

Question Number	Scheme	Marks
<b>1 (a)</b>	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3 \dots$	
<b>(i)</b>	$\frac{1}{2}k = \frac{1}{8} \Rightarrow k = \frac{1}{4}$	M1A1
<b>(ii)</b>	$A = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times "k" ^2 = -\frac{1}{128}$ $B = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times "k" ^3 = \frac{1}{1024}$	M1 A1 A1
<b>(b)</b>	Substitutes $x = 0.6 \Rightarrow \sqrt{1.15} = 1 + \frac{1}{8} \times 0.6 - \frac{1}{128} \times 0.6^2 + \frac{1}{1024} \times 0.6^3 = 1.072398$	M1 A1
		<b>(5)</b>
		<b>(2)</b>
		<b>(7 marks)</b>
<b>1(b) alt</b>	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) - \left(\frac{1}{8}\right) \times (kx)^2 + \left(\frac{1}{16}\right) \times (kx)^3$ Substitutes " $kx$ " = 0.15 $\Rightarrow \sqrt{1.15} = 1 + \frac{1}{2} \times 0.15 - \frac{1}{8} \times 0.15^2 + \frac{1}{16} \times 0.15^3 = 1.072398$	M1A1
		<b>(2)</b>

(a)(i)

M1: Sets  $\frac{1}{2}k = \frac{1}{8}$  or  $\frac{1}{2}kx = \frac{1}{8}x$  and proceeds to find  $k$ . Implied by a correct value for  $k$

A1:  $k = \frac{1}{4}$  oe such as  $\frac{2}{8}$  or 0.25

(a)(ii)

M1: Correct attempt at 3rd or 4th term. Eg.  $Ax^2 = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2$  or  $Bx^3 = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3$  with

an attempt at substituting in their value for  $k$  to find a value for  $A$  or a value for  $B$ . Condone a missing bracket around the  $kx$  terms so allow with  $k$  instead of  $k^2$  or  $k^3$  respectively. These may be simplified so award for  $A = -\frac{1}{8} \times ("k")^2$  or  $B = \frac{1}{16} \times ("k")^3$  again, condoning  $k$  instead of  $k^2$  or  $k^3$  respectively.

A1:  $A = -\frac{1}{128}$  o.e. There is no requirement to simplify the fraction

A1:  $B = \frac{1}{1024}$  o.e You may occasionally see  $B = \frac{3}{3072}$  which is fine.

(b)

M1: For an attempt to substitute

- either  $kx = 0.15$  into an expansion of the form  $1 \pm p \times (kx) \pm q \times (kx)^2 \pm r \times (kx)^3$
- or  $x = \frac{0.15}{"k"}$  into their  $1 + \frac{1}{8}x + "A"x^2 + "B"x^3$

A1: 1.072398 Must be to 6 decimal places.

Question Number	Scheme	Marks
2	<p>Achieves a lower limit of 2</p> <p>Attempts <math>\alpha \int y^2 dx = \alpha \int \frac{81}{(2x-3)^{2.5}} dx = \alpha \times \frac{-27}{(2x-3)^{1.5}}</math></p> <p>Correct attempt at volume of solid generated <b>under</b> curve =</p> $= \pi \int_{"2"}^6 y^2 dx = \left[ \frac{k}{(2x-3)^{1.5}} \right]_{"2"}^6 = \left( \left( -\frac{27}{27} \right) - \left( -\frac{27}{1} \right) \right)$ <p style="text-align: center;">Volume = <math>26\pi</math></p> <p>Correct attempt at volume of solid = <math>\pi \times 9^2 \times (6 - "2") - "26\pi"</math></p> <p style="text-align: center;">= <math>298\pi</math></p>	<p>B1</p> <p>M1, A1</p> <p>dM1</p> <p>A1</p> <p>ddM1</p> <p>A1</p> <p style="text-align: right;"><b>(7 marks)</b></p>

B1: Finds the lower limit. This may be awarded anywhere. Accept on the diagram or in the limits of an integral

M1: Integrates an expression of the form  $\int \frac{\dots}{(2x-3)^{2.5}} dx$  and achieves  $\frac{\dots}{(2x-3)^{1.5}}$  oe

A1: Correct integration of  $\alpha \int y^2 dx = \alpha \int \frac{81}{(2x-3)^{2.5}} dx$  giving a solution  $\alpha \times \frac{-27}{(2x-3)^{1.5}}$  o.e. following through on their  $\alpha$ . Typical values of  $\alpha$  will be 1,  $\pi$  or  $2\pi$ . No need to simplify here

dM1: Attempts to find the volume of the solid generated by rotating the area under the curve.

Look for  $\left[ \frac{\dots}{(2x-3)^{1.5}} \right]_{"2"}^6 = \dots$  It is dependent upon the previous M1.

The top limit must be 6 and the bottom limit must be their solution to  $\frac{9}{(2x-3)^{1.25}} = 9$

A1: Correct exact volume for the solid generated by rotating the area under the curve. Volume =  $26\pi$

This may be the second part of a larger/complete expression. E.g.  $\dots - 26\pi$

ddM1: Correct method for the volume of the solid formed by rotating the area above the curve.

Accept  $\pi \times 9^2 \times (6 - "2") - "26\pi"$

It is dependent both previous M's

A1:  $298\pi$

Question Number	Scheme	Marks
3 (a)	Attempts to find $\frac{1}{3} \times 20^2 \times 24$ 160 = 20 seconds	M1 A1  (2)
(b)	Attempts $\frac{dV}{dh} = h^2 + \frac{8}{3}h$ Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$ Substitutes $h = 5 \Rightarrow 160 = \left(5^2 + \frac{40}{3}\right) \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{96}{23}$ (4.2) cm s <sup>-1</sup>	M1 M1 A1 dM1 A1  (5)
		(7 marks)

(a)

M1: Attempts to find the volume, using the given formula, at  $h = 20$  and dividing by 160. Implied by 20  
Condone slips. For example they may have incorrectly multiplied out/calculated the expression for  $V$

A1: 20 seconds. This requires the correct units as well

(b)

M1: For an attempt to differentiate  $V$

Scored for an attempt to multiply out and then differentiate term by term to achieve  $\frac{dV}{dh} = \alpha h^2 + \beta h$

or via the product rule to achieve  $V = \frac{1}{3}h^2(h+4) \Rightarrow \frac{dV}{dh} = (h+4) \times ph + qh^2$

M1: Attempts to use  $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$  (or equivalent) with  $\frac{dV}{dt} = 160$  and their  $\frac{dV}{dh}$ .

A1: Correct expression involving  $\frac{dh}{dt}$  and  $h$  E.g.  $160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$

dM1: Dependent upon the previous M only. It is for substituting  $h = 5$  and proceeding to a value for  $\frac{dh}{dt} = \dots$

A1: Either  $\frac{96}{23}$  or awrt 4.2 cm s<sup>-1</sup> There is no requirement for the units here

Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}} dx$ $u = \sqrt{x} \Rightarrow x = u^2 \Rightarrow \frac{dx}{du} = 2u$ $\int \frac{10}{5x+2x\sqrt{x}} dx = \int \frac{10}{5u^2+2u^3} 2udu$ $= \int \frac{20}{5u+2u^2} du = \int \frac{4}{u} - \frac{8}{5+2u} du$ $= 4 \ln u - 4 \ln(5+2u)$ $\text{Limits} = [4 \ln u - 4 \ln(5+2u)]_1^2 = 4 \ln 2 - 4 \ln 9 + 4 \ln 7 = \dots$ $= 4 \ln \left( \frac{14}{9} \right)$	<p>B1</p> <p>M1 A1</p> <p>dM1 A1</p> <p>ddM1</p> <p>M1</p> <p>A1 <b>(8 marks)</b></p>

B1: For  $\frac{dx}{du} = 2u$  or  $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$  or equivalent

M1: Attempts to write all terms in the integral in terms of  $u$  (inc the  $dx$ )

Condone slips for this mark BUT the  $dx$  CANNOT just be replaced by  $du$ .

Look for either  $x \rightarrow u^2$  or  $x\sqrt{x} \rightarrow u^3$  with either  $dx \rightarrow f(u)du$  or  $dx \rightarrow k \times du, k \neq 1$

A1: Correct integrand in terms of just  $u$  which may be unsimplified. E.g  $\int \frac{10}{5u^2+2u^3} 2udu$

dM1: Attempts to use PF and writes their integrand in terms of its component fractions.

Look for an integral of the form  $\int \frac{P}{Qu+Ru^2} du \rightarrow \int \frac{p}{u} + \frac{q}{Q+Ru} du$

A1: Correct PF  $\int \frac{4}{u} - \frac{8}{5+2u} (du)$

ddM1: For  $\dots \ln u \pm \dots \ln(5+2u)$  but follow through on their PF's which must be of a similar form

M1: Uses the limits 1 and 2 within their attempted integral. The integration may be incorrect.

Alternatively substitutes  $u = \sqrt{x}$  and uses the limits 1 and 4 within their attempted integral

A1:  $4 \ln \left( \frac{14}{9} \right)$

Question Number	Scheme	Marks
5 (a)	$\underline{\underline{2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3}}$	B1 M1 A1
	$(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} *$	A1* (4)
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$	B1
	Attempts tangent at $(0,0)$ <b>or</b> $(0,1)$ $y = 3x$ <b>or</b> $y = -5x + 1$	M1 A1
	Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	dM1 A1 (5)
		<b>(9 marks)</b>

(a) Originally scored M1 B1 A1 A1, now B1 M1 A1 A1

B1: Correct differentiation using the chain rule  $y^2 \rightarrow 2y \frac{dy}{dx}$ .

M1: Attempts to apply the product rule of differentiation on  $ye^{-2x}$  to give  $e^{-2x} \frac{dy}{dx} \pm \dots ye^{-2x}$

A1: Correct differentiation  $2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3$

Allow  $2y dy = e^{-2x} dy - 2ye^{-2x} dx - 3dx$

A1\*: Proceeds to the given answer via an intermediate line equivalent to  $(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3$  with correct bracketing.

(b)

B1: Deduces or implies that  $x = 0, y = 1$  at  $P$ . May state or use coordinates of  $P(0,1)$

M1: Scored for an attempt to find the equation of the tangent at  $O$  **or** the equation of the tangent at  $P$

E.g. Substitutes  $(0,0)$  into  $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} = \beta$  and states  $y = \beta x$

Alternatively substitutes  $(0, "1")$  into  $\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} \rightarrow "k"$  and attempts  $y - 1 = "k" x$  or equivalent

A1: Achieves a correct equation for **either** tangent. Look for **either**  $y = 3x$  **or**  $y = -5x + 1$  o.e.

dM1: Correct attempt **at both** tangents with an attempt to solve simultaneously.

For the attempt to solve accept  $y = 3x$ ,  $y = -5x + 1 \Rightarrow x = \dots, y = \dots$

A1: Correct coordinates for  $R = \left(\frac{1}{8}, \frac{3}{8}\right)$  oe

Question Number	Scheme	Marks
<p><b>6 (a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	$\text{Area } R = \int y \frac{dx}{dt} dt = \int 4 \sin t \times -4 \sin 2t dt$ $= - \int 32 \sin^2 t \cos t dt$ $x=0 \Rightarrow t = \frac{\pi}{4} \text{ and } y=0 \Rightarrow t=0 \Rightarrow \text{Area} = - \int_{\frac{\pi}{4}}^0 32 \sin^2 t \cos t dt = \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t dt \quad *$ $\text{Area} = \left[ \frac{32}{3} \sin^3 t \right]_0^{\frac{\pi}{4}} = \frac{32}{3} \times \frac{2\sqrt{2}}{8} = \frac{8\sqrt{2}}{3}$ <p>Attempts to use <math>\cos 2t = 1 - 2 \sin^2 t \Rightarrow \frac{x}{2} = 1 - 2 \left( \frac{y}{4} \right)^2</math></p> $y = \sqrt{8 - 4x}$ <p>Range is <math>0 \leq f \leq 4</math></p>	<p>M1 A1</p> <p>dM1</p> <p>A1*</p> <p><b>(4)</b></p> <p>M1 A1</p> <p><b>(2)</b></p> <p>M1A1</p> <p>A1</p> <p><b>(3)</b></p> <p>B1</p> <p><b>(1)</b></p> <p><b>(10 marks)</b></p>

(a)(i)

M1: Attempts to multiply  $y$  by  $\frac{dx}{dt}$  to achieve an integrand of the form  $\pm k \sin t \sin 2t$

Look for area  $R = \int y \frac{dx}{dt} (dt) = \int (4) \sin t \times \pm k \sin 2t (dt)$  but condone a missing 4 or  $dt$

A1: Correct integrand. Achieves area  $R = \int y \frac{dx}{dt} (dt) = \int 4 \sin t \times -4 \sin 2t (dt)$

Condone a missing  $dt$  and do not be concerned by the limits. Allow unsimplified.

dM1: Substitutes  $\sin 2t = 2 \sin t \cos t \rightarrow$  seen or implied  $= \pm \int A \sin^2 t \cos t dt$

Dependent upon previous M. Condone a missing  $dt$  and do not be concerned by the limits.

A1\*: Achieves area  $\left( = \int_0^2 y dx \right) = - \int_{\frac{\pi}{4}}^0 32 \sin^2 t \cos t dt \rightarrow \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t dt$

This is a given answer so you must see

- evidence of the  $dt$  in at least one line other than the given answer

- correct application of the limits as seen above with as a minimum  $-\int_{\frac{\pi}{4}}^0 \dots dt \rightarrow \int_0^{\frac{\pi}{4}} \dots dt$

(a)(ii)

M1: Integrates to a form  $\left[ A \sin^3 t \right]$  with some attempt to apply the limit(s)

You may see a substitution  $u = \sin t$  which is fine. Just look for  $Au^3$  with some attempt to apply the adapted limits

A1: Correct answer  $\frac{8\sqrt{2}}{3}$  o.e. following correct **algebraic** integration.

Cannot be scored without the M mark.

(b)

M1: Attempts to use a double angle formula of the form  $\cos 2t = \pm 1 \pm 2 \sin^2 t$  with the parametric equations to get a Cartesian equation.

If the parametric equations are substituted into the given form of the answer  $y = \sqrt{ax+b}$ , marks are only scored when the double angle formula is used

A1: Any correct un-simplified equation  $\frac{x}{2} = 1 - 2 \left( \frac{y}{4} \right)^2$

On the alternative method, this A mark is scored when the candidate writes down

$$a + 2b = 0. -4b = 16 \Rightarrow a = \dots, b = \dots$$

A1:  $y = \sqrt{8-4x}$  or  $y = \sqrt{-4x+8}$  ONLY

(c)

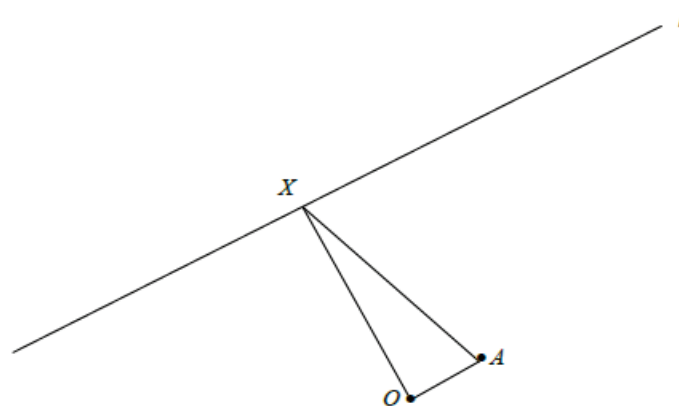
B1: Range is  $0 \leq f \leq 4$

Allow other acceptable forms such as  $0 \leq y \leq 4$ ,  $0 \leq f(x) \leq 4$  and  $[0,4]$

Examples of unacceptable forms are  $0 \leq x \leq 4$ ,  $[0,4)$

Question Number	Scheme	Marks
7 (a)	Finds $\left  \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \right  = \sqrt{4^2 + 4^2 + 2^2} = 6$ and attempts $\frac{1}{"6"} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$	M1
	$\overline{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ oe	A1
<b>(2)</b>		
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$	M1
	$\overline{OX} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0 \Rightarrow 4(1+4\lambda) + 4(-10+4\lambda) + 2(-9+2\lambda) = 0$	dM1
	$36\lambda = 54 \Rightarrow \lambda = 1.5$ $X = (7, -4, -6)$	ddM1 A1 A1
<b>(5)</b>		
(c)	Finds $OX = \sqrt{7^2 + (-4)^2 + (-6)^2} = \sqrt{101}$ and $OA = 1$	M1
	Area $OXA = \frac{1}{2} \times 1 \times \sqrt{101} = \frac{\sqrt{101}}{2}$	dM1 A1
<b>(3)</b>		
<b>(10 marks)</b>		

Handy diagram



(a)

M1: Correct attempt at the unit vector. Look for an attempt at  $\sqrt{4^2 + 4^2 + 2^2}$  and use of  $\frac{\mathbf{r}}{|\mathbf{r}|}$  where  $\mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$

or equivalent vector form such as  $\mathbf{r} = (4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ . Condone an attempt to find the coordinates of A



A1:  $\overline{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$  or equivalent such as  $\overline{OA} = \frac{1}{6} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$  or  $\overline{OA} = \frac{1}{6}(4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$  but **must be in vector form and**

**not in coordinate form.**

(b)

M1: For an attempt at the co-ordinates or position vector of point  $X = (1+4\lambda, -10+4\lambda, -9+2\lambda)$  or  $\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix}$

dM1: For using  $\overline{OX} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0$  to set up an equation in  $\lambda$

Alternatively finds  $OX^2 = (1+4\lambda)^2 + (-10+4\lambda)^2 + (-9+2\lambda)^2$  and attempts to differentiate and set = 0

May use  $OX^2 + OA^2 = AX^2$  to set up an equation in  $\lambda$

ddM1: Solves for  $\lambda$  This is dependent upon having scored both previous M's

A1: Correct value for  $\lambda = 1.5$

A1: Correct coordinates for  $X = (7, -4, -6)$ . Condone position vector form  $7\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$  o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance  $OX$  or  $OX^2$  AND distance  $OA$  or  $OA^2$  (which is 1)

dM1: Correct method of finding the area of  $OXA = \frac{1}{2} \times OX \times 1$

A1: Area =  $\frac{\sqrt{101}}{2}$  o.e

There are various alternatives for part (c). Amongst others are;

**Alt I via vector product.**

$$\frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| 8\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right| = \frac{1}{2} \times \sqrt{\left(\frac{8}{3}\right)^2 + \left(-\frac{19}{3}\right)^2 + \left(\frac{22}{3}\right)^2} = \frac{\sqrt{101}}{2}$$

M1: For an attempt at  $\frac{1}{2} \times \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & -6 \\ 2/3 & 2/3 & 1/3 \end{vmatrix} = \frac{1}{2} \times \left| 8\mathbf{i} - \frac{19}{3}\mathbf{j} + \frac{22}{3}\mathbf{i} \right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

**Alt II via scalar products**

M1: Attempts to find all three components required to find the area triangle  $OXA$

E.g. Angle  $OXA$  with length of side  $OX$  and  $XA$

Alternatively angle  $OAX$  with length of side  $OA$  and  $XA$

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$  to find  $\cos\theta$  or  $\theta$

dM1: Full attempt at area of triangle using  $\frac{1}{2}|OX||XA|\sin(OXA)$  or equivalent

Question Number	Scheme	Marks
8 (a)	$\int y^{-\frac{1}{3}} dy = \int 6xe^{-2x} dx$	B1
	$\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} + \int 3e^{-2x} dx$	M1 M1
	$\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + c$	dM1 A1
	Substitutes $(0,1) \Rightarrow c = 3$	M1
	$y^2 = (-2xe^{-2x} - e^{-2x} + 2)^3$	A1
		(7)
(b)	As $x \rightarrow \infty, e^{-2x} \rightarrow 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$	M1 A1
		(2)
		<b>(9 marks)</b>

(a)

B1: Separates the variables either  $\int y^{-\frac{1}{3}} dy = \int 6xe^{-2x} dx$  or  $\int \frac{1}{6}y^{-\frac{1}{3}} dy = \int xe^{-2x} dx$

Condone with missing integral signs but the dx and dy **must be present and in the correct positions**

M1: For integrating the lhs  $y^{-\frac{1}{3}} \rightarrow y^{\frac{2}{3}}$

M1: For integrating the rhs by parts the right way around. Look for  $\int xe^{-2x} dx \rightarrow \dots xe^{-2x} \pm \int \dots e^{-2x} dx$

dM1: For fully integrating the rhs to obtain  $\dots xe^{-2x} \pm \dots e^{-2x}$ . Depending upon the previous M

A1: Correct integration with or without '+ c'.

Look for  $\frac{3}{2}y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2}e^{-2x} + c$  or equivalent such as  $3y^{\frac{2}{3}} = -6xe^{-2x} - 3e^{-2x} + a$

dM1: Must have a "+c" now. Substitutes  $(0,1) \Rightarrow c = \dots$

It is dependent upon having a correct attempt to integrate one side so M1 or M2 must have been awarded.

A1: CSO  $y^2 = (-2xe^{-2x} - e^{-2x} + 2)^3$

(b)

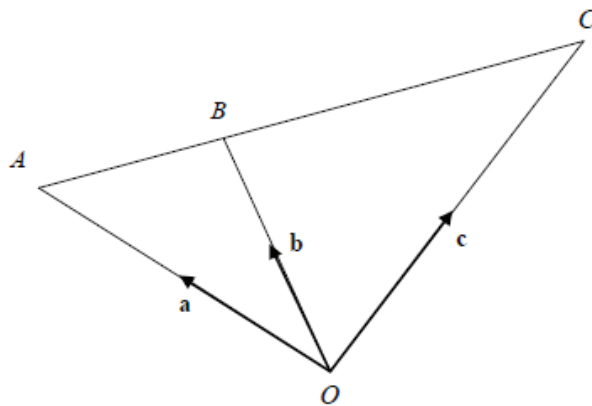
M1: For  $e^{-2x} \rightarrow 0$   $y^2 = ("2")^3$

Follow through on their  $g(x)$  in  $y^2 = g(x)$  but  $g(x)$  must be a function of  $e^{-kx}$  with  $g(0) \neq 0$

Implied by a correct decimal answer for their  $y^2 = g(x)$

A1:  $y = 2^{\frac{3}{2}}$  o.e such as  $y = \sqrt{8}$  cso. ISW after sight of this. Condone  $y = \pm 2^{\frac{3}{2}}$  o.e.

Question Number	Scheme	Marks
9 (i)	Attempts two of $\overline{AB} = \mathbf{b} - \mathbf{a}$ , $\overline{AC} = \mathbf{c} - \mathbf{a}$ and $\overline{BC} = \mathbf{c} - \mathbf{b}$ either way around Attempts $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a})$ oe such as $\mathbf{c} - \mathbf{a} = 3 \times (\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ *	M1 dM1 A1 * (3)
9 (ii)	Assume that there exists a number $n$ that isn't a multiple of 3 yet $n^2$ is a multiple of 3 If $n$ is not a multiple of 3 then $m = 3p + 1$ or $m = 3p + 2$ ( $p \in \mathbb{N}$ ) giving $m^2 = (3p + 1)^2 = 9p^2 + 6p + 1$ Or $m^2 = (3p + 2)^2 = 9p^2 + 12p + 4 = 3(3p^2 + 4p + 1) + 1$ $(3p + 1)^2 = 9p^2 + 6p + 1 (= 3(3p^2 + 2p) + 1)$ is one more than a multiple of 3 $(3p + 2)^2 = 9p^2 + 12p + 4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) Hence if $n$ is a multiple of 3 then $n^2$ is a multiple of 3	B1 M1 M1 A1 A1 (5) (8 marks)



- (i)
- M1: Attempts any two of  $\overline{AB}$ ,  $\overline{AC}$  and  $\overline{BC}$ .  
 Condone the wrong way around but it must be subtraction.  
 Allow marked in the correct place on a diagram
- dM1: Uses the given information.  
 Accept  $\overline{AB} = \frac{1}{3}\overline{AC}$ ,  $\overline{BC} = 2 \times \overline{AB}$ ,  $\overline{BC} = \frac{2}{3} \times \overline{AC}$  etc condoning slips as in previous M1.
- A1\*: Fully correct work inc bracketing leading to the given answer  $\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$   
 Expect to see the brackets multiplied out. So  $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a}) \Rightarrow \mathbf{c} - \mathbf{b} = 2\mathbf{b} - 2\mathbf{a} \Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$  is fine.

(ii)

B1: For setting up the contradiction.

Eg Assume that there exists a number  $n$  that isn't a multiple of 3, yet  $n^2$  is a multiple of 3

As a minimum accept something like " define a number  $n$  such that  $n$  is not a multiple of 3 but  $n^2$  is"

M1: States that  $m = 3p + 1$  or  $m = 3p + 2$  and attempts to square.

Alternatives exist such as  $m = 3p + 1$  or  $m = 3p - 1$

Using modulo 3 arithmetic it would be  $1 \rightarrow 1$  and  $2 \rightarrow 4 = 1$

M1: States that  $m = 3p + 1$  AND  $m = 3p + 2$  and attempts to square o.e.

A1: Achieves forms that can be argued as to why they are NOT a multiple of 3

$$\text{E.g. } m^2 = (3p+1)^2 = 3(3p^2 + 2p) + 1 \text{ or even } 9p^2 + 6p + 1$$

$$\text{and } m^2 = (3p+2)^2 = 3(3p^2 + 4p + 1) + 1 \text{ or even } 9p^2 + 12p + 4$$

A1: Correct proof which requires

- Correct calculations
- Correct reasons. E.g.  $9p^2 + 12p + 4$  is not a multiple of 3 as 4 is not a multiple of 3  
There are many ways to argue these. E.g.  $m^2 = (3p+1)^2 = 3(3p^2 + 2p) + 1$  is sufficient as long as followed (or preceded by) "not a multiple of 3"
- Minimal conclusion such as  $\checkmark$ . Note that B0 M1 M1 M1 A1 is possible