Question Number	Scheme	Marks
1	Attempt at chain rule $y^2 \to ...y\dfrac{dy}{dx}$	M1
	Attempt at product rule $3x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$	M1
	Correct differentiation $2 - 8y\dfrac{dy}{dx} + 6xy + 3x^2\dfrac{dy}{dx} = 8x$	A1
	Substitutes $x = 3, y = 2 \Rightarrow \dfrac{dy}{dx} = -\dfrac{14}{11}$	M1, A1
	Correct method for normal at $(3,2) \Rightarrow y - 2 = \dfrac{11}{14}(x-3)$	dM1
	$11x - 14y - 5 = 0$	A1
		(7)
		(7 marks)

M1: Attempts at chain rule $y^2 \to ...y\dfrac{dy}{dx}$ where ... is a constant

M1: Attempt at product rule $3x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$ where ... are constants

A1: Correct differentiation $2x - 4y^2 + 3x^2y = 4x^2 + 8 \to 2 - 8y\dfrac{dy}{dx} + 6xy + 3x^2\dfrac{dy}{dx} = 8x$ o.e.

You may see $2x - 4y^2 + 3x^2y = 4x^2 + 8 \to 2dx - 8ydy + 6xydx + 3x^2dy = 8xdx$ o.e. which is fine

Note that $\dfrac{dy}{dx} = 2 - 8y\dfrac{dy}{dx} + 6xy + 3x^2\dfrac{dy}{dx} - 8x$ is A0 unless recovered

M1: Substitutes $x = 3, y = 2$ into a suitable equation and finds a value for $\dfrac{dy}{dx}$.

It is dependent upon having exactly two $\dfrac{dy}{dx}$ terms, one from differentiating each of y^2 and x^2y

A1: $\dfrac{dy}{dx} = -\dfrac{14}{11}$ o.e.

dM1: Dependent upon the previous M. It is for a correct method of finding the equation of the normal at (3,2). Look for $y - 2 = "\dfrac{11}{14}"(x-3)$ with the negative reciprocal of their $-\dfrac{14}{11}$ being used.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: $11x - 14y - 5 = 0$ but accept any integer multiple of this. The "= 0" must be seen

Question Number	Scheme	Marks
2	$\frac{dy}{dx}=\frac{4y^2}{\sqrt{4x+5}}\Rightarrow\int\frac{1}{y^2}dy=\int\frac{4}{\sqrt{4x+5}}dx$	B1
	$-\frac{1}{y}=2\sqrt{4x+5}\,(+c)$	M1, A1
	Substitutes $y=\frac{1}{3},\ x=-\frac{1}{4}\Rightarrow -3=4+c\Rightarrow c=...$	dM1
	Rearranges $\frac{a}{y}=b\sqrt{4x+5}+c$ to $y=....$	ddM1
	$y=\frac{1}{7-2\sqrt{4x+5}}$	A1
		(6) **(6 marks)**

B1: Separates the variables. Note that the "4" may be on either side but must be in the correct place
The dy and dx **must** be present **and** in the correct place. Condone missing integral signs

M1: Integrates one side to a correct form. No requirement for $+c$

Look for $\int\frac{1}{y^2}dy\to\frac{a}{y}$ or $\int\frac{1}{\sqrt{4x+5}}dx\to k\sqrt{4x+5}$ or equivalent

A1: Correct integration for both sides. Allow unsimplified but there is no requirement for $+c$

Look for $-\frac{1}{y}=2\sqrt{4x+5}$ or equivalent such as $-\frac{1}{4y}=\frac{\sqrt{4x+5}}{4\times\frac{1}{2}}$

dM1: Substitutes $y=\frac{1}{3},\ x=-\frac{1}{4}$ into their integrated form to find a value for c

It is dependent upon having integrated one side to a correct form.
Condone this being done following poor re-arrangement

ddM1: Rearranges $\frac{a}{y}=b\sqrt{4x+5}+c$ to $y=....$ using a correct method. Do not allow each term to be inverted.

It is dependent upon

- integrating BOTH sides to a correct form
- substituting $y=\frac{1}{3},\ x=-\frac{1}{4}$ into the correct integrated form to find a value for c
- rearranging $\frac{a}{y}=b\sqrt{4x+5}+c$ to $y=....$ using a correct method but condone sign slips

A1: $y=\frac{1}{7-2\sqrt{4x+5}}$ or exact equivalent. E.g. $y=\frac{-1}{2(4x+5)^{0.5}-7}$

Do not isw. So if the candidate then writes $y=\frac{1}{7}-\frac{1}{2\sqrt{4x+5}}$ it is A0

Question Number	Scheme	Marks
3 (a)	Attempts a correct identity $3x^3+8x^2-3x-6 \equiv (Ax+B)x(x+3)+C(x+3)+Dx$ o.e.	M1
	Correct method for two constants e.g. $x=0, x=-3 \Rightarrow C=..., D=...$	dM1
	Two correct constants e.g. $C=-2$ and $D=2$	A1
	Correct method to find all four constants (e.g. by comparing coefficients)	ddM1
	$A=3, B=-1, C=-2, D=2$	A1
		(5)
(b)	$g(x)=3x-1-\dfrac{2}{x}+\dfrac{2}{x+3}$	
	$\Rightarrow g'(x)=3+\dfrac{2}{x^2}-\dfrac{2}{(x+3)^2}$	M1 A1ft
		(2)
(c)	Explains that (if $x>0$) $\dfrac{2}{x^2}>\dfrac{2}{(x+3)^2}$ so $\dfrac{2}{x^2}-\dfrac{2}{(x+3)^2}>0$ and $\Rightarrow g'(x)>3$	B1
		(1) **(8 marks)**
3 (a) ALT	Via division	
	Attempts to divide $3x^3+8x^2-3x-6$ by x^2+3x forming a linear quotient	M1
	Correct method for two constants implied by quotient of $3x+...$	dM1
	Correct quotient $3x-1$	A1
	Correct method to find all four constants (e.g. uses PF on $\dfrac{rem}{x(x+3)}$)	ddM1
	$A=3, B=-1, C=-2, D=2$	A1
		(5)

(a)
M1: Establishes a method of starting the problem

- either by attempting a correct identity
- or attempting to divide the cubic by x^2+3x to obtain a linear quotient

Condone slips in the cubic expression but the intention must be correct

dM1: Attempts a correct method to find two constants. An attempt must be made at a correct equation
For the main method this could be via substituting both $x=0, x=-3 \Rightarrow C$ and D
They may set up 4 simultaneous equations and solve to find two of the constants.
FYI the equations are $A=3$, $3A+B=8$, $3B+C+D=-3$ and $3C=-6$
For division it will be arriving at a linear quotient of the form $3x\pm...$

A1: Achieves **any** two correct constants.
Allow $3x-1$ as the quotient via the alt method. Condone this just written down, i.e $A=3$, $B=-1$

ddM1: Attempts a full method to find all four constants.
In the main method this may be achieved by substituting both $x=0, x=-3$ followed by comparing coefficients of x^3 and x^2 for instance. Other methods can involve substituting other values of x.
In the alternative method it is for dividing and then setting $\dfrac{\text{remainder}}{x(x+3)} \equiv \dfrac{C}{x}+\dfrac{D}{x+3}$ before finding values of C and D

A1: Correct values for all four constants $A=3, B=-1, C=-2, D=2$
Allow for these to be embedded within the identity

(b)
M1: Attempts to differentiate and achieves $\dfrac{C}{x}+\dfrac{D}{x+3} \to \dfrac{...}{x^2}+\dfrac{...}{(x+3)^2}$

A1ft: Differentiates $Ax+B+\dfrac{C}{x}+\dfrac{D}{x+3}$ to $A-\dfrac{C}{x^2}-\dfrac{D}{(x+3)^2}$ following through on their constants.
Candidates should simplify their coefficients.
You may award as above with constants A, C and D instead of the numerical values.

(c)
B1: This requires a correct part (a) and a correct (b). Look for a reason and a conclusion

Accept as a minimum $\dfrac{2}{x^2}-\dfrac{2}{(x+3)^2}>0$ so $g'(x)>3$

Or $\dfrac{2}{x^2}>\dfrac{2}{(x+3)^2}$ so $g'(x)>3$

Also $3+\dfrac{2}{x^2}-\dfrac{2}{(x+3)^2}=3+\dfrac{2(6x+9)}{x^2(x+3)^2}$ and explains that this is $3+(+ve)>3$

Question Number	Scheme	Marks
4(a)	$\sqrt{1-4x^2}=1-\frac{1}{2}\times 4x^2$	B1
	$\dfrac{\frac{1}{2}\times-\frac{1}{2}\times\left(-4x^2\right)^2}{2}$ or $\dfrac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}\times\left(-4x^2\right)^3}{3!}$	M1
	$=1-2x^2-2x^4-4x^6+...$	A1, A1
		(4)
(b)	Substitutes $x=\frac{1}{4}$ into both sides of (a) e.g. $\sqrt{\frac{3}{4}}\approx 1-2\times\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)^4-4\left(\frac{1}{4}\right)^6$	M1
	$\sqrt{3}\approx 1.7324$ cao	A1
		(2)
		(6 marks)

(a)

B1: Correct first two terms which does not need to be simplified, so $1+\frac{1}{2}\times-4x^2$ is fine.

M1: Correct attempt at the binomial expansion for term 3 **or** term 4.

Look for $\dfrac{\frac{1}{2}\times-\frac{1}{2}\times\left(\pm 4x^2\right)^2}{2}$ or $\dfrac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}\times\left(\pm 4x^2\right)^3}{3!}$ but condone a failure to square or cube the 4 (i.e. missing brackets)

Also award this mark for candidates who mistakenly attempt the binomial expansion of

$\sqrt{1-4x}$ to produce either $\dfrac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\pm 4x\right)^2$ or $\dfrac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\pm 4x\right)^3$

Or $\sqrt{1-x^2}$ to produce either $\dfrac{\frac{1}{2}\times-\frac{1}{2}}{2}\left(\pm x^2\right)^2$ or $\dfrac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\left(\pm x^2\right)^3$

A1: Two correct and simplified terms of $-2x^2, -2x^4, -4x^6$

A1: $1-2x^2-2x^4-4x^6$ or exact simplified equivalent such as $1+(-2)x^2+(-2)x^4+(-4)x^6$

Ignore any additional terms. This may be given separately as a list. $1, -2x^2, -2x^4, -4x^6$

(b)

M1: Substitutes $x=\frac{1}{4}$ into both sides of (a) and achieves LHS of $\sqrt{\frac{3}{4}}$ or $\frac{\sqrt{3}}{2}$

It would be implied by $\left(\sqrt{3}=\right)\ 2\times$ their"$\left(1-2\times\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)^4-4\left(\frac{1}{4}\right)^6\right)$"

A1: 1.7324 Correct answer only here. This is not awrt

Note that the calculator answer is 1.7321

Alt (a) It is possible to attempt part (a) as follows but it would be very unusual to get the terms up to x^6

$$\sqrt{1-4x^2} = (1-2x)^{\frac{1}{2}} \times (1+2x)^{\frac{1}{2}}$$

$$= \left(1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 \ldots\right) \times \left(1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 \ldots.\right) = 1\ldots..$$ Score M1 for such an attempt and award the other marks as in the main scheme. As is the main scheme look for a correct attempt at a term 3 or 4 in either bracket followed by an attempt to multiply out

Question Number	Scheme	Marks
5 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{16\sec^2 t\tan t}{2\sec^2 t} = 8\tan t$	M1 A1
	At $x = 3, \tan t = -1 \Rightarrow$ Gradient = –8	dM1 A1
		(4)
(b)	Attempts to use $1+\tan^2 t = \sec^2 t \Rightarrow 1+\frac{(x-5)^2}{4} = \frac{y}{8}$	M1 A1
	$y = 2(x-5)^2 + 8$	A1
		(3)
(c)	8 剟f 32	M1 A1
		(2)
		(9 marks)

(a)

M1: Attempts to use the rule $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$. Condone incorrect attempts on $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$

A1: A correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16\sec^2 t\tan t}{2\sec^2 t}$ or unsimplified equivalent. Note that there may be many different versions of this including ones that have used double angle formulae. Look carefully

dM1: Dependent upon the previous M. It is for

- either substituting $\tan t = -1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(t)$ to find the gradient
- or substituting $t = -\frac{\pi}{4}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(t)$ to find the gradient. Condone $t = \text{awrt} -0.785$

A1: CSO Gradient = – 8. This cannot be awarded from differentiation of $y = 2(x-5)^2 + 8$

(b)

M1: Attempts to use $\pm 1 \pm \tan^2 t = \pm \sec^2 t$ with $\tan t$ being replaced by an expression in x and $\sec^2 t$ being replaced by an expression in y

A1: A correct unsimplified equation $1 + \frac{(x-5)^2}{4} = \frac{y}{8}$ o.e.

A1: $y = 2(x-5)^2 + 8$. $\mathrm{f}(x) = 2(x-5)^2 + 8$ is also fine

NB 1: It is possible to use part (a), find $\frac{\mathrm{d}y}{\mathrm{d}x} = 4(x-5)$, and then integrate using a point such as (3, 16) to find "c"

NB2: It is possible to use points to generate f (x).

For the M1 you should expect to see $\mathrm{f}(x) = ax^2 + bx + c$, or equivalent, and the use 3 points to set up 3 simultaneous equations in a, b and c which must be then solved. At least one of three points used must be correct. Examples of possible points that can be used are $(5, 8)$, $(3, 16)$ $(5 - 2\sqrt{3}, 32)$ and $(7, 16)$

For the A1 you need to see a correct equation, e.g. $y = 2x^2 - 20x + 58$

The final A1 will be $y = 2(x-5)^2 + 8$.

If you see a solution worthy of merit and you cannot see how to award the marks then send to review.

(c)

M1: One correct end found, condoning strict inequalities.

Look for the lower value of the range to be 8 (or their c) or the higher value to be 32

A1: 8 剟f 32 or equivalent such as $[8, 32]$

Question Number	Scheme	Marks
6	$u = 3+4\sin x \Rightarrow \frac{du}{dx} = 4\cos x$	B1
	$\int \frac{16\sin 2x}{(3+4\sin x)^2}\,dx = \int \frac{32\sin x\cos x}{(3+4\sin x)^2}\,dx = \int \frac{2(u-3)}{u^2}\,du$	M1 A1
	$= \int \frac{2}{u} - \frac{6}{u^2}\,du = 2\ln u + \frac{6}{u}$	dM1 A1
	Uses limits of 5 and 7 $\Rightarrow 2\ln 7 + \frac{6}{7} - 2\ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln\frac{49}{25}$	M1 A1
		(7 marks)

B1: States or uses $\frac{du}{dx} = 4\cos x$ o.e. This may be seen within the integrand.

M1: Attempts to write all in terms of u.

Look for $\int \frac{16\sin 2x}{(3+4\sin x)^2}\,dx = \int \frac{...\sin x\cos x}{(3+4\sin x)^2}\,dx = \pm\int \frac{...(u\pm 3)}{u^2}\,du$

It is common to lose the square on the u^2 which would be M0.

Only the penultimate mark would then be available.

A1: Correct $\int \frac{2(u-3)}{u^2}\,du$ o.e. Allow this to be unsimplified.

Condone a missing du this can be implied by further work.

dM1: Integrates $\int \frac{...(u\pm 3)}{u^2}\,du$ to $...\ln u \pm \frac{...}{u}$ o.e Condone a missing du this can be implied by further work.

Note that $\int \frac{...(u\pm 3)}{u^2}\,du \to ...\ln u^2 \pm \frac{...}{u}$ is acceptable

You may see attempts using parts.

$$\int \frac{1}{u^2}\times(2u-6)\,du = -\frac{1}{u}\times(2u-6) - \int -\frac{1}{u}\times 2\,du = -\frac{1}{u}\times(2u-6) + 2\ln u$$

A1: Correct $2\ln u + \frac{6}{u}$. Note that $\ln u^2 + \frac{6}{u}$ or $-\frac{1}{u}\times(2u-6) + 2\ln u$ is also completely correct

M1: Uses limits 5 and 7 within their attempted integral and subtracts . Condone poor attempts at the integration

Alternatively converts their answer in u back to x's using the correct substitution and uses the given limits

A1: $-\frac{12}{35} + \ln\frac{49}{25}$ or equivalent such as $\ln 1.96 - \frac{12}{35}$

Question Number	Scheme	Marks
7 (a)	Co-ordinates or position vector of a point on l $= \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$	
	$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$	M1
	Uses $\overrightarrow{AX}.\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow\ 20+16\lambda-15+9\lambda-25+25\lambda=0 \Rightarrow \lambda = \frac{2}{5}$	dM1 A1
	(i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$	dM1 A1
	(ii) $\overrightarrow{AX} = -\frac{33}{5}\mathbf{i} + \frac{19}{5}\mathbf{j} - 3\mathbf{k}$	
	Shortest distance $= \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} = \sqrt{67}$	M1 A1
		(7)
(b)	Uses $\overrightarrow{AX} = \overrightarrow{XB}$ or similar correct method	M1
	point B has position vector $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$	A1
		(2) **(9 marks)**
7 (a) ALT way	Co-ordinates or position vector of a point on l $= \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$	
	$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$	M1
	Uses $d^2 = (-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ which is smallest when its gradient =0	
	$\Rightarrow -8(-5-4\lambda) - 6(5-3\lambda) + 10(-5+5\lambda) = 0 \Rightarrow \lambda = \frac{2}{5}$	dM1 A1
	(Note that this could be done by completing the square)	
	(i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$	dM1 A1

Question Number	Scheme	Marks
	(ii) Shortest distance = $\left(-5-4\times\frac{2}{5}\right)^2+\left(5-3\times\frac{2}{5}\right)^2+\left(-5+5\times\frac{2}{5}\right)^2 \Rightarrow d=\sqrt{67}$	M1 A1 **(7)**

(a)

M1: States or uses a general point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ and attempt

$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$ either way around with their general point.

Condone slips

dM1: Uses a correct method to find a value for λ

This could be

- via scalar products $\overrightarrow{AX}.\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$
- via differentiation using minimum distance of $(-5-4\lambda)^2+(5-3\lambda)^2+(-5+5\lambda)^2$ in this method condone slips on the differentiation

A1: $\lambda = \frac{2}{5}$

(i)

dM1: Uses their λ to find the coordinates (or position vector) of X This is dependent upon the previous M

A1: Finds coordinates for $X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ Condone use of position vector

(ii)

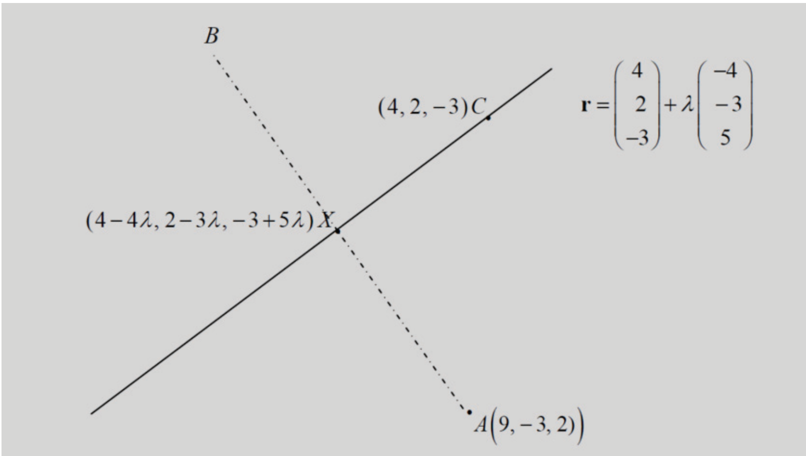
M1: Uses a correct method to find distance AX or distance AX^2 using A and their X.
Award if a correct method is seen for two of the three coordinates
Usually look for an attempt at $(-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ with their λ.

A1: $\sqrt{67}$ Note that $d = \sqrt{67}$ so $d = 67$ is quite common. This is fine as d is the constant given in the question

(b)

M1: Uses a correct method to find B.
The method can be implied by two correct coordinates for their coordinates for X and A.

A1: Correct position vector for B. $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$. Condone use of coordinates



Note that it is possible (though unlikely) to attempt this questions via scalar product using CA and CX. FYI

$$\left|\overrightarrow{CX}\right| = \left|\overrightarrow{CA}\right|\cos\theta = \frac{\overrightarrow{CA}.\overrightarrow{CX}}{\left|\overrightarrow{CX}\right|} = 2\sqrt{2}$$

Hence $\lambda = \frac{2\sqrt{2}}{\sqrt{50}} = \frac{2}{5}$

Question Number	Scheme	Marks
8 (a)	$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx$	M1
	$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 (+c)$	dM1 A1
		(3)
(b)	$\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \int \frac{1}{3}x^3 \times 2\ln x \times \frac{1}{x} \, dx$	M1
	$= \frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right)$	A1ft
	Volume $= \int_1^e \pi x^2 \ln^2 x \, dx = \pi \times \left[\frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right)\right]_1^e$	dM1
	$= \frac{5}{27}\pi e^3 - \frac{2}{27}\pi$	A1
		(4) **(7 marks)**

(a) Note that no marks are available for integrating incorrect functions

M1: Attempts to integrate by parts the correct way around. Look for

$\int x^2 \ln x \, dx = ...x^3 \ln x - \int ...x^3 \times \frac{1}{x} \, dx$ oe

dM1: And then integrates again to a form $px^3 \ln x - qx^3$ where p and q are positive constants.

A1: $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ Condone a missing $+ c$.

Candidates who go on to write $\ln x \times \frac{x^3}{3} - \frac{1}{9}x^3 = \ln \frac{x^4}{3} - \frac{1}{9}x^3$ o.e. withhold this final A1

(b)

M1: Integrates $\int x^2 \ln^2 x \, dx$ by parts, the correct way around to reach a form

$...x^3 \ln^2 x - ...\int x^3 \times \ln x \times \frac{1}{x} \, dx$

You may see a π or 2π in front of this expression which is fine.

A1ft: Achieves $\int x^2 \ln^2 x \, dx = \frac{1}{3}x^3 \ln^2 x - \frac{2}{3} \times$ their answer to (a).

Accept $\int \pi x^2 \ln^2 x \, dx = \frac{1}{3}\pi x^3 \ln^2 x - \frac{2}{3} \times \pi \times$ their answer to (a) or other incorrect factors such as

$\int 2\pi x^2 \ln^2 x \, dx = \frac{2}{3}\pi x^3 \ln^2 x - \frac{4}{3} \times \pi \times$ their answer to (a)

FYI The correct answer is $\frac{1}{3}x^3 \ln^2 x - \frac{2}{3}\left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right)$

dM1: Applies the limits 1 and e to a function of the form $\pi\times\left[\alpha x^3\ln^2 x-\beta x^3\ln x\pm\chi x^3\right]_1^{e}$ and subtracts where α,β,χ are positive constants. The π must now be seen.

A1: $\frac{5}{27}\pi e^3-\frac{2}{27}\pi$ or exact simplified equivalent such as $\frac{\pi}{27}\left(5e^3-2\right)$

Alt (b)

(b)	$\int x^2\ln^2 x\,dx=\int \ln x\times x^2\ln x\,dx$		
	$="\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)"\ln x-\int "\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)"\times\frac{1}{x}\,dx$	M1	
	$="\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)"\ln x-\int \left(\frac{1}{3}x^2\ln x-\frac{1}{9}x^2\right)dx$		
	$="\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)"\ln x-\frac{1}{3}"\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)"+\frac{1}{27}x^3$	A1ft	
	Volume $=$		
	$\int_1^{e}\pi x^2\ln^2 x\,dx=\pi\times\left[\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)\ln x-\frac{1}{3}\left(\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right)+\frac{1}{27}x^3\right]_1^{e}$	dM1	
	$=\frac{5}{27}\pi e^3-\frac{2}{27}\pi$	A1	
			(4)

Question Number	Scheme	Marks
9 (a)	States or uses $V=\pi\times4^2\times h\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}h}=16\pi$	B1
	States or uses $\dfrac{\mathrm{d}V}{\mathrm{d}t}=0.6\pi-0.15\pi h$	B1
	Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow0.6\pi-0.15\pi h=16\pi\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$	M1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{12-3h}{320}$ *	A1*
		(4)
(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{12-3h}{320}\Rightarrow\displaystyle\int\frac{\mathrm{d}h}{12-3h}=\int\frac{1}{320}\mathrm{d}t$	
	$-\dfrac{1}{3}\ln(12-3h)=\dfrac{1}{320}t+c$	M1 A1
	Uses $t=0,\ h=0.5$ $\quad-\dfrac{1}{3}\ln\left(\dfrac{21}{2}\right)=c\Rightarrow-\dfrac{1}{3}\ln(12-3h)=\dfrac{1}{320}t-\dfrac{1}{3}\ln\left(\dfrac{21}{2}\right)$	M1 A1
	Substitutes $h=3.5$ $\quad-\dfrac{1}{3}\ln\left(\dfrac{3}{2}\right)=\dfrac{1}{320}t-\dfrac{1}{3}\ln\left(\dfrac{21}{2}\right)\Rightarrow t=\ldots$	dM1
	208 minutes cso	A1
		(6)
		(10 marks)

(a)

B1: States or uses $V=\pi\times4^2\times h\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}h}=16\pi$ o.e.

B1: States or uses $\dfrac{\mathrm{d}V}{\mathrm{d}t}=0.6\pi-0.15\pi h$

M1: Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$ or equivalent with their $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ and $\dfrac{\mathrm{d}V}{\mathrm{d}t}$

A1*: Proceeds to $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{12-3h}{320}$ from a correct statement with no errors seen. The main scheme is acceptable.

Note that the answer is given and there may be many "fudged" attempts.

If you see $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{1}{16\pi}\times\left(0.6\pi-0.15\pi h\right)=\dfrac{12-3h}{320}$ or similar without any explanation or sight of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ etc you can award B1 (implied), B1 (implied). M1 (implied), A0* (not fully shown)

(b)

M1: Separates variables and integrates both sides. Look for $\pm a\ln(12-3h) = \pm bt + c$ or equivalent.

Note that $\pm a\ln(4-h) = \pm bt + k$ is also correct. There is no need for a constant at this stage

Alternatively $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{320}{12-3h} \Rightarrow t = \ldots\ln(12-3h) + c$

Watch for variations on this such as $t = \ln(4-h) + k$

A1: $-\frac{1}{3}\ln(12-3h) = \frac{1}{320}t + c$ with or without the $+ c$

M1: Uses $t = 0,\ h = 0.5$ to find c.

Condone poor integration and poor attempts to re arrange the equation for this mark.

A1: Correct equation linking h and t.

This may be implied by an equation with h replaced by 0.5. E.g.

$$\ln(12-3\times 3.5) = -\frac{3}{320}t + \ln 10.5$$

Condone c being a decimal (3sf) for this mark. So $\ln(12-3h) = -\frac{3}{320}t + 2.35$ is fine

dM1: Substitutes $h = 3.5$ to find t. Dependent upon previous M

Note that $\left[-\frac{1}{3}\ln(12-3h)\right]_{0.5}^{3.5} = \left[\frac{1}{320}t\right]_0^T$ can score 3 marks at once provided they reach a value for T

A1: States 207- 208 minutes with correct units following correct work.

Accept any value between and including these. Condone $\frac{320}{3}\ln 7$ **minutes**

$\int \frac{1}{12-3h}\,dh = \int \cancel{d}\,\frac{1}{320}\,dt$

$-\frac{1}{3}\ln|12-3h| = \frac{1}{320}t + C$

~~Or~~

$t = -\frac{320}{3}\ln|12-3h| - 320C$

when $h = 0\cancel{.5\ 3.5 = 0.5} = 3$ 3.5

$t_1 = -\frac{320}{3}\ln\frac{3}{2} - 320c$

when $h = 0.5$

$t_2 = -\frac{320}{3}\ln\frac{21}{2} - 320c$

$t_1 - t_2 = 207.56$ mins.

This method is similar to

$$\left[-\frac{1}{3}\ln(12-3h)\right]_{0.5}^{3.5} = \left[\frac{1}{320}t\right]_0^T$$

and can score 3 marks on the final line when they subtract

Question Number	Scheme	Marks
10 (a)	Writes $2\left(4p^3+6p^2+3p\right)+1$ which is odd	B1
		(1)
(b)	Assumption:	
	E.g. States that there exists integers p and q such that $\sqrt[3]{2}=\frac{p}{q}$ (where $\frac{p}{q}$ is in its simplest form) and then cubes to get $2=\frac{p^3}{q^3}$	M1
	$2=\frac{p^3}{q^3}\Rightarrow p^3=2q^3$ and concludes that p^3 is even so therefore p is even	A1
	If p is even then it can be written $p=2m$ so $(2m)^3=2q^3$	M1
	States that $q^3=4m^3$ and concludes that q^3 is even so therefore q is even	A1
	This contradicts our initial statement, as if they both have a factor of 2 it means that $\frac{p}{q}$ is not in its simplest form, so $\sqrt[3]{2}$ is irrational *	A1*
		(5)
		(6 marks)

(a)

B1: See scheme. Requires correct reason/algebra and a statement of the expression being odd.

Allow even + even + even +1 = odd. Allow $2p\left(4p^2+6p+3\right)+1=\text{odd}$

(b)

M1: Sets up the contradiction AND cubes. Condone the omission of the fact that $\frac{p}{q}$ is in its simplest form for this mark. Condone as a minimum $\sqrt[3]{2}=\frac{p}{q}$ followed by $2=\frac{p^3}{q^3}$ o.e.

A1: States that $p^3=2q^3$ and concludes both that p^3 is even so therefore p is even.

Accept other equivalent statements to even such as "multiple of 2"

Condone poor explanations so long as they state that both p^3 and p are even

M1: Writes $p=2m$ so $(2m)^3=2q^3$ and then attempts to find $q^3=\ldots$

A1: States that $q^3=4m^3$ and concludes that both q^3 is even so therefore q is even

Accept other equivalent statements to even such as "multiple of 2"

Condone poor explanations so long as they state that both q^3 and q are even

A1*: Completely correct proof and conclusion with no missing statements.

To score this final mark the statements **now** need to be the correct way around .

E.g. q^3 is even so therefore q is even

It requires $\frac{p}{q}$ to be in simplest form (or equivalent such as no common factor) in the initial assumption.