Question	Scheme	Marks
1	$(1-4x)^{-3} = 1\underline{\pm 3}\times 4x \pm \underline{\frac{-3\times -4}{2}}(\ldots x)^2 \pm \underline{\frac{-3\times -4\times -5}{6}}(\ldots x)^3 + \ldots$	M1
	$(1-4x)^{-3} = 1+3\times 4x + \frac{-3\times -4}{2}(-4x)^2 + \frac{-3\times -4\times -5}{6}(-4x)^3 + \ldots$	A1
	$= 1+12x+96x^2+640x^3+\ldots$	A1A1
		(4)
		(4 marks)

Notes:

M1: Attempts the binomial expansion of $(1-4x)^{-3}$ with correct attempts at the (unsimplified, and may be in terms of factorials) binomial coefficients for **at least two** of the x, x^2, x^3 terms. The "–4" may be missing or have incorrect sign and allow for missing brackets. (May be scored if the x^3 term is omitted.)
A1: For a correct unsimplified expansion. (May be in terms of factorials.)
A1: Any two terms correct and simplified (of the four, so includes the 1). The M must have been scored.
A1: Fully correct and all terms simplified. Ignore any higher order terms. ISW after a correct simplified.
Note M1A0A1A0 is possible e.g. if the x and x^3 terms have the wrong signs.
Note: listing terms can score maximum of M1A1A1A0.

Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x = 2 \to 10 = A \times 25 \Rightarrow A = \ldots$ or $x = -\frac{1}{2} \to \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \ldots$ or $x^2 : 0 = 4A + 2B, x : 3 = 4A - 3B + C, x^0 : 4 = A - 2B - 2C \to A = \ldots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \ldots, B = \ldots$ and $C = \ldots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)
(b)	$\int \frac{p}{x-2}\,dx = \alpha \ln(x-2)$ or $\int \frac{q}{2x+1}\,dx = \beta \ln(2x+1)$ Alt: $\int \frac{8x+9}{(2x+1)^2}\,dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2}\,dx = \gamma \ln(2x+1)^2 + \ldots$	M1
	$\int \frac{k}{(2x+1)^2}\,dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,dx = \left[\frac{"2"}{5}\ln(x-2) - \frac{"4"}{5} \times \frac{1}{2}\ln(2x+1) + \frac{"1"}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5}\ln(10) - \frac{2}{5}\ln(25) + \frac{1}{50}\right) - \left(\frac{2}{5}\ln(5) - \frac{2}{5}\ln(15) + \frac{1}{30}\right) = \ldots$	DM1
	$= \frac{2}{5}\ln\frac{10}{25} + \frac{1}{50} - \frac{2}{5}\ln\frac{5}{15} - \frac{1}{30} = \frac{2}{5}\ln\frac{10 \times 15}{25 \times 5} + \ldots$	M1
	$\frac{2}{5}\ln\frac{6}{5} - \frac{1}{75}$ (oe)	A1
		(6)
		(10 marks)

Notes:
(a) **M1:** Attempts a correct method leading to one of the constants. E.g. Multiplies through to $3x+4=A(2x+1)^2+B(x-2)(2x+1)+C(x-2)$ (allowing for minor slips) and substitutes $x=-\frac{1}{2}$ or $x=2$ or other method such as comparing coefficients. Implied by a correct value for A by cover-up rule if no incorrect working is seen. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then allow M1 here for the attempt. **A1:** Any one of the three constants correct (probably A or C). **M1:** For a full and correct method to obtain all three of the constants. If e.g. multiplies through by a wrong term such as $(2x+1)^3$ then M0 here. Implied by correct values if no method shown following a correct identity (otherwise M0 for just incorrect values stated). **A1:** For a correct full partial expression form or correct values stated. Note: if by comparing coefficients allow both M's for achieving any 3 values following three equations from the attempt at comparing with at least one correct. **(b)** **M1:** For integrating to obtain at least one ln term. Note $\alpha\ln\left(5x-10\right)$ and $\beta\ln\left(10x+5\right)$ are equally fine, as are other constant multiples of the inner brackets. **M1:** For $\frac{k}{(2x+1)^2}\to\frac{K}{(2x+1)}$ (any K) oe. **A1ft:** Fully correct integration following through on their non-zero constants. **DM1:** Depends on at least one the first two M's being scored. Substitutes the limits and subtracts either way round. **M1:** Achieves a single log term with evidence of a correct use of at least one log law (e.g. power or addition law). (Note log terms may initially have been combined earlier correctly, in which case look for reaching a single log term.) **A1:** Correct answer accept equivalents in the correct form (including if the modulus signs are left in). Use of constants (if no values for A, B and C are found) can score the M's but not the A's.

Question	Scheme	Marks
3(a)	$y^2x+3y=4x^2+k$	
	$y^2x\to ...\times y\dfrac{dy}{dx}(+...)$ or $3y\to 3\dfrac{dy}{dx}$	M1
	$y^2x\to 2xy\dfrac{dy}{dx}+y^2$	A1
	$...+3y=4x^2+k\to ...+3\dfrac{dy}{dx}=8x$	A1
	$2xy\dfrac{dy}{dx}+y^2+3\dfrac{dy}{dx}=8x\Rightarrow\dfrac{dy}{dx}=...$	dM1
	$\dfrac{dy}{dx}=\dfrac{8x-y^2}{2xy+3}$ oe	A1
		(5)
(b)	$\dfrac{dy}{dx}=0\Rightarrow 8x-4=0\Rightarrow x=...$	M1
	$p=\dfrac{1}{2}$	A1
	$k=2^2\times\dfrac{1}{2}+3\times 2-4\times\dfrac{1}{4}=...$	M1
	$k=7$	A1
		(4)
Alt (b)	$(p,2)$ on $C\Rightarrow 4p+6=4p^2+k\Rightarrow 4p^2-4p+k-6=0$ $\Rightarrow 4\left(p-\dfrac{1}{2}\right)^2-1+k-6=0$ or $b^2-4ac=16-16(k-6)$	M1
	(Min, single solution) $\Rightarrow p=\dfrac{1}{2}$ or $16-16(k-6)=0\Rightarrow k=7$	A1
	$\Rightarrow k=7-4\left(\dfrac{1}{2}-\dfrac{1}{2}\right)^2$ or $4p^2-4p+7-6=0\Rightarrow p=...$	M1
	The other of $p=\dfrac{1}{2}$ and $k=7$	A1
		(4)
		(9 marks)

Notes:

(a)

M1: Some evidence of an attempt at implicit differentiation. Look for one of the terms being differentiated with either the $3\dfrac{dy}{dx}$ or a term with $kxy\dfrac{dy}{dx}$ or $y^2\to ky\dfrac{dy}{dx}$ as part of a product rule seen appropriately.

A1: y^2x differentiated correctly

A1: $3y \to 3\frac{\mathrm{d}y}{\mathrm{d}x}$ and $4x^2 + k \to 8x$

dM1: Make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject with correct terms factored (but allow e.g. sign errors rearranging or slips in copying). There must be at least two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their expression.

A1: Correct expression, accept any equivalent.

(b) Note: allow all marks in (b) from a correct numerator in (a) if the denominator is incorrect.

M1: Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and substitutes $y = 2$ and solves for x. They must have a y term in their numerator to be able to access this mark. If the numerator also contains a k they will need to also use the equation of the curve to find a second equation in p and k and solve simultaneously – look for them reaching a value for p.

A1: Correct value for p or x

M1: Must have been attempting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to find p. Substitutes back into the curve to find k. May be scored if their numerator was in terms of x only.

A1: Correct value for k

Alt (b)

M1: Substitutes (p, 2) into the equation of the curve and attempts to complete the square or considers the discriminant (or other valid method to be able to deduce the values using a single root)

A1: Correct value for p or k

M1: Substitutes back into the curve to find the remaining unknown or other complete method to find the other value.

A1: Correct value for the other of p and k (so both should be correct).

Question	Scheme	Marks
4(a)	$(l=)\sqrt{25+r^2}$	B1
		(1)
(b)	$(S=\pi r^2+)\pi r\sqrt{25+r^2} \Rightarrow \left(\frac{\mathrm{d}S}{\mathrm{d}r}=2\pi r+\right)\underline{\pi\sqrt{25+r^2}+\pi r.\frac{1}{2}(25+r^2)^{-\frac{1}{2}}.2r}$ Or $(S=\pi r^2+)\pi\sqrt{25r^2+r^4} \to \left(\frac{\mathrm{d}S}{\mathrm{d}r}=2\pi r+\right)\underline{\pi.\frac{1}{2}(25r^2+r^4)^{-\frac{1}{2}}\times(50r+4r^3)}$	M1 A1
	$\boxed{(S=\pi(l^2-25))+\pi l\sqrt{l^2-25} \Rightarrow \left(\frac{\mathrm{d}S}{\mathrm{d}l}=2\pi l+\right)\underline{\pi\sqrt{l^2-25}+\pi l.\frac{1}{2}(l^2-25)^{-\frac{1}{2}}.2l}}$	(M1 A1)
	$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=..\times 3$	M1
	$=81.5\ (\mathrm{cm}^2/\mathrm{min})$	A1
		(4)
		(5 marks)

Notes:

(a)

B1: Correct expression. Accept $(l=)\sqrt{5^2+r^2}$ and isw.

(b)

M1: Differentiates their surface area with use of the product and chain rules on a term of the form $Kr\sqrt{\pm L\pm r^2}$ - look the form (oe) $P\sqrt{"25"+r^2}+Qr^2("25"+r^2)^{-\frac{1}{2}}$. The base area may be missing for this mark. Alternatively they may work in terms of l and the scheme marks equivalently – see box in main scheme for the derivative in this case.

Alt: Taking r inside the square root look for $K\sqrt{25r^2+r^4} \to K.\frac{1}{2}(25r^2+r^4)^{-\frac{1}{2}}\times(\alpha r+\beta r^3)$

A1: Correct differentiation for the curved surface area. Need not be simplified.

Alt: With r inside the square root $\pi\sqrt{25r^2+r^4} \to \pi.\frac{1}{2}(25r^2+r^4)^{-\frac{1}{2}}\times(50r+4r^3)$

M1: Applies the chain rule correctly with $\frac{\mathrm{d}r}{\mathrm{d}t}=3$ and their expression for $\frac{\mathrm{d}S}{\mathrm{d}r}$ (which may be in terms of r and l) or their value for $\frac{\mathrm{d}S}{\mathrm{d}r}$ at $r=1.5$ if they find this first. This mark is for the chain rule being applied, so their previous expressions need not have been correct. If they are working in terms of l they will need to use $\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}S}{\mathrm{d}l}\times\frac{\mathrm{d}l}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$.

A1: For awrt 81.5, do not need the units. Accept $\left(9+\dfrac{177\sqrt{109}}{109}\right)\pi$ oe (FYI it is 81.535404…)		
(b) **Alt 1**	$\dfrac{dr}{dt}=3\Rightarrow r=3t(+c)\Rightarrow S=9\pi(t(+c'))^2+3\pi(t(+c'))\sqrt{25+9(t(+c'))^2}$ $\Rightarrow\dfrac{dS}{dt}=18\pi(t)+\underline{3\pi\sqrt{25+9(t)^2}+3\pi(t).\dfrac{1}{2}\left(25+9(t)^2\right)^{-\frac{1}{2}}.18(t)}$ Or $\dfrac{dS}{dt}=18\pi(t)+\underline{3\pi.\dfrac{1}{2}\left(25(t)^2+9(t)^4\right)^{-\frac{1}{2}}\times\left(50(t)+36(t)^3\right)}$ where ((t) may be $(t+(c'))$ throughout)	**M1** **A1**
	$r=1.5\Rightarrow t(+c')=0.5\Rightarrow\dfrac{dS}{dt}=...$	**M1**
	$=81.5\ (\text{cm}^2/\text{min})$	**A1**
		(4)
Notes:		

Alt 1 (b)

M1: Uses $\dfrac{dr}{dt}=3$ and integrates to $r=3t+c$, where c may be omitted (without loss of generality the time frame can be set to t = 0 when r =0), and substitutes into the surface area formula, with their l to form an expression of the correct form and proceeds to differentiate and find $\dfrac{dS}{dt}$ achieving a suitable form for the curved surface area part (as per main scheme, the only difference is the constant multiples).

A1: Correct differentiation for the curved surface area. Need not be simplified. Accept equivalents and may be working to t or $t+c$ throughout.

M1: Finds t (+ c) when r = 1.5 and substitutes into their $\dfrac{dS}{dt}$.

A1: For awrt 81.5, or as per main scheme.

(b) **Alt 2**	$\dfrac{dl}{dt}=\dfrac{dl}{dr}\dfrac{dr}{dt}=3\times\dfrac{1}{2}(r^2+25)^{-\frac{1}{2}}\times 2r$	**M1** **A1**
	$S=\pi r^2+\pi rl\Rightarrow\dfrac{dS}{dt}=2\pi r\dfrac{dr}{dt}+\pi\dfrac{dr}{dt}l+\pi r\dfrac{dl}{dt}$	**M1**
	$=81.5\ (\text{cm}^2/\text{min})$	**A1**
		(4)
Notes:		

Alt 2 (b)

M1: Uses the chain rule and their expression from (a) to find $\dfrac{dl}{dt}$ and uses $\dfrac{dr}{dt}$ = 3.

A1: Correct derivative.

M1: Uses implicit differentiation to find $\frac{\mathrm{d}S}{\mathrm{d}t}$ in terms of $\frac{\mathrm{d}r}{\mathrm{d}t}$ and $\frac{\mathrm{d}l}{\mathrm{d}t}$.

A1: As main scheme.

Question	Scheme	Marks
5(a)	$\int x^2\cos 2x\,\mathrm{d}x = x^2.\frac{1}{2}\sin 2x - \int 2x.\frac{1}{2}\sin 2x\,\mathrm{d}x$	M1 A1
	$= x^2.\frac{1}{2}\sin 2x - \left(-\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,\mathrm{d}x\right)$	M1
	$= \frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x\ (+c)$	A1
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{t^2\cos^2 t}{y^2} \Rightarrow$ e.g. $\int y^2\frac{\mathrm{d}y}{\mathrm{d}t}\mathrm{d}t$ or $\int y^2\,\mathrm{d}y = \int t^2\cos^2 t\,\mathrm{d}t$	B1
	$\Rightarrow \frac{y^3}{3} = \ldots \quad ; \quad \ldots = \frac{1}{2}\int t^2 + t^2\cos 2t\,\mathrm{d}t$	M1 ; M1
	$\ldots = \frac{1}{6}t^3 + \frac{1}{2}\left(\frac{1}{2}t^2\sin 2t + \frac{1}{2}t\cos 2t - \frac{1}{4}\sin 2t\right) \quad (+c)$	M1
	$\Rightarrow y^3 = \frac{1}{2}t^3 + \frac{3}{4}t^2\sin 2t + \frac{3}{4}t\cos 2t - \frac{3}{8}\sin 2t + c$	A1ft
		(5)
		(9 marks)

Notes:

(a)

M1: Attempts parts in the correct direction – look for $Ax^2\sin 2x \pm \int Bx\sin 2x\,\mathrm{d}x$ unless they explicitly state an incorrect formula (in which case it is M0).

A1: Correct first application of parts.

M1: Attempts parts again in the same direction. $\int x\sin 2x\,\mathrm{d}x \to Kx\cos 2x \pm L\int \cos 2x\,\mathrm{d}x$

A1: Correct answer with or without constant of integration. ISW after a correct answer.

(b)

Note: accept use of x in place of t (or even mixed x's and t's) for the method marks if the intention is clear, but the final A mark needs to be correct in terms of t.

B1: Correct separation of variables with indication to integrate (e.g. statement as shown in scheme, or an attempt to integrate one side).

M1: For $y^n \to \ldots y^{n+1}$ on the left hand side.

M1: Applies double angle formula to right hand side. Accept $\cos^2 t \to \frac{1}{2}(\pm 1 \pm \cos 2t)$ for this mark.

M1: Applies their result from (a) (allow with t or x here) or redoes integration by parts and reaches the correct form or same form as their (a), and integrates the $t^{\text{"2"}}$ term (no need for $+c$)

A1ft: Achieves the correct result, accepting equivalent forms, but must include a constant of integration (+ c or other labelling is fine). Follow through on their answer to (a) *providing* isw was not applied. So for $y^3 = \frac{1}{2}t^3 + \frac{3}{2} \times \text{their answer to (a)} + c$

5(a) By DI method

S	D	I
+	x^2	$\cos 2x$
-	$2x$	$\frac{1}{2}\sin 2x$
+	2	$-\frac{1}{4}\cos 2x$
-	0	$-\frac{1}{8}\sin 2x$

M1 A1

$$\int x^2 \cos 2x \,\mathrm{d}x = x^2 \times \frac{1}{2}\sin 2x - 2x \times -\frac{1}{4}\cos 2x + 2 \times -\frac{1}{8}\sin 2x$$ **M1**

$$= \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x \ (+c)$$ **A1**

Notes

M1: Sets up a correct table or equivalent method with x^2 and $\cos x$ as the leads.

A1: Correct tables of derivatives and integrals.

M1: Extracts the answer from the table achieving the correct form.

A1: Correct answer.

Note an answer of the form $Ax^2 \sin 2x + Bx\cos 2x + C\sin 2x$ with no incorrect working seen can score M1A0M1A0 if the coefficients are incorrect.

Question	Scheme	Marks
6(a)	$(3\mathbf{i}+p\mathbf{j}+7\mathbf{k})+\lambda(2\mathbf{i}-5\mathbf{j}+4\mathbf{k})=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$ $3+2\lambda=8+4\mu \quad (1)$ $\Rightarrow p-5\lambda=-2+\mu \quad (2)$ $7+4\lambda=5+2\mu \quad (3)$	M1
	e.g. $(3)-2\times(1): 1=-11-6\mu \Rightarrow \mu=-2 \quad \left(\text{or } \lambda=-\frac{3}{2}\right)$	M1
	$\mu=-2\ ,\lambda=-\frac{3}{2} \Rightarrow p=-2-2-\frac{15}{2}=...$	M1
	$p=-\frac{23}{2}$	A1
		(4)
(b)	Intersect at $(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+-2(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=..$	M1
	$=-4\mathbf{j}+\mathbf{k}$	A1
		(2)
(c)	$(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=2\times4-5\times1+4\times2=...$	M1
	$\cos\theta=\frac{"11"}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=...\left(=\frac{11}{3\sqrt{105}}=0.3578...\right)$	M1
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	A1
		(3)
(d)	$\lambda=2\Rightarrow\overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	B1ft
	$\overrightarrow{AB}=\pm\left((8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right)$	M1
	$\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	M1
	$\overrightarrow{OB}=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+"-\frac{1}{6}"(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=...$	dM1
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	A1
		(5)
		(14 marks)

Notes: Accept alternative notations, such as column or row vector notation, throughout. Condone if **i**, **j** and **k** are left in columns etc. except for in the final answers to (b) and (d).
(a) Note: work for (a) must be shown in part (a) not recovered from later parts. **M1:** Equates equations of the lines and extracts at least two simultaneous equations (usually (1) and (3)) by equating coefficients. **M1:** Solves the equations to find at least one of λ or μ **M1:** Uses their values or other suitable method to find p allowing for slips. **A1:** Correct value.
(b) **M1:** Uses either parameter to find the point of intersection. May be implied by two correct entries if no method shown. **A1:** Correct point. Accept as coordinates or in vector form. **(c)** **M1**: Attempts the scalar product with the two direction vectors (or any multiples of them). Allow if there are miscopies etc if the intent of the two correct vectors is clear. Note that $-3\mathbf{i}+\frac{15}{2}\mathbf{j}-6\mathbf{k}$ and $-8\mathbf{i}-2\mathbf{j}-4\mathbf{k}$ are commonly seen correct direction vectors. **M1:** Applies the scalar product formula to find the value for cos θ. Allow if wrong "direction" vectors are used, or if a slip is made but the intent is clear. This mark is for demonstrating a correct use of the formula with any vectors. **A1:** Correct angle, awrt 69.0°, and isw if they give go on to give the obtuse angle. But A0 if another incorrect angle is found. **(d)** **B1ft:** Correct coordinates for A, follow through their p **M1:** Attempts to find $\overrightarrow{AB}$ in terms of μ with their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown. Note, if $\overrightarrow{AB}=(x-7)\mathbf{i}+\left(y+"\frac{43}{2}"\right)\mathbf{j}+(z-15)\mathbf{k}$ is used this mark is not scored until the point where they identify x, y and z in terms of μ. **M1:** Takes scalar product of their $\overrightarrow{AB}$ with direction of l_2, sets equal to zero and solves to find μ **dM1:** Depends on previous M mark. Uses their parameter in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. **A1:** Correct answer. Accept as coordinates or in vector form.

Alt (d)	$\lambda = 2 \Rightarrow \overrightarrow{OA} = \left(7\mathbf{i} - \dfrac{43}{2}\mathbf{j} + 15\mathbf{k}\right)$	**B1ft**
E.g.	$\overrightarrow{XA} = \pm\left(\left(7\mathbf{i} - \dfrac{43}{2}\mathbf{j} + 15\mathbf{k}\right) - \left(-4\mathbf{j} + \mathbf{k}\right)\right)$ $\overrightarrow{XB} = \pm\left(\left(8+4\mu\right)\mathbf{i} + \left(-2+\mu\right)\mathbf{j} + \left(5+2\mu\right)\mathbf{k} - \left(-4\mathbf{j}+\mathbf{k}\right)\right)$	**M1**
	$AX\cos\theta = BX \Rightarrow \left(7^2 + \left(-\dfrac{35}{2}\right)^2 + 14^2\right)\left(\dfrac{121}{945}\right) = \left(8+4\mu\right)^2 + \left(2+\mu\right)^2 + \left(4+2\mu\right)^2$ $\Rightarrow 36\mu^2 + 144\mu + 23 = 0 \Rightarrow \mu = ..\left(= -\dfrac{1}{6}, -\dfrac{23}{6}\right)$	**M1**
	$\overrightarrow{OB} = \left(8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}\right) + "-\dfrac{1}{6}"\left(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right) = ...$	**dM1**
	$= \dfrac{22}{3}\mathbf{i} - \dfrac{13}{6}\mathbf{j} + \dfrac{14}{3}\mathbf{k}$ or $\left(\dfrac{22}{3}, -\dfrac{13}{6}, \dfrac{14}{3}\right)$	**A1**
		(5)

Notes

Alt (d) – by right triangle

B1ft: Correct coordinates for A, follow through their p

M1: Attempts to find at least two sides of the triangle AXB where X is the intersection point. (Note if no further progress in this method is made the main scheme can apply to allow M1 for just $\overrightarrow{AB}$) in terms of μ where relevant, using their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown.

M1: For a full, correct method leading to a value for μ. There are variation to that shown in the scheme, such as use of Pythagoras on all three sides, or via $AX\sin\theta = AB$. The correct side must be used for the hypotenuse, but allow use of decimals in the trig work but must be working in the correct mode to 3s.f..

dM1: Depends on previous M mark. Uses one of their parameters in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. They may use either solution to their quadratic for this mark.

A1: Correct answer only. Must have selected the root which corresponds to A and reject the other. Accept as coordinates or in vector form. Allow awrt 3 s.f. for the coordinates.

Question	Scheme	Marks
7(a)	$u = 4x + 2\sin 2x \Rightarrow \dfrac{du}{dx} = 4 + 4\cos 2x$ oe	M1 A1
	$\dfrac{du}{dx} = 4 + 4\cos 2x = 8\cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} e^{4x+2\sin 2x}\cos^2 x\, dx = \int_{(0)}^{(2\pi)} \dfrac{e^u}{8}\, du$	M1
	$= \left["\dfrac{1}{8}" e^u \right]_0^{2\pi}$ or $\left["\dfrac{1}{8}" e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	A1ft
Alt by inspection for first 4 marks	$\int e^{4x+2\sin 2x}\cos^2 x\, dx = \int e^{4x+2\sin 2x} \times \dfrac{1}{2}(1 \pm \cos 2x)\, dx$	(M1 A1)
	$= ke^{4x+2\sin 2x};= \left["\dfrac{1}{8}" e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	(M1; A1ft)
	$= \left[\dfrac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\dfrac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}} = \dfrac{1}{8}\left(e^{2\pi} - 1\right)$ *	A1*cso
		(5)
(b)	$V \left[= \pi \int_0^{\frac{\pi}{2}} y^2\, dx = \pi \int_0^{\frac{\pi}{2}} \left(6e^{2x+\sin 2x}\cos x\right)^2 dx \right] = 36\pi \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\, dx$	B1
	$K \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\, dx = \dfrac{K}{8}\left(e^{2\pi} - 1\right)$	M1
	$= \dfrac{9\pi}{2}\left(e^{2\pi} - 1\right)$	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative of u. Look for $\sin 2x \to ...\cos 2x$ May apply double angle formula first, look for $u = 4x + k\sin x\cos x \to \dfrac{du}{dx} = (4+)k\left(\cos^2 x - \sin^2 x\right)$ oe. Form should be correct but there may be sign errors.

A1: Correct derivative (any form).

M1: Applies a double angle formula – must be seen – and carries out the full substitution to get an integral in terms of u. Must include an attempt to replace dx. Do not be concerned with the limits for this mark.

A1ft: Integrates to achieve the correct form with their constant with correct limits assigned (either in u or reverted in x)– must have scored the M1.

A1*cso: Correct answer from fully correct working. Must have identified **correct** limits in terms of u or have reverted back to x with the original limits before giving the final answer.

Alt for first 4 marks, by inspection:

M1: Applies double angle formula for $\cos^2 x$ (allowing for sign error) in the integrand in the form shown – must be this form to be able to apply recognition.

A1: Correct expression for the integrand in this form.

M1: Integrates, achieving the correct form.

A1ft: Achieves the correct integral from correct work with correct limits assigned – the double angle formula must have been correct (follow through if e.g they lose the ½ from the double angle formula before integrating).

(b)

B1: Applies correct volume formula with π seen somewhere in the answer and squares correctly to achieve the correct y^2. The limits may be missing for this mark. May be implied by the correct answer.

M1: Makes the connection with part (a) and multiplies the answer to (a) by their constant formed from their attempt at y^2. The use of (a) may be via reuse of the substitution. May be implied by an expression $M\left(e^{2\pi}-1\right),\ M \neq \frac{1}{8}$ if no incorrect working is shown for this (but a clearly incorrect method seen is M0).

A1: cao. As shown or $\frac{9\pi}{2}e^{2\pi}-\frac{9\pi}{2}$

Question	Scheme	Marks
8	$y = 2x + x^3 + \cos x$	
	Assume (there is a stationary point so) $\frac{dy}{dx} = 0$ for some value of x	B1
	So for this x $\quad \frac{dy}{dx} = 2 + 3x^2 - \sin x$	M1
	$\frac{dy}{dx} = 0 \Rightarrow \sin x = 3x^2 + 2 \geqslant 2$	dM1
	This is a contradiction as $\lvert \sin x \rvert \leqslant 1$ for all x, hence the assumption is false and so the function has no stationary points.	A1cso
		(4)
		(4 marks)

Notes:

B1: For a clear, suitable assumption with reference to $\frac{dy}{dx} = 0$ or implication that it means $\frac{dy}{dx} = 0$ at some point in their proof. Need not mention "for some x", but stating "for all x" is B0. Accept just "Assume $\frac{dy}{dx} = 0$"

M1: Attempts the derivative achieving $2 + 3x^2 \pm \sin x$

dM1: Makes progress with a correct method leading to a contradiction. E.g. set their derivative equal to 0, makes $\sin x$ the subject and deduces greater than or equal to 2. Other methods are possible, e.g. states $x^2 \geq 0$ and $\sin x \leq 1$ and deduces $2 + 3x^2 - \sin x \geq 1$ (hence cannot be zero). It must be clear what the contradiction will be but allow if there are some inaccuracies in details e.g. condone if a strict inequality is used for this mark. There must be reference to both $(3)x^2 \geqslant 0$ and the range of $\sin x$ in some way.

Do not accept "this equation has no solutions" without proof as to why there can be no solutions.

Note: A sketch is not a proof. An algebraic approach is required, but accept verbal arguments (e.g. stating "the minimum of $2+3x^2$ is 2" and "$\sin x$ cannot exceed one" etc), which may accompany a sketch.

A1cso: Fully correct proof, with derivative correct and contradiction made clear and conclusion drawn referring back to no stationary points. All necessary details must have been given. Condone cases where strict inequalities have been given instead of unstrict ones.

Question	Scheme	Marks
9(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \sec t \tan t$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{3}\sec^2\left(t+\frac{\pi}{3}\right)$	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right)\frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\sqrt{3}\sec^2\left(t+\frac{\pi}{3}\right)}{\sec t \tan t}$ oe	M1 A1
		(3)
(a) Alt	$x = \sec t \qquad y = \sqrt{3}\frac{\tan t + \sqrt{3}}{1-\sqrt{3}\tan t}$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \sec t \tan t$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{3}\frac{\left(1-\sqrt{3}\tan t\right)\sec^2 t - \left(\tan t+\sqrt{3}\right)\left(-\sqrt{3}\sec^2 t\right)}{\left(1-\sqrt{3}\tan t\right)^2}$ (oe)	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{3}\frac{\left(1-\sqrt{3}\tan t\right)\sec^2 t - \left(\tan t+\sqrt{3}\right)\left(-\sqrt{3}\sec^2 t\right)}{\sec t \tan t\left(1-\sqrt{3}\tan t\right)^2}$ oe	M1 A1
(b)	$t = \frac{\pi}{3} \Rightarrow x = 2, y = -3$	B1
	$t = \frac{\pi}{3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{3}\sec^2\frac{2\pi}{3}}{\sec\frac{\pi}{3}\tan\frac{\pi}{3}} = \ldots(=2)$	M1
	Tangent is $y+3 = 2(x-2) \Rightarrow y = 2x-7$ cso	M1A1
		(4)
(c)	$y = \sqrt{3}\frac{\tan t \pm \tan\frac{\pi}{3}}{1 \pm \tan t \tan\frac{\pi}{3}}$	M1
	$x^2 = \sec^2 t = 1+\tan^2 t \Rightarrow \tan t = \sqrt{x^2-1} \Rightarrow y = \sqrt{3}\frac{\sqrt{x^2-1}+\sqrt{3}}{1-\sqrt{3}\sqrt{x^2-1}}$	M1 A1
	$= \sqrt{3}\frac{\sqrt{x^2-1}+\sqrt{3}}{1-\sqrt{3}\sqrt{x^2-1}} \times \frac{1+\sqrt{3}\sqrt{x^2-1}}{1+\sqrt{3}\sqrt{x^2-1}} = ..$	
	$= \sqrt{3}\frac{\sqrt{x^2-1}+\sqrt{3}\left(x^2-1\right)+\sqrt{3}+3\sqrt{x^2-1}}{1-\left(3x^2-3\right)} = \frac{..\sqrt{x^2-1}+..x^2}{4-3x^2}$	M1
	$= \frac{3x^2+4\sqrt{3x^2-3}}{4-3x^2}$	A1
		(5)

(12 marks)
Notes:
(a) **B1:** Both derivatives given correctly. See Alt for an alternative form for y **M1:** Divides their $\frac{dy}{dt}$ by their $\frac{dx}{dt}$ **A1:** Correct expression and isw after a correct answer, accept any equivalent – e.g. see the Alt form in scheme.
(b) **B1:** Correct values for x and y stated or implied. **M1:** Some evidence of substituting in $t=\frac{\pi}{3}$ to their gradient expression to find the gradient of the tangent. Must be evaluated. You may need to check their value matches their $\frac{dy}{dx}$ if no substitution or other evidence is shown. **M1:** Correct method for the equation of the tangent, using their evaluated gradient. Must use their $\frac{dy}{dx}$ for the gradient but allow a slip in sign on one side for the coordinates in the formula. If they use $y = mx + c$ they need to proceed as far as finding c. **A1cso:** Correct equation in the form stated. Must have come from a correct $\frac{dy}{dx}$.
(c) **M1:** Attempts the compound angle formula on y, accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c). **M1:** Uses $\sec^2 t = 1+\tan^2 t$ to find $\tan t$ or $\tan^2 t$ in terms of x and substitutes to produce an equation linking y and x. Allow if there are errors rearranging after a correct formula. **A1:** Any correct Cartesian formula, with trig terms evaluated. **M1:** Full method to reach an answer in the required form, so multiplies through numerator and denominator by appropriate factor for their denominator and expands the numerator. The denominator work may be implied. **A1**: Achieves the correct answer.

Alt (c)	$y=\sqrt{3}\dfrac{\tan t\pm\tan\frac{\pi}{3}}{1\pm\tan t\tan\frac{\pi}{3}}$	**M1**
	$\left(\Rightarrow y\left(1\pm\sqrt{3}\tan t\right)=\sqrt{3}\tan t\pm 3\Rightarrow \tan t=\dfrac{\pm y\pm 3}{\pm y\sqrt{3}\pm\sqrt{3}}\right)$ $x^2=\sec^2 t=1+\tan^2 t=1+\left(\dfrac{y-3}{\sqrt{3}\left(y+1\right)}\right)^2$	**M1** **A1**
	$\Rightarrow 3x^2-3=\left(\dfrac{y-3}{y+1}\right)^2\Rightarrow \dfrac{y-3}{y+1}=\sqrt{3x^2-3}\;\left(\text{as } y<-1\Rightarrow \dfrac{y-3}{y+1}>0\right)$	
	$\Rightarrow y=\dfrac{3+\sqrt{3x^2-3}}{1-\sqrt{3x^2-3}}=\dfrac{\left(3+\sqrt{3x^2-3}\right)\left(1+\sqrt{3x^2-3}\right)}{1-3x^2+3}=\dfrac{...x^2+...\sqrt{3x^2-3}}{4-3x^2}$	**M1**
	$=\dfrac{3x^2+4\sqrt{3x^2-3}}{4-3x^2}$	**A1**
		(5)

Alt (c)

M1: Attempts the compound angle formula on y, accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c).

M1: Rearranges to find tan t and uses $\sec^2 t=1+\tan^2 t$ to find x or x^2 in terms of tan t with their attempt at y to achieve an equation linking y and x. Allow if there are errors rearranging after a correct formula and the $\sqrt{3}$ may have been missing.

A1: Any correct Cartesian formula with all trig terms evaluated.

M1: Full method to reach an answer in the required form, takes the 1 across, square roots (no need for justification of which sign, and allow if wrong sign is chosen), makes y the subject, multiplies numerator and denominator by appropriate factor for their denominator and expands numerator. The denominator work may be implied.

A1: Achieves the correct answer.

Alt (c)	$y=\sqrt{3}\dfrac{\sin\left(t+\frac{\pi}{3}\right)}{\cos\left(t+\frac{\pi}{3}\right)}=\sqrt{3}\dfrac{\sin t\cos\frac{\pi}{3}+\cos t\sin\frac{\pi}{3}}{\cos t\cos\frac{\pi}{3}-\sin t\sin\frac{\pi}{3}}$	**M1**
	$\left(\Rightarrow y\left(\cos t-\sqrt{3}\sin t\right)=\sqrt{3}\sin t+3\cos t\right)\Rightarrow y\left(\dfrac{1}{x}-\sqrt{3}\sqrt{1-\dfrac{1}{x^2}}\right)=\sqrt{3}\sqrt{1-\dfrac{1}{x^2}}+\dfrac{3}{x}$	**M1** **A1**

Alt (c) By sine and cosine, or variants, apply as follows.

M1: Writes tan in terms of sin and cos and attempts the compound angle formula on y, accept with sign errors. Allow if the $\sqrt{3}$ is missing. Must be seen or used in part (c).

M1: Applies $\sin^2 t = 1 - \cos^2 t$ throughout to get an equation linking y and x. The $\sqrt{3}$ may have been missing.
A1: Any correct Cartesian formula with all trig terms evaluated.
M1A1: Per main scheme.