Question Number	Scheme	Marks
1	$(2+5x)^{-2}$	
(a)	$(2+5x)^{-2}=\frac{1}{4}\left(1+\frac{5}{2}x\right)^{-2}$	B1
	$\left(1+\frac{5}{2}x\right)^{-2}=1+(-2)\times\left(\frac{5}{2}x\right)+\frac{(-2)\times(-3)}{2}\times\left(\frac{5}{2}x\right)^2+\frac{(-2)\times(-3)\times(-4)}{3!}\times\left(\frac{5}{2}x\right)^3$	M1A1
	$(2+5x)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2-\frac{125}{8}x^3$	A1A1
		(5)
(b)	$\lvert x\rvert<\frac{2}{5}$	B1
		(1)
(c)	$\left(\frac{4}{2+5x}\right)^2\approx 4-20x+75x^2$ or $a=4, b=-20$ and $c=75$	B1ft
		(1)
		(7 marks)

(a) Note that B1M1A0A1A0 is a possible mark trait

B1: For taking out a common factor of $\frac{1}{4}$ (or 0.25 or accept 2^{-2}) so score for $\frac{1}{4}(1+kx)^{-2}$ or $0.25(1+kx)^{-2}$. It cannot be for just $\frac{1}{4}$ on its own

M1: Attempts the binomial expansion with index -2

Score for the correct binomial coefficient for the 3rd or 4th term with the correct power of *x*

Look for $\frac{(-2)\times(-3)}{2}\times(...x)^2$ oe or $\frac{(-2)\times(-3)\times(-4)}{3!}\times(...x)^3$ oe If no working is shown then the coefficient must be consistent with their *k*. Condone invisible or poor bracketing for this mark.

Do not allow notation such as $\binom{-2}{1},\binom{-2}{2}$ unless interpreted correctly in further work.

A1: Correct binomial expansion for $\left(1+\frac{5}{2}x\right)^{-2}$ up to the 4th term. This may be left unsimplified. May be implied by four correct terms. Isw if they subsequently simplify incorrectly. Invisible brackets may be implied by further work.

A1: Three correct and **simplified terms** from $\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2-\frac{125}{8}x^3$ May be written as decimal equivalents e.g. $0.25-1.25x+4.6875x^2-15.625x^3$. May be seen as a list or written on different lines. Condone $-\frac{5}{4}x^1$ for this mark.

A1: $\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2-\frac{125}{8}x^3$ oe See notes above for decimal equivalent forms. May be seen as a list or written on different lines. Isw once the four correct terms are seen unless they attempt to multiply by e.g. 16 Do not accept $-\frac{5}{4}x^1$

Alt(a) Direct expansion

$$(2\pm5x)^{-2}=2^{-2}+(-2)\left(2^{-3}\right)\left(\pm5x\right)+\frac{(-2)(-3)}{2}\left(2^{-4}\right)\left(\pm5x\right)^2+\frac{(-2)(-3)(-4)}{6}\left(2^{-5}\right)\left(\pm5x\right)^3+...$$

B1: For 2^{-2} oe seen as the constant term e.g. $=\frac{1}{4}+...x+...$

M1: For the correct 3rd or 4th term of the expansion. Condone missing brackets on $(\pm5x)^2$ or $(\pm5x)^3$

A1: Correct expansion up to the 4th term which may be left unsimplified. May be implied by four correct terms. Isw if they subsequently simplify incorrectly. Invisible brackets may be implied by further work.

A1: As above in main scheme notes

A1: As above in main scheme notes

(b)

B1: $|x| < \frac{2}{5}$ oe e.g. $-\frac{2}{5} < x < \frac{2}{5}$ or e.g. $\left(-\frac{2}{5}, \frac{2}{5}\right)$ isw once a correct inequality is seen.

Condone $|x| < \left|-\frac{2}{5}\right|$

(c)

B1ft: $\left(\frac{4}{2+5x}\right)^2 \approx 4 - 20x + 75x^2$ but follow through on $16\times"\left(\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2\right)"$. Must be a three- term quadratic but ignore any terms in x^3 or higher.

May just state e.g. $a = "4", b = "-20", c = "75"$

They must all be values (not unevaluated expressions)

Question Number	Scheme	Marks
2	Volume = $\int_{(2)}^{(8)} \frac{25\pi(x-1)}{9}(\mathrm{d}x)$ oe e.g. $\frac{25\pi}{9}\int_{(2)}^{(8)} x-1(\mathrm{d}x)$	B1
	$(\pi)\int \frac{25(x-1)}{9}\mathrm{d}x = (\pi)\frac{25(x-1)^2}{18}$ or $(\pi)\left(\frac{25x^2}{18}-\frac{25x}{9}\right)$	M1A1
	Volume = $\left[\frac{25\pi(x-1)^2}{18}\right]_2^8 = \left(\frac{25\pi(8-1)^2}{18}-\frac{25\pi(2-1)^2}{18}\right) = \frac{200}{3}\pi$	M1A1
		(5) **(5 marks)**

B1: Correct expression for the volume ignoring limits

May see e.g. $\frac{25\pi}{9}\int x-1(\mathrm{d}x)$ or $\int \frac{25\pi x}{9}-\frac{25\pi}{9}(\mathrm{d}x)$ May be unsimplified so allow $\pi\int\left(\frac{5}{3}\sqrt{x-1}\right)^2(\mathrm{d}x)$ Condone omission of dx. It cannot be scored if they make errors before the integral expression is written. π may appear later in a correct integral or integrand. Alternatively may use substitution e.g. $u^2 = x-1$ and proceeding to $\pi\int\left(\frac{50}{9}u^3\right)(\mathrm{d}u)$ oe

M1: Correct attempt at integrating $(x-1)$. Look for $A(x-1)^2$ or $Bx^2 - Cx$ where *A*, *B* and *C* are non zero constants (indices do not need to be processed for this mark). Do not award this mark if this is only part of a sum of integrals. If using the substitution $u^2 = x-1$ then look for $u^3 \to u^4$

A1: $(\pi)\frac{25(x-1)^2}{18}$ or $(\pi)\left(\frac{25x^2}{18}-\frac{25x}{9}\right)$ oe (or using the substitution $u^2 = x-1 \Rightarrow$ $(\pi)\frac{25}{18}u^4$ oe)

Ignore any reference to π which could even appear as 2π

M1:Uses the limits of 2 and 8 within their integrated expression of the form $...(x-1)^2$ or $...x^2 - ...x$ and subtracts either way round. Allow $\left[\frac{25\pi(x-1)^2}{18}\right]_2^8 = (\pm)\frac{200}{3}\pi$ to score this mark but if their integration is incorrect then we must see evidence of using limits not just a final answer. May be partially evaluated which you may need to check. Ignore the absence of π

If using the substitution $u^2 = x-1$ then they need to use the limits $\sqrt{7}$ and 1 on an expression of the form $... u^4$

A1: $\frac{200}{3}\pi$ or exact equivalent e.g. $66\frac{2}{3}\pi$. If they subtract the wrong way round to achieve $-\frac{200}{3}\pi$ they cannot ignore the negative sign but must subsequently state e.g. volume = $\frac{200}{3}\pi$. Ignore units.

Question Number	Scheme	Marks
3 (a)	$x=\dfrac{t+15}{t+4}\Rightarrow t=\dfrac{15-4x}{x-1}$	M1
	$y=\dfrac{5}{t+2}\Rightarrow y=\dfrac{5}{\dfrac{15-4x}{x-1}+2}$	M1
	$y=\dfrac{5(x-1)}{15-4x+2(x-1)}$	dM1
	$g(x)=\dfrac{5x-5}{13-2x}\qquad 1<x\leqslant\dfrac{15}{4}$	A1B1
		(5)
(b)	$0<g(x)\leqslant\dfrac{5}{2}$	M1A1
		(2)
		(7 marks)

(a)

M1: Attempts to get *t* in terms of *x* or *t* in terms of *y*. FYI $y=\dfrac{5}{t+2}\Rightarrow t=\dfrac{5}{y}-2$

Look for an equation of the form $t=\dfrac{A+Bx}{C+Dx}$ oe or $t=\dfrac{E}{y}+F$ oe where *A*,*B*, *C*, *D*, *E*, $F\neq 0$

If they get *t* in terms of *x* in one and *t* in terms of *y* in the other then just look for one of the equations to be of the required forms.

M1: Substitutes their *t* in terms of *x* (or *y*) in *y* (or *x*) however poorly manipulated and proceeds to an equation linking *x* and *y*. Condone slips substituting in provided the intention is clear.

FYI: $y=\dfrac{5}{t+2}\quad x=\dfrac{\dfrac{5}{y}-2+15}{\dfrac{5}{y}-2+4}=\dfrac{\dfrac{5}{y}+13}{\dfrac{5}{y}+2}$

Rearranging both equations to get *t* in terms of *x* in one and *t* in terms of *y* in the other and then equating scores this mark. Condone $y=...$ or $x=...$ to be implied as long as it is clearly their *y* or *x*

dM1: Proceeds to an equation of the form $y=\dfrac{ax+b}{cx+d}$ **which may be unsimplified.**

e.g. $y=\dfrac{5(x-1)}{15-4x+2(x-1)}$. It is dependent on the previous two method marks. There should be no fractions in the numerator or denominator but brackets do not need to be multiplied out. Condone slips provided they achieved the required form. Condone $y = \ldots$ to be implied as long as it is clearly their y

A1: $\mathrm{g}(x) = \frac{5x-5}{13-2x}$ o.e. Must be a single fraction in the correct form e.g. $\mathrm{g}(x) = \frac{5-5x}{2x-13}$ Condone $y = \ldots$ Note the fraction may not be in its lowest terms e.g. $\mathrm{g}(x) = \frac{10x-10}{26-4x}$

The $y = \ldots$ or $\mathrm{g}(x) = \ldots$ can be seen on an earlier line

B1: States the values separately $e = 1,\ f = 3.75$. Accept $1 < x \leqslant \frac{15}{4}$ oe (condone $1 < x < \frac{15}{4}$ as this was given in the question

(b)

M1: One correct end. e.g. $\mathrm{g}(x) \leqslant \frac{5}{2}$ oe. Do not be concerned over the inequality being strict or inclusive.

A1: $0 < \mathrm{g}(x) \leqslant \frac{5}{2}$ o.e. Condone use of *y* or g but not *x* e.g. $0 < y \leqslant \frac{5}{2}$ and alternative equivalent notation e.g. $\left(0, \frac{5}{2}\right]$ $0 < \mathrm{g}(x), \frac{5}{2} \geqslant \mathrm{g}(x)$ $0 < \mathrm{g}(x) \cap \frac{5}{2} \geqslant \mathrm{g}(x)$ or $0 < \mathrm{g}(x)\ \mathrm{AND}\ \frac{5}{2} \geqslant \mathrm{g}(x)$ but do not accept "OR" or use of $\cup$

Special case: $0 < x \leqslant \frac{5}{2}$ oe score M1A0

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \to ...y\frac{dy}{dx}$	M1
	Attempt at product rule $2xy \to \alpha x\frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y-2x)\frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} = \right) \frac{12-4x+y}{y-x}$ oe	A1
		(5)
(b)	Sets $\frac{12-4x+y}{y-x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	A1
		(5) **(10 marks)**

(a) **Allow use of y' for $\frac{dy}{dx}$**

M1: Attempt at chain rule $y^2 \to ...y\frac{dy}{dx}$ where ... is a constant

M1: Attempt at product rule $2xy \to \alpha x\frac{dy}{dx} + \beta y$ where α, β ... are non-zero constants.

Condone $-2xy \to \pm\alpha x\frac{dy}{dx} \pm \beta y$

A1: Correct differentiation $8x + 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 24$ oe

M1: Collects their two different terms in $\frac{dy}{dx}$ which must have come from differentiating y^2 and $2xy$ (however poorly the differentiation was carried out) on one side of the equation with the remaining terms on the other side proceeding to $(...\pm...)\frac{dy}{dx} = ...$. May be implied by a correct answer.

Ignore a spurious $\frac{dy}{dx}$ e.g. on the left hand side as long as it is not included in the collecting of terms.

A1: $\left(\frac{dy}{dx} =\right) \frac{12-4x+y}{y-x}$ oe including $\left(\frac{dy}{dx} =\right) \frac{4x-y-12}{x-y}$ **Must be in simplest form and seen in (a)**

(b) May work in terms of *a* and *b* which is acceptable

M1: Sets their $\frac{dy}{dx} = 2$ and proceeds to a linear equation in *x* and *y*. May be unsimplified. Alternatively, substitutes in $\frac{dy}{dx} = 2$ into the given equation $8x+2y\times2-2y-2x\times2=24 \ (\Rightarrow 4x+2y=24)$

dM1: Substitutes their linear equation in *x* and *y* in the equation for C **and proceeds to a quadratic equation in *x* or *y***. Do not be concerned by the mechanics of their rearrangement. Terms should be collected but all terms do not need to be on the same side of the equation. (if all on one side then = 0 must be seen or may be implied by further work). It is dependent on the previous method mark.

A1: Correct quadratic in *x* or *y*. $x^2-8x+12=0$ oe or $y^2-8y=0$ oe terms should be collected but all terms do not need to be on the same side of the equation. (if all on one side then = 0 may be implied by further work)

ddM1: Solves their quadratic in *x* or *y* and proceeds to a pair of coordinates for *P*. Allow any valid method (usual rules apply). If via a calculator to find *x* then they must have rearranged to a three term quadratic with all terms on one side. You may need to check their roots are correct for their quadratic. It is dependent on both of the previous method marks.

Note when finding a value for *b* they may substitute into C resulting in false solutions.
e.g. ("6", "0"), ("6", "6") or ("2", "8"), ("2", "−4") which is acceptable for this mark.

A1: $P(2,8)$ only but allow other forms such as $a=2, b=8$. Condone if labelled as *x* and *y*.

If they find the coordinates $(6,0)$ then they must clearly identify $(2,8)$ as their chosen answer.

Question Number	Scheme	Marks
5. (i)	Attempts to solve $2+\lambda=-1+2\mu$ and $3+2\lambda=6+7\mu$	M1
	Correct solution $\lambda=-9,\ \mu=-3$	A1
	Substitutes their $\lambda=-9,\ \mu=-3$ into $-1+4\lambda=\ \beta-\mu$	dM1
	$\beta=-40$	A1
(ii)	E.g. Substitutes their $\lambda=-9$ into $\mathbf{r}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\2\end{pmatrix}$	dM1
	$(-7,-37,-15)$	A1
		(6)
		(6 marks)

(i)

M1: Attempts to write down and solve equations (1) and (3) to find the value of either λ or μ
Do not be concerned by the mechanics of the rearrangement in solving.
May be implied by $\lambda=-9$ or $\mu=-3$.

A1: Correct solution **for both** $\lambda=-9,\ \mu=-3$

dM1: Substitutes their values for λ and μ into equation (2) and solves to find a value for β
Do not be concerned by the mechanics of the rearrangement in solving. It is dependent on the previous method mark.

A1: $\beta=-40$

(ii)

dM1: Scored for either substituting their $\lambda=-9$ into $\mathbf{r}=\begin{pmatrix}2\\-1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\2\end{pmatrix}$ or

substituting their $\mu=-3$ and $\beta=-40$ into $\mathbf{r}=\begin{pmatrix}-1\\\beta\\6\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\7\end{pmatrix}$ **and**

proceeding to find coordinates for *P*. Do not allow this mark to be scored if they then do some further manipulation.

It is dependent upon the attempt to find either λ or both of μ and β. May be implied by 2 correct coordinates for their λ or both of μ and β.

A1: $(-7, -37, -15)$ or position vector e.g. $-7\mathbf{i}-37\mathbf{j}-15\mathbf{k}$ or $\begin{pmatrix} -7 \\ -37 \\ -15 \end{pmatrix}$ condone

$(-7\mathbf{i}, -37\mathbf{j}, -15\mathbf{k})$

Allow $x=-7,\ y=-37,\ z=-15$ or condone $-7, -37, -15$. Do not isw if they calculate two different sets of coordinates using μ and β for one and then λ for the other unless they select the correct answer.

Question Number	Scheme	Marks
6	$u=3+\cos\theta \Rightarrow \dfrac{du}{d\theta}=-\sin\theta$	B1
	$\displaystyle\int \frac{\sin 2\theta}{\sqrt{3+\cos\theta}}\,d\theta = \int -\frac{2(u-3)}{\sqrt{u}}\,du$	M1 A1
	$= -\dfrac{4}{3}u^{\frac{3}{2}} + 12u^{\frac{1}{2}}$	dM1A1
	Uses limits of 3 and 4 within their attempted integration	ddM1
	$= 8\sqrt{3} - \dfrac{40}{3}$	A1
		(7 marks)

Condone the omission of d*u* to be implied

B1: States or uses $\dfrac{du}{d\theta}=-\sin\theta$ o.e. This may be seen within the integrand. Note that B0 will mean that no A marks will be scored.

M1: Simplifies $\dfrac{\sin 2\theta}{\sqrt{3+\cos\theta}}\,d\theta$ to a function in terms of *u* using

- the double angle formula for $\sin 2\theta$ Allow/condone $\sin 2\theta = \kappa \sin\theta\cos\theta$
- the given substitution condoning $\cos\theta = \pm 3 \pm u$
- $\dfrac{du}{d\theta} = \pm\sin\theta$
- $\sqrt{u}$ in the denominator

Expect to see a form $\displaystyle\int k\frac{(\pm u \pm 3)}{\sqrt{u}}(du)$ oe condone missing d*u* provided the integral is of the required form. Condone $d\theta$ for d*u* as a slip for this mark. Condone $k=\pm 1$

A1: $\displaystyle\int -\frac{2(u-3)}{\sqrt{u}}(du)$ from correct work. Ignore limits for this mark but allow $\displaystyle\int_3^4 \frac{2(u-3)}{\sqrt{u}}(du)$ if the limits are written on the integral (3 and 4) and are the opposite way round. Condone missing d*u* but do not allow $d\theta$

Do not withhold this mark once a correct expression is seen from correct work.

dM1: Integrates to a form $\alpha u^{\frac{3}{2}} + \beta u^{\frac{1}{2}}$. It is dependent on the previous method mark.

A1: Correct integration, $-\frac{4}{3}u^{\frac{3}{2}} + 12u^{\frac{1}{2}}$ oe Ignore limits but allow e.g. $\frac{4}{3}u^{\frac{3}{2}} - 12u^{\frac{1}{2}}$ if the limits (3 and 4) are the opposite way round– send to review if unsure. May have a constant of integration. **All previous marks should have been scored condoning only $\mathrm{d}\theta$ instead of d*u* as a slip on their integral expression.**

ddM1: Uses limits of 3 and 4 (or alternative 0 and $\frac{\pi}{2}$ if substituted $u = 3 + \cos\theta$) and subtracts either way round within their attempted integration. The expression is sufficient. We must see some evidence of the use of limits for this mark. May be partially evaluated but cannot just be the final answer. It is dependent on both of the previous method marks.

A1: $8\sqrt{3} - \frac{40}{3}$ **provided all previous marks scored**. Do not accept $\frac{24\sqrt{3} - 40}{3}$ as this is not in the required form.

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA}\square\overrightarrow{BC}$ $\begin{pmatrix}-6\\2\\-3\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\-5\end{pmatrix}=24+6+15=45$	M1
	Attempts to apply $\mathbf{a}\bullet\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta\Rightarrow 45=7\times\sqrt{50}\cos\theta$	dM1
	$\cos\theta=\dfrac{9\sqrt{2}}{14}$ oe	A1
		(3)
(b)	Attempts $\sin\theta=\sqrt{1-\left(\dfrac{9\sqrt{2}}{14}\right)^2}$	M1
	Attempts to apply $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta=7\times\sqrt{50}\times\dfrac{\sqrt{34}}{14}$	M1
	Area *ABCD* = $5\sqrt{17}$	A1
		(3) **(6 marks)**

(a)

M1: Attempts $\overrightarrow{BA}\bullet\overrightarrow{BC}$. Look for $\pm\begin{pmatrix}6\\-2\\3\end{pmatrix}\bullet\pm\begin{pmatrix}4\\-3\\5\end{pmatrix}=\pm24\pm6\pm15$ The expression is sufficient but may be implied by ±45. Condone slips but at least two products should be correct.

dM1: Uses $\mathbf{a}\bullet\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ with either of $\lvert\mathbf{a}\rvert$ or $\lvert\mathbf{b}\rvert$ found using a correct method. Either length may be implied by "$\sqrt{49}$" oe or "$\sqrt{50}$" oe. It is dependent on the previous method mark.

A1: $\cos\theta=\dfrac{9\sqrt{2}}{14}$ o.e such as $\cos\theta=\dfrac{9}{7\sqrt{2}}$ **Must be exact**

Note that $\cos\theta=-\dfrac{9\sqrt{2}}{14}$ or $\cos\theta=\dfrac{9\sqrt{2}}{14}$ from $\mathbf{a}\bullet\mathbf{b}=-45$ is M1dM1A0

Alt (a) using the cosine rule

M1: Attempts to find $\lvert\overrightarrow{AC}\rvert$, $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{BC}\rvert$ (or $\lvert\overrightarrow{AC}\rvert^2$, $\lvert\overrightarrow{AB}\rvert^2$ and $\lvert\overrightarrow{BC}\rvert^2$) using a correct method with at least two lengths correct. May be seen within the cosine rule formula.

dM1: Uses the cosine rule to form a correct equation using their lengths in the correct positions. It is dependent on the previous method mark.

e.g. $\cos\theta = \dfrac{7^2 + (5\sqrt{2})^2 - 3^2}{2(7)(5\sqrt{2})}$

A1: $\cos\theta = \dfrac{9\sqrt{2}}{14}$ o.e such as $\cos\theta = \dfrac{9}{7\sqrt{2}}$

(b) Note that finding e.g. cos *BAC* $\left(=\dfrac{4}{21}\right)$ in (a) can still lead to full marks in (b)

M1: Attempts to find the **exact value** of $\sin\theta$ examples include:

- using $\sin\theta = \sqrt{1 - \left(\text{"}\dfrac{9\sqrt{2}}{14}\text{"}\right)^2}$ oe May be implied by their $\dfrac{\sqrt{34}}{14}$ or exact equivalent or score for a correct expression for $\sin\theta$ with their $\cos\theta = \dfrac{9\sqrt{2}}{14}$ substituted. Condone invisible brackets.
- drawing a right-angled triangle to find the opposite side and then using $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

If an exact expression is not seen somewhere in their solution then M0M0A0 but allow a candidate to write exact values but then use decimals in intermediate working provide exact values are recovered before the final answer.

Do not accept $\sin\theta = \sin\left(\cos^{-1}\left(\text{"}\dfrac{9\sqrt{2}}{14}\text{"}\right)\right)$ for this mark.

M1: Attempts to apply $|\mathbf{a}||\mathbf{b}|\sin\theta$. The expression is sufficient. Look for e.g.

$\text{"}7\text{"}\times\text{"}\sqrt{50}\text{"}\times\text{"}\dfrac{\sqrt{34}}{14}\text{"}$

Condone $\text{"}7\text{"}\times\text{"}\sqrt{50}\text{"}\times\sin\left(\cos^{-1}\left(\text{"}\dfrac{9\sqrt{2}}{14}\text{"}\right)\right)$ for this mark or

$\text{"}7\text{"}\times\text{"}\sqrt{50}\text{"}\times\sin(\text{awrt } 24.6)$

Note there are other methods to find the area. Send to review if you see something that you think is credit worthy.

A1: $5\sqrt{17}$ following an exact value for $\sin\theta$ **seen**, both previous method marks scored and $\cos\theta = \pm\dfrac{9\sqrt{2}}{14}$ o.e. (units not required)

Note: There are other valid methods which if seen then send to review:

Heron's formula

$$\left(s = \frac{7+5\sqrt{2}+3}{2} = 5 + \frac{5\sqrt{2}}{2}\right) \Rightarrow A = 2\sqrt{s(s-7)(s-5\sqrt{2})(s-3)} = 5\sqrt{17}$$

Cross product e.g.

$$\left\|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 2 & -3 \\ -4 & 3 & -5 \end{vmatrix}\right\| = \left|-\mathbf{i} - 18\mathbf{j} - 10\mathbf{k}\right| = \sqrt{(-1)^2 + (-18)^2 + (-10)^2} = 5\sqrt{17}$$

Question Number	Scheme	Marks
8 (a)	$\int te^{-t}\,\mathrm{d}t = -te^{-t} + \int e^{-t}\,\mathrm{d}t, = -te^{-t} - e^{-t}$	M1dM1
	$\frac{\mathrm{d}V}{\mathrm{d}t} = 12te^{-t} \Rightarrow V = -12te^{-t} - 12e^{-t} + c$	A1
	$t = 0, V = 6 \Rightarrow 6 = 0 - 12 + c \Rightarrow c = \ldots$	M1
	$V = 18 - 12te^{-t} - 12e^{-t}$	A1
		(5)
(b)	No, because e.g. upper limit of volume is 18 (litres)	B1 ft
		(1)

(a) **Note that differentiating using the product rule scores 0 marks**

M1: Uses integration by parts to achieve $\int te^{-t}\,\mathrm{d}t = \alpha te^{-t} \pm \beta \int e^{-t}\,(\mathrm{d}t)$ where $\alpha, \beta \neq 0$ but may be equal to each other (you do not need to be concerned about how they arrive at this) May be implied by further work.

dM1: Uses a correct method to integrate an expression of the form $\int te^{-t}\,(\mathrm{d}t) = \alpha te^{-t} \pm \gamma e^{-t}$ where $\beta, \gamma \neq 0$. It is dependent on the previous method mark.

A1: Solves the differential equation with or without a constant of integration e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 12te^{-t} \Rightarrow V = -12te^{-t} - 12e^{-t}\,(+c)$ o.e. May be written in an equivalent form e.g. $\ldots V = \ldots\left(te^{-t} + e^{-t}\right)(+c)$

Watch out for the DI method

	D	I
$+$	$12t$	e^{-t}
$-$	12	$-e^{-t}$
$+$	0	e^{-t}

Giving correct integration e.g. $\int t\mathrm{e}^{-t}\ \mathrm{d}t = -12t\mathrm{e}^{-t} - 12\mathrm{e}^{-t}$

M1: Ether states $t = 0, V = 6$ and states a value for c or uses the initial conditions for their integral to find the value of c. You do not need to be concerned by the mechanics of the rearrangement. Whilst not dependent on the earlier method marks, there must have been some attempt to solve the differential equation however poor (but not clearly differentiation)
May alternatively substitute in limits *V* and 6 and *t* and 0 onto each side of their integrated expression so the constant of integration is within the calculation.

A1: $V = 18 - 12t\mathrm{e}^{-t} - 12\mathrm{e}^{-t}$ oe Condone if seen in (b)

(b)

B1ft: No, because upper limit of volume is 18 (litres) oe Ft on their
$V = "18" - At\mathrm{e}^{-t} - B\mathrm{e}^{-t}$
where *A*, *B* > 0

May set $V = 20$ and show that the equation cannot be solved.
e.g. $-2 = 12\mathrm{e}^{-t}(t+1)$ and explains that $\mathrm{e}^{-t} > 0$ and $t + 1 > 0$ so cannot be negative
If their $"18" > 20$ then they need to conclude yes with a reason.

Question Number	Scheme	Marks
9 (a)	$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{-1}{(t+1)^2}$, $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2}{2t+5}$ $\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{2t+5}{2(t+1)^2}$	M1A1
	Sets $-\frac{2t+5}{2(t+1)^2}=-4\Rightarrow 8t^2+14t+3=0$	M1A1
	$\Rightarrow(4t+1)(2t+3)=0\Rightarrow t=-\frac{1}{4}\Rightarrow(y=)\ \frac{4}{3}$	dM1A1
		(6)
(b) (i)	$a=2, b=5$	B1
	Area = $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int_2^5\frac{1}{t+1}\times\frac{2}{2t+5}\,\mathrm{d}t=\int_2^5\frac{2}{(t+1)(2t+5)}\,\mathrm{d}t$	M1A1
(ii)	$\frac{2}{(t+1)(2t+5)}\equiv\frac{A}{(t+1)}+\frac{B}{(2t+5)}\Rightarrow A=...,B=...$	M1
	$=\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}$	A1
	$\int\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}\,\mathrm{d}t=\frac{2}{3}\ln\lvert t+1\rvert-\frac{2}{3}\ln\lvert 2t+5\rvert$	M1A1ft
	$\int_2^5\frac{2}{(t+1)(2t+5)}\,\mathrm{d}t=\left(\frac{2}{3}\ln 6-\frac{2}{3}\ln 15\right)-\left(\frac{2}{3}\ln 3-\frac{2}{3}\ln 9\right)$ $=\frac{2}{3}\ln\left(\frac{6}{5}\right)=\frac{1}{3}\ln\left(\frac{36}{25}\right)$	dM1A1
		(9) **(15 marks)**

(a)

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ by dividing their $\frac{\mathrm{d}y}{\mathrm{d}t}$ by their $\frac{\mathrm{d}x}{\mathrm{d}t}$. Both must be attempts at differentiation, however poor. Do not allow e.g. $\frac{2}{2t+5}\to...\ln(2t+5)$ as this is clearly integration.

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{2t+5}{2(t+1)^2}$ which may be unsimplified and may be implied by further work

M1: Sets their "$-\frac{2t+5}{2(t+1)^2}$" $=\pm4$ and proceeds to a 3TQ in t. Do not be concerned by the mechanics of the rearrangement.

A1: $8t^2+14t+3=0$ o.e. e.g. $t^2+\frac{7}{4}t+\frac{3}{8}=0$ The = 0 may be implied by further work if the three terms are on the same side of the equation.

dM1: Dependent upon both previous M's, score for solving their 3TQ AND finding a value for *y*. Allow any valid attempt to solving the quadratic including via a calculator. You may need to check this if just a value for *t* is stated. Condone a *y* value being found from a value of *t* such that $t<-1$.

A1: $(y=)\ \frac{4}{3}$ oe only (ignore the *x* coordinate if found)

See alt below – send to review if unsure

9 (a)	$y=\frac{2}{e^x-3} \quad \Rightarrow \frac{dy}{dx}=\frac{-2e^x}{(e^x-3)^2}$	M1A1
	Sets $\frac{-2e^x}{(e^x-3)^2}=-4 \Rightarrow 2(e^x)^2-13e^x+18=0=0$	M1A1
	$\Rightarrow \left(2e^x-9\right)\left(e^x-2\right)=0 \Rightarrow e^x=\frac{9}{2} \Rightarrow t=-\frac{1}{4} \Rightarrow y=\frac{4}{3}$	dM1A1
		(6)

(b)(i)

B1: Correct values for *a* and *b*. Must be correctly paired or indicated on the integral. If there is a contradiction between the two then the values on the integral sign take precedence.

M1: For attempting $y\frac{dx}{dt}$ with their $\frac{dx}{dt}$ in the form $\frac{k}{2t+5}$ where *k* is a constant. Must see either the general expression $y\frac{dx}{dt}$ stated or written as e.g. $\frac{1}{t+1}\times\frac{k}{2t+5}$ oe. There needs to be an intermediate stage of working before writing $\frac{a}{(t+1)(2t+5)}$.

It cannot be implied by $\int_a^b \frac{2}{(t+1)(2t+5)}\,\mathrm{d}t$

A1: Area $R = \int_a^b \frac{2}{(t+1)(2t+5)}\,\mathrm{d}t$ with or without correct values for *a* and *b*. **The dt only needs to be present in the final answer.** Allow if seen at the start of (b)(ii). Allow slips in working to be recovered provided it is before the final answer. This mark cannot be scored without the previous M mark being scored i.e. we need to see an intermediate stage of working.

Note allow $\int_a^b \frac{-2}{(t+1)(2t+5)}\,\mathrm{d}t$ only if their limits are clearly stated as being the other way round.

(b)(ii)

M1: Attempts to write $\frac{k}{(t+1)(2t+5)}$ as $\frac{A}{(t+1)}+\frac{B}{(2t+5)}$. May be implied by a correct value for *A* or *B* (but not e.g. $A=1,\ B=1$ without a valid method seen) May have taken a factor outside of the integral.

A1: $\frac{2}{(t+1)(2t+5)} \equiv \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}$. May have taken a factor outside of the integral so allow e.g. $\frac{1/3}{(t+1)} - \frac{2/3}{(2t+5)}$ oe to score. Must see the partial fractions written or implied by their integration.

M1: Correct attempt at integration. Look for $\pm\alpha\ln|t+1|\pm\beta\ln|2t+5|$ oe where $\alpha,\ \beta \neq 0$ They must have separated into partial fractions first. Condone brackets instead of moduli and condone the omission of brackets to be implied by further work. Allow this mark to be scored if they miscopy their partial fractions (e.g. a sign slip or a coefficient slip) so they will still integrate to a similar form.

A1ft: $\int \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}\,\mathrm{d}t = \frac{2}{3}\ln|t+1| - \frac{2}{3}\ln|2t+5|$ oe so allow $\frac{1}{3}\ln|t+1| - \frac{1}{3}\ln|2t+5|$ oe but ft on their *A* and *B*. May have taken a factor of 2 outside of the integral. Condone brackets instead of moduli and invisible brackets to be implied by further work. Ignore any constant of integration if present.

dM1: Attempts to use their limits and subtract either way round. They must use at least one law to combine at least two terms. They cannot proceed directly to the final answer. It is dependent on the previous method mark.

A1: $\frac{1}{3}\ln\left(\frac{36}{25}\right)$ o.e. in the required form $\frac{1}{3}\ln a$ where *a* is a rational number. Do not accept $\frac{1}{3}\ln\left(\frac{6}{5}\right)^2$

Question Number	Scheme	Marks
10 (a)	Requires a minimum of • $x^2 = (2m)^2 = 4m^2$ • which is even so contradiction, so proven	B1 dB1 **(2)**
(b)	e.g. $a^2 - 4b = 27 \Rightarrow a^2 = 27 + 4b$ which is odd e.g. If a^2 is odd then *a* is odd and can be written in the form $a = 2m+1$ e.g. $(2m+1)^2 = 27 + 4b \Rightarrow b = ...$ Requires a full explanation and conclusion e.g. • $b = m^2 + m - \frac{13}{2}$ which is not an integer • so contradiction, hence proven *	M1 A1 dM1 A1* **(4)** **(6 marks)**

(a)

B1: States that $x^2 = 4m^2$

dB1: Dependent on the previous B mark. Explains or concludes that $4m^2$ is **even** (or condone not odd) and either states contradiction/so proven or rewrites statement X (condone omission of some of the words provided the general conclusion is clear). Do not allow if the contradiction is not based on the assumption that x^2 is odd and $4m^2$ is even.

(b) **Note that there many ways of showing this – send to review if you think it might be credit worthy**

M1: $a^2 - 4b = 27 \Rightarrow a^2 = 27 + 4b$ oe with an indication that this is odd. Allow writing in the form $2(2b+13)+1$ to imply this.

A1: **States** a^2 is odd (then *a* is odd) and writes in the form $a = 2m+1$

dM1: Attempts $(2m+1)^2 = 27 + 4b$ and rearranges to $b = ...$ or $2b = ...$ or a form where a contradiction can be deduced such as $m^2 + m = b + \frac{13}{2}$ where m^2 and *m* will both be integers but $b + \frac{13}{2}$ is not. It is dependent on the previous method mark. Expect to see the brackets multiplied out and terms collected condoning slips.

A1*: cso requires a correct statement showing that there is a contradiction with a conclusion