Question	Scheme	Marks
1	$\int x\cos 3x\,dx = \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$	M1
	$= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$	dM1 A1
	$\int_0^{\frac{\pi}{6}} x\cos 3x\,dx = \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}} = \frac{1}{3}\frac{\pi}{6}\sin\frac{\pi}{2} + \frac{1}{9}\cos\frac{\pi}{2} - \left(0 + \frac{1}{9}\right)$	M1
	$= \frac{\pi}{18} - \frac{1}{9}$	A1
		(5)
		(5 marks)

M1: Applies integration by parts in the correct direction to achieve $\alpha x\sin 3x \pm \beta\int \sin 3x\,dx$

So do not penalise the appearance of a '+' sign unless they state $\int vu'\,dx = uv + \int uv'\,dx$ o.e.

dM1: Completes the process by integrating the sin3*x* term, achieving $\pm\alpha x\sin 3x \pm \beta\cos 3x$

A1: Correct integration which may be left un simplified. E.g. $\frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x\right)$

May have been done in stages (allow if $\frac{1}{3}x\sin 3x$ and $+\frac{1}{9}\cos 3x$ are seen on separate lines.)

Ignore spurious notation as long as the intention is clear.

M1: Applies both the limits to their integral which must be of the form $\pm\alpha x\sin 3x \pm \beta\cos 3x$. The substitution must be seen, or clearly implied by the correct evaluation of their $\pm\alpha x\sin 3x \pm \beta\cos 3x$

If embedded limits are not seen first, then $\left[\pm\alpha x\sin 3x \pm \beta\cos 3x\right]_0^{\frac{\pi}{6}} = (...) - \underline{\underline{0}}$ is M0

Condone the subtraction being done either way around.

A1: Correct simplified answer. ISW after a correct answer. Allow $\frac{\pi - 2}{18}$

Alt methods may be seen. E.g. D & I method

Differentiation		Integration
x	+	$\cos 3x$
1	−	$\frac{1}{3}\sin 3x$
0		$-\frac{1}{9}\cos 3x$

Score M1 dM1 for sight of $\pm\alpha x\sin 3x \pm \beta\cos 3x$

Question	Scheme	Marks
2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\quad\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=...$	M1
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	A1
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=...$	M1
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	dM1
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=...$	ddM1
	$a=-1,\ 6$	A1
		(4)
		(6 marks)

(a)

M1: Attempts to add $\overrightarrow{OA}$ and $\overrightarrow{AB}$, which may be implied by two correct coordinates out of three if there is no direct statement as in the main scheme.

A1: Correct coordinates, in either vector or Cartesian form. (See main scheme).

Withhold the mark for hybrid versions such as $(5\mathbf{i},6\mathbf{j},-2\mathbf{k})$ but isw after you have seen a correct answer

(b)

M1: Attempts $\overrightarrow{BC}$ by subtracting their $\overrightarrow{OB}$ (look at their answer to (a)) and $\overrightarrow{OC}$, subtracted either way round, which may be implied by two correct coordinates out of three following through on their (a).

dM1: Applies the scalar product with $\overrightarrow{OC}$ and their $\overrightarrow{BC}$, sets equal to 0 and forms a quadratic equation in *a*. Dependent upon the previous M.

An alternative method is via Pythagoras' theorem $OC^2+BC^2=OB^2\Rightarrow$

$(a^2+5^2+1^2)+((a-5)^2+1^2+1^2)=5^2+6^2+(-2)^2\Rightarrow 2a^2-10a-12=0$

ddM1: Depends on both previous Ms. Solves their 3- term quadratic equation (which must have real roots) by any valid means including a calculator (check roots if done this way).

A1: Correct values for *a*. If the candidate goes on to reject a value it is A0.

Question	Scheme	Marks
3	$8x^3-3y^2+2xy=9$	
	Either $3y^2 \to ky\frac{dy}{dx}$ **or** $2xy \to \alpha y+\beta x\frac{dy}{dx}$	M1
	Both $3y^2 \to ky\frac{dy}{dx}$ **and** $2xy \to \alpha y+\beta x\frac{dy}{dx}$	dM1
	$8x^3-3y^2+2xy=9 \to 24x^2-6y\frac{dy}{dx}+2y+2x\frac{dy}{dx}=0$	A1
	$(2,5): 96-30\frac{dy}{dx}+10+4\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=...\left(=\frac{53}{13}\right)$	M1
	$\frac{dy}{dx}=\frac{53}{13}$ or $\frac{dx}{dy}=\frac{13}{53}$ o.e.	A1
	Normal: $y-5="-\frac{13}{53}"(x-2)$	M1
	$13x+53y-291=0$	A1
		(7)
		(7 marks)

M1: Differentiates one term in *y* to a correct form. Award for either $-3y^2 \to \pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2xy \to \alpha y + \beta x\frac{\mathrm{d}y}{\mathrm{d}x}$ Accept equivalents if they differentiate with respect to *y* instead.

dM1: Differentiates both *y* terms to a correct form

Award for **Both** $-3y^2 \to \pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $2xy \to \alpha y + \beta x\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Fully correct differentiation. (o.e. if with respect to *y*). Allow y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$

You may see $24x^2\,\mathrm{d}x - 6y\,\mathrm{d}y + 2y\,\mathrm{d}x + 2x\,\mathrm{d}y = 0$ which is fine.

Note that $\frac{\mathrm{d}y}{\mathrm{d}x} = 24x^2 - 6y\frac{\mathrm{d}y}{\mathrm{d}x} + 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ is A0 unless the extra $\frac{\mathrm{d}y}{\mathrm{d}x} =$ is subsequently omitted

M1: For

- either rearranging to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or $\frac{\mathrm{d}x}{\mathrm{d}y}$) in terms of **both** *x* and *y* and then substituting **both** (2, 5)
- or substituting **both** (2, 5) then rearranging to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ (or $\frac{\mathrm{d}x}{\mathrm{d}y}$)

FYI $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{24x^2 + 2y}{6y - 2x}$ o.e.

It is dependent upon having exactly two terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$, one each coming from differentiating the $3y^2$ and the $2xy$.

A1: Correct value for either $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{53}{13}$ (accept awrt 4.08) or $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{13}{53}$ (awrt 0.245)

M1: Correct method for the equation of the normal at (2, 5). E.g. $y - 5 = "-\frac{13}{53}"(x-2)$

It is dependent upon them having attempted differentiation (may be only one term in $\frac{\mathrm{d}y}{\mathrm{d}x}$) and using $x = 2, y = 5$ to find the value of $-\frac{\mathrm{d}x}{\mathrm{d}y}$ which must be the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$

If the form $y = mx + c$ is used then the method must proceed as far as $c = ...$

A1: Correct answer or any integer multiple of this. The = 0 must be seen

Question	Scheme	Marks
4(a)	$A=\frac{1}{2}5^2\left(\theta-\sin\theta\right) \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{25}{2}\left(1\pm\cos\theta\right)$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{25}{2}\left(1-\cos\theta\right)$	A1
		(2)
(b)	$\frac{\mathrm{d}\theta}{\mathrm{d}t}=0.1$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}t}$ (oe)	B1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{25}{2}\left(1-\cos\frac{\pi}{3}\right)\times 0.1$	M1
	$=\frac{5}{8}$ (cm^2 per second)	A1
		(4)
		(6 marks)

(a)

M1: Attempts formula for area of segment with $r = 5$ and differentiates with respect to θ.

Look for $\alpha 5^2\left(\theta - \sin\theta\right) \Rightarrow \frac{dA}{d\theta} = \beta\left(1 \pm \cos\theta\right)$ Allow with either $\alpha = \frac{1}{2}$ or 1

You may see $A = \alpha 5^2\theta - \alpha 5^2 \sin\theta \Rightarrow \frac{dA}{d\theta} = \beta \pm \beta\cos\theta$ again with $\alpha = \frac{1}{2}$ or 1

A1: Reaches correct result with no errors seen. Allow with 12.5.

Must follow M1 (the form is given). It cannot just appear without sight of some correct working

(b)

B1: States or uses $\frac{d\theta}{dt} = 0.1$

This may be seen anywhere in the question, even in part (a) or next to the Figure

B1: States or uses a correct version of the chain rule for the given problem.

May be seen in (a) or in the question itself.

E.g $\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ or $\frac{d\theta}{dA} \times \frac{dA}{dt} = \frac{d\theta}{dt}$ are two correct versions

This may be implied by an equation involving their $\frac{dA}{d\theta}$ or $\frac{dA}{d\theta} = K\left(1 - \cos\theta\right)$ and their $\frac{d\theta}{dt}$

M1: Attempts the chain rule with their (a), a correct $\frac{d\theta}{dt} = 0.1$ and $\theta = \frac{\pi}{3}$ to find a value for $\frac{dA}{dt}$

If they don't attempt (a), or use a made up K, condone $\frac{dA}{dt} = 0.1 \times \frac{1}{2}'K' = \frac{1}{20}'K'$, but $\frac{dA}{d\theta}$ must be of the form $K\left(1 - \cos\theta\right)$ via this route.

A1: CSO. units not required. Decimal answer is 0.625.

Cannot be scored following an incorrect part (a) but allow if they have

(a) left as $\frac{dA}{d\theta} = \frac{r^2}{2}\left(1 - \cos\theta\right)$ followed by (b) $\frac{dA}{dt} = \frac{25}{2}\left(1 - \cos\frac{\pi}{3}\right) \times 0.1 = \frac{5}{8}$ which scores 00 1111.

Question	Scheme	Marks
5(a)	$y=\dfrac{2}{t(3-t)}=1\Rightarrow 2=3t-t^2\Rightarrow t=...$	M1
	$\left(t^2-3t+2=(t-1)(t-2)=0\Rightarrow\right)a=1,b=2$	A1
		(2)
(b)	Area under curve $=\displaystyle\int_{t="1"}^{t="2"} y\,\mathrm{d}x=\int_1^2 y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int_1^2\frac{2}{t(3-t)}\times(2t+2)\,\mathrm{d}t$	M1 A1
	$t=1\Rightarrow x=3,\quad t=2\Rightarrow x=8$	B1
	So area of R = $1\times(8-3)-\displaystyle\int_1^2\frac{2}{t(3-t)}\times(2t+2)\,\mathrm{d}t$	M1
	$=5-4\displaystyle\int_1^2\frac{t+1}{t(3-t)}\,\mathrm{d}t$	A1
		(5)
(c)(i)	$\dfrac{t+1}{t(3-t)}=\dfrac{A}{t}+\dfrac{B}{3-t}$	M1
	$\Rightarrow t+1=A(3-t)+Bt$ E.g. $t=0\Rightarrow 1=3A\Rightarrow A=\dfrac{1}{3}$; $t=3\Rightarrow 4=3B\Rightarrow B=\dfrac{4}{3}$	M1
	$\dfrac{t+1}{t(3-t)}=\dfrac{1}{3t}+\dfrac{4}{3(3-t)}$	A1
(ii)	$\displaystyle\int\frac{t+1}{t(3-t)}\,\mathrm{d}t=\int\frac{1}{3t}+\frac{4}{3(3-t)}\mathrm{d}t=\frac{1}{3}\ln t-\frac{4}{3}\ln(3-t)$	M1
	$\text{Area}=5-4\left[\dfrac{1}{3}\ln t-\dfrac{4}{3}\ln(3-t)\right]_1^2=5-4\left(\dfrac{1}{3}\ln 2-\dfrac{4}{3}\ln 1-\dfrac{1}{3}\ln 1+\dfrac{4}{3}\ln 2\right)$ $=5-4\left(\dfrac{1}{3}\ln 2+\dfrac{4}{3}\ln 2\right)$	dM1
	$=5-\dfrac{20}{3}\ln 2$	A1
		(6)
		(13 marks)

(a)

M1: Sets the y equation equal to 1 (condoning slips) and then attempts to solve the resulting quadratic equation (usual rules). May be implied by correct values if a calculator is used.

A1: Correct values. That is $a=1$ and $b=2$. Condone ($t=$) 1 and 2 due to the demand of the question. Correct answers with no (incorrect) working scores both marks but must be seen in part (a).

If they do state $a=3$ and $b=8$ following $t=1$ and $t=2$ score A0

(b) M1: Attempts $\frac{dx}{dt}$ and multiplies by y condoning slips (e.g. missing brackets).

No requirement to see any integral.

A1: Correct unsimplified expression for the area under the curve $\int_{"1"}^{"2"} \frac{2}{t(3-t)} \times (2t+2)\,dt$

Allow with their a and b or even with a and b (as letters). May be seen within an expression.
Condone the omission of the dt for this mark. Allow missing brackets to be recovered.

B1: Sight of correct (x) values 3 and 8 at the ends of the given curve. May be awarded from work done in (a). Award for sight of 5 as the width of the rectangle or for $5-\int$ as their attempt at the area of R

M1: Full attempt at finding the area of region R. Be aware that the form is given.
Look for the area of the rectangle subtract the area under the curve. dt's not required here
It is dependent upon

Use of the correct x_a and x_b or use of their t values from (a) to find x_a and x_b which may be implied by the width of the rectangle

Use of $1\times\left(x_b - x_a\right) - \int_{"1"}^{"2"} y\frac{dx}{dt}\,dt$ to find area of R.

May be achieved by $\int_{t="1"}^{t="2"} (1-y)\,dx = \int_1^2 (1-y)\frac{dx}{dt}\,dt = \int_1^2 (2t+2) - \frac{4(t+1)}{t(3-t)}\,dt = \left[t^2+2t\right]_1^2 - \int_1^2 \frac{4(t+1)}{t(3-t)}\,dt$

A1: Correct answer achieved. All aspects including the dt must be present at least once before the final solution

(c)(i)

M1: Correct form for partial fractions which may be implied

M1: Correct method to find at least one of the constants.
If they use cover up rule etc it may be implied by one correct constant or one correct pf

A1: Correct partial fractions, not just correct constants. May be awarded in (c)(ii)

Allow fractions like $\frac{1/3}{t} + \frac{4/3}{(3-t)}$. Ignore any … which may be set equal to this.

(c)(ii)

M1: Integrates to correct form $p\ln t + q\ln(3-t)$ o.e.

Be aware that $p\ln ct + q\ln k(3-t)$ where c and k are constants is also correct

dM1: Substitutes their t limits (either way around) and uses the answer from (b) to find the area.
There must be some correct ln work seen. E.g $\ln 1 = 0$ or attempts to combine terms
To score this there must be non-zero values for M, k, a and b.

Condone with M and k leading to a correct expression in M and k. E.g. $M - K\left(\frac{1}{3}\ln 2 + \frac{4}{3}\ln 2\right)$

It is dependent upon the previous M1 and must be of the form $M - K"\left(\frac{1}{3}\ln 2 + \frac{4}{3}\ln 2\right)"$

A1: Correct answer or equivalent simplest form. E.g. $\frac{15-20\ln 2}{3}$. ISW after a correct answer.

Condone other simple equivalents such as $5-\frac{5}{3}\ln 16$ or $5-\frac{4}{3}\ln 32$

Question	Scheme	Marks
6(a)	$\left\|\overrightarrow{OA}\right\|^2=(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2$	M1
	$\left\|\overrightarrow{OA}\right\|=5\sqrt{10}\Rightarrow(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2=250$	M1
	$\Rightarrow 64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ $\Rightarrow 81\lambda^2+52\lambda-220=0$ *	A1*
		(3)
(b)	$81\lambda^2+52\lambda-220=0\Rightarrow(81\lambda-110)(\lambda+2)=0\Rightarrow\lambda=...\left(-2,\frac{110}{81}\right)$ $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1+8\times\text{"their}\lambda\text{"}\\2-\text{"their}\lambda\text{"}\\5+4\times\text{"their}\lambda\text{"}\end{pmatrix}$	M1
	E.g $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1\\2\\5\end{pmatrix}-2\begin{pmatrix}8\\-1\\4\end{pmatrix}=\begin{pmatrix}-15\\4\\-3\end{pmatrix}$ * ; or $\overrightarrow{OA}=\begin{pmatrix}1+8\times\frac{110}{81}\\2-\frac{110}{81}\\5+4\times\frac{110}{81}\end{pmatrix}=\begin{pmatrix}\frac{961}{81}\\\frac{52}{81}\\\frac{845}{81}\end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos\theta=\pm\dfrac{\begin{pmatrix}-15\\4\\-3\end{pmatrix}.\begin{pmatrix}8\\-1\\4\end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}}=...$	M1
	$=\pm\dfrac{-136}{45\sqrt{10}}\;(=\pm0.9557...)$ or e.g. $\theta=$ awrt $0.299(rad)/17.1°$	A1
	Area $OAB=\frac{1}{2}5\sqrt{10}\times4\sqrt{10}\sin\theta=...$	M1
	= awrt 29.4	A1
		(4)
		(10 marks)

(a)

M1: Attempts to find the distance, or distance squared, from O to a general point on the line, or to A. Condone slips. It is acceptable to use a different variable rather than λ.

M1: Attempts to use $\left|\overrightarrow{OA}\right|=5\sqrt{10}$ to set up a quadratic equation in λ, need not be expanded for this mark.

Condone slips on the 250 for OA^2 on this mark. Allow it to appear as $5\times10=50$

A1*: Expands and reaches the correct equation from correct work.

An intermediate line equivalent to $64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ must be written out before the final given answer is seen.

(b)

M1: Correct attempt at solving the quadratic equation, (allow a calculator), and substitutes at least one of their values for λ into the equation for the line to find a possible position vector. The $(-15, 4, -3)$ is given so they cannot just write $\lambda = -2$, $A = (-15, 4, -3)$ without some evidence. However, sight of a correct second coordinate $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ would imply M1

A1*: Correct value $\lambda = -2$ found, and substituted to reach the given position vector.

Scored for sight of $\overrightarrow{OA} = \begin{pmatrix} 1+8\times-2 \\ 2-(-2) \\ 5+4\times-2 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}$ o.e.

Also allow the coordinate $(-15, 4, -3)$ but not an incorrect hybrid version. E.g. $(-15\mathbf{i}, 4\mathbf{j}, -3\mathbf{k})$

A1: Correct second possible position vector but condone in coordinate form. $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ Must be exact. Do not allow hybrid versions but if this form has been penalised already, allow here.

(c)

M1: Attempts the scalar product of $\pm\overrightarrow{OA} = \pm(-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ and the direction vector of the line $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ to find $\cos\theta$ for the angle between $\overrightarrow{OA}$ and l_2. Condone slips but the intention must be clear. If the method of finding modulus is not shown award if one of the two is correct. For example, $4\sqrt{10}$ is a common error/slip on the modulus of $\pm(8\mathbf{i} - 1\mathbf{j} + 4\mathbf{k})$ as this length is given in the question.

Note that it is acceptable to find the scalar product of $\pm\overrightarrow{OA}$ with $\pm(8\mu\mathbf{i} - 1\mu\mathbf{j} + 4\mu\mathbf{k})$ as long as μ is used consistently within the calculation on the numerator. Note any variable may be used here.

A1: Correct value for $\cos\theta$ (either sign acceptable). Implied by use of $\theta =$ awrt 0.299 rads or awrt 17.1°

M1: Uses formula for area of triangle in a complete method to find the area required. It is dependent upon having found $\cos\theta$ via a scalar product of the two correct vectors.

A1: Correct area, awrt 29.4

Alt: There are alternative methods most of which involve finding the perpendicular distance between the two lines. An example of such a method is shown below.

Alt (c)	Midpoint of possible A's is $M = \frac{1}{81}\begin{pmatrix} -127 \\ 188 \\ 301 \end{pmatrix}$ and $OM = \frac{1}{81}\sqrt{127^2 + 188^2 + 301^2} = \ldots$	**M1**
	$= 4.6534\ldots$	**A1**
	Area $= \frac{1}{2} \times 4\sqrt{10} \times 4.6534\ldots ;= 29.43\ldots$	**M1; A1**
		(4)

M1: Finds the midpoint for the two possible positions for A and proceeds to find the distance from O to this point.

A1: Correct distance between lines.

M1: Uses formula for area of triangle in a complete method to find the area required.

A1: Correct area, awrt 29.4

There may be longer alternatives, such as finding point B first and using a different angle, but the scheme will apply in like manner.

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int \frac{1}{k-3x}\,dx = \int dt$ oe	M1
	$\Rightarrow -\frac{1}{3}\ln(k-3x) = t(+c)$	M1 A1
	$t = 0, x = 0 \Rightarrow -\frac{1}{3}\ln k = c$	M1
	$-\ln(k-3x) = 3t - \ln k \Rightarrow \ln\frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = e^{-3t} \Rightarrow x = \ldots$	M1
	$x = \frac{k}{3}\left(1 - e^{-3t}\right)$	A1
		(6)
(b)	$t \to \infty \Rightarrow e^{-3t} \Rightarrow 0 \Rightarrow x \to \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = \ldots$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7\left(1 - e^{-3t}\right) = 5 \Rightarrow e^{-3t} = \ldots$	M1
	$\Rightarrow t = -\frac{1}{3}\ln\frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
		(11 marks)

Alt (a): Use the same criteria for the first 3 marks. See notes

7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int_0^x \frac{1}{k-3x}\,dx = \int_0^t dt$ oe	M1
	$\Rightarrow \left[-\frac{1}{3}\ln(k-3x)\right]_0^x = [t]_0^t$	M1 A1
	$-\frac{1}{3}\ln(k-3x) + \frac{1}{3}\ln k = t$	M1
	$\ln k - \ln(k-3x) = 3t \Rightarrow \ln\frac{k}{k-3x} = 3t \Rightarrow \frac{k}{k-3x} = e^{3t} \Rightarrow k = ke^{3t} - 3xe^{3t} \Rightarrow x = \ldots$	M1
	$x = \frac{k}{3}\left(1 - e^{-3t}\right)$	A1

Mark as one complete question. For example, it is possible to get (b) without doing (a)

(a)

M1: Separates the variables correctly, with or without the integration signs shown. The dx and dt must be shown in the correct positions

M1: Attempts the integration of the LHS with $\frac{1}{k-3x} \to \pm a\ln(k-3x)$ A missing bracket should be penalised unless recovered later in the solution

A1: Fully correct integration, with or without the + c for this mark.

A missing bracket should be penalised unless recovered later in the solution

M1: Attempts to find the constant of integration following **any** attempt at integration to an equation involving a constant. This involves $t=0, x=0 \Rightarrow c=...$ where c is non zero. For a definite integral we would require the x = 0 to be clearly substituted to obtain a constant term.

M1: Correct log work to make x the subject. May be scored before the previous M if the constant is found later. Allow if the constant is zero, or is missing. Condone slips on signs.

You may see versions of the following showing correct exponential work:

$$-\frac{1}{3}\ln\left(k-3x\right)=t+c \Rightarrow \ln\left(k-3x\right)=-3t+d \Rightarrow k-3x=\mathrm{e}^{-3t+d} \Rightarrow k-3x=A\mathrm{e}^{-3t} \Rightarrow x=\frac{k-A\mathrm{e}^{-3t}}{3}$$

A1: Correct answer or exact equivalent. E.g. $x=\frac{k\left(\mathrm{e}^{3t}-1\right)}{3\mathrm{e}^{3t}}$.

(b)

M1: Correctly identifies $\mathrm{e}^{-3t} \to 0$ (which may be implied) and deduces the long-term limit for their expression and sets their long-term limit to 7 and solves for k. The equation must be of the form $x=p-q\mathrm{e}^{-3t}$ with p in terms of k and q > 0. Alternatively, if they have an equation of the form $x=\frac{k\left(\mathrm{e}^{3t}-\beta\right)}{\delta\mathrm{e}^{3t}}$ they would need to solve $7=\frac{k}{\delta}$ to find the value of k.

Condone work on equations of the same form where they have lost the 3 and used another positive value, usually 1. So, score for equations $x=p-q\mathrm{e}^{-\omega t}$ or $x=\frac{k\left(\mathrm{e}^{\omega t}-\beta\right)}{\delta\mathrm{e}^{\omega t}}$ $\omega>0$ with the same conditions.

A1: k = 21 following a correct equation.

Correct answer following correct equation with no working scores both marks

Alt (b)

M1: Correctly identifies $\frac{\mathrm{d}x}{\mathrm{d}t}=k-3x=0$ and substitutes in x = 7 leading to a value for k

A1: k = 21

(c)

M1: Substitutes x = 5 and their value for k into their equation from (a), then rearranges to $\mathrm{e}^{\pm 3t}=...$

This is dependent upon having a **solvable** equation of the form $x=\pm p \pm q\mathrm{e}^{-3t}$ o.e such as $x=\frac{\pm\mu\mathrm{e}^{3t}\pm\kappa}{\mathrm{e}^{3t}}$

Condone work on equations of the same form where they have lost the 3 and used another positive value, usually 1. So, score for equations $x=\pm p \pm q\mathrm{e}^{-\omega t}$ or $x=\frac{\pm\mu\mathrm{e}^{\omega t}\pm\kappa}{\mathrm{e}^{\omega t}}$ $\omega>0$ with the same conditions.

dM1: Solves an equation of the form $\mathrm{e}^{\pm 3t}=k, k>0$ to find a value for t. Condone work in solving equations of the form $\mathrm{e}^{\pm\omega t}=k, k>0$ following the loss of the 3

A1: cso 0.42 (seconds)

Alt (c)

M1: Substitutes x = 5 into their **solvable** equation of the form $\pm\beta\ln\left(k-3x\right)=t\pm\delta\ln k$ with their k

dM1: ... And finds a value for t

A1: cso awrt 0.42 (seconds)

Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}}=16\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k\neq 3$	M1
	"Correct" term 3 or term 4 in $(1-kx)^{\frac{4}{3}}=1\pm\frac{4}{3}kx+\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2+\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3+\ldots$	M1
	Two terms correct of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+\ldots$ following M1, M1 and $k=\pm\frac{3}{8}$	A1
	$=16-8x+\frac{x^2}{2}+\frac{x^3}{24}+\ldots$	A1
		(4)
(b)	Assume the curves meet so $8x-\frac{15x^2}{2}+8=\text{"}16-8x+\frac{x^2}{2}+\frac{x^3}{24}\text{"}$ (for some x)	M1
	$\Rightarrow 0=8-16x+8x^2+\frac{x^3}{24}+\ldots\Rightarrow 0=8\left(x^2-2x\right)+8+\frac{x^3}{24}+\ldots\Rightarrow 8\left(x-1\right)^2\pm..+..$	M1
	$\Rightarrow 0=8\left(x-1\right)^2+\frac{x^3}{24}$	A1
	Requires • Argument that $8\left(x-1\right)^2+\frac{x^3}{24}>0$. E.g. $8\left(x-1\right)^2\geqslant 0$ and $\frac{x^3}{24}>0$ (for $0<x<\frac{8}{3}$) so $8\left(x-1\right)^2+\frac{x^3}{24}>0$ • Minimal statement (contradiction) and conclusion	A1
		(4)
		(8 marks)

(a)

M1: Attempts to takes out a factor of 16 or $8^{\frac{4}{3}}$. Look for $(8-3x)^{\frac{4}{3}}=16(1-kx)^{\frac{4}{3}}$, $k\neq 3$ or $(8-3x)^{\frac{4}{3}}=8^{\frac{4}{3}}(1-kx)^{\frac{4}{3}}$, $k\neq 3$ Condone incorrect values of k other than 3

M1: Attempts the binomial expansion on $\left(1-kx\right)^{\frac{4}{3}}$ achieves at least a "correct" term 3 or term 4 (which may be unsimplified) for their k. Even allow with k =1

Look for either $\frac{\frac{4}{3}\times\frac{1}{3}}{2}(\pm kx)^2$ or $\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}(\pm kx)^3$ with the correct binomial coefficient and the correct power of x. Condone missing brackets on the $\left(kx\right)$ terms.

If they expand $\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ without showing the unsimplified terms it is implied by $\pm\frac{x^3}{384}$

A1: For two correct and simplified terms of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+\ldots$ following M1, M1 and $k=\pm\frac{3}{8}$

A1: Correct expansion from correct work. Terms may be as a list for both accuracy marks

Alt (a) by direct expansion. $(8\pm3x)^{\frac{4}{3}}=8^{\frac{4}{3}}+\frac{4}{3}\left(8^{\frac{1}{3}}\right)(\pm3x)+\frac{\frac{4}{3}\times\frac{1}{3}}{2}\left(8^{-\frac{2}{3}}\right)(\pm3x)^2+\frac{\frac{4}{3}\times\frac{1}{3}\times\left(-\frac{2}{3}\right)}{6}\left(8^{-\frac{5}{3}}\right)(\pm3x)^3+...$

M1: For $8^{\frac{4}{3}}$ or 16 oe seen as the constant term E.g. $=16\pm...x+...$
M1: For the correct term 3 or term 4 of the expansion. Condone missing brackets on the $(\pm3x)^2$, $(\pm3x)^3$ terms

A1: For two correct and simplified terms of $16-8x+\frac{x^2}{2}+\frac{x^3}{24}+...$ Allow as a list

A1: Correct expansion from correct work. Allow as a list
(b)
M1: Sets up the contradiction hypothesis using the result of their (a) and setting equal to the quadratic equation. There must be some words so look for a minimum of

Assume they meet (or simply let) $8x-\frac{15x^2}{2}+8="16"-8x+\frac{x^2}{2}+\frac{x^3}{"24"}$

M1: Rearranges terms and attempts to complete the square (or factorises as $k(x-1)(x-1)$) on the quadratic terms or other means to show the quadratic part is always positive (e.g. considers discriminant). The work on the quadratic terms of the expression, which must be of the form $8x^2-16x+(A-8)$ should take the form of one of

- $8x^2-16x+(A-8)=8(x-1)^2+...$
- $8x^2-16x+(A-8)$ and attempts $b^2-4ac=16^2-4\times8(A-8)=...$

A1: Correctly completes the square to form equation $0=8(x-1)^2+\frac{x^3}{24}$.

Via the discriminant you would need $b^2-4ac=16^2-4\times8\times8=0$,
A1: Completes the argument by drawing together the contradiction.

States that $8(x-1)^2\geqslant0$ and $\frac{x^3}{24}>0$ (in the interval), hence $0\neq8(x-1)^2+\frac{x^3}{24}$ so proven

Stating that $8(x-1)^2+\frac{x^3}{24}>0$ is insufficient without some explanation. (See above)

Stating that both terms are positive (as $x>0$) is not enough as $8(x-1)^2=0$ at $x=1$

For discriminant must mention $b^2-4ac=0$, hence single root (at 1) and positive quadratic or similar, $\frac{x^3}{24}>0$ hence $0\neq8(x-1)^2+\frac{x^3}{24}$ so proven.

…………………………………………………………………………………………………
There will be alternatives for the middle two marks in (b), so consider carefully what is attempted.

For example, candidates could work on the cubic $8-16x+8x^2+\frac{x^3}{24}$. Condone use of calculus which would involve some calculator work that proves that the function has a minimum value at x = awrt 0.99 y = awrt 0.04 and hence $8-16x+8x^2+\frac{x^3}{24}\neq0$ (for $0<x<\frac{8}{3}$) proving that there is a contradiction. M1: Differentiates, sets = 0 and solves to find x and y in the interval. A1: Correct with proof of minimum

Note that merely solving $8-16x+8x^2+\frac{x^3}{24}=0$ on a calculator to get $x=-194$ and stating this is not in the range is M0.

Question	Scheme	Marks
9(a)	Volume, $V = \pi\int_{\frac{1}{3}}^{3} y^2\,dx = \pi\int_{\frac{1}{3}}^{3} \frac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan\left(\sqrt{x}\right)\right)^2}\,dx$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ oe	M1 A1
	$\Rightarrow V = (\pi)\int \frac{2\times\frac{1}{2}x^{-\frac{1}{2}}}{\left(\arctan\left(\sqrt{x}\right)\right)^2(1+x)}\,dx = (\pi)\int\frac{2}{u^2\left(1+\tan^2 u\right)}\sec^2 u\,du$	dM1
	$= \pi\int\frac{2}{u^2\cancel{\sec^2 u}}\cancel{\sec^2 u}\,du = 2\pi\int\frac{1}{u^2}\,du$	A1
	$\left.\begin{array}{l} x=3 \Rightarrow u = \arctan\sqrt{3} = \frac{\pi}{3} \\ x=\frac{1}{3} \Rightarrow u = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}\end{array}\right\} \Rightarrow V = 2\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{u^2}\,du$	B1
		(6)
(b)	$V = (2\pi)\left[-\frac{1}{u}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi)\left[-\frac{1}{\pi/3} - \left(-\frac{1}{\pi/6}\right)\right]$ Or $= (2\pi)\left[-\frac{1}{\arctan\sqrt{x}}\right]_{\frac{1}{3}}^{3} = (2\pi)\left(-\frac{1}{\arctan\sqrt{3}} - \left(-\frac{1}{\arctan\sqrt{1/3}}\right)\right)$	dM1
	$= 2\pi\left(-\frac{3}{\pi}+\frac{6}{\pi}\right) = 6$	A1
		(3)
		(9 marks)

B1: For a fully correct integral in x or u with π, dx or du, correct limits and bracketing.

May be implied by a fully correct (simplified or un simplified) integral in u

May be seen at any stage in the working following correct work.

The limits must be correct for the variable they are working in. So for an integral in u the limits must be in radians

Look for $V=\pi\int_{\frac{1}{3}}^{3}\frac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan\left(\sqrt{x}\right)\right)^2}\,\mathrm{d}x$ OR $V=\pi\int_{\frac{1}{3}}^{3}\left(\frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\arctan\left(\sqrt{x}\right)}\right)^2\mathrm{d}x$

If the correct integral in x is not seen score for $V=\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2}{u^2\left(1+\tan^2 u\right)}\sec^2 u\,\mathrm{d}u$ o.e.

M1: Attempts the differentiation of the given substitution achieving the correct form with suitable placement for du and dx.

Examples;

- $\tan u=\sqrt{x}\Rightarrow\sec^2 u\,\mathrm{d}u=\alpha x^{-\frac{1}{2}}\,\mathrm{d}x$
- $x=\tan^2 u\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=\ldots\tan u\sec^2 u$
- $u=\arctan\sqrt{x}\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{1+x}\times\ldots x^{-\frac{1}{2}}$

A1: Correct differentiation, any form. Score when first seen, don't penalise an incorrect rearrangement.

Examples

- $\tan u=\sqrt{x}\Rightarrow\sec^2 u\,\mathrm{d}u=\frac{1}{2}x^{-\frac{1}{2}}\,\mathrm{d}x$
- $x=\tan^2 u\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=2\tan u\sec^2 u$
- $u=\arctan\sqrt{x}\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{1+x}\times\frac{1}{2}x^{-\frac{1}{2}}$

dM1: Makes a complete substitution for x into $\int y^2\,\mathrm{d}x$.

No need for the π or limits for this mark. It is dependent upon the previous M

Look for the form $\int\frac{c}{u^k\left(1+\tan^2 u\right)}\sec^2 u\,(\mathrm{d}u)$ (which may be un-simplified) where $k=1$ or 2

Condone a missing du. It can be implied if the substitution for dx has been attempted.

Condone missing brackets if the intention is clear.

A1: Correct simplified integral in terms of u including the π from correct working.

Allow recovery from missing brackets. No need for limits for this mark.

Condone a missing du. It can be implied if the substitution for dx has been attempted

B1: Correct limits for the integral found. Allow where seen, even in part (b) and even if not attached to the integral. Must be in radians

(b)

M1: Carries out the integration.

The π etc may be missing or incorrect for this mark. Accept $\frac{1}{u^2} \to \pm\frac{a}{u}$

dM1: Substitutes appropriate limits into their integral and subtracts either way round.
If the integral is in terms of u the limits must be in radians, not degrees.
If in terms of x they should be using $\arctan\sqrt{3}$ and $\arctan\frac{1}{\sqrt{3}}$

A1: cso. Cannot be scored if k is guessed or found incorrectly