Pearson Edexcel International A Level Mathematics

Pure Mathematics 3

Past Paper Collection (from 2020)

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Last updated: July 1, 2024

Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Wednesday 22 January 2020

Morning (Time: 1 hour 30 minutes)	Paper Reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have: Mathematical Formulae and Statistical Tables (Lilac), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Leave blank

1. A population of a rare species of toad is being studied.

The number of toads, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \qquad t \geqslant 0, t \in \mathbb{R}$$

According to this model,

(a) calculate the number of toads in the population at the start of the study,

(1)

(b) find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places.

(4)

(c) Explain why, according to this model, the number of toads in the population can never reach 500

(1)

Leave blank

Question 1 continued

(Total 6 marks)

Q1

Leave blank

2. The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$

(2)

(b) Find f^{-1}

(3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

(3)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

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3.

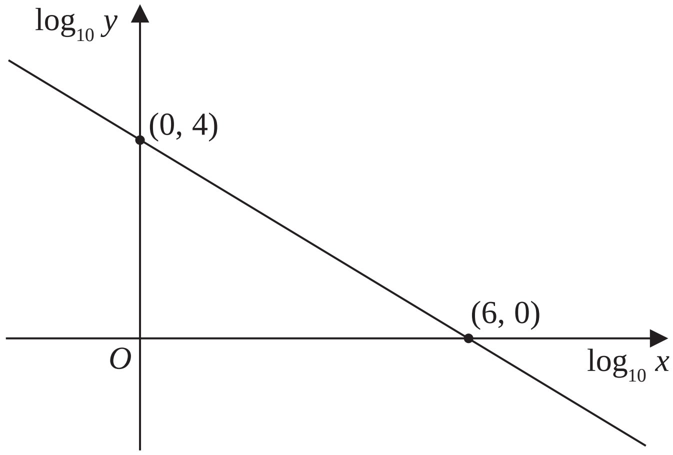

Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (0, 4) and (6, 0) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q, where p and q are constants to be found.

(3)

Leave blank

Question 3 continued

Q3

(Total 5 marks)

Leave blank

4. (i)

$$f(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $f'(x)$ in the form $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(6)

(ii)

$$g(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = g(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

Leave blank

5. (a) Use the substitution $t = \tan x$ to show that the equation

$$12\tan 2x + 5\cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 360°$, the equation

$$12\tan 2x + 5\cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6.

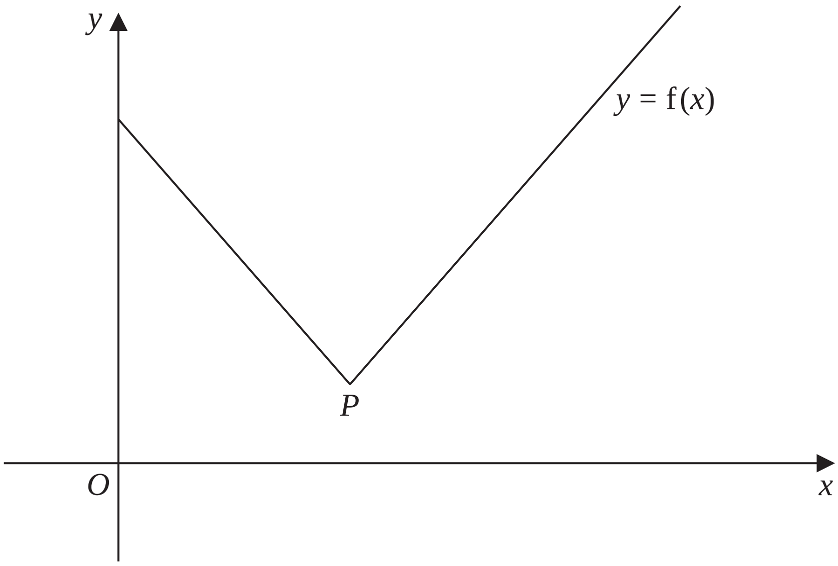

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P.

(2)

(b) Solve the equation $\mathrm{f}(x) = 3x - 2$

(4)

Given that the equation

$$\mathrm{f}(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7.

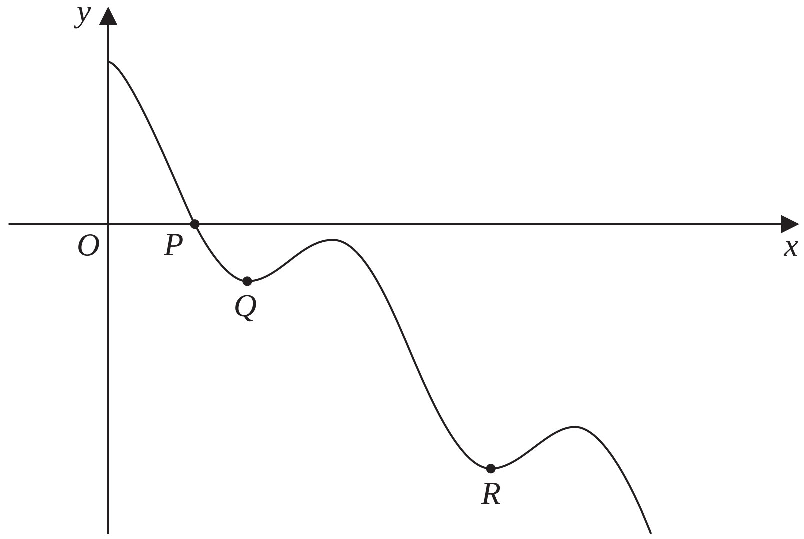

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad x > 0$$

where x is measured in radians.

The curve crosses the x-axis at the point P, as shown in Figure 3.

Given that the x coordinate of P is α,

(a) show that α lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for α.

(b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) x_2

(ii) x_5

(3)

The point Q and the point R are local minimum points on the curve, as shown in Figure 3.

Given that the x coordinates of Q and R are β and λ respectively, and that they are the two smallest values of x at which local minima occur,

(c) find, using calculus, the exact value of β and the exact value of λ.

(6)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x$$

giving your answer in simplest form.

(4)

(ii)
$$\mathrm{h}(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $\mathrm{h}(x) = Ax + B + \dfrac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int \mathrm{h}(x)\,\mathrm{d}x$$

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

Leave blank

9. $$f(\theta) = 5\cos\theta - 4\sin\theta \qquad \theta \in \mathbb{R}$$

(a) Express $f(\theta)$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos\theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

(b) (i) describe fully the transformation that is a stretch,

(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \qquad \theta \in \mathbb{R}$$

(c) find the range of g.

(2)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Thursday 08 October 2020

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **WMA13/01**

Mathematics
International Advanced Level
Pure Mathematics P3

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Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

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Turn over

Leave blank

1. Solve, for $0 \leqslant x < 360°$, the equation

$$2\cos 2x = 7\cos x$$

giving your solutions to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. A scientist monitored the growth of bacteria on a dish over a 30-day period.

The area, N mm^2, of the dish covered by bacteria, t days after monitoring began, is modelled by the equation

$$\log_{10} N = 0.0646t + 1.478 \qquad 0 \leqslant t \leqslant 30$$

(a) Show that this equation may be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of a to the nearest integer and give the value of b to 3 significant figures.

(4)

(b) Use the model to find the area of the dish covered by bacteria 30 days after monitoring began. Give your answer, in mm^2, to 2 significant figures.

(2)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

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3.

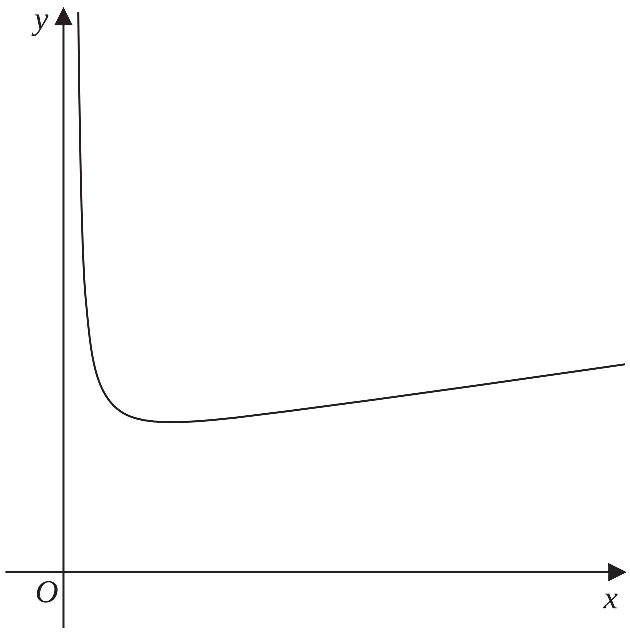

Figure 1

Figure 1 shows a sketch of a curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x+3}{\sqrt{4x-1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $\mathrm{f}'(x)$.

(4)

(b) Hence find the range of f.

(3)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4.

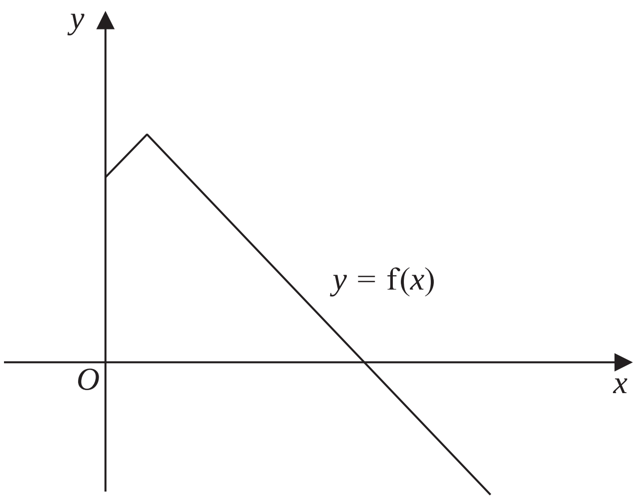

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find $\mathrm{ff}(6)$

(2)

(b) Solve the equation $\mathrm{f}(x) = 5x$

(2)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k.

(2)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x - b)$

The vertex of the graph with equation $y = a\mathrm{f}(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$$

(4)

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6.

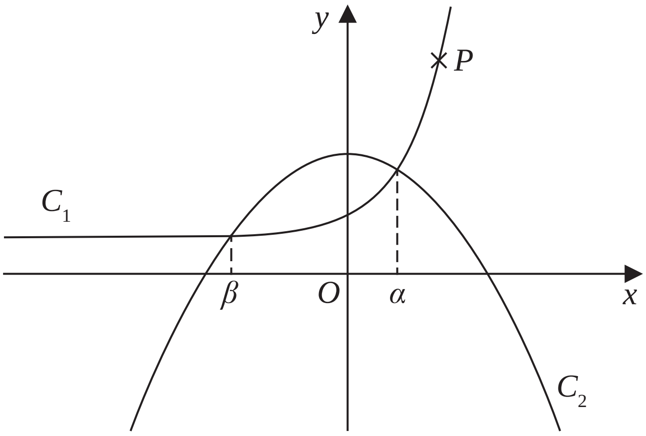

Figure 3

Figure 3 shows a sketch of curve C_1 with equation $y = 5e^{x-1} + 3$

and curve C_2 with equation $y = 10 - x^2$

The point P lies on C_1 and has y coordinate 18

(a) Find the x coordinate of P, writing your answer in the form $\ln k$, where k is a constant to be found.

(3)

The curve C_1 meets the curve C_2 at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

(3)

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to β.

Using this iterative formula with $x_1 = -3$

(c) find the value of x_2 and the value of β, giving each answer to 6 decimal places.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7. (a) Express $\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

A scientist is studying the behaviour of seabirds in a colony.

She models the height above sea level, H metres, of one of the birds in the colony by the equation

$$H = \frac{24}{3 + \cos\left(\frac{1}{2}t\right) + 4\sin\left(\frac{1}{2}t\right)} \qquad 0 \leqslant t \leqslant 6.5$$

where t seconds is the time after it leaves the nest.

Find, according to the model,

(b) the minimum height of the seabird above sea level, giving your answer to the nearest cm,

(2)

(c) the value of t, to 2 decimal places, when $H = 10$

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. (i) The curve C has equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \mathrm{e}^{3x} \sec 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

(a) Find $\mathrm{g}'(x)$

(2)

(b) Hence find the x coordinate of the stationary point of C.

(3)

(ii) A different curve has equation

$$x = \ln(\sin y) \qquad 0 < y < \frac{\pi}{2}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^x}{\mathrm{f}(x)}$$

where $\mathrm{f}(x)$ is a function of e^x that should be found.

(4)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

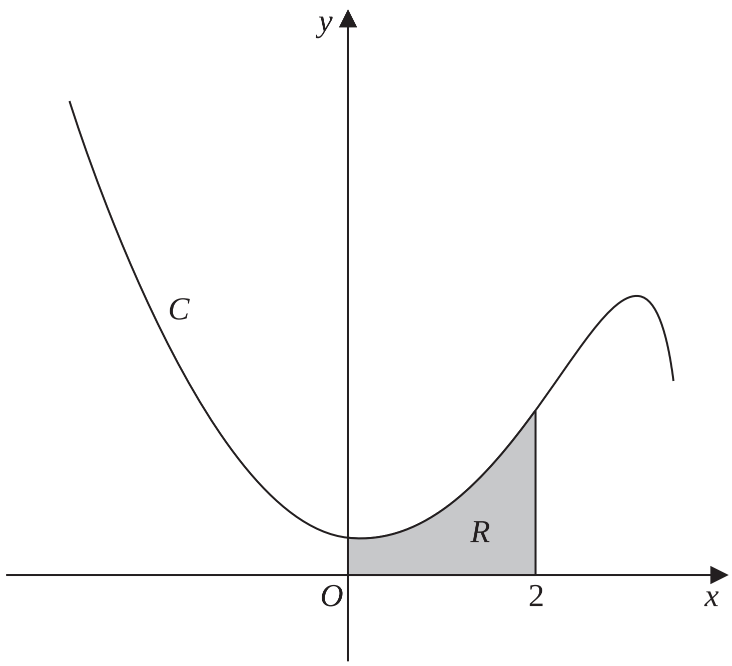

Figure 4

Figure 4 shows a sketch of the curve C.
The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 14 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

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Turn over

Leave blank

1. Find

$$\int \frac{x^2 - 5}{2x^3}\,\mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

Leave blank

Question 1 continued

Q1

(Total 3 marks)

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2.

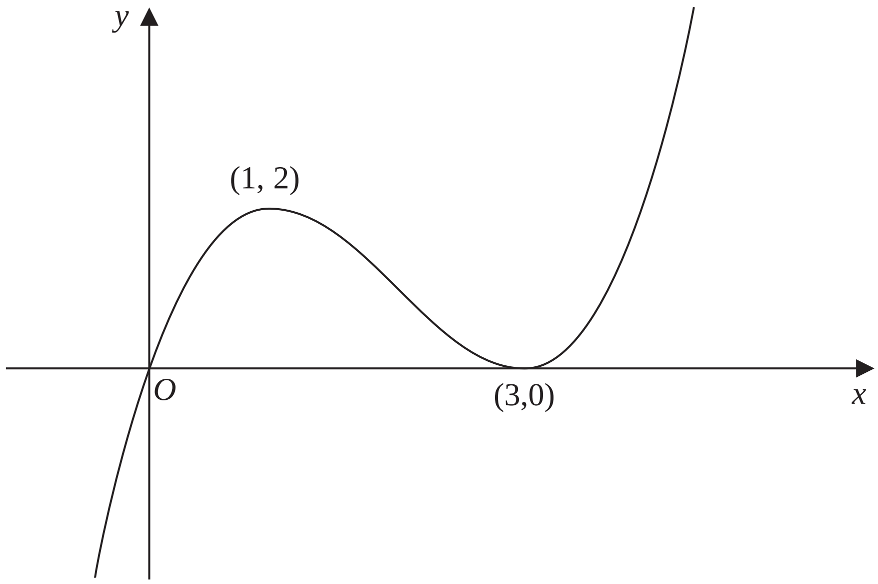

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where $x \in \mathbb{R}$ and $\mathrm{f}(x)$ is a polynomial.

The curve passes through the origin and touches the x-axis at the point (3, 0)

There is a maximum turning point at (1, 2) and a minimum turning point at (3, 0)

On separate diagrams, sketch the curve with equation

(i) $y = 3\mathrm{f}(2x)$

(3)

(ii) $y = \mathrm{f}(-x) - 1$

(3)

On each sketch, show clearly the coordinates of

- the point where the curve crosses the y-axis
- any maximum or minimum turning points

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3.

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2 - 3x - 5} \qquad x > 4$$

(a) Show that

$$f(x) = \frac{ax+b}{cx+d} \qquad x > 4$$

where a, b, c and d are integers to be found.

(4)

(b) Hence find $f^{-1}(x)$

(2)

(c) Find the domain of f^{-1}

(2)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4.

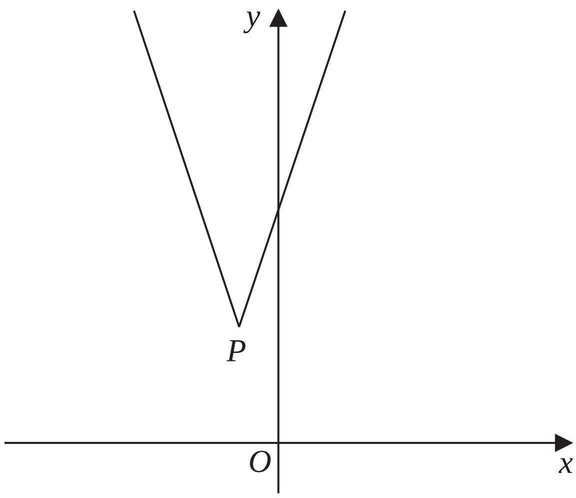

Figure 2

Figure 2 shows a sketch of the graph with equation $y = \text{f}(x)$, where

$$\text{f}(x) = |3x + a| + a$$

and where a is a positive constant.

The graph has a vertex at the point P, as shown in Figure 2.

(a) Find, in terms of a, the coordinates of P.

(2)

(b) Sketch the graph with equation $y = \text{g}(x)$, where

$$\text{g}(x) = |x + 5a|$$

On your sketch, show the coordinates, in terms of a, of each point where the graph cuts or meets the coordinate axes.

(2)

The graph with equation $y = \text{g}(x)$ intersects the graph with equation $y = \text{f}(x)$ at two points.

(c) Find, in terms of a, the coordinates of the two points.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

(Total 9 marks)

Q4

Leave blank

5. The temperature, θ °C, inside an oven, t minutes after the oven is switched on, is given by

$$\theta = A - 180e^{-kt}$$

where A and k are positive constants.

Given that the temperature inside the oven is initially 18 °C,

(a) find the value of A.

(2)

The temperature inside the oven, 5 minutes after the oven is switched on, is 90 °C.

(b) Show that $k = p\ln q$ where p and q are rational numbers to be found.

(4)

Hence find

(c) the temperature inside the oven 9 minutes after the oven is switched on, giving your answer to 3 significant figures,

(2)

(d) the rate of increase of the temperature inside the oven 9 minutes after the oven is switched on. Give your answer in °C min^{-1} to 3 significant figures.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 11 marks)

Leave blank

6.

$$\mathrm{f}(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $\mathrm{f}'(x)$

(2)

(b) Show that the equation $\mathrm{f}'(x) = 0$ can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where k is an integer to be found.

(2)

(c) Starting with $x_1 = 2.5$ use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of k found in part (b), to calculate the values of x_2 and x_6 giving your answers to 3 decimal places.

(2)

(d) Using a suitable interval and a suitable function that should be stated, show that a root of $\mathrm{f}'(x) = 0$ is 2.581 correct to 3 decimal places.

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

7. (a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate.

(6)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

8. The percentage, P, of the population of a small country who have access to the internet, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after the start of 2005

Using the data for the years between the start of 2005 and the start of 2010, a graph is plotted of $\log_{10} P$ against t.

The points are found to lie approximately on a straight line with gradient 0.09 and intercept 0.68 on the $\log_{10} P$ axis.

(a) Find, according to the model, the value of a and the value of b, giving your answers to 2 decimal places.

(4)

(b) In the context of the model, give a practical interpretation of the constant a.

(1)

(c) Use the model to estimate the percentage of the population who had access to the internet at the start of 2015

(2)

Leave blank

Question 8 continued

Q8

(Total 7 marks)

Leave blank

9. Find

(i) $$\int \frac{3x - 2}{3x^2 - 4x + 5}\, dx$$

(2)

(ii) $$\int \frac{e^{2x}}{(e^{2x} - 1)^3}\, dx \qquad x \neq 0$$

(2)

Leave blank

Question 9 continued

Q9

(Total 4 marks)

Leave blank

10. The curve C has equation

$$x = 3\sec^2 2y \qquad x > 3 \qquad 0 < y < \frac{\pi}{4}$$

(a) Find $\dfrac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{p}{qx\sqrt{x-3}}$$

where p is irrational and q is an integer, stating the values of p and q.

(3)

(c) Find the equation of the normal to C at the point where $y = \dfrac{\pi}{12}$, giving your answer in the form $y = mx + c$, giving m and c as exact irrational numbers.

(5)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

Leave blank

1. The curve C has equation

$$y = x^2 \cos\left(\frac{1}{2}x\right) \qquad 0 < x \leqslant \pi$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = 2\arctan\left(\frac{4}{x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right) \qquad x_1 = 2$$

(b) find the value of x_2 and the value of x_6, giving your answers to 3 decimal places.

(3)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. (a) Show that

$$\frac{1-\cos 2x}{2\sin 2x} \equiv k\tan x \qquad x \neq (90n)^\circ \qquad n \in \mathbb{Z}$$

where k is a constant to be found.

(3)

(b) Hence solve, for $0 < \theta < 90^\circ$

$$\frac{9(1-\cos 2\theta)}{2\sin 2\theta} = 2\sec^2\theta$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. (i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\dfrac{4x+3}{x+2}$ in the form

$$A + \frac{B}{x+2} \quad \text{where } A \text{ and } B \text{ are constants to be found}$$

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

(6)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. The functions f and g are defined by

$$f(x) = \frac{4x+6}{x-5} \qquad x \in \mathbb{R},\ x \neq 5$$

$$g(x) = 5 - 2x^2 \qquad x \in \mathbb{R},\ x \leqslant 0$$

(a) Solve the equation

$$fg(x) = 3$$

(4)

(b) Find f^{-1}

(3)

(c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$. Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes.

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

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Question 4 continued

Q4

(Total 10 marks)

Leave blank

5.

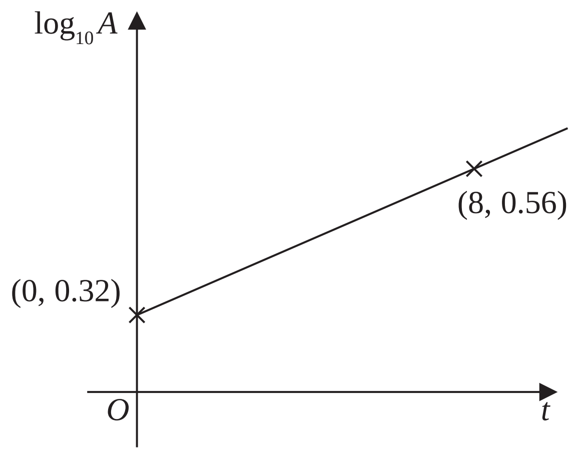

Figure 1

The growth of duckweed on a pond is being studied.

The surface area of the pond covered by duckweed, $A\,\text{m}^2$, at a time t days after the start of the study is modelled by the equation

$$A = pq^t \qquad \text{where } p \text{ and } q \text{ are positive constants}$$

Figure 1 shows the linear relationship between $\log_{10} A$ and t.

The points (0, 0.32) and (8, 0.56) lie on the line as shown.

(a) Find, to 3 decimal places, the value of p and the value of q.

(4)

Using the model with the values of p and q found in part (a),

(b) find the rate of increase of the surface area of the pond covered by duckweed, in m^2/day, exactly 6 days after the start of the study.
Give your answer to 2 decimal places.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left|2x - \frac{k}{3}\right|$

Show on each sketch the coordinates, in terms of k, of each point where the graph meets or cuts the axes.

(4)

(b) Hence find, in terms of k, the values of x for which

$$\left|2x - \frac{k}{3}\right| = k - 2|x|$$

giving your answers in simplest form.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. Given that

$$x = 6\sin^2 2y \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{A\sqrt{(Bx - x^2)}}$$

where A and B are integers to be found.

(5)

Leave blank

Question 7 continued

Q7

(Total 5 marks)

Leave blank

8. A scientist is studying a population of fish in a lake. The number of fish, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{600\mathrm{e}^{0.3t}}{2 + \mathrm{e}^{0.3t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c), (d) and (e).

(a) Find the number of fish in the lake at the start of the study.

(1)

(b) Find the upper limit to the number of fish in the lake.

(1)

(c) Find the time, after the start of the study, when there are predicted to be 500 fish in the lake. Give your answer in years and months to the nearest month.

(4)

(d) Show that

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{A\mathrm{e}^{0.3t}}{\left(2 + \mathrm{e}^{0.3t}\right)^2}$$

where A is a constant to be found.

(3)

Given that when $t = T$, $\dfrac{\mathrm{d}N}{\mathrm{d}t} = 8$

(e) find the value of T to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 13 marks)

Leave blank

9. (a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians, to 3 decimal places.

(3)

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \theta > 0$$

Find

(b) (i) the minimum value of $g(\theta)$

(ii) the smallest value of θ at which the minimum value occurs.

(3)

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

(c) Find the range of h.

(2)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Leave blank

1. The function f is defined by

$$\mathrm{f}(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \qquad x > 0$$

(a) Show that $\mathrm{f}(x) = \dfrac{5x}{x + 3}$

(3)

(b) Find f^{-1}

(3)

(c) (i) Find, in simplest form, $\mathrm{f}'(x)$.

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer.

(3)

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Question 1 continued

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Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 9 marks)

Leave blank

2.

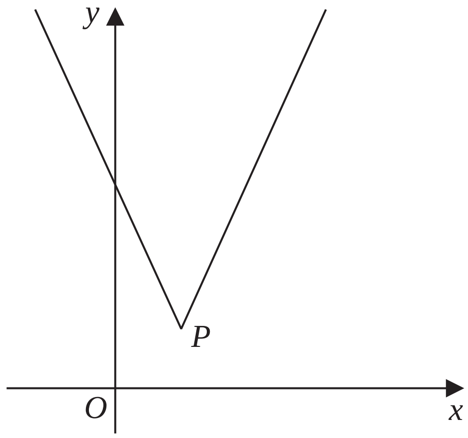

Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |3x - 13| + 5 \qquad x \in \mathbb{R}$$

The vertex of the graph is at point P, as shown in Figure 1.

(a) State the coordinates of P.

(2)

(b) (i) State the range of f.

(ii) Find the value of ff(4)

(2)

(c) Solve, using algebra and showing your working,

$$16 - 2x > |3x - 13| + 5$$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x + b)$

The vertex of the graph with equation $y = a\mathrm{f}(x + b)$ is (4, 20)

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

(Total 10 marks)

Q2

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3.

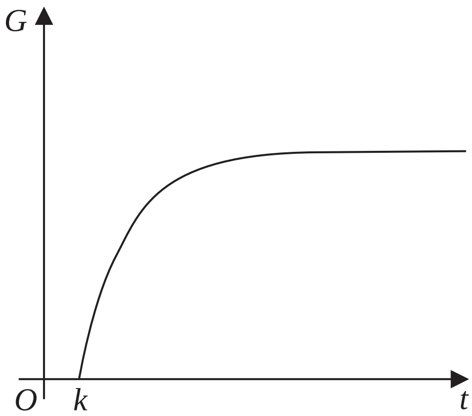

Figure 2

The total mass of gold, G tonnes, extracted from a mine is modelled by the equation

$$G = 40 - 30e^{1-0.05t} \qquad t \geqslant k \qquad G \geqslant 0$$

where t is the number of years after 1st January 1800.

Figure 2 shows a sketch of G against t.

Use the equation of the model to answer parts (a), (b) and (c).

(a) (i) Find the value of k.

(ii) Hence find the year and month in which gold started being extracted from the mine.

(3)

(b) Find the total mass of gold extracted from the mine up to 1st January 1870.

(2)

There is a limit to the mass of gold that can be extracted from the mine.

(c) State the value of this limit.

(1)

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. **In this question you should show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\sin(\theta - 30°) = 5\cos\theta$$

can be written in the form

$$\tan\theta = 2\sqrt{3}$$

(4)

(b) Hence, or otherwise, solve for $0 \leqslant x \leqslant 360°$

$$2\sin(x - 10°) = 5\cos(x + 20°)$$

giving your answers to one decimal place.

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,dx$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7\,dx$$

(2)

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Leave blank

6. (i) The curve C_1 has equation

$$y = 3\ln(x^2 - 5) - 4x^2 + 15 \qquad x > \sqrt{5}$$

Show that C_1 has a stationary point at $x = \frac{\sqrt{p}}{2}$ where p is a constant to be found.

(4)

(ii) A different curve C_2 has equation

$$y = 4x - 12\sin^2 x$$

(a) Show that, for this curve,

$$\frac{dy}{dx} = A + B\sin 2x$$

where A and B are constants to be found.

(b) Hence, state the maximum gradient of this curve.

(4)

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7 The mass, M kg, of a species of tree can be modelled by the equation

$$\log_{10} M = 1.93 \log_{10} r + 0.684$$

where r cm is the base radius of the tree.

The base radius of a particular tree of this species is 45 cm.

According to the model,

(a) find the mass of this tree, giving your answer to 2 significant figures.
(2)

(b) Show that the equation of the model can be written in the form

$$M = pr^q$$

giving the values of the constants p and q to 3 significant figures.
(3)

(c) With reference to the model, interpret the value of the constant p.
(1)

Leave blank

Question 7 continued

Q7

(Total 6 marks)

Leave blank

8. A curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \arcsin\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2 \qquad -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$$

(a) Sketch C.

(1)

(b) Given $x = 2\sin y$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{A - x^2}}$$

where A is a constant to be found.

(3)

The point P lies on C and has y coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

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Question 8 continued

Q8

(Total 7 marks)

Leave blank

9.

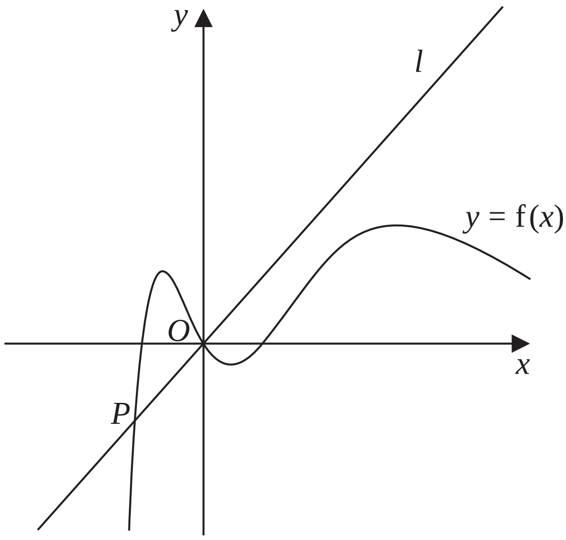

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x(x^2 - 4)\mathrm{e}^{-\frac{1}{2}x}$$

(a) Find $\mathrm{f}'(x)$.

(2)

The line l is the normal to the curve at O and meets the curve again at the point P.

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x}}$$

(4)

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + \mathrm{e}^{\frac{1}{2}x_n}} \qquad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of x_2

(ii) the x coordinate of P.

(3)

Leave blank

Question 9 continued

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Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

Leave blank

10.

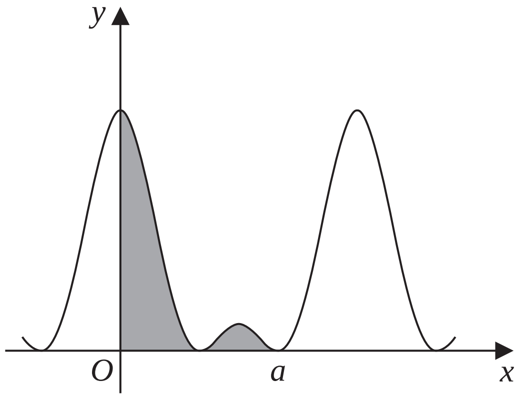

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = (1 + 2\cos 2x)^2$$

(a) Show that

$$(1 + 2\cos 2x)^2 \equiv p + q\cos 2x + r\cos 4x$$

where p, q and r are constants to be found.

(2)

The curve touches the positive x-axis for the second time when $x = a$, as shown in Figure 4.

The regions bounded by the curve, the y-axis and the x-axis up to $x = a$ are shown shaded in Figure 4.

(b) Find, using algebraic integration and making your method clear, the exact total area of the shaded regions. Write your answer in simplest form.

(5)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Leave blank

1. Find, using calculus, the x coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. (a) Show that the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

can be written in the form

$$\sin 2\theta = k$$

where k is a constant to be found.

(3)

(b) Hence find the smallest positive solution of the equation

$$8\cos\theta = 3\operatorname{cosec}\theta$$

giving your answer, in degrees, to one decimal place.

(2)

Leave blank

Question 2 continued

Q2

(Total 5 marks)

Leave blank

3. (i) Find, in simplest form,

$$\int (2x - 5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1 + 2\cos x} \, dx = \ln a$$

where a is a rational constant to be found.

(4)

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

where p is a positive constant and t is the number of days after the start of the study.

Given that

- $30\,\text{m}^2$ of the surface of the pond was covered by the weed at the start of the study
- $50\,\text{m}^2$ of the surface of the pond was covered by the weed T days after the start of the study

(a) show that $p = 2.4$

(2)

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on graphical or numerical methods are not acceptable*.)

(4)

The weed grows until it covers the surface of the pond.

(c) Find, according to the model, the maximum possible surface area of the pond.

(1)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

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5.

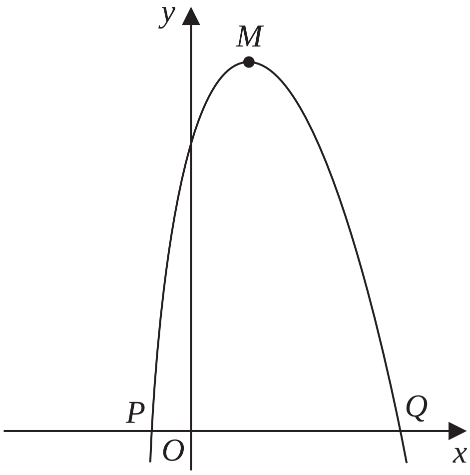

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x+3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative x-axis at the point P, as shown in Figure 1.

(a) Show that the x coordinate of P lies in the interval [–1.25, –1.2]

(2)

The curve cuts the positive x-axis at the point Q, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by continued iteration, the x coordinate of Q. Give your answer to 4 decimal places.

(3)

The curve has a maximum turning point at M, as shown in Figure 1.

(c) Using calculus and showing each stage of your working, find the x coordinate of M.

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Leave blank

6. The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function.

(3)

(b) Find f^{-1}

(3)

(c) (i) Show that $ff(x) = \dfrac{ax + b}{x + c}$ where a, b and c are constants to be found.

(ii) Deduce the range of ff.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 11 marks)

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7.

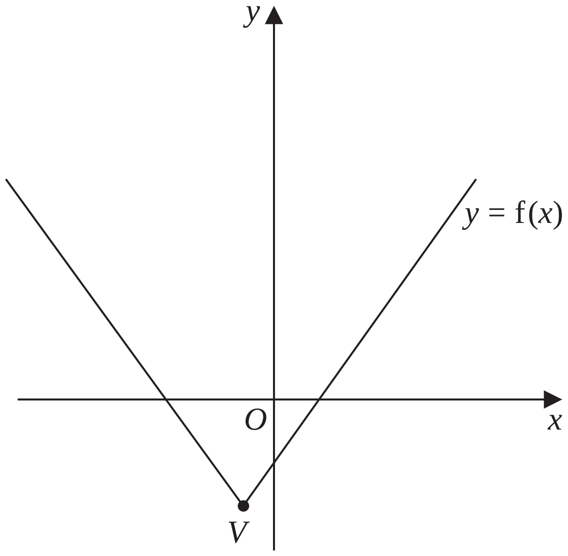

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, V, of the graph.

(2)

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geqslant \frac{1}{3}x + 1$$

(4)

(c) Sketch the graph with equation

$$y = |\mathrm{f}(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 10 marks)

Leave blank

8. A dose of antibiotics is given to a patient.

The amount of the antibiotic, x milligrams, in the patient's bloodstream t hours after the dose was given, is found to satisfy the equation

$$\log_{10} x = 2.74 - 0.079t$$

(a) Show that this equation can be written in the form

$$x = pq^{-t}$$

where p and q are constants to be found. Give the value of p to the nearest whole number and the value of q to 2 significant figures.

(4)

(b) With reference to the equation in part (a), interpret the value of the constant p.

(1)

When a different dose of the antibiotic is given to another patient, the values of x and t satisfy the equation

$$x = 400 \times 1.4^{-t}$$

(c) Use calculus to find, to 2 significant figures, the value of $\frac{dx}{dt}$ when $t = 5$

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 8 marks)

Leave blank

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$2\sec^2 x - 3\tan x = 2$$

giving the answers, as appropriate, to 3 significant figures.

(4)

(ii) Prove that

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv 2$$

(4)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

(Total 8 marks)

Q9

Leave blank

10.

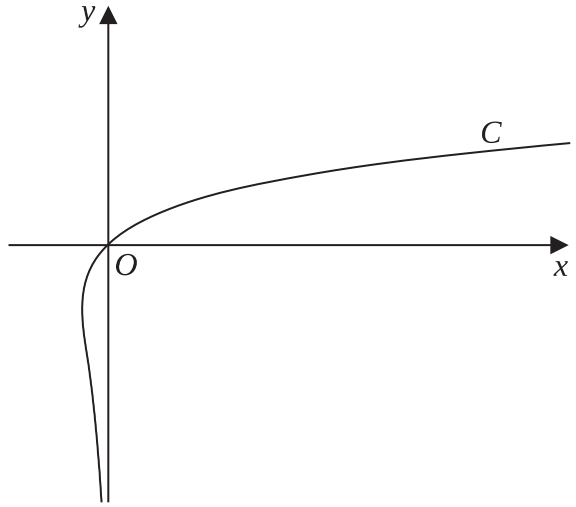

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

(4)

Given that the straight line with equation $x = k$, where k is a constant, cuts C at exactly two points,

(b) find the range of possible values for k.

(3)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Leave blank

1. The curve C has equation

$$y = (3x - 2)^6$$

(a) Find $\frac{dy}{dx}$

(2)

Given that the point $P\left(\frac{1}{3}, 1\right)$ lies on C,

(b) find the equation of the normal to C at P. Write your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. The functions f and g are defined by

$$f(x) = \frac{5 - x}{3x + 2} \qquad x \in \mathbb{R}, x \neq -\frac{2}{3}$$

$$g(x) = 2x - 7 \qquad x \in \mathbb{R}$$

(a) Find the value of fg(5)

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$f\left(\frac{1}{a}\right) = g(a + 3)$$

(4)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k}\,\mathrm{d}x$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k}\,\mathrm{d}x = \ln 8$$

(b) find the value of k

(4)

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4.

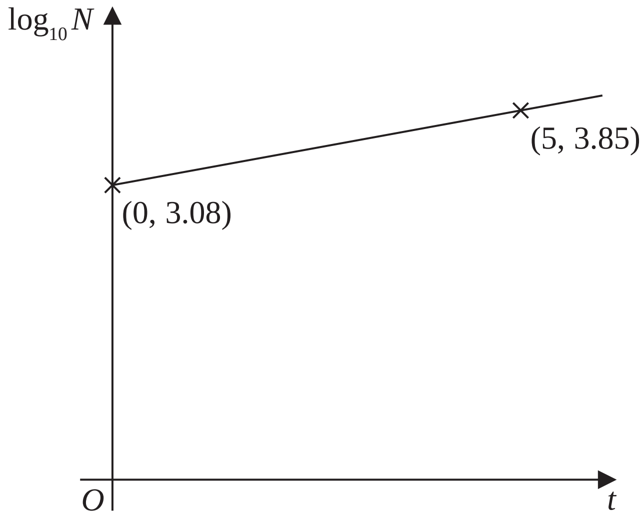

Figure 1

The number of subscribers to an online video streaming service, N, is modelled by the equation

$$N = ab^t$$

where a and b are constants and t is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between t and $\log_{10} N$

The line passes through the points (0, 3.08) and (5, 3.85)

Using this information,

(a) find an equation for this line.

(2)

(b) Find the value of a and the value of b, giving your answers to 3 significant figures.

(3)

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of T

(2)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5.

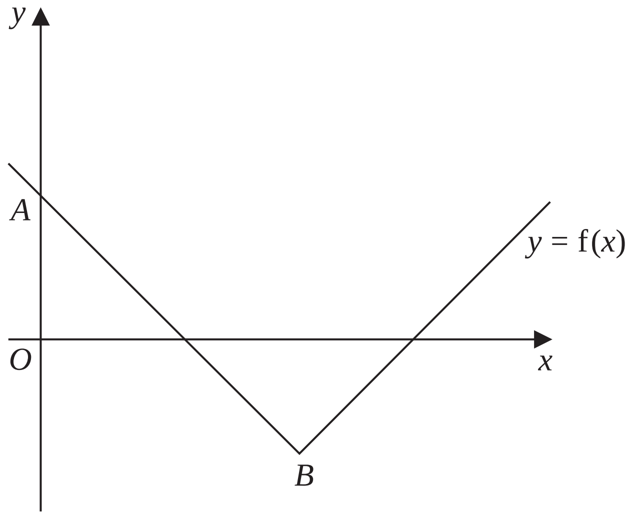

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |kx - 9| - 2 \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph intersects the y-axis at the point A and has a minimum point at B as shown.

(a) (i) Find the y coordinate of A

(ii) Find, in terms of k, the x coordinate of B

(2)

(b) Find, in terms of k, the range of values of x that satisfy the inequality

$$|kx - 9| - 2 < 0$$

(3)

Given that the line $y = 3 - 2x$ intersects the graph $y = \mathrm{f}(x)$ at two distinct points,

(c) find the range of possible values of k

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6.

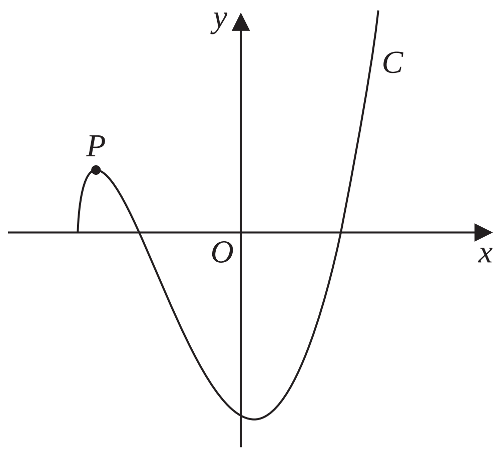

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$\mathrm{f}(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $\mathrm{f}'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 10 marks)

Q6

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

can be written as

$$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$$

(4)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

giving your answers to 3 significant figures.

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Leave blank

8.

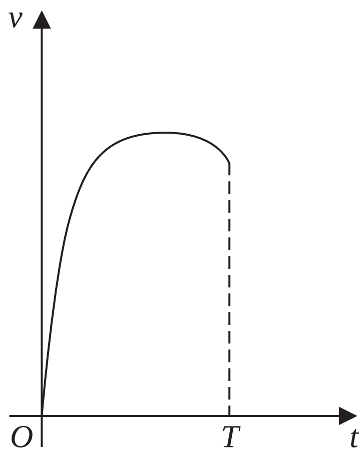

Figure 4

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v\,\mathrm{m\,s^{-1}}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \qquad t \geqslant 0$$

where t seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.
(5)

Given that the sprinter runs 100 m in T seconds, such that

$$\int_0^T v\,dt = 100$$

(b) show that T is a solution of the equation

$$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$$
(4)

The iteration formula

$$T_{n+1} = \frac{1}{12}\left(116 - 16e^{-0.75T_n} + e^{T_n-10} - e^{-10}\right)$$

is used to find an approximate value for T

Using this iteration formula with $T_1 = 10$

(c) find, to 4 decimal places,

(i) the value of T_2

(ii) the time taken by the sprinter to run the race, according to the model.
(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 12 marks)

Leave blank

9.

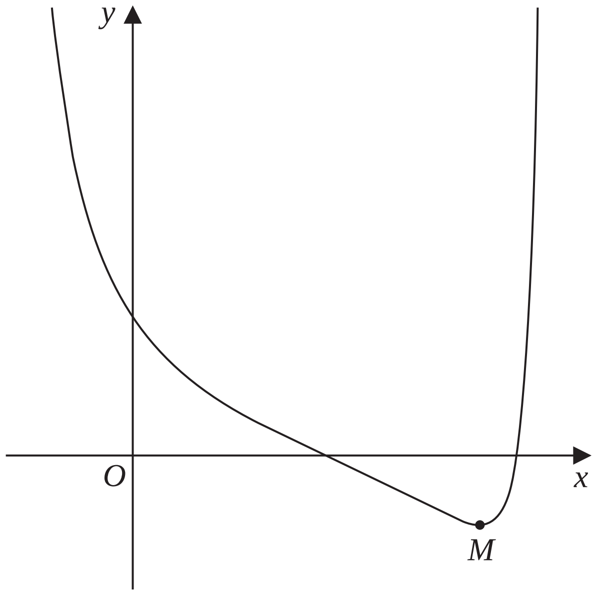

Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2\cos x}{1 + \sin x} \qquad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M, shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2\sin x + \cos x = -2$$

(4)

(b) Hence find, to 3 significant figures, the x coordinate of M.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number	Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Leave blank

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x+3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 9 marks)

Leave blank

2. The functions f and g are defined by

$$f(x) = 5 - \frac{4}{3x+2} \qquad x \geqslant 0$$

$$g(x) = \left| 4\sin\left(\frac{x}{3} + \frac{\pi}{6}\right) \right| \qquad x \in \mathbb{R}$$

(a) Find the range of f

(2)

(b) (i) Find $f^{-1}(x)$

(ii) Write down the domain of f^{-1}

(3)

(c) Find $fg(-\pi)$

(2)

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

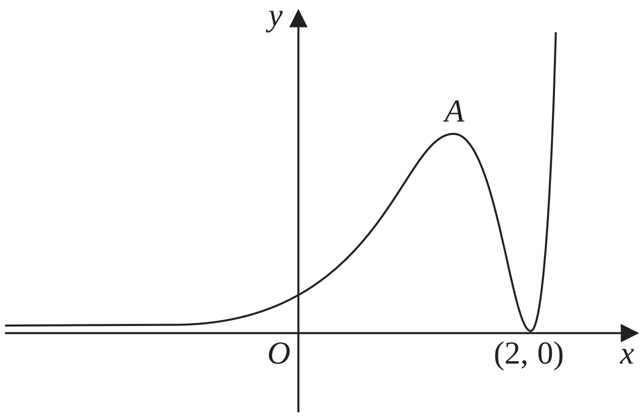

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x - 2)^2 e^{3x} \qquad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at (2, 0)

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $f(x) = k$, where k is a constant, has **at least** two distinct roots,

(b) state the range of possible values for k.

(2)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. $$y = \log_{10}(2x + 1)$$

(a) Express x in terms of y.

(2)

(b) Hence, giving your answer in terms of x, find $\dfrac{dy}{dx}$

(3)

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Question 4 continued

Q4

(Total 5 marks)

Leave blank

5.

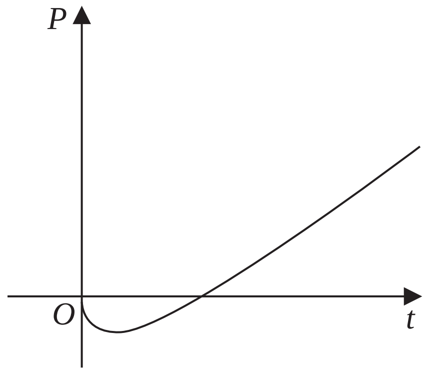

Figure 2

The profit made by a company, £P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of P against t is shown in Figure 2.

According to the model,

(a) show that exactly one year after it started trading, the company had made a loss of approximately £830 000

(2)

A manager of the company wants to know the value of t for which $P = 0$

(b) Show that this value of t occurs in the interval $[6, 7]$

(2)

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

(2)

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of t_2 and the value of t_6, giving your answers to 3 decimal places.

(3)

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit.

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 11 marks)

Q5

Leave blank

6.

$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$

Show that

$$\frac{dy}{dx} = \frac{a\tan x + b\sec x + c}{\sec x + 2\sin x}$$

where a, b and c are integers to be found.

(6)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 6 marks)

Leave blank

7.

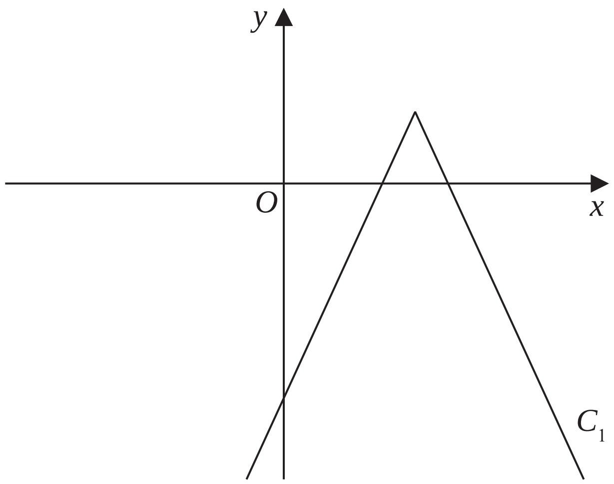

Figure 3

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

(i) the vertex of C_1

(ii) the intersection of C_1 with the y-axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x-axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

(d) Find the coordinates of the points of intersection of C_1 and C_2
(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

Leave blank

Question 7 continued

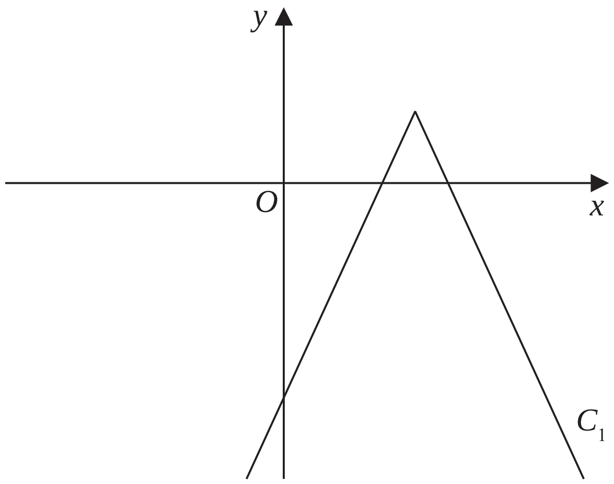

Diagram 1

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 12 marks)

Leave blank

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express $8\sin x - 15\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.
(3)

$$\mathrm{f}(x) = \frac{15}{41 + 16\sin x - 30\cos x} \qquad x > 0$$

(b) Find

(i) the minimum value of $\mathrm{f}(x)$

(ii) the smallest value of x at which this minimum value occurs.
(4)

(c) State the y coordinate of the minimum points on the curve with equation

$$y = 2\mathrm{f}(x) - 5 \qquad x > 0$$
(1)

(d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -\mathrm{f}(2x) \qquad x > 0$$
(1)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that $\cos 2\theta - \sin 3\theta \neq 0$

(a) prove that

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} \equiv \frac{1 + \sin\theta}{1 - 2\sin\theta - 4\sin^2\theta}$$

(4)

(b) Hence solve, for $0 < \theta \leqslant 360°$

$$\frac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = 2\operatorname{cosec}\theta$$

Give your answers to one decimal place.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = \frac{3}{2x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Write down the range of f

(1)

(b) Find the value of fg(1.5)

(2)

(c) Find g^{-1}

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

$$f(x) = \cos x + 2\sin x$$

(a) Express $f(x)$ in the form $R\cos(x - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $g(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

3.

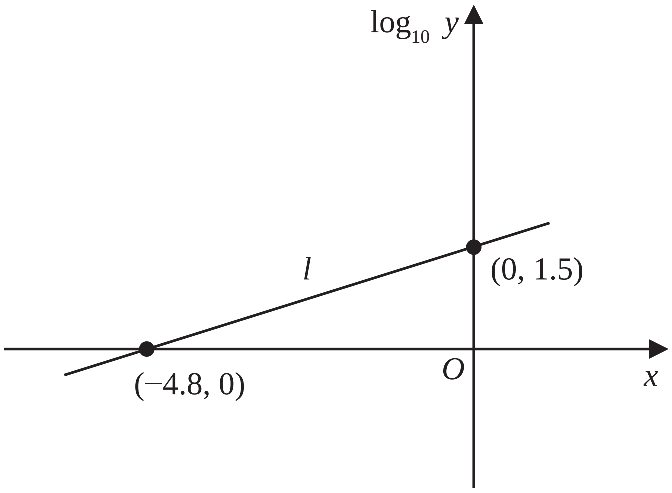

Figure 1

The line l in Figure 1 shows a linear relationship between $\log_{10} y$ and x.

The line passes through the points (0, 1.5) and (−4.8, 0) as shown.

(a) Write down an equation for l.

(2)

(b) Hence, or otherwise, express y in the form kb^x, giving the values of the constants k and b to 3 significant figures.

(3)

Question 3 continued

(Total for Question 3 is 5 marks)

4.
$$f(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int f(x)\,dx$$

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6.

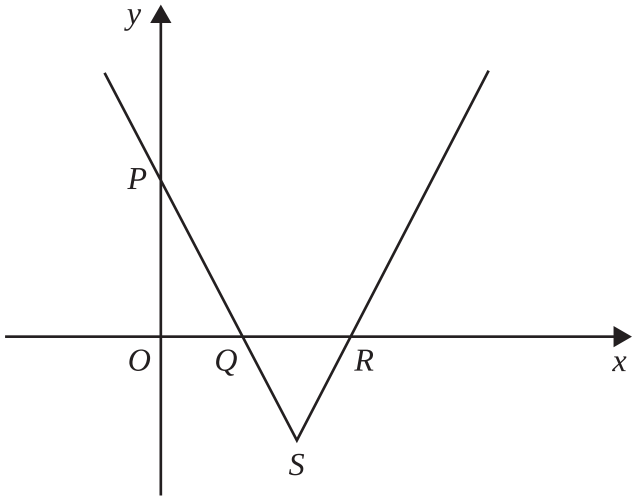

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y-axis at the point P
- cuts the x-axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a, the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a, the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. The curve C has equation

$$x = 3\tan\left(y - \frac{\pi}{6}\right) \qquad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C.

Given that the tangent to C at P crosses the x-axis at the point Q.

(b) find, in simplest form, the exact x coordinate of Q.

(5)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

Question 8 continued

(Total for Question 8 is 5 marks)

9.

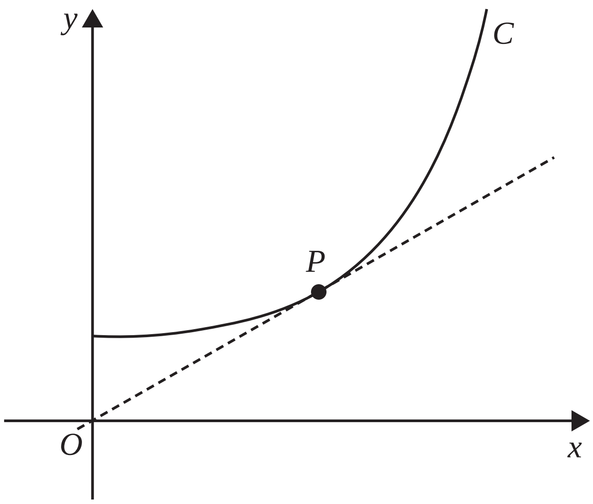

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C.

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

(b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

(c) Hence show that α lies between 1 and 2

(2)

(d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α.

(e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 11 marks)

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A population of fruit flies is being studied.

The number of fruit flies, F, in the population, t days after the start of the study, is modelled by the equation

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

where k is a constant.

Use the equation of the model to answer parts (a), (b) and (c).

(a) Find the number of fruit flies in the population at the start of the study.

(1)

Given that there are 200 fruit flies in the population 15 days after the start of the study,

(b) show that $k = \frac{1}{15}\ln 12$

(3)

Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

(c) find the possible values of T, giving your answers to one decimal place.

(5)

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 31 May 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1.

$$g(x) = x^6 + 2x - 1000$$

(a) Show that $g(x) = 0$ has a root α in the interval $[3, 4]$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α.
Give your answer to 4 decimal places.

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

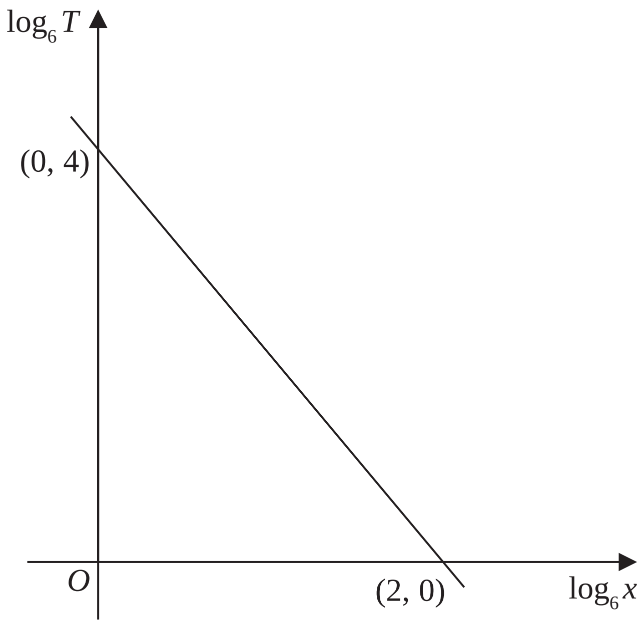

Figure 1

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points (0, 4) and (2, 0) as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of T when $x = 216$

(3)

(b) Find an equation, not involving logs, linking T with x

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4\right)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

Question 4 continued

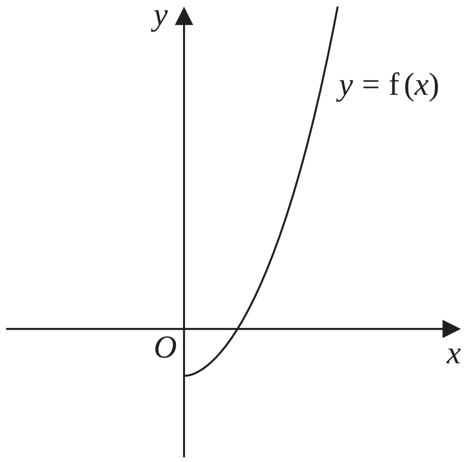

Diagram 1

(Total for Question 4 is 6 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x < \pi$

$$(x-2)\left(\sqrt{3}\sec x + 2\right) = 0$$

(3)

(ii) Solve, for $0 < \theta < 360°$

$$10\sin\theta = 3\cos 2\theta$$

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 7 marks)

6.

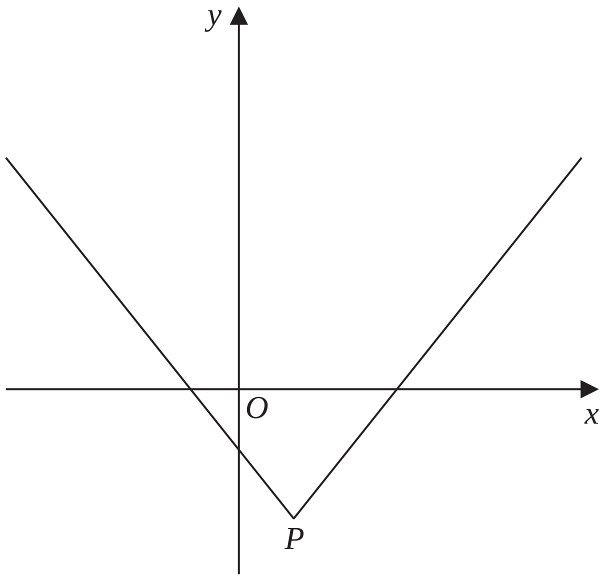

Figure 2

Figure 2 shows a sketch of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P, shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $\mathrm{ff}(0)$

(2)

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$\mathrm{f}\left(|x|\right) = 0$$

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \qquad t \geqslant 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

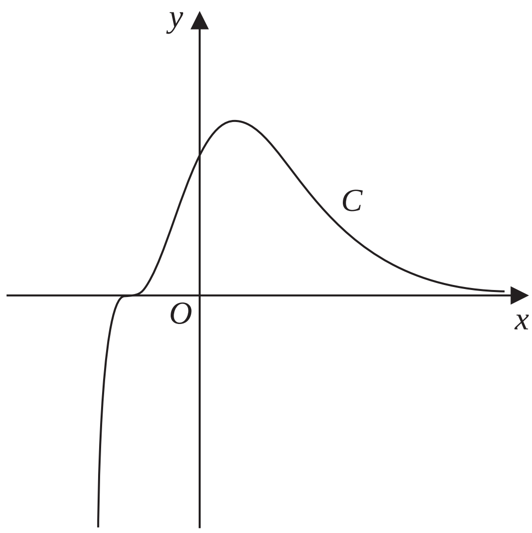

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, \mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

(2)

Question 9 continued

Question 9 continued

Question 9 continued

V

(Total for Question 9 is 10 marks)

10.

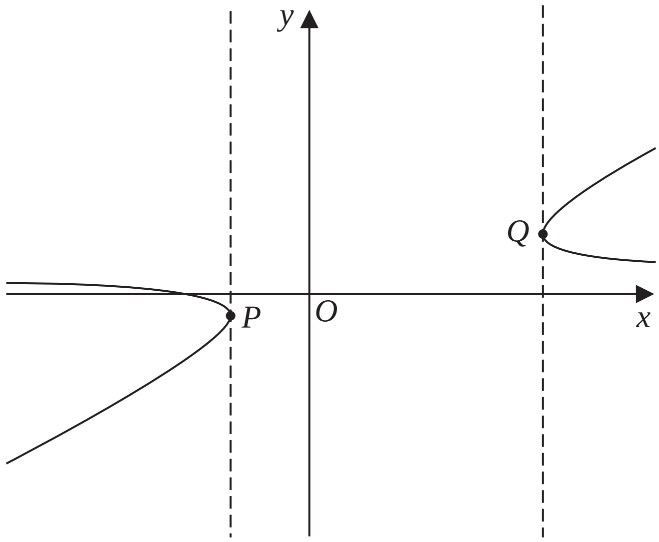

Figure 4

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

(a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y-axis, as shown in Figure 4.

(b) Use the answer to part (a) to find the equations of these two tangents.

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 18 October 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. A curve has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 - 5x + \mathrm{e}^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root, α, in the interval $[1, 2]$

(2)

The iterative formula

$$x_{n+1} = \sqrt{5x_n - \mathrm{e}^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root α.

(b) (i) Find the value of x_2 to 4 decimal places.

(ii) Find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6)

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

(4)

Question 3 continued

(Total for Question 3 is 6 marks)

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A.

(1)

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places.

(3)

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

(1)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5. The curve C has equation

$$y = \frac{\ln\left(x^2 + k\right)}{x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax\left(B - \ln\left(x^2 + k\right)\right)}{\left(x^2 + k\right)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k.

(1)

Question 5 continued

(Total for Question 5 is 7 marks)

6. An area of sea floor is being monitored.

The area of the sea floor, $S\,\text{km}^2$, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$

(2)

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant q

(1)

Question 6 continued

(Total for Question 6 is 6 marks)

7.

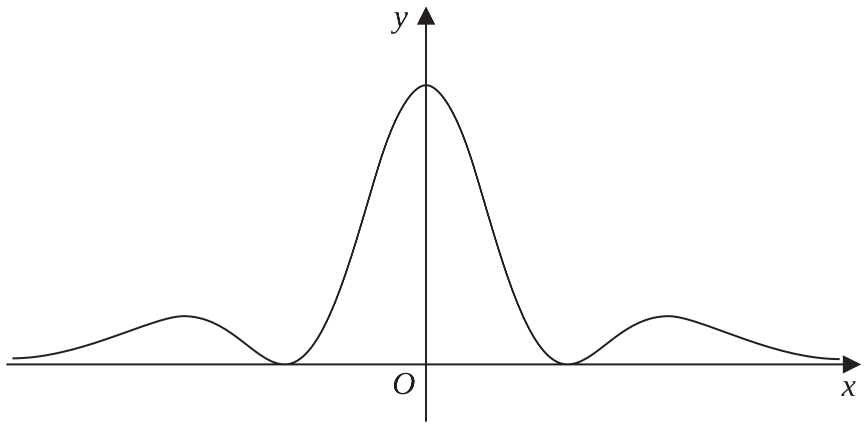

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}\left(2x^2 - 3\right)^2$$

(a) Find the range of f

(2)

(b) Show that

$$\mathrm{f}'(x) = 2x\left(2x^2 - 3\right)\mathrm{e}^{-x^2}\left(A - Bx^2\right)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. (a) Prove that

$$2\operatorname{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2\theta$$

(4)

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\operatorname{cosec}^2 2x(1 - \cos 2x) = 4 + 3\sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

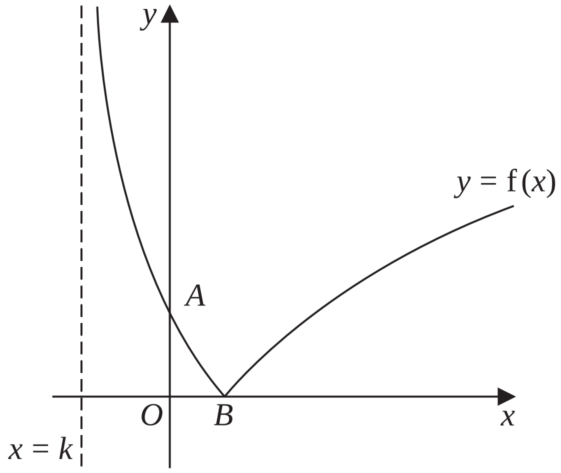

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x+1)| \qquad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y-axis at point A
- meets the x-axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x+1)| > 3$$

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\frac{dx}{dy}$

(2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x+s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point.

(2)

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Monday 8 January 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. The point $P(-4, -3)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(2x)$

(1)

(b) $y = 3f(x - 1)$

(2)

(c) $y = |f(x)|$

(1)

Question 1 continued

(Total for Question 1 is 4 marks)

2. A curve has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^4 - 5x^2 + 4x - 7 \qquad x \in \mathbb{R}$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root, α, in the interval [2, 3]

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

(1)

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^{\,2} - 4x_n + 7}{x_n}}$$

is used to find α

(c) Starting with $x_1 = 2$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £D, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of T, giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The function f is defined by

$$\mathrm{f}(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $\mathrm{f}(x) = \dfrac{2x}{3x-5}$

(3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$\mathrm{g}(x) = 3 + 2\ln x \qquad x \geqslant 1$$

(c) Find g^{-1}

(3)

(d) Find the exact value of a for which

$$\mathrm{gf}(a) = 5$$

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 13 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, T °C, of the air in a room t minutes after a heat source is switched off, is modelled by the equation

$$T = 10 + Ae^{-Bt}$$

where A and B are constants.

Given that the temperature of the air in the room at the instant the heat source was switched off was 18 °C,

(a) find the value of A

(1)

Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16 °C,

(b) find the value of B to 3 significant figures.

(3)

Using the values for A and B,

(c) find, according to the model, the rate of change of the temperature of the air in the room exactly two minutes after the heat source was switched off.
Give your answer in °C min^{-1} to 3 significant figures.

(2)

(d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5 °C

(1)

Question 5 continued

(Total for Question 5 is 7 marks)

6.

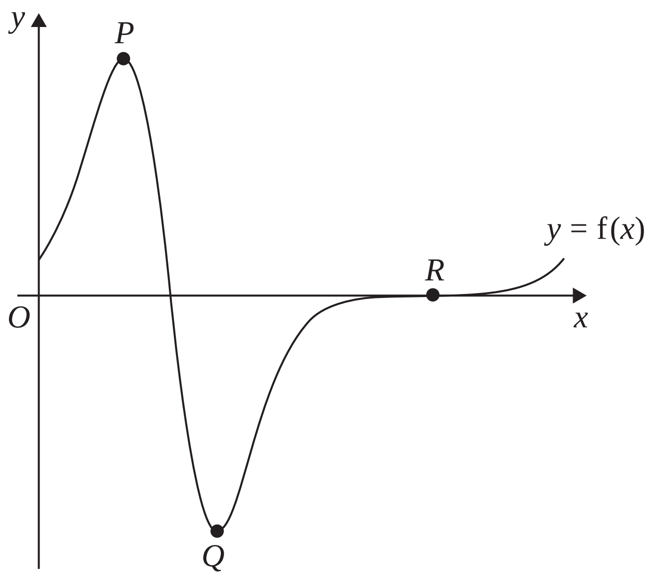

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2\mathrm{e}^{3\sin x}\cos x \qquad 0 \leqslant x \leqslant 2\pi$$

The curve intersects the x-axis at point R, as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q, also shown in Figure 1.

(b) Show that, at points P and Q,

$$a\sin^2 x + b\sin x + c = 0$$

where a, b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q, giving your answer to 3 decimal places.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \qquad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of k.

(2)

The point P with x coordinate 1 lies on C.

Given that the gradient of the curve at P is –12

(b) find the two possible values of k

(3)

Given also that $k < 3$

(c) find the equation of the normal to C at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x - k)}\,dx = \lambda \ln 10$$

where λ is a constant to be found.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8.

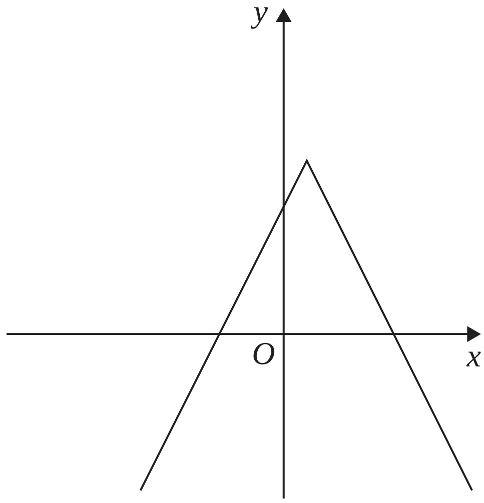

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b,

(i) the coordinates of the maximum point of the graph,

(ii) the coordinates of the point of intersection of the graph with the y-axis,

(iii) the coordinates of the points of intersection of the graph with the x-axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)

Question 8 continued

Question 8 continued

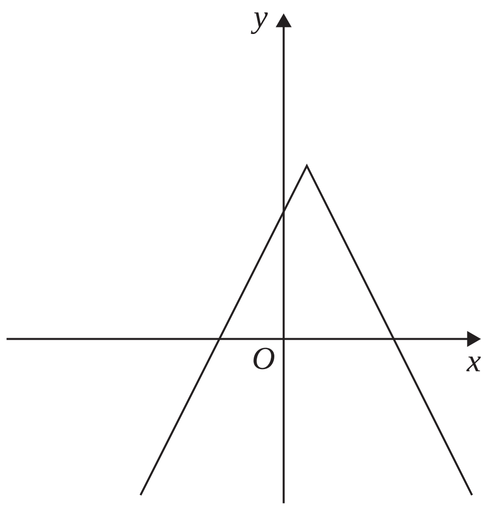

Diagram 1

Question 8 continued

(Total for Question 8 is 11 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{\cos\theta+\sin\theta} = (2+\sec 2\theta)(\cos\theta-\sin\theta)$$

can be written in the form

$$3\sin 2\theta - 4\cos 2\theta = 2$$

(3)

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3\sin x\cos x}{\cos x+\sin x} = (2+\sec 2x)(\cos x-\sin x)$$

giving the answer to 3 significant figures.

(5)

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Thursday 30 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1.

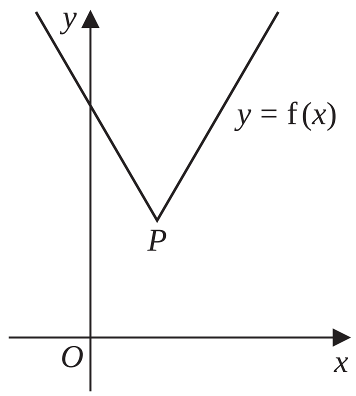

Figure 1

Figure 1 shows a sketch of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2|x - 5| + 10$$

The point P, shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of P

(2)

(b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

(c) Find the point to which P is mapped, when the graph with equation $y = \mathrm{f}(x)$ is transformed to the graph with equation $y = 3\mathrm{f}(x - 2)$

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

$$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A, B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. (i) The variables x and y are connected by the equation

$$y = \frac{10^6}{x^3} \qquad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

(3)

(ii)

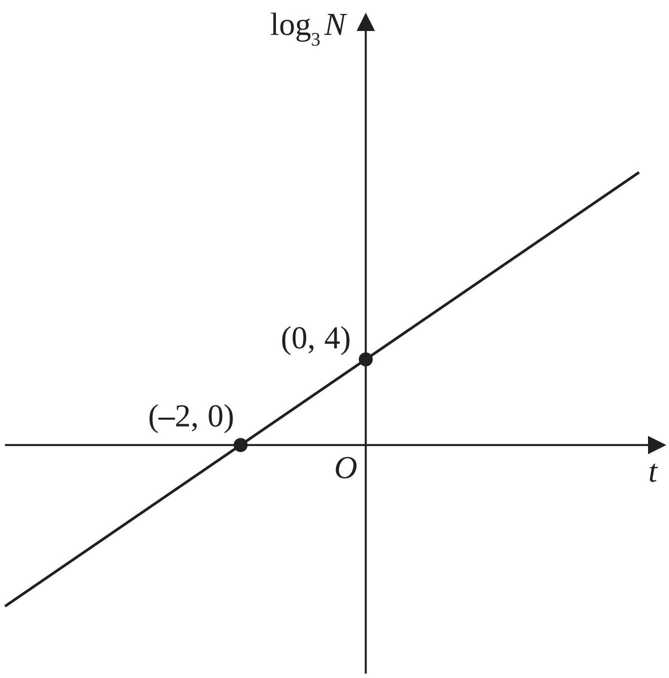

Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t.

Show that $N = ab^t$ where a and b are constants to be found.

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. $$\mathrm{f}(x) = 8\sin x\cos x + 4\cos^2 x - 3$$

(a) Write $\mathrm{f}(x)$ in the form

$$a\sin 2x + b\cos 2x + c$$

where a, b and c are integers to be found.

(3)

(b) Use the answer to part (a) to write $\mathrm{f}(x)$ in the form

$$R\sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

(c) Hence, or otherwise,

(i) state the maximum value of $\mathrm{f}(x)$

(ii) find the **second** smallest positive value of x at which a maximum value of $\mathrm{f}(x)$ occurs. Give your answer to 3 significant figures.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. The functions f and g are defined by

$$f(x) = 2 + 5\ln x \qquad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6.

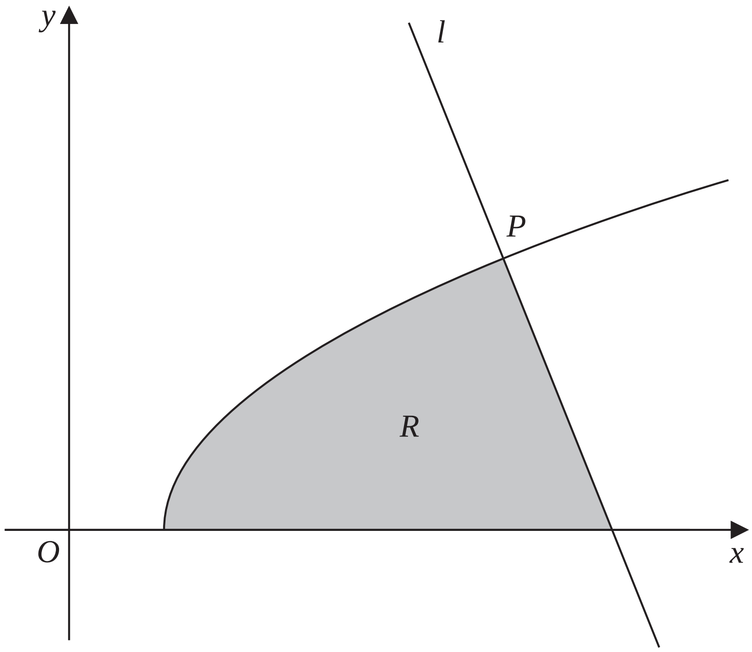

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$\sqrt{2}\sin(x + 45°) = \cos(x - 60°)$$

show that

$$\tan x = -2 - \sqrt{3}$$

(4)

(b) Hence or otherwise, solve, for $0 \leqslant \theta < 180°$

$$\sqrt{2}\sin(2\theta) = \cos(2\theta - 105°)$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

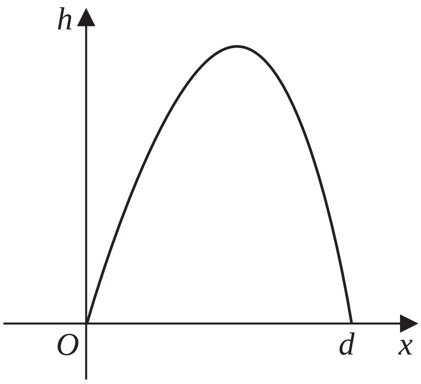

Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,\mathrm{e}^{0.02x} \qquad 0 \leqslant x \leqslant d$$

Use the model to answer parts (a), (b) and (c).

(a) Find the value of d, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(b) Show that the maximum value of h occurs when

$$x = 50\ln\left(\frac{150}{x+50}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n+50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of x_2 to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9.

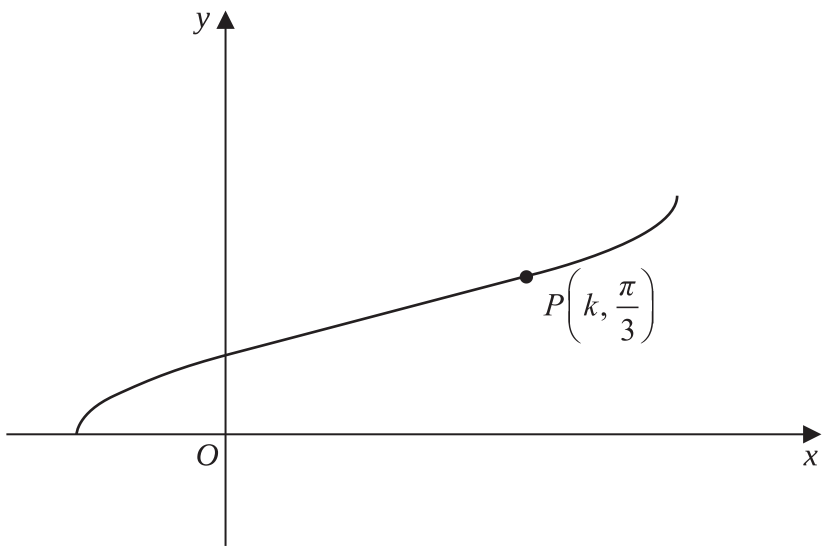

Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y

(ii) Hence show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS