Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}}=1+\left(-\tfrac{3}{2}\right)(-2x)+\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2+\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(-2x)^3+\dots$	M1 A1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8+24x+60x^2+140x^3+\dots$	A1, A1
		(5)
(b)	$n=2$	B1
		(1)
(c)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^2=\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2$	B1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\left(\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2\right)\left(8+24x+60x^2+140x^3+\dots\right)$ $=8\times\frac{1}{16}+24x\times\frac{1}{16}+60x^2\times\frac{1}{16}-8\times\frac{1}{4}x-24x\times\frac{1}{4}x+8\times\frac{1}{4}x^2$	M1
	$=\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+\dots$	A1
		(3)
		Total 9

(a)

B1: Takes out the **correct and simplified** common factor to obtain $8(1\pm\dots)^{-\frac{3}{2}}$ Implied by an expansion 8 + ...

M1: Attempts the binomial expansion of $(1\pm kx)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.
The correct binomial coefficient must be combined with the correct power of x.
Look for $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(\pm kx)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(\pm kx)^3$ with or without the brackets on the kx
Even allow with $k=1$

A1: Correct simplified or unsimplified expansion for $(1-2x)^{-\frac{3}{2}}$. (NB simplified is $1+3x+\frac{15}{2}x^2+\frac{35}{2}x^3+\dots$)

A1: Two correct and simplified terms of $8+24x+60x^2+140x^3$

A1: All correct and simplified $8+24x+60x^2+140x^3$ Ignore extra terms of x^4 and above

(b)

B1: Correct value, $n = 2$. Do NOT allow incomplete answers such as $\frac{4}{2}$

(c) **Hence**

B1: Correct expansion of $\left(\frac{1}{4}-\frac{1}{2}x\right)^2$, simplified or unsimplified

M1: Attempts correct strategy to find expansion.
Terms do not need to be collected. There may be other terms as well, for example terms in x^3
Look for an attempt to find 4 or more terms from the following (condoning slips)
$\left(A+Bx+Cx^2\right)\left(E+Fx+Gx^2+Hx^3+\dots\right)=AE+AFx+AGx^2+BEx+BFx^2+CEx^2+\dots$

A1: Correct simplified expansion. Ignore extra terms of x^3 and above

(c) **Otherwise:** Applies binomial expansion to $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$

B1: Correct simplified expression $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\frac{1}{2}(1-2x)^{\frac{1}{2}}$

M1: Correct structure for 3rd term: $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-2x)^2$ but allow $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(\pm kx)^2$ with or without brackets. Even allow with $k=1$

A1: $\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+...$ Ignore extra terms of x^3 and above

Direct expansion in (a)

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}\right)^{-\frac{3}{2}}+\left(-\frac{3}{2}\right)\left(\frac{1}{4}\right)^{-\frac{5}{2}}\left(-\frac{1}{2}x\right)+\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{-\frac{7}{2}}\left(-\frac{1}{2}x\right)^2+\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{-\frac{9}{2}}\left(-\frac{1}{2}x\right)^3$$

B1: Obtains an expansion with a constant term of 8

M1: Attempts the binomial expansion of $\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}$ to get the third and/or fourth term with an acceptable structure.

Look for $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{2}\left(\frac{1}{4}\right)^{-\frac{7}{2}}\left(\pm\frac{1}{2}x\right)^2$ or $\frac{\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)\times\left(-\frac{7}{2}\right)}{3!}\left(\frac{1}{4}\right)^{-\frac{9}{2}}\left(\pm\frac{1}{2}x\right)^3$

with or without the brackets on the $\left(\pm\frac{1}{2}x\right)$

A1: Correct simplified or unsimplified expansion .
Expression at the top of the page is acceptable

A1: Two correct and simplified terms of $8+24x+60x^2+140x^3$

A1: All correct and simplified $8+24x+60x^2+140x^3$

Direct expansion in (c)

B1: First two terms which may be unsimplified $\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}\left(-\frac{1}{2}x\right)^1+....$

M1: Correct form for the third term $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}\left(-\frac{1}{2}x\right)^2$

A1: Correct simplified expansion $\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+...$

Question Number	Scheme	Marks
2(a)	$x=2\Rightarrow 4-8y+y^2=13\Rightarrow\left(y^2-8y-9=0\Rightarrow\right)y=...$	M1
	$y=9$	A1
		(2)
(b)	$2^x\rightarrow 2^x\ln 2$	B1
	$-4xy\rightarrow -4x\frac{dy}{dx}-4y$ **OR** $y^2\rightarrow 2y\frac{dy}{dx}$	M1
	$2^x\ln 2-4x\frac{dy}{dx}-4y+2y\frac{dy}{dx}=0$	A1
	$\frac{dy}{dx}(2y-4x)=4y-2^x\ln 2\Rightarrow\frac{dy}{dx}=...$	M1
	$\frac{dy}{dx}=\frac{4y-2^x\ln 2}{2y-4x}$ or $\frac{dy}{dx}=\frac{2^x\ln 2-4y}{4x-2y}$ or $\frac{dy}{dx}=\frac{\ln 2e^{x\ln 2}-4y}{4x-2y}$	A1
		(5)
(c)	$(2,9)\rightarrow\frac{dy}{dx}=\frac{4(9)-2^2\ln 2}{2(9)-4(2)}$ $\Rightarrow y-"9"="\frac{36-4\ln 2}{10}"(x-2)$	M1
	$y=0\Rightarrow 0-"9"="\frac{36-4\ln 2}{10}"(x-2)\Rightarrow x=...$	dM1
	$x=\frac{4\ln 2+9}{2\ln 2-18}$ o.e. e.g. $x=\frac{-8\ln 2-18}{-4\ln 2+36}$.	A1
		(3)
		Total 10

(a)
M1: Substitutes $x=2$ into the equation for C, forms a quadratic equation in y and then solves to obtain at least one value for y
A1: $y=9$ only

(b) It is acceptable to use $y'\leftrightarrow\frac{dy}{dx}$ in this question

B1: Correct differentiation of 2^x. Allow also $2^x=e^{x\ln 2}\rightarrow e^{x\ln 2}\ln 2$

M1: Differentiates $-4xy\rightarrow\pm Ax\frac{dy}{dx}\pm By$ **OR** differentiates $y^2\rightarrow k\,y\frac{dy}{dx}$

A1: Fully correct differentiation. Allow versions such as $2^x\ln 2dx-4xdy-4ydx+2ydy=0$

M1: Attempts to make $\frac{dy}{dx}$ the subject with the 2 terms in $\frac{dy}{dx}$ coming from differentiating y^2 and $-4xy$

A1: Any correct expression for $\frac{dy}{dx}$. Note that you can isw after a correct answer. $\frac{2y-2^{x-1}\ln 2}{y-2x}$ **is** correct

(c)
M1: Uses $x = 2$ and their y value from part (a) to find the gradient at P and attempts to form the equation of the tangent at P. Condone poor attempts at differentiation but $\frac{dy}{dx}$ must have both x and y terms.
If the form $y = mx + c$ is used they must proceed as far as $c = ...$
dM1: Substitutes $y = 0$ into their tangent equation and rearranges to find x. Dependent upon previous M
It is possible for a method to combine this with the previous M.
Look for methods like the following, which score M1, dM1
$-"9" = "m"(x-2) \Rightarrow x = ...$ where 'm' is the value of their $\frac{dy}{dx}$ at $x = 2$
or $\frac{"9"}{2-x} = "m" \Rightarrow x = ...$ where 'm' is the value of their $\frac{dy}{dx}$ at $x = 2$

A1: Correct expression in the required form. Must come from a correct $\frac{dy}{dx}$
Do not be concerned about the order of the terms on the numerator and denominator

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1}+\frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3)+B(2x-1) \Rightarrow A=\ldots\ B=\ldots$	M1
	$A=1$ or $B=2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1}+\frac{2}{4x-3}$	A1
		(3)
(b)	$\int\left(\frac{1}{2x-1}+\frac{2}{4x-3}\right)\mathrm{d}x = \frac{1}{2}\ln\|2x-1\|+\frac{1}{2}\ln\|4x-3\|(+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(4x-3)\right]_k^{3k} = \frac{1}{2}\ln(6k-1)(12k-3)-\frac{1}{2}\ln(2k-1)(4k-3)$ $=\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2}\ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2-170k+57=0$	dM1A1
	$88k^2-170k+57=0 \Rightarrow k=\ldots$	ddM1
	$k=\frac{3}{2}$	A1
		(5)
		Total 11

(a)

M1: "Correct" P.F form (condoning slips) with strategy to find at least one constant or partial fraction.

Look for an attempt at the correct form (condoning slips) $5-8x = A(4x-3)+B(2x-1)$ followed by values for A or B

A1: One correct fraction or constant

A1: Correct partial fractions which may be awarded for sight of correct PF in part (b).

Award if the correct answer is just written down with little or no working.

Award if the correct partial fractions are seen in (b)

(b)

M1: For $\int \frac{\alpha}{2x-1}\,\mathrm{d}x = \beta\ln|2x-1|$ **OR** $\int \frac{\alpha}{4x-3}\,\mathrm{d}x = \beta\ln|4x-3|$

Allow for example brackets instead of moduli, e.g. $\ln(2x-1) \leftrightarrow \ln|2x-1|$

Condone the omission of moduli/brackets, for example $\int \frac{\alpha}{2x-1}\,\mathrm{d}x = \beta\ln 2x-1$ for this mark only

A1ft: For $\int \frac{A}{2x-1}\,\mathrm{d}x = \frac{A}{2}\ln|2x-1|$ **OR** $\int \frac{B}{4x-3}\,\mathrm{d}x = \frac{B}{4}\ln|4x-3|$ Follow through their A and B

Allow for example brackets instead of moduli, e.g . $\ln(2x-1) \leftrightarrow \ln|2x-1|$

A1ft: For $\int \frac{A}{2x-1}\,dx + \int \frac{B}{4x-3}\,dx = \frac{A}{2}\ln|2x-1| + \frac{B}{4}\ln|4x-3|$

or $\int \frac{A}{2x-1}\,dx + \int \frac{B}{4x-3}\,dx = \frac{A}{2}\ln(2x-1) + \frac{B}{4}\ln(4x-3)$

Follow through their numerical A and B. There is no requirement for the $+c$
ISW after a correct answer. This can be awarded (but not implied) from part (c)

Note that $\frac{1}{2}\ln(2x-1)(4x-3)(+c)$ and $\ln\sqrt{(2x-1)(4x-3)}(+c)$ are also correct

(c)

M1: Attempts the use of the correct limits and uses correct log work to combine terms to obtain a single ln term.
It is dependent upon having an answer to part (b) involving two ln terms.
Condone slips/ bracketing errors .

E.g $\left\{\frac{1}{2}\ln(6k-1)+\frac{1}{2}\ln(12k-3)\right\}-\left\{\frac{1}{2}\ln(2k-1)+\frac{1}{2}\ln(4k-3)\right\}=\frac{1}{2}\ln(6k-1)+\frac{1}{2}\ln(12k-3)-\frac{1}{2}\ln(2k-1)+\frac{1}{2}\ln(4k-3)$

$$=\frac{1}{2}\ln\frac{(6k-1)(12k-3)(4k-3)}{(2k-1)}$$

dM1: Eliminates ln's, multiplies up and collects terms to form a polynomial equation in k.
It is dependent upon the previous M

A1: Correct 3TQ. The = 0 may be implied by subsequent work

ddM1: Solves 3TQ. Dependent upon both previous M's.
Can be solved via a graphical calculator (you may need to check the answers)

A1: Correct value and no others. Condone $x=\frac{3}{2}$

Solution with limited working in (c)

Case I

$\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=\frac{1}{2}\ln 20 \Rightarrow x \text{ or } k=\frac{3}{2}$ SC 11000

Case II

$\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=20 \Rightarrow x \text{ or } k=\frac{3}{2}$ SC 11100

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r}=4\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+6\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g $\overrightarrow{PC}=\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}$	M1
	Uses $\overrightarrow{PC}.(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ E.g. $\Rightarrow 1(2+\lambda)-2(10-2\lambda)+2\times2\lambda=0\Rightarrow\lambda=...$	dM1
	E.g I $\lambda=2\Rightarrow\mathbf{c}=(4+2)\mathbf{i}+(8-4)\mathbf{j}+(1+4)\mathbf{k}$ E.g II $\mu=1\Rightarrow\mathbf{c}=(5+1)\mathbf{i}+(6-2)\mathbf{j}+(3+2)\mathbf{k}$	ddM1
	$(6,\ 4,\ 5)$	A1
		(4)
(c)	$\overrightarrow{OP'}=\mathbf{p}+2\overrightarrow{PC}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}+2(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$	M1
	$(10,\ 10,\ 9)$	A1
		(2)
(d)	$\lvert\overrightarrow{PP'}\rvert=2\lvert\overrightarrow{PC}\rvert=2\sqrt{4^2+6^2+4^2}=...$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

General rule in this question: If no method is shown look for two "correct" components for their vector

(a) **There are many correct versions so please check the candidates answer carefully**

M1: Attempts the direction of *l* by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.

If no method is shown look for two correct components of $(\pm1\mathbf{i}\pm2\mathbf{j}\pm2\mathbf{k})$ or a multiple of this

A1: Any correct equation including the lhs of '$\mathbf{r}=$'

Allow in the form $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ but $l=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and is $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}\mathrm{i}\\-2\mathrm{j}\\2\mathrm{k}\end{pmatrix}$ A0

(b)

M1: Attempts the general vector from *P* to *l* forming $\overrightarrow{PC}$ where *C* is a general point on *l*.

Look for an attempt to subtract their "$\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$" and $\begin{pmatrix}2\\-2\\1\end{pmatrix}$ either way around

dM1: Attempts $\overrightarrow{PC}$.their$(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ and solves for λ.

It is dependent upon the previous M and the scalar product of this vector must be attempted with their gradient for the line *l* to produce and solve a linear equation for λ.

ddM1: Uses their λ to find *C*. Scored for substituting their correctly found λ into their equation for *l*

A1: Correct coordinates or vector.

Only penalise an error such as $\begin{pmatrix}6\mathrm{i}\\4\mathrm{j}\\5\,\mathrm{k}\end{pmatrix}$ once, the first time that it is made. Hence award if a mark has already been withheld for such a mistake.

(c)
M1: Correct strategy for $\overrightarrow{OP'}$ Can be implied by two correct coordinates using their P and C
For example look for $\overrightarrow{OP'} = \overrightarrow{OP} + 2\times\overrightarrow{PC}$ using the coordinates of C found in part (b).
A1: Correct coordinates or vector
(d)
M1: Correct method for $|\overrightarrow{PP'}|$ using their values for P and P' or P and C or P' and C. E.g $|\overrightarrow{PP'}| = 2\times|\overrightarrow{PC}|$
There are many ways to do this but it must be a complete method, not a distance squared.
A1: CAO

Alternative part (b) using shortest distance
M1: Attempts the general vector from P to l.
Look for an attempt to subtract their "$\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$" and $\begin{pmatrix}2\\-2\\1\end{pmatrix}$ either way around
dM1: Finds an expression for d or d^2, differentiates and sets = 0 to find λ.
As it is a quadratic expression an equivalent method would be via completing the square .

$$\text{FYI}: D^2 = (2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 = 9\lambda^2 - 36\lambda + 104 \Rightarrow \frac{dD^2}{d\lambda} = 18\lambda - 36 = 0 \Rightarrow \lambda = 2$$

ddM1: Uses their λ to find C.
A1: Correct coordinates or vector

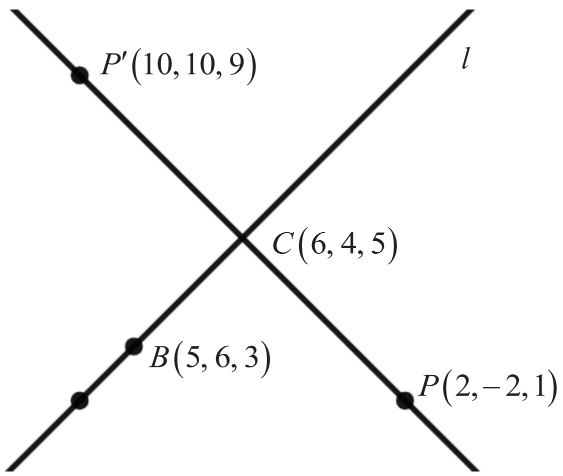
Alternative part (b) using Pythagoras' Theorem

M1: Attempts the general vector from P to l.
Look for an attempt to subtract their "$\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$" and $\begin{pmatrix}2\\-2\\1\end{pmatrix}$ either way around
dM1: Uses $PC^2 + CB^2 = PB^2$ or $PC^2 + CA^2 = PA^2$ to set up and solve an equation in λ.
E.g Using $PC^2 + CB^2 = PB^2$ the equations are

$$(2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 + (\lambda-1)^2 + (2-2\lambda)^2 + (2\lambda-2)^2 = 77$$

$$\Rightarrow 18\lambda^2 - 54\lambda + 36 = 0 \Rightarrow \lambda = 2(1)$$

ddM1: Uses their λ to find C.
A1: Correct coordinates or vector



Question Number	Scheme	Marks
5(i)	$\int x^2e^x\,dx = x^2e^x - \int 2xe^x\,dx(+c)$	M1A1
	$\int xe^x\,dx = xe^x - \int e^x\,dx(+c)$	M1
	$\int x^2e^x\,dx = x^2e^x - 2xe^x + 2e^x(+c)$ Also allow $\int x^2e^x\,dx = x^2e^x - (2xe^x - 2e^x)(+c)$	A1
		(4)
(ii)	$u = (1-3x)^{\frac{1}{2}} \Rightarrow u^2 = 1-3x \Rightarrow 2u\frac{du}{dx} = -3$ or $u = (1-3x)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}}\,dx = 27\int \frac{\frac{1-u^2}{3}}{u}\left(-\frac{2u}{3}\right)du$ or $\int \frac{27x}{\sqrt{1-3x}}\,dx = 27\int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}}\left(-\frac{2}{3}\sqrt{1-3x}\right)du$	M1
	$6\int (u^2-1)\,du$ or $-6\int (1-u^2)\,du$	A1
	$6\left(\frac{u^3}{3} - u\right)(+k)$	A1ft
	$6\left(\frac{(1-3x)^{\frac{3}{2}}}{3} - (1-3x)^{\frac{1}{2}}\right)(+k) = 2(1-3x)^{\frac{1}{2}}(1-3x-3)(+k)$	M1
	$= -2(1-3x)^{\frac{1}{2}}(2+3x)(+k)$ or $= -2(1-3x)^{\frac{1}{2}}(3x+2)(+k)$	A1
		(6)
		Total 10

(i) Condone missing dx's here

M1: For applying parts to obtain $x^2e^x - k\int xe^x\,dx(+c)$ where $k > 0$

A1: Correct expression. No requirement for $+c$

M1: Applies parts again to $\int xe^x\,dx$ and obtains $\int xe^x\,dx = xe^x - \alpha\int e^x\,dx(+c)$ where $\alpha > 0$

Note that if the whole expression is written out then this mark is implied by
$\int x^2e^x\,dx = x^2e^x - \alpha xe^x \pm \int \beta e^x\,dx$ or $\int x^2e^x\,dx = x^2e^x - \alpha xe^x \pm \beta e^x$ where $\alpha > 0$

A1: Correct answer (+ c not required)

Watch for $\int x^2\mathrm{e}^x\,\mathrm{d}x = x^2\mathrm{e}^x - \left(2x\mathrm{e}^x - 2\mathrm{e}^x\right)(+c)$ which scores all 4 marks. ISW after sight of this

Watch for D & I method which may be seen. You will just see the answer here.

D		**I**
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

$= x^2\mathrm{e}^x - 2x\mathrm{e}^x + 2\mathrm{e}^x$

Score M2 for $x^2\mathrm{e}^x \pm 2x\mathrm{e}^x \pm 2\mathrm{e}^x$
and then A2 for $x^2\mathrm{e}^x - 2x\mathrm{e}^x + 2\mathrm{e}^x\,(+c)$

(ii)

B1: Any correct expression involving $\frac{\mathrm{d}u}{\mathrm{d}x}$ which may be "split" E.g. $\mathrm{d}u = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}\mathrm{d}x$

M1: Substitutes $x = \mathrm{f}\left(u^2\right)$ to fully change the integrand from $\frac{27x}{\sqrt{1-3x}}\mathrm{d}x$ to $k\frac{\mathrm{f}\left(u^2\right)}{u}u\,\mathrm{d}u$ o.e.

Note that the u's may have been cancelled.
Condone the du not being present but it cannot appear as dx

A1: Correct simplified integral. Look for the u's being cancelled and some simplification/collection of the constant terms

Look for, for example. integrals such as $6\int\left(u^2-1\right)\mathrm{d}u$, $\int 6u^2 - 6\,\mathrm{d}u$, $-6\int\left(1-u^2\right)\mathrm{d}u$ or even

$-\frac{2}{3}\int\left(9-9u^2\right)\mathrm{d}u$ The brackets may be implied by subsequent work

Condone the du not being present as long as it is implied by subsequent integration

A1ft: Correct follow through integration on their simplified integral.
It is dependent upon having achieved an integral of the form above

E.g. $\alpha\int\left(u^2-1\right)\mathrm{d}u$ or $\beta\int\left(1-u^2\right)\mathrm{d}u$ $\alpha,\beta > 0$. No requirement for + k

M1: Achieves an integral of $Pu^3 + Qu$ and

- "correctly" takes out a common factor of u or $(1-3x)^{\frac{1}{2}}$ for their $Pu^3 + Qu$ or $\alpha(1-3x)^{\frac{3}{2}} + \beta(1-3x)^{\frac{1}{2}}$ AND proceeds to the form $(1-3x)^{\frac{1}{2}}\left(\delta(1-3x)+\varepsilon\right)$ or better. Condone slips on coefficients
- substitutes $u = (1-3x)^{\frac{1}{2}}$

A1: CAO. No requirement for + k

Question Number	Scheme	Marks
6(a)	$\frac{d\theta}{dt}=-k(\theta-15)^2 \Rightarrow \int \frac{d\theta}{(\theta-15)^2} = \int -k\,dt$	B1
	$\int \frac{d\theta}{(\theta-15)^2} = -(\theta-15)^{-1}$	M1
	$-\frac{1}{\theta-15}=-kt+c$	A1
	$t=0,\theta=85 \Rightarrow -\frac{1}{70}=c$	M1
	$t=10,\theta=40 \Rightarrow \frac{1}{25}=10k+\frac{1}{70} \Rightarrow k=...\left(\frac{9}{3500}\right)$	M1
	$\frac{1}{\theta-15}=\frac{9t}{3500}+\frac{1}{70} \Rightarrow \theta=...$	M1
	$\theta=\frac{135t+4250}{9t+50}$	A1
		(7)
(b)	$20=\frac{135t+4250}{9t+50} \Rightarrow t=...$	M1
	$t=$ awrt 72	A1
		(2)
		Total 9

(a) **Note that candidates cannot work backwards from the answer using differentiation**

B1: Correct separation of variables.
The integral signs do not have to be present, but the $d\theta$ and dt do, and be in the correct positions

M1: Integrates $\int \frac{1}{(\theta-15)^2}\,d\theta$ to $\frac{\alpha}{\theta-15}$

A1: Correct equation including a k and another **different** constant.

M1: Uses $t=0,\theta=85$ to find "c". May be awarded after incorrect integration. May be also awarded if the constant k had been assigned a value using given values of θ and t.
Their initial equation may have been (incorrectly) adapted but it must be solvable with $t=0,\theta=85$ used leading to c =...

M1: Uses t = 10 and θ = 40 with their value of "c" to find k.
May be awarded after incorrect integration but there must have been two constants. Their initial equation may have been adapted but it must be solvable with t = 10, θ = 40 AND their c =...

M1: Rearranges using correct algebra to obtain θ in terms of t.
For this to be awarded the integral must be in the correct form and both constants found using a correct method . Condone slips in working but the overall process should be sound.
Look for $\frac{1}{\theta-15}=\alpha t+\beta \Rightarrow \theta=\frac{ct+d}{et+f}$

A1: Correct expression. Allow integer multiples of the given answer E.g. $\theta=\frac{297500+9450t}{3500+630t}$

(b)

M1: Uses their answer to part (a) or equivalent (or possibly an earlier equation) to find t when $\theta = 20$.

For this to be awarded, substitute $\theta = 20$ into a "correct" form such as $\frac{1}{\theta - 15} = \alpha t + \beta \Rightarrow t = ...$

Or substitute $\theta = 20$ into a "correct" form such as $\theta = \frac{ct+d}{et+f} \Rightarrow t = ...$ where $c, d, e, f \neq 0$

They cannot just make values up for the c, d, e and f for example. It must follow their work in (a)

A1: Correct value which must have come from a correct equation. Exact value is $\frac{650}{9}$ but allow awrt 72

Of course candidates can move the "k" or "$-k$" term over to the LHS or use limits .

6(a)	$\frac{d\theta}{dt} = -k(\theta-15)^2 \Rightarrow \int \frac{d\theta}{-k(\theta-15)^2} = \int dt$	B1
	$\int \frac{d\theta}{(\theta-15)^2} = \alpha(\theta-15)^{-1}$	M1
	$\frac{1}{k(\theta-15)} = t + \beta$	A1
	$t = 0, \theta = 85 \Rightarrow \frac{1}{70k} = \beta$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25k} = \beta + 10 \Rightarrow k = ...\left(\frac{9}{3500}\right) \quad \beta = ...\left(\frac{50}{9}\right)$	M1
	$\frac{3500}{9(\theta-15)} = t + \frac{50}{9} \Rightarrow \theta = ...$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1

6(a)	$\frac{d\theta}{dt} = -k(\theta-15)^2 \Rightarrow \int \frac{d\theta}{(\theta-15)^2} = \int -k\,dt$	B1
	$\int \frac{d\theta}{(\theta-15)^2} = -(\theta-15)^{-1}$	M1
	Correct equation using limits $\left[-\frac{1}{\theta-15}\right]_{85}^{40} = [-kt]_0^{10}$ Or $\left[\frac{1}{\theta-15}\right]_{85}^{40} = [kt]_0^{10}$	A1
	Correct attempt to find k $\quad -\frac{1}{25} + \frac{1}{70} = -10k \Rightarrow k = ...\left(\frac{9}{3500}\right)$	M1
	Correct attempt to find equation linking θ and t E.g. $\left[-\frac{1}{\theta-15}\right]_{85}^{\theta} = [-kt]_0^{t}$	M1
	$-\frac{1}{\theta-15} + \frac{1}{70} = -\frac{9t}{3500} \Rightarrow \theta = ...$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1

Question Number	Scheme	Marks
7	Assume that $\sqrt{7}$ is rational so that $\sqrt{7}=\frac{a}{b} \Rightarrow 7=\frac{a^2}{b^2}$ (where a and b have no factors in common)	M1
	$7=\frac{a^2}{b^2} \Rightarrow 7b^2=a^2$ So a^2 is a multiple of 7 which means a is a multiple of 7	A1
	$a=7k \Rightarrow 7b^2=49k^2 \Rightarrow b^2=7k^2$	M1
	So b^2 is a multiple of 7 which means b is a multiple of 7. As a and b are both multiples of 7, this contradicts the fact that a and b have no factors in common. Hence $\sqrt{7}$ must be irrational.	A1
		(4)
		Total 4

Any variables may be used

M1: Starts the proof by contradicting the given statement and squares both sides.

Condone the omission of the statement about common factors here.
As a minimum allow "assume $\sqrt{7}=\frac{a}{b} \Rightarrow 7=\frac{a^2}{b^2}$" or similar "let $\sqrt{7}=\frac{p}{q} \Rightarrow p^2=7q^2$"

A1: Reaches for example $7b^2=a^2$ and deduces that a^2 is a multiple of 7 which means a is a multiple of 7
Condone these being the "wrong way around" as long as they state both a and a^2 are multiples of 7
Look for

- $7b^2=a^2$
- states a is a multiple of 7 (which may be implied by setting $a=7k$ for example)
- states a^2 is a multiple of 7

M1: Sets $a=7k$ and proceeds to $b^2=7k^2$

A1: Fully Correct proof. This must include

- an initial statement that included that a and b had no common factors. Allow equivalent statements such as $\frac{a}{b}$ is a fraction in simplest form or co-prime, relatively prime, mutually prime
- the statements and conclusions the correct way around.
  E.g. a^2 is a multiple of 7 which means a is a multiple of 7 (not the other way around) and b^2 is a multiple of 7 which means b is a multiple of 7 (not the other way around)
- a minimal conclusion

Be tolerant of slight issues with terminology/language used as this may not be the students first language.

Question Number	Scheme	Marks
8(a)	$x=2$	B1
		(1)
(b)	$(2.5, 1.5)$	B1
		(1)
(c)	$\dfrac{dy}{dx}=\dfrac{1+\frac{1}{t^2}}{1-\frac{1}{t^2}}$	M1
	$t=2\Rightarrow\dfrac{dy}{dx}=\dfrac{5}{3}\ \Rightarrow y-1.5=-\dfrac{3}{5}(x-2.5)$	dM1
	$3x+5y=15*$	A1*
		(3)
(d)	l crosses x-axis at $x=5$	B1
	$\dfrac{1}{3}\pi\times1.5^2(5-2.5)\left(=\dfrac{15}{8}\pi\right)$	M1
	$V=\pi\displaystyle\int y^2dx=\pi\int y^2\frac{dx}{dt}\,dt=\pi\int\left(t-\frac{1}{t}\right)^2\left(1-\frac{1}{t^2}\right)dt$	M1
	$\pi\displaystyle\int\left(t^2-2+\frac{1}{t^2}\right)\left(1-\frac{1}{t^2}\right)dt=\pi\int\left(t^2+\frac{3}{t^2}-\frac{1}{t^4}-3\right)dt=\ldots$	M1
	$\pi\left[\dfrac{t^3}{3}-\dfrac{3}{t}+\dfrac{1}{3t^3}-3t\right]$	A1
	$\pi\left[\dfrac{t^3}{3}-\dfrac{3}{t}+\dfrac{1}{3t^3}-3t\right]_1^2=\pi\left(\left(\dfrac{8}{3}-\dfrac{3}{2}+\dfrac{1}{24}-6\right)-\left(\dfrac{1}{3}-3+\dfrac{1}{3}-3\right)\right)$	M1
	$V=\dfrac{13}{24}\pi+\dfrac{15}{8}\pi=\dfrac{29}{12}\pi$ Cao	A1
		(7)
		Total 12

(a)
B1: Correct value. Just look for 2 and condone $Q=2$

(b)
B1: Correct coordinates. Allow these to be given separately $x=2.5,\quad y=1.5$

(c)

M1: Attempts to differentiate both parameters and use $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}$ to achieve $\dfrac{1\pm\frac{1}{t^2}}{1\pm\frac{1}{t^2}}$ or equivalent via quotient rule

dM1: Uses $t=2$ in their $\dfrac{dy}{dx}$ and uses the negative reciprocal gradient with their point P to form equation of normal.It is dependent upon the previous M

A1*: Obtains the printed answer with no errors and sufficient working shown.
What is shown in the MS is sufficient

(d)

B1: Correct x intercept for l. This may appear on the x-axis on Figure 2.
It may be implied by the value appearing in the volume of the cone but not the volume under the curve

M1: Fully correct method for the cone volume $\frac{1}{3}\pi \times "1.5"^2("5"-"2.5") =$

May be awarded for $\int_{2.5}^{5} \pi(3-0.6x)^2\,dx = \left[\frac{\pi(3-0.6x)^3}{-1.8}\right]_{2.5}^{5} = \frac{\pi(3-0.6\times 2.5)^3}{1.8}$

Condone slips. If they expand $(3-0.6x)^2$ look for a quadratic expression integrating to a cubic of the correct form followed by use of the limits 5 and their 2.5

M1: Correct strategy for the volume generated by rotating the area under the curve

Look for $V = \pi\int y^2 dx = \pi\int y^2 \frac{dx}{dt}\,dt$ leading to $\pi\int\left(t-\frac{1}{t}\right)^2\left(1\pm\frac{k}{t^2}\right)dt$, where k is a constant

Condone a sign slip on $\frac{dx}{dt}$ so allow $\left(1\pm\frac{1}{t^2}\right)$. The π may be added later in the solution.

The dt may be omitted

M1: Attempt to expand $\left(t-\frac{1}{t}\right)^2\left(1\pm\frac{k}{t^2}\right)$ and integrate term by term.

Look for $\left(t-\frac{1}{t}\right)^2\left(1\pm\frac{k}{t^2}\right) \to$ polynomial in t containing positive and negative indices followed by some correct integration for both positive and negative indices

A1: Correct integration of $\left(t-\frac{1}{t}\right)^2\left(1-\frac{1}{t^2}\right) \to \frac{t^3}{3}-\frac{3}{t}+\frac{1}{3t^3}-3t$ which does not need to be simplified

M1: Applies the t limits "1" and 2 to an attempt at the integral of $(\pi)\int\left(t-\frac{1}{t}\right)^2\left(1\pm\frac{k}{t^2}\right)dt$

The π does not need to be included.

A1: $\frac{29}{12}\pi$ or exact equivalent.

....

Part (c) using a calculator
Whilst it is possible to do the integration for the area under the curve using a calculator, it will not score all of the marks.

E.g. $\pi\int_1^2\left(t-\frac{1}{t}\right)^2\left(1-\frac{1}{t^2}\right)dt = 0.541\dot{6}\pi$

The marks in (c) available to such a candidate are:
B1(for $x = 5$),
M1 (for volume of cone),
M1 (Sight of correct integral),
M0 (no expansion),
A0 ,
M1(sight of limits),
A0

....

See below for Cartesian approach for (c) and (d)

(c)	$x = t + \frac{1}{t},\ \ y = t - \frac{1}{t} \Rightarrow x^2 - y^2 = 4 \Rightarrow 2x - 2y\frac{dy}{dx} = 0$ or $y^2 = x^2 - 4 \Rightarrow y = \left(x^2 - 4\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = x\left(x^2 - 4\right)^{-\frac{1}{2}}$ Attempts to differentiate a Cartesian equation of the form $x^2 - y^2 = k$ to obtain either $\alpha x + \alpha y\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = kx\left(x^2 - 4\right)^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx} = \frac{x}{y} = \frac{t + \frac{1}{t}}{t - \frac{1}{t}} = \frac{2.5}{1.5}$ or $\frac{dy}{dx} = 2.5\left(2.5^2 - 4\right)^{-\frac{1}{2}}$ $\Rightarrow y - 1.5 = -\frac{3}{5}(x - 2.5)$ Uses $t = 2$ in their $\frac{dy}{dx}$ and uses the negative reciprocal gradient with their point P. If using explicit differentiation, must attempt x using $t = 2$ first	M1
	$3x + 5y = 15*$ Obtains the printed answer with no errors and sufficient working shown.	A1*
		(3)
(d)	l crosses x-axis at $x = 5$	B1
	$\frac{1}{3}\pi \times 1.5^2\,(5 - 2.5)\left(= \frac{15}{8}\pi\right)$ Fully correct method for the cone volume	M1
	$V = \pi\int y^2 dx = \pi\int \left(x^2 - 4\right) dx$ Correct strategy for the other volume Condone for this $V = \pi\int y^2 dx = \pi\int \left(x^2 - k\right) dx$	M1
	$\pi\int \left(x^2 - 4\right) dx = \pi\left[\frac{x^3}{3} - 4x\right]$ M1: Attempt to integrate $\int \left(x^2 - k\right) dx$ A1: Correct integration of $\int \left(x^2 - 4\right) dx$	M1A1
	$\pi\left[\frac{x^3}{3} - 4x\right]_2^{2.5}$ Applies the limits "2" and their x from $t = 2$ to an attempted integral of $(\pi)\int \left(x^2 - k\right) dx$	M1
	$V = \frac{13}{24}\pi + \frac{15}{8}\pi = \frac{29}{12}\pi$ Cao	A1
		(7)