AND $\frac{dV}{dr}=\mu r^2$

$\Rightarrow -C=\mu r^2\times\frac{dr}{dt}$

See the first M1 for conditions on $\frac{dV}{dt}$ and $\frac{dV}{dr}$

A1*: Proceeds to $\frac{dr}{dt}=-\frac{k}{r^2}$ via a correct method. Look for $-C=4\pi r^2\times\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=-\frac{C}{4\pi r^2}=-\frac{k}{r^2}$

Condone a proof such as $-C=\mu r^2\times\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=-\frac{C}{\mu r^2}=-\frac{k}{r^2}$

If candidates had originally used k they must explain why $-\frac{k}{4\pi r^2}$ can be replaced by $-\frac{k}{r^2}$

If the formula for V is used it must be correct $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr}=4\pi r^2$

(b)

Case 1: If candidates use any value for k e.g 0.75 (apart from the correct one) they can only score the first two marks in (b). Candidates using $k = 351$ can score all marks.

Case 2: Some candidates have an extra 4π and start (b) with $\frac{dr}{dt} = -\frac{k}{4\pi r^2} \Rightarrow \int 4\pi r^2 dr = \int -k \, dt$

This should not affect the solution, so condone with any multiple of a constant for all marks

M1: Separates the variables (condoning lack of integral sign) and integrates to a form $...r^3 = ...t(+c)$

Condone the lack of the constant of integration.

A1: Achieves $\frac{1}{3}r^3 = -kt + c$ or equivalent including the $+ c$

Note that $r^3 = -3kt + c$ is also correct

M1: Substitutes $t = 0$ and $r = 30$ in their $r^3 = -3kt + c$ to find a non zero value for c.

This may be awarded following any attempt to integrate.

So $t = 0,\ r = 30$ in their $\Rightarrow f(r) = g(t) + c \Rightarrow c = ...$

ddM1: Substitutes $r = 12$ and $t = 24$ with their non zero value of c to find a non zero value for k.

It is dependent upon BOTH previous M marks. Condone transcription errors on the value of c

A1: $r = \sqrt[3]{27000 - 1053t}$

………

You may see k and c found in a different order.

For example, $\int_{30}^{12} r^2 dr = \int_0^{24} -k \, dt \Rightarrow \frac{1}{3}(12^3 - 30^3) = -k(24 - 0) \Rightarrow k = 351$

In cases like this the 2nd M mark would be awarded for the calculation of k

Hence equation would be $\frac{1}{3}r^3 = -351t + c \Rightarrow r^3 = -1053t + c$

………

(c)

M1: Sets $r = 0$ and finds a value for T.

The equation must be of the form $r = \sqrt[n]{A - Bt}$ o.e. with A, $B > 0$ and n a positive integer

A1: Awrt 25.6 .

Alt (c) Using the fact that the volume decreases at a constant rate

M1: Finds the rate of decrease in volume $\frac{36000\pi - 2304\pi}{24} = (1404\pi)$ and uses this to find the time $\frac{36000\pi}{1404\pi}$

A1: Awrt 25.6

Question Number	Scheme	Marks
9 (a)	$k=\dfrac{\pi}{2}$	B1
	Attempts $y^2\dfrac{dx}{dt}=(5\sin 2t)^2\,6\cos t=150\sin^2 2t\cos t$	M1
	$=600\sin^2 t\cos^3 t$	dM1
	$V=600\pi\displaystyle\int_0^{\frac{\pi}{2}}\sin^2 t\cos^3 t\,dt$	A1
		(4)
(b)	$\displaystyle\int \sin^2 t\cos^3 t\,dt=\int \sin^2 t(1-\sin^2 t)\cos t\,dt$	
	$=\displaystyle\int \sin^2 t\cos t-\sin^4 t\cos t\,dt$	M1
	$=\dfrac{1}{3}\sin^3 t-\dfrac{1}{5}\sin^5 t$	dM1
	$V=600\pi\left[\dfrac{1}{3}\sin^3 t-\dfrac{1}{5}\sin^5 t\right]_0^{\frac{\pi}{2}}=600\pi\left(\dfrac{1}{3}-\dfrac{1}{5}\right)=80\pi$	ddM1, A1
		(4)
		(8 marks)

(a)

B1: States or uses $k=\dfrac{\pi}{2}$. This may be scored from work in part (b) if not seen in (a)

M1: Attempts $y^2\dfrac{dx}{dt}=(5\sin 2t)^2\,...\cos t=...\sin^2 2t\cos t$

Condone missing brackets

dM1: Attempts to use $\sin 2t=2\sin t\cos t$ in an expression of the form $...\sin^2 2t\cos t$ and reaches $y^2\dfrac{dx}{dt}=...\sin^2 t\cos^3 t$. Condone $\sin^2 2t\to 2\sin^2 t\cos^2 t$ here

A1: $V=600\pi\displaystyle\int_0^{\frac{\pi}{2}}\sin^2 t\cos^3 t\,dt$ but allow this to be awarded with any k or in fact a constant k

The dt must be present on the final line

(b)

M1: Writes $\sin^2 t\cos^3 t$ in the form $\pm\sin^2 t\cos t \pm \sin^4 t\cos t$

dM1: And integrates to the form $\pm...\sin^3 t \pm ...\sin^5 t$

ddM1: And uses the limits 0 and $\frac{\pi}{2}$ in an expression of the form $\left[\pm\beta\sin^3 t \pm \delta\sin^5 t\right]$

A1: 80π

Alternative method via substitution. Let $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$

M1: Uses $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \pm\cos t \Rightarrow \int \sin^2 t\cos^3 t \,\mathrm{d}t = \int u^2\left(\pm 1 \pm u^2\right)\mathrm{d}u$

dM1: $\int u^2\left(\pm 1 \pm u^2\right)\mathrm{d}u = \int \pm u^2 \pm u^4 \,\mathrm{d}u = \pm\alpha u^3 \pm \beta u^5$

ddM1: And uses limits 0 and 1 in an expression of the form $\left[\pm\chi u^3 \pm \delta u^5\right]$

A1: 80π

Alternative method via $x = 6\sin t$ using $\frac{\mathrm{d}x}{\mathrm{d}t} = 6\cos t$ can be marked as above

$$V = 600\pi\int_0^{\frac{\pi}{2}} \sin^2 t\cos^3 t \,\mathrm{d}t = 100\pi\int_0^6 \frac{x^2}{36}\left(1 - \frac{x^2}{36}\right)\mathrm{d}x = \frac{100\pi}{36\times 36}\int_0^6 \left(36x^2 - x^4\right)\mathrm{d}x = \frac{100\pi}{36\times 36}\left[12x^3 - \frac{1}{5}x^5\right]_0^6 = 80\pi$$

M1: $\alpha\int x^2\left(\pm 1 \pm \frac{x^2}{\beta}\right)\mathrm{d}x$

dM1: Integrates to a form $\left[\pm\chi x^3 \pm \delta x^5\right]$

ddM1: Uses limits 0 and 6 in an expression of the form $\left[\pm\chi x^3 \pm \delta x^5\right]$

A1: 80π