

Question Number	Scheme	Marks
<b>1(a)</b>	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \frac{1}{2}(\dots)$	B1
	$= (1 - 20x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (-20x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (-20x)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (-20x)^3 \dots$	M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1 A1
	<b>Special case:</b> If the final answer is left as $\frac{1}{2}(1 - 10x - 50x^2 - 500x^3 + \dots)$ Award SC B1M1A1A1A0	
	<b>(5)</b>	
<b>Alternative by direct expansion</b>		
	$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1 + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1A1
<b>(b)</b>	$\left(\frac{1}{4} - \frac{5}{100}\right)^{\frac{1}{2}} = \left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{2} - 5 \times \frac{1}{100} - 25\left(\frac{1}{100}\right)^2 - 250\left(\frac{1}{100}\right)^3 + \dots$	M1
	$\frac{\sqrt{5}}{5} \approx \frac{1789}{4000} \quad \text{or} \quad \frac{1}{\sqrt{5}} \approx \frac{1789}{4000}$	
	$\Rightarrow \sqrt{5} \approx 5 \times \frac{1789}{4000} \quad \text{or} \quad \sqrt{5} \approx 1 \div \frac{1789}{4000}$	A1
	$\sqrt{5} \approx \frac{1789}{800} \quad \text{or} \quad \frac{4000}{1789}$	
	<b>(2)</b>	
		<b>(7 marks)</b>

**(a)**

B1: For taking out a factor of  $\left(\frac{1}{4}\right)^{\frac{1}{2}}$  or  $\frac{1}{2}$  or 0.5 etc.

M1: Expands  $(1 + kx)^{\frac{1}{2}}$ ,  $k \neq \pm 1$  with the correct structure for the third or fourth term

e.g.  $\pm \frac{\frac{1}{2}(-\frac{1}{2})}{2!} \times (kx)^2$  or  $\pm \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!} \times (kx)^3$  with or without the bracket around the  $kx$

A1: For either term three or term four being correct in any form.

E.g.  $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (20x)^2$  or  $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (-20x)^2$  or  $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (-20x)^3$  or  $-\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (20x)^3$

The brackets must be present unless they are implied by subsequent work. This mark is independent of the B mark.

A1: Two terms correct and simplified of  $\frac{1}{2} - 5x - 25x^2 - 250x^3$ . Allow if any of the ‘-’ signs are written as “+--”.

A1: All four terms correct and simplified of  $\frac{1}{2} - 5x - 25x^2 - 250x^3$ . Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the ‘-’ signs are written as “+--” score A0.

**Alternative:**

B1: For a first term of  $\left(\frac{1}{4}\right)^{\frac{1}{2}}$  or  $\frac{1}{2}$  or 0.5 etc.

M1: For the correct structure for the third or fourth term. E.g.  $\frac{1}{2} \times \frac{-1}{2} \left(\frac{1}{4}\right)^{-\frac{3}{2}} (kx)^2$  or  $\frac{1}{2} \times \frac{-1}{2} \times \frac{-3}{2} \left(\frac{1}{4}\right)^{-\frac{5}{2}} (kx)^3$

where  $k \neq \pm 1$

A1: For either term three or term four being correct in any form.

e.g.  $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times \left(\frac{1}{4}\right)^{-\frac{3}{2}} (-5x)^2$  or  $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times \left(\frac{1}{4}\right)^{-\frac{5}{2}} (-5x)^3$

The brackets must be present unless they are implied by subsequent work.

A1: Two terms correct and simplified of  $\frac{1}{2} - 5x - 25x^2 - 250x^3$ . Allow if any of the ‘-’ signs are written as “+--”.

A1: All four terms correct and simplified of  $\frac{1}{2} - 5x - 25x^2 - 250x^3$ . Allow the terms to be listed.

Ignore any extra terms and apply isw if necessary. If any of the ‘-’ signs are written as “+--” score A0.

(b)

M1: Attempts to substitute  $x = \frac{1}{100}$  into their part (a) and either multiplies by 5 or finds reciprocal.

A1:  $(\sqrt{5} =) \frac{1789}{800}$  or  $\frac{4000}{1789}$

Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA} \cdot \overrightarrow{BC} = -6 \times 2 + 2 \times 5 - 3 \times 8 = (-26)$	M1
	Uses $\overrightarrow{BA} \cdot \overrightarrow{BC} =  \overrightarrow{BA}   \overrightarrow{BC}  \cos \theta \Rightarrow -26 = \sqrt{49} \times \sqrt{93} \cos \theta \Rightarrow \theta = \dots$	dM1
	$\theta = 112.65^\circ$	A1
		(3)
(b)	Attempts to use $ \overrightarrow{BA}   \overrightarrow{BC}  \sin \theta$ with their $\theta$	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

M1: Attempts the scalar product of  $\pm \overrightarrow{AB} \cdot \pm \overrightarrow{BC}$  condone slips as long as the intention is clear

**Or** attempts the vector product  $\pm \overrightarrow{AB} \times \pm \overrightarrow{BC}$  (see alternative 1)

**Or** attempts vector AC (see alternative 2)

dM1: Attempts to use  $\pm \overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{AB}| |\overrightarrow{BC}| \cos \theta$  AND proceeds to a value for  $\theta$

Expect to see at least one correct attempted calculation for a modulus.

For example  $\sqrt{2^2 + 5^2 + 8^2} (= \sqrt{93})$  or  $\sqrt{6^2 + 2^2 + 3^2} (= 7)$

**Note that we condone poor notation such as:**  $\cos \theta = \frac{26}{7\sqrt{93}} = 67.35^\circ$  **Depends on the first mark.**

**Must be an attempt to find the correct angle.**

A1:  $\theta =$  awrt  $112.65^\circ$  Versions finishing with  $\theta =$  awrt  $67.35^\circ$  will normally score M1 dM1 A0

Angles given in radians also score A0 (NB  $\theta = 1.9661\dots$  or acute  $1.1754\dots$ )

Allow e.g.  $\theta = 67.35^\circ \Rightarrow \theta = 180 - 67.35^\circ = 112.65$  and allow  $\cos \theta = \frac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$

### **1. Alternative using the vector product:**

M1: Attempts the vector product  $\pm \overrightarrow{AB} \times \pm \overrightarrow{BC} = \pm \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \times \pm \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix} = \pm \begin{pmatrix} -31 \\ -42 \\ 34 \end{pmatrix}$  condone slips as long as the intention is

clear

dM1: Attempts to use  $\pm \overrightarrow{AB} \times \overrightarrow{BC} = |\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$  AND proceeds to a value for  $\theta$

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example  $\sqrt{2^2 + 5^2 + 8^2}$  or  $\sqrt{6^2 + 2^2 + 3^2}$  and  $\sqrt{31^2 + 42^2 + 34^2} (= \sqrt{3881})$

**Note that we condone poor notation such as:**  $\sin \theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35^\circ$  **Depends on the first mark.**

**Must be an attempt to find the correct angle.**

A1:  $\theta =$  awrt  $112.65^\circ$  Versions finishing with  $\theta =$  awrt  $67.35^\circ$  will normally score M1 dM1 A0

**2. Alternative using cosine rule:**

M1: Attempts  $\pm \overrightarrow{AC} = \pm (\overrightarrow{AB} + \overrightarrow{BC}) = \pm (8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$  condone slips and poor notation as long as the intention is

clear e.g. allow  $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use  $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos \theta$  AND proceeds to a value for  $\theta$

**Must be an attempt to find the correct angle.**

A1:  $\theta = \text{awrt } 112.65^\circ$

(b)

M1: Attempts to use  $|\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$  with their  $\theta$ . You may see  $\frac{1}{2} |\overrightarrow{AB}| |\overrightarrow{BC}| \sin \theta$  found first before it is doubled.

or attempts the magnitude of their vector product e.g.  $\sqrt{3881}$

A1: Area = awrt 62.3. If this is achieved from an angle of  $\theta = \text{awrt } 67.35^\circ$  full marks can be scored

**Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.**

Question Number	Scheme	Marks
3	States the largest odd number and an odd number that is greater E.g. odd number $n$ and $n + 2$	M1
	Fully correct proof including <ul style="list-style-type: none"> <li>• the assumption: there exists a greatest odd number "<math>n</math>"</li> <li>• a correct statement that their second odd number is greater than their assumed greatest odd number</li> <li>• a minimal conclusion " this is a contradiction, hence proven"</li> </ul> <p>You can ignore any spurious information e.g. <math>n &gt; 0</math>, <math>n + 2 &gt; 0</math> etc.</p>	A1*
		(2)
		(2 marks)

M1: For starting the proof by **stating** an odd number and a larger odd number.

Examples of an allowable start are

- **odd number** " $n$ " with " $n + 2$ "
- **odd number** " $n$ " with " $n^2$ "
- " $2k + 1$ " with " $2k + 3$ "
- " $2k + 1$ " with " $(2k + 1)^3$ "
- " $2k + 1$ " with " $2k + 1 + 2k$ "

Note that stating  $n = 2k$ , even when accompanied by the statement that " $n$ " is odd is M0

A1\*: A fully correct proof using contradiction

This must consist of

- 1) An assumption E.g. "(Assume that) there exists a greatest odd number  $n$ "  
"Let " $2k + 1$ " be the greatest odd number"
- 2) A minimal statement showing their second number is greater than the first,  
E.g. If " $n$ " is odd and " $n + 2$ " is greater than  $n$

$$\text{If } "n" \text{ is odd and } n^2 > n$$

$$2k + 3 > 2k + 1$$

$$2k + 2k + 1 > 2k + 1$$

Any algebra (e.g. expansions) must be correct. So  $(2k + 1)^2 = 4k^2 + 2k + 1$  would be A0

- 3) A minimal conclusion which could be

"hence there is no greatest odd number", "hence proven", or simply ✓

Question Number	Scheme	Marks
4(a)	$k = 2$ or $x > 2$	B1
	$t = \frac{1}{x-2} \Rightarrow y = \frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}}$	M1 A1
	$\frac{1 - \frac{2}{x-2}}{3 + \frac{1}{x-2}} = \frac{x-2-2}{\dots}$ or $\frac{\dots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y = \frac{x-4}{3x-5}$	A1
		(5)
(b)	$-2 < g < \frac{1}{3}$	M1 A1
		(2)
		(7 marks)

(a)

B1: States that  $k = 2$  or else states that the domain is  $x > 2$ . **Must be seen in part (a).**

M1: Attempts to find  $t$  in terms of  $x$  and substitutes into  $y$ .

Condone poor attempts but you should expect to see  $t = f(x)$  found from  $x = \frac{1}{t} + 2$  substituted into

$$y = \frac{1-2t}{3+t} \text{ condoning slips.}$$

A1: A correct unsimplified equation involving just  $x$  and  $y$

A1(M1 on EPEN): Correct numerator or denominator with fraction removed (allow unsimplified)

A1:  $y = \frac{x-4}{3x-5}$  or  $g(x) = \frac{x-4}{3x-5}$  (must be  $y = \dots$  or  $g(x) = \dots$  but allow this mark as long as the  $y = \dots$  or  $g(x) = \dots$  is present at some point)

**Alternative 1 for part (a)**

M1: Assume  $g(x) = \frac{ax+b}{cx+d}$  and substitute in  $x = \frac{1}{t} + 2$

$$A1: g(x) = \frac{a + (b+2a)t}{c + (d+2c)t}$$

A1(M1 on EPEN): Correct numerator or denominator

A1:  $y = \frac{x-4}{3x-5}$  or  $g(x) = \frac{x-4}{3x-5}$  (must be  $y = \dots$  or  $g(x) = \dots$  but allow this mark as long as the  $y = \dots$  or  $g(x) = \dots$  is present at some point)

**Alternative 2 for part (a)**

M1: Attempts to find  $t$  in terms of  $y$  and substitutes into  $x$ .

Condone poor attempts but you should expect to see  $t = f(y)$  found from  $y = \frac{1-2t}{3+t}$  substituted into

$$x = \frac{1}{t} + 2 \text{ condoning slips. (NB } t = \frac{1-3y}{y+2} \Rightarrow x = \frac{y+2}{1-3y} + 2)$$

A1: A correct unsimplified equation involving just  $x$  and  $y$

A1(M1 on EPEN): Correct numerator or denominator

All  $y = \frac{x-4}{3x-5}$  or  $g(x) = \frac{x-4}{3x-5}$  (must be  $y = \dots$  or  $g(x) = \dots$  but allow this mark as long as the  $y = \dots$  or  $g(x) = \dots$  is present at some point)

(b)

M1: For obtaining one of the 2 boundaries (just look for values) e.g.  $-2$  or  $\frac{1}{3}$  or for attempting  $g(2)$  for their

$g$  or for attempting  $\frac{\text{their } a}{\text{their } c}$ . Note that for this mark they must be attempting values of  $y$  (or  $g(x)$ ).

A1: Correct range: Allow  $-2 < g < \frac{1}{3}$ ,  $-2 < g(x) < \frac{1}{3}$ ,  $-2 < y < \frac{1}{3}$ ,  $\left(-2, \frac{1}{3}\right)$ ,  $g > -2$  and  $g < \frac{1}{3}$

Question Number	Scheme	Marks
5	$u = 3 + \sqrt{2x-1} \Rightarrow x = \frac{(u-3)^2 + 1}{2} \Rightarrow \frac{dx}{du} = u - 3$	M1 A1

Question Number	Scheme	Marks
	<b>or</b> $u = 3 + \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{2x-1}} = \frac{1}{u-3}$	
	$\int \frac{4}{3+\sqrt{2x-1}} dx = \int \frac{4}{u} \times (u-3) du$	M1
	$\int \frac{4}{u} \times (u-3) du = \int \left(4 - \frac{12}{u}\right) du$	dM1
	$\int \left(4 - \frac{12}{u}\right) du = 4u - 12 \ln u \quad \text{or} \quad k(4u - 12 \ln u)$	ddM1 A1ft
	$\int_1^{13} \frac{4}{3+\sqrt{2x-1}} dx = [4u - 12 \ln u]_4^8 = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$	M1
	<b>or</b> $\int_1^{13} \frac{4}{3+\sqrt{2x-1}} dx = [4(3+\sqrt{2x-1}) - 12 \ln(3+\sqrt{2x-1})]_1^{13} = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$	
	$= 16 - 12 \ln 2$	A1
		<b>(8 marks)</b>

M1: Differentiates to get  $\frac{du}{dx}$  in terms of  $x$  and then obtains  $\frac{dx}{du}$  in terms of  $u$

Need to see  $\frac{du}{dx} = k(2x-1)^{-\frac{1}{2}} \rightarrow \frac{du}{dx} = \frac{1}{au+b}$  or  $\frac{dx}{du} = au+b$

**or**

Attempts to change the subject of  $u = 3 + \sqrt{2x-1}$  and differentiates to get  $\frac{dx}{du}$  in terms of  $u$

Need to see  $x = \frac{(u \pm 3)^2 \pm 1}{2} \rightarrow \frac{dx}{du} = au+b$

A1:  $\frac{dx}{du} = u-3$  or e.g.  $\frac{du}{dx} = \frac{1}{u-3}$ ,  $du = \frac{dx}{u-3}$ ,  $dx = (u-3)du$

M1: Attempts to write the integral completely in terms of  $u$ .

Need to see  $\int \dots \times \text{their } \frac{dx}{du} du$  with or without the “ $du$ ” but **not** e.g.  $\int \dots \times \frac{1}{\frac{dx}{du}} du$

dM1: Divides to reach an integral of the form  $\int \left( A + B \times \frac{1}{u} \right) du$ . **Depends on both previous M's**

dM1: Integrates to a form  $Au + B \ln u$ . **Depends on the previous M.**

An alternative for the previous 2 marks is to use integration by parts:

E.g.  $\int \frac{4}{u} \times (u-3) du = 4(u-3) \ln u - \int 4 \ln u du = 4u \ln u - 12 \ln u - 4u \ln u + 4u = 4u - 12 \ln u$

Score dM1 for  $\int \frac{k}{u} \times (Au+B) du = k(Au+B) \ln u - \int k \ln u du$  and dM1 for integrating to a form  $Au + B \ln u$ .



Att:  $4u - 12 \ln u$  or  $k(4u - 12 \ln u)$  following through on  $\frac{dx}{du} = k(u-3)$  only.

M1: Substitutes 8 and 4 into their  $4u - 12 \ln u$  and subtracts **or** substitutes 13 and 1 into their  $4u - 12 \ln u$  with  $u = 3 + \sqrt{2x-1}$  and subtracts. This mark depends on there having been an attempt to integrate, however poor.

A1:  $16 - 12 \ln 2$

Question Number	Scheme	Marks
<b>6(a)</b>	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ oe	M1 A1
		<b>(5)</b>
<b>(b)</b>	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2}$ oe	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{3}{2}$	A1
		<b>(4)</b>
		<b>(9 marks)</b>

(a)

B1: Differentiates  $4y^2 + 3x$  to obtain  $8y \frac{dy}{dx} + 3$ . Allow unsimplified forms such as  $4 \times 2y \frac{dy}{dx} + 3$

M1: Uses the product rule on  $6ye^{-2x}$  to obtain an expression of the form  $Aye^{-2x} + Be^{-2x} \frac{dy}{dx}$

A1: Differentiates  $6ye^{-2x}$  to obtain  $-12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$

M1: Collects two terms in  $\frac{dy}{dx}$  (one from attempting to differentiate  $4y^2$  and one from attempting to differentiate  $6ye^{-2x}$ ) and proceeds to make  $\frac{dy}{dx}$  the subject.

A1:  $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$  or equivalent e.g.  $\frac{dy}{dx} = \frac{2e^{-2x} \times 6y + 3}{6e^{-2x} - 8y}$  or  $\frac{dy}{dx} = \frac{12y + 3e^{2x}}{6 - 8ye^{2x}}$

You can ignore any spurious " $\frac{dy}{dx} =$ " at the start and allow  $y'$  for  $\frac{dy}{dx}$ .

(b)

B1: Uses  $x = 0$  to obtain  $y = \frac{3}{2}$  oe e.g.  $\frac{6}{4}$  (ignore any reference to  $y = 0$ )

M1: Substitutes  $x = 0$  and their  $y$  at  $x = 0$  which has come from substituting  $x = 0$  into the original equation into their  $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$  to find a numerical value. Working is normally shown here but you may need to check for evidence. Use of  $x = 0$  and  $y = 0$  is M0.

dM1: Uses the negative reciprocal of " $\frac{7}{-2}$ " for the gradient of the normal and uses this and their value of  $y$  at  $x = 0$  to form the equation of the normal. **Depends on the previous M.**

Note that the use of (0, 0) for  $P$  will generally lose the final 3 marks in (b)

Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$	M1
	$= \dots - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	A1
	$\frac{5}{4} \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$	M1
	$= \dots + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	A1
	$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
(b)	$\left( \frac{2}{5} e^{2\pi} \sin \pi - \frac{1}{5} e^{2\pi} \cos \pi \right) - \left( \frac{2}{5} e^0 \sin 0 - \frac{1}{5} e^0 \cos 0 \right) = \dots$	M1
	$= \frac{1}{5} e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi} + 1}{5} *$	A1*
		(2)
		(7 marks)

Note that you can condone the omission of the dx's throughout.

(a) Way 1

M1: Attempts integration by parts with  $u = \sin x$  and  $v' = e^{2x}$  to obtain

$$\int e^{2x} \sin x \, dx = A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx \quad A > 0$$

dM1: Attempts integration by parts again with  $u = \cos x$  and  $v' = e^{2x}$  on  $B \int e^{2x} \cos x \, dx$  to obtain

$$B \int e^{2x} \cos x \, dx = \pm C e^{2x} \cos x \pm D \int e^{2x} \sin x \, dx$$

Depends on the previous mark.

A1: For  $\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$

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Allow unsimplified e.g.  $\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \left\{ \frac{1}{4} e^{2x} \cos x + \int \frac{1}{4} e^{2x} \sin x \, dx \right\}$

**ddM1: Dependent upon having scored both M's.**

It is for collecting  $\int e^{2x} \sin x \, dx$  terms together and making it the subject of the formula

A1:  $\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$  (allow with or without "+ c")

(a) **Way 2**

M1: Attempts integration by parts with  $u = e^{2x}$  and  $v' = \sin x$  to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

dM1: Attempts integration by parts again with  $u = e^{2x}$  and  $v' = \cos x$  on  $B \int e^{2x} \cos x \, dx$  to obtain

$$B \int e^{2x} \cos x \, dx = \pm C e^{2x} \sin x \pm D \int e^{2x} \sin x \, dx$$

**Depends on the previous mark.**

A1: For  $\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$

Allow unsimplified e.g.  $\int e^{2x} \sin x \, dx = -e^{2x} \cos x - \left\{ -2e^{2x} \sin x - \int -4e^{2x} \sin x \, dx \right\}$

**ddM1: Dependent upon having scored both M's.**

It is for collecting  $\int e^{2x} \sin x \, dx$  terms together and making it the subject of the formula

A1:  $\int e^{2x} \sin x \, dx = \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$  (allow with or without "+ c")

(a) **Way 3**

M1: Attempts integration by parts with  $u = \sin x$  and  $v' = e^{2x}$  to obtain

$$\int e^{2x} \sin x \, dx = A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx \quad A > 0$$

or attempts integration by parts with  $u = e^{2x}$  and  $v' = \sin x$  to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

dM1: Attempts integration by parts with  $u = \sin x$  and  $v' = e^{2x}$  to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \sin x \pm B \int e^{2x} \cos x \, dx$$

and attempts integration by parts with  $u = e^{2x}$  and  $v' = \sin x$  to obtain

$$\int e^{2x} \sin x \, dx = \pm A e^{2x} \cos x \pm B \int e^{2x} \cos x \, dx$$

A1:  $I_1 = \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$  AND  $I_2 = \int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$

ddM1: E.g.  $4I_1 + I_2 = 2e^{2x} \sin x - e^{2x} \cos x = 5I \Rightarrow I = \dots$  Correct attempt to eliminate  $\int e^{2x} \cos x \, dx$  term.

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A1:  $\int e^x \sin x \, dx = \frac{2}{5} e^x \sin x - \frac{1}{5} e^x \cos x + c$  (allow with or without "+ c")

(b)

M1: For applying the limits 0 and  $\pi$  to an expression containing at least one term of the form  $Ae^{2x} \sin x$  and at least one term of the form  $Be^{2x} \cos x$ . **There must be some evidence that both limits have been used.**

A1\*:  $\frac{e^{2\pi} + 1}{5}$  found correctly **from the correct answer in part (a)** via at least one intermediate line

which could be  $\frac{e^{2\pi}}{5} + \frac{1}{5}$

Note a correct answer in (a) and evidence of use of the limits 0 and pi followed by  $\frac{e^{2\pi} + 1}{5}$  with no intermediate

line scores M1A0

Question Number	Scheme	Marks
8	$\begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix} \Rightarrow \begin{array}{l} -1 + 2\lambda = 2 + 4\mu \quad (1) \\ 5 - \lambda = -2 - 3\mu \quad (2) \\ 4 + 5\lambda = -5 + \mu b \quad (3) \end{array}$	
	Uses equations (1) and (2) to find either $\lambda$ <b>or</b> $\mu$ e.g. $(1) + 2(2) \Rightarrow \mu = \dots$ or $3(1) + 4(2) \Rightarrow \lambda = \dots$	M1
	Uses equations (1) and (2) to find both $\lambda$ and $\mu$	dM1
	$\mu = -\frac{11}{2} \text{ and } \lambda = -\frac{19}{2}$	A1
	$4 + 5\lambda = -5 + \mu b \Rightarrow 4 + 5 \times -\frac{19}{2} = -5 - \frac{11}{2} b$ <p style="text-align: center;">or</p> $4 + 5\lambda = -5 + 7\mu \Rightarrow 4 + 5 \times -\frac{19}{2} = -5 - \frac{11}{2} \times 7$	ddM1
	$\Rightarrow 11b = 77 \Rightarrow b = 7 \text{ or obtains } -\frac{87}{2} = -\frac{87}{2}$	A1
	States that when $b = 7$ , lines intersect or when $b \neq 7$ , lines do not intersect Lines are not parallel so when $b \neq 7$ lines are skew. *	A1 Cso
		(6)
<b>Alternative assuming <math>b = 7</math>:</b>		
	$\begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix} \Rightarrow \begin{array}{l} -1 + 2\lambda = 2 + 4\mu \quad (1) \\ 5 - \lambda = -2 - 3\mu \quad (2) \\ 4 + 5\lambda = -5 + 7b \quad (3) \end{array}$	
	Uses any 2 equations to find either $\lambda$ <b>or</b> $\mu$	M1
	Uses any 2 equations to find both $\lambda$ and $\mu$	dM1
	$\mu = -\frac{11}{2} \text{ and } \lambda = -\frac{19}{2}$	A1
	Checks in the 3 <sup>rd</sup> equation e.g. equation 3: $4 + 5\left(-\frac{19}{2}\right) = -5 + 7\left(-\frac{11}{2}\right) = \dots$ equation 1: $-1 + 2\left(-\frac{19}{2}\right) = 2 + 4\left(-\frac{11}{2}\right) = \dots$ equation 2: $5 - \left(-\frac{19}{2}\right) = -2 - 3\left(-\frac{11}{2}\right) = \dots$	ddM1
	Equation 3: $-\frac{87}{2}$ Equation 1: $-20$ Equation 2: $\frac{29}{2}$	A1
	States that when $b = 7$ , lines intersect or when $b \neq 7$ , lines do not intersect Lines are not parallel so when $b \neq 7$ lines are skew. *	A1 Cso
		(6 marks)

M1: For attempting to solve equations (1) and (2) to find **either**  $\lambda$  **or**  $\mu$

dM1: For attempting to solve equations (1) and (2) to find **both**  $\lambda$  **and**  $\mu$  **Depends on the first M.**

$$A1: \mu = -\frac{11}{2} \text{ and } \lambda = -\frac{19}{2}$$

ddM1: Attempts to solve  $4 + 5\lambda = -5 + \mu b$  for their values of  $\lambda$  and  $\mu$ . Or uses  $b = 7$  with their  $\lambda$  and  $\mu$  in an attempt to show equality. **Depends on both previous M's.**

A1: Achieves (without errors) that they will intersect when  $b = 7$

**Note that the previous 3 marks may be scored without explicitly seeing the values of both parameters e.g.**

$$\mu = -\frac{11}{2}, (2) \rightarrow \lambda = 3\mu + 7 \rightarrow 4 + 5(3\mu + 7) = -5 + \mu b \rightarrow b = 7$$

A1\*:Cso States that when  $b = 7$ , lines intersect and since lines are not parallel it shows that when  $b \neq 7$  lines are skew.

**Alternative:**

M1: Uses  $b = 7$  and attempts to solve 2 equations to find **either**  $\lambda$  **or**  $\mu$

dM1: For attempting to solve 2 equations to find **both**  $\lambda$  **and**  $\mu$  **Depends on the first M.**

$$A1: \mu = -\frac{11}{2} \text{ and } \lambda = -\frac{19}{2}$$

ddM1: Attempts to show that the 3<sup>rd</sup> equation is true for their values of  $\lambda$  and  $\mu$

**Depends on both previous M's.**

A1: Achieves (without errors) that the 3<sup>rd</sup> equation gives the same values for (or equivalent)

A1\*: Cso States that when  $b = 7$ , lines intersect and since lines are not parallel it shows that when  $b \neq 7$  lines are skew.

To score the final mark there must be some statement that the lines intersect (or equivalent e.g. meet at a point, cross, etc.) when  $b = 7$  or that they do not intersect if  $b \neq 7$  **and** that the lines are not parallel which may appear anywhere (reason not needed but may be present) so lines are skew when  $b \neq 7$ .

Ignore any work attempting to show that the lines are perpendicular or not.



Question Number	Scheme	Marks
9(a)	$\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3}$ (or $60^\circ$ ) (Allow $x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$ (or $60^\circ$ ))	B1
	$V = (\pi) \int y^2 dx = (\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$ oe	M1A1
	$4(\pi) \int \sin^2 2\theta \sec^2 \theta d\theta = 4(\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta$	dM1
	$= 16(\pi) \int \sin^2 \theta d\theta$ oe e.g. $16(\pi) \int (1 - \cos^2 \theta) d\theta$	A1
	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \Rightarrow 16(\pi) \int \sin^2 \theta d\theta = 16(\pi) \int \frac{1 - \cos 2\theta}{2} d\theta$	dM1
	Volume = $\int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta$	A1 Cso
		(7)
(b)	$\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$	B1
	Volume = $\int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta = [8\pi\theta - 4\pi \sin 2\theta]_0^{\frac{\pi}{3}} = \frac{8}{3}\pi^2 - 2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

(a)

B1: States or uses  $\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3}$  (Allow  $60^\circ$  here). May be implied by their integral.

Allow if seen anywhere in the question either stated or used as their upper limit.

M1: Attempts volume =  $(A\pi) \int y^2 dx = (A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$  with or without  $\pi$  or “d $\theta$ ”.

Condone bracketing errors

A1: For a volume of  $(A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$  with or without  $\pi$  or “d $\theta$ ”. The brackets must be present but may be implied by subsequent work.

dM1: Uses  $\sin 2\theta = 2 \sin \theta \cos \theta$  and proceeds to Volume =  $B \int \sin^2 \theta d\theta$  with or without “d $\theta$ ”. (No requirement for limits yet). Note that if  $(2 \sin 2\theta)^2$  becomes  $2 \sin^2 \theta \cos^2 \theta$  with no evidence of a correct identity then score dM0

**Depends on the first M.**

A1: Volume =  $(A\pi) \int 16 \sin^2 \theta d\theta$  oe e.g.  $(A\pi) \int 16(1 - \cos^2 \theta) d\theta$  with or without  $\pi$  or “d $\theta$ ”. (No requirement for limits yet)

dM1: Attempts to use  $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$  or  $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$  and obtains Volume =  $\int (P \pm Q \cos 2\theta) d\theta$

A1: CSO  $\int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta$ . Fully correct integral with both limits and the “dθ” but the 8 and/or the π can be either side of the integral sign.

**Note this alternative solution for part (a):**

$$V = (A\pi) \int y^2 dx = (A\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta = (A\pi) \int \frac{4 \sin^2 2\theta}{\cos^2 \theta} d\theta \quad \mathbf{M1 \ A1 \ as \ above}$$

$$= (A\pi) \int \frac{4 \sin^2 2\theta}{\frac{1}{2}(1 + \cos 2\theta)} d\theta$$

**dM1:** uses  $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$  in the denominator. **A1:** Correct integral

$$= 8(A\pi) \int \frac{1 - \cos^2 2\theta}{1 + \cos 2\theta} d\theta = 8(A\pi) \int \frac{(1 + \cos 2\theta)(1 - \cos 2\theta)}{1 + \cos 2\theta} d\theta$$

**dM1:** Uses  $\sin^2 2\theta = 1 - \cos^2 2\theta$  and the difference of 2 squares in the numerator and cancels

$$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta \quad \mathbf{A1 \ CSO}$$

Note that a Cartesian approach in part (a) essentially follows the main scheme e.g.

$$V = (\pi) \int y^2 dx = (A\pi) \int (4x \cos^2 \theta)^2 \sec^2 \theta d\theta = 4(A\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta \text{ etc.}$$

**If in doubt whether such attempts deserve credit send to review.**

(b)

B1: States or uses  $\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$

M1: Volume =  $\int_0^{\frac{\pi}{3}} p(1 - \cos 2\theta) d\theta = [p\theta \pm kp \sin 2\theta]_0^{\frac{\pi}{3}}$  and uses the limit  $\frac{\pi}{3}$  (not 60°).

(The limit of 0 may not be seen)

A1:  $\frac{8}{3}\pi^2 - 2\sqrt{3}\pi$  oe e.g.  $8\pi\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)\frac{8}{3}$ ,  $\pi^2 - 2\sqrt{3}\pi$ ,  $\frac{2\pi}{3}(4\pi - 3\sqrt{3})$ ,  $\frac{8\pi^2 - 6\sqrt{3}\pi}{3}$

Question Number	Scheme	Marks
<b>10(a)</b>	$\frac{1}{(H-5)(H+3)} = \frac{A}{H-5} + \frac{B}{H+3} \Rightarrow A = \dots \text{ or } B = \dots$	M1
	$A = \frac{1}{8} \text{ or } B = -\frac{1}{8}$	A1
	$\frac{1}{(H-5)(H+3)} = \frac{1}{8(H-5)} - \frac{1}{8(H+3)} \text{ or } \frac{\frac{1}{8}}{(H-5)} - \frac{\frac{1}{8}}{(H+3)} \text{ or } \frac{\frac{1}{8}}{(H-5)} + \frac{-\frac{1}{8}}{(H+3)}$ $\text{or } \frac{1}{8H-40} - \frac{1}{8H+24}$	A1
		<b>(3)</b>
<b>(b)</b>	$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$ $\int \frac{40}{(H-5)(H+3)} dH = \int -1 dt \text{ or e.g. } \int \frac{1}{(H-5)(H+3)} dH = \int -\frac{1}{40} dt$ $\int \frac{5}{(H-5)} - \frac{5}{(H+3)} dH = \int -1 dt \text{ or e.g. } \frac{1}{8} \int \frac{1}{(H-5)} - \frac{1}{(H+3)} dH = \int -\frac{1}{40} dt$	M1
	$5 \ln H-5  - 5 \ln H+3  = -t(+c) \text{ oe e.g. } \frac{1}{8} \ln H-5  - \frac{1}{8} \ln H+3  = -\frac{1}{40} t(+c)$ Or e.g. $5 \ln(8H-40) - 5 \ln(8H+24) = -t(+c)$ etc.	M1 A1ft
	Substitutes $t=0, H=13 \Rightarrow c = \dots$	M1
	Note that this may happen at a later stage e.g. may attempt to remove logs and then substitute to find the constant	M1
	$5 \ln H-5  - 5 \ln H+3  = -t + 5 \ln\left(\frac{1}{2}\right) \text{ oe e.g.}$ $\frac{1}{8} \ln H-5  - \frac{1}{8} \ln H+3  = -\frac{1}{40} t + \frac{1}{8} \ln\left(\frac{1}{2}\right)$	A1
	$5 \ln\left(2 \frac{H-5}{H+3}\right) = -t \Rightarrow \frac{H-5}{H+3} = \frac{1}{2} e^{-0.2t} \Rightarrow H = \dots$	dddM1
	$H = \frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}} *$	A1*
		<b>(7)</b>
<b>(c)</b>	Sets $\frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}} = 8 \Rightarrow e^{-0.2t} = \left(\frac{6}{11}\right)$	M1
	$\Rightarrow t = -5 \ln\left(\frac{6}{11}\right) = \text{awrt } 3.03 \text{ days}$	dM1 A1
		<b>(3)</b>
<b>(d)</b>	$k = 5$	B1
		<b>(1)</b>
		<b>(14 marks)</b>

(a)

M1: Attempts any correct method to find either constant. It is implied by one correct constant

A1: One correct constant

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 A1: Correct partial fractions  $\frac{1}{8(H-5)} - \frac{1}{8(H+3)}$ . Note that this mark is not just for the correct constants, it is for the

correctly stated fractions either in part (a) or used in part (b). Allow 0.125 for 1/8.

(b)

M1: Separates the variables and uses part (a) to reach:  $\int \frac{P}{(H-5)} + \frac{Q}{(H+3)} dH = \int \pm k dt$  with or without the integral signs

M1: Attempts to integrate both sides to reach:  $\alpha \ln|H-5| + \beta \ln|H+3| = kt$

or e.g.  $\alpha \ln|8H-40| + \beta \ln|8H+24| = kt$  Condone  $| \leftrightarrow ( )$  and condone the omission of brackets e.g. allow

$\alpha \ln H-5 + \beta \ln H+3 = kt$  or e.g.  $\alpha \ln 8H-40 + \beta \ln 8H+24 = kt$

A1ft: Correct integration of both sides following through on their PF in (a). Condone  $| \leftrightarrow ( )$  and condone the omission of + c but brackets must be present unless they are implied by subsequent work.

Also follow through on a MR of  $\frac{dH}{dt} = \frac{(H-5)(H+3)}{40}$  for  $\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$

E.g. obtains  $\frac{1}{8} \ln|H-5| - \frac{1}{8} \ln|H+3| = \frac{1}{40} t (+c)$

M1: Substitutes  $t=0, H=13 \Rightarrow c = \dots$  For this to be scored there must have been a + c and depends on some attempt at integration of both sides however poor.

Alternatively attempts  $\int_{13}^H \frac{1}{(H-5)} - \frac{1}{(H+3)} dH = \int_0^t -\frac{1}{5} dt \Rightarrow \left[ \ln \frac{H-5}{H+3} \right]_{13}^H = \left[ -\frac{1}{5} t \right]_0^t \Rightarrow \ln \frac{H-5}{H+3} - \ln \frac{1}{2} = -\frac{1}{5} t$

A1: For a correct equation in H and t. Condone  $| \leftrightarrow ( )$  but brackets must be present unless they are implied by subsequent work.

dddM1: A correct attempt to make H the subject of the formula. **All previous M's in (b) must have been scored.**

A1\*:  $H = \frac{10+3e^{-0.2t}}{2-e^{-0.2t}}$  cso with sufficient working shown and no errors.

**Note that marks in (b) may need to be awarded retrospectively:**

E.g. First 3 marks gained to reach  $\ln|H-5| - \ln|H+3| = -\frac{1}{5}t + c$  and then:

$$\ln \frac{H-5}{H+3} = -\frac{1}{5}t + c \Rightarrow \frac{H-5}{H+3} = Ae^{-0.2t} \Rightarrow H = \frac{5+3Ae^{-0.2t}}{1-Ae^{-0.2t}}$$

$$H=13, t=0 \Rightarrow 13 = \frac{5+3A}{1-A} \Rightarrow A = \frac{1}{2} \Rightarrow H = \frac{5 + \frac{3}{2}e^{-0.2t}}{1 - \frac{1}{2}e^{-0.2t}} = \frac{10+3e^{-0.2t}}{2-e^{-0.2t}} *$$

**The M3 can be awarded when they attempt to find "A", the dddM4 can be awarded for a correct attempt to make H the subject and then A2 and A3 can be awarded together at the end.**

(c)

M1: Sets  $\frac{10+3e^{-0.2t}}{2-e^{-0.2t}} = 8$  or possibly an earlier version of H or possibly their t in terms of H and reaches

$$Ae^{\pm 0.2t} = p, \quad p > 0$$

dM1: Correct processing of an equation of the form  $Ae^{\pm 0.2t} = p$  with correct log work leading to  $t = \dots$

**Depends on the first M.**

(d)

B1:  $k = 5$  (Allow  $H = 5$  or just "5")