Question Number	Scheme	Mark
1(a)	$(-2, -3)$	B1
		(1)
(b)	$(1, -9)$	B1B1
		(2)
(c)	$(4, 0)$	B1B1
		(2)
		(5 marks)

Condone omission of brackets throughout the question.
If the parts are not labelled, then mark the responses in the order they appear.

(a)
B1: Correct coordinates. Allow $x=-2,\ y=-3$

(b)
B1: One correct coordinate. Either $x=1$ or $y=-9$
B1: Both coordinates correct. Allow $x=1,\ y=-9$

(c)
B1: One correct coordinate. Either $x=4$ or $y=0$
B1: Both coordinates correct. Allow $x=4,\ y=0$

SC1: Allow a special case (b) B1B0 (c) B1B0 for **both** (b) $(-9, 1)$ (c) $(0, 4)$ but not for just one of these.
SC2: If they write their coordinates as vectors throughout then penalise the first occurrence only, and allow e.g. (a) B0, (b) B1B1, (c) B1B1.

Question Number	Scheme	Mark
2(a)	$\frac{1}{2}$	B1
		(1)
(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}\ (3+2x)\ln 2x+\frac{1}{x}(3x+x^2)$ o.e.	M1A1
	$(3+2x)\ln 2x+\frac{1}{x}(3x+x^2)=0 \Rightarrow \ln 2x=...$	dM1
	$\ln 2x=-\frac{3+x}{3+2x} \Rightarrow x=\frac{1}{2}\mathrm{e}^{-\left(\frac{x+3}{2x+3}\right)}$ *	A1*
		(4)
(c)(i)	$\{x_2=\}\ \frac{1}{2}\mathrm{e}^{-\frac{0.5+3}{2\times 0.5+3}}=0.2084310...$	M1
	awrt 0.2084	A1
(ii)	0.1948	A1
		(3)
		(8 marks)

(a)

B1: cao $\frac{1}{2}$ or 0.5. Condone (0.5, 0). There is no need to see $x=\ldots$ Ignore any solutions for $x \leqslant 0$

Must be clear that this is their answer to part (a) and not just $\frac{1}{2}$ or 0.5 seen anywhere.

(b)

M1: Attempts to use the product rule achieving $(A+Bx)\ln 2x+\frac{C}{x}(3x+x^2)$, A, B and C non-zero.

You may see e.g. $(3+x)\ln 2x+x\ln 2x+\frac{1}{x}x(3+x)$ if they do not expand the brackets before differentiating. Condone invisible brackets for this mark even if they are not recovered.

A1: $(3+2x)\ln 2x+3+x$ or unsimplified equivalent e.g. $(3+2x)\ln 2x+\frac{2}{2x}(3x+x^2)$

There is no need to see a LHS. Allow invisible brackets provided they are recovered.

dM1: Sets an expression of the form $(A+Bx)\ln 2x+\frac{C}{x}(3x+x^2)$ o.e. = 0, with A, B and C non-zero, and isolates $\ln 2x$ or $\ln x$ on one side. Dependent on the previous method mark.

A1*: Proceeds to the given answer with no errors seen.
Invisible brackets earlier in their work are penalised here.

Must be in this form but condone $x = \frac{1}{2}\mathrm{e}^{-\frac{x+3}{2x+3}}$ or $x = \frac{1}{2}\mathrm{e}^{-\frac{3+x}{3+2x}}$ or $x = \frac{1}{2}\mathrm{e}^{\frac{-(x+3)}{2x+3}}$

Do not accept e.g. $x = \frac{1}{2}\mathrm{e}^{\frac{-x-3}{2x+3}}$

Note that you may see some that work backwards from the given result $x = \frac{1}{2}\mathrm{e}^{-\left(\frac{x+3}{2x+3}\right)}$ to $(3+2x)\ln 2x + 3 + x = 0$. This approach scores no marks unless there is clear evidence of differentiating the given equation $y = x(3+x)\ln 2x$ and must have a conclusion to score the final A1*. If unsure, send to review.

(c)(i)
M1: Substitutes 0.5 into the iterative formula. May be implied by awrt 0.208 or awrt 0.196
A1: awrt 0.2084

(c)(ii)
A1: cao 0.1948 following award of M1. This is not scored for awrt 0.1948

Question Number	Scheme	Mark
3(a)	$\left\{\frac{dx}{dy}=\right\}\ \frac{2(y-1)^2-2(2y-1)(y-1)}{(y-1)^4}$	M1
	$=\frac{-2y^2+2y}{(y-1)^4}$ **or** $=\frac{2(y-1)-2(2y-1)}{(y-1)^3}$	M1
	$=-\frac{2y}{(y-1)^3}$	A1
		(3)
(b)	e.g. $\frac{dy}{dx}="-\frac{(2-1)^3}{2\times2}"=...\left\{-\frac{1}{4}\right\}$ **and** $x=\frac{2\times2-1}{1}=...\{3\}$	M1
	$y-2="-\frac{1}{4}"(x-"3")$	dM1
	$x+4y-11=0$	A1
		(3)
		(6 marks)

(a)

M1: Attempts the quotient rule (or product rule) – see below for the required forms.

Allow for $\left\{\frac{dx}{dy}=\right\}\ \frac{\alpha(y-1)^2-\beta(2y-1)(y-1)}{(y-1)^4}$ or $\left\{\frac{dx}{dy}=\right\}\ \alpha(y-1)^{-2}-\beta(2y-1)(y-1)^{-3}$ o.e.

where $\alpha,\beta>0$ but condone $\left\{\frac{dx}{dy}=\right\}\ \frac{\alpha(y-1)^2-\beta(2y-1)(y-1)}{(y-1)^2}$ where $\alpha,\beta>0$

Note the correct differentiation using the product rule is $\left\{\frac{dx}{dy}=\right\}\ 2(y-1)^{-2}-2(2y-1)(y-1)^{-3}$

Attempts using implicit differentiation or partial fractions should be sent to review.

M1: Attempts to expand and simplify their numerator to a quadratic expression with brackets removed **or** cancels a factor of $(y-1)$ correctly and simplifies to a linear numerator, which *would* simplify to $ay+b$ where a **or** b may be 0 (there is no need to expand the brackets for this mark in this approach).

Using the product rule, they must attempt to factor out $(y-1)^{-3}$ and have a linear expression as their other factor (as above) **or** create a common denominator of $(y-1)^3$ with a linear numerator (as above).

e.g. $2(y-1)^{-2}-2(2y-1)(y-1)^{-3}\rightarrow(y-1)^{-3}\left[2(y-1)-2(2y-1)\right]$ **or** $\frac{2(y-1)-2(2y-1)}{(y-1)^3}$

Dependent on a reasonable attempt to use the quotient rule or product rule but condone slips in applying these so that the forms specified above might not be achieved e.g. sign errors or miscopied terms.

A1: $-\frac{2y}{(y-1)^3}$ or simplified equivalent. FYI $-\frac{2y}{y^3-3y^2+3y-1}$ is also acceptable.

ISW if they go on to attempt to expand the denominator incorrectly.

(b)

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ when $y = 2$ **and** attempts to find the x coordinate when $y = 2$

Ignore labelling of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$

dM1: Attempts to find the equation of the tangent using their value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. the reciprocal of $\frac{\mathrm{d}x}{\mathrm{d}y}$

(**not** $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-\frac{\mathrm{d}y}{\mathrm{d}x}$) **and** their attempt at x when $y = 2$

If using $y = mx + c$ they must proceed as far as $c = \ldots$

Note that they may use $x - x_1 = \frac{\mathrm{d}x}{\mathrm{d}y}(y - y_1)$ directly (or $x = ny + d$ with n = their value of $\frac{\mathrm{d}x}{\mathrm{d}y}$)

A1: $x + 4y - 11 = 0$ or any integer multiple of this. Requires the = 0. The order of the terms does not matter.

Note: allow $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{4}$ coming from an incorrect $\frac{\mathrm{d}x}{\mathrm{d}y}$ or from a restart to score full marks in part (b).

Question Number	Scheme	Mark
4(a)	$p=10^{1.3}$ **or** $q=10^{\frac{3.4-1.3}{10}}$	M1
	$p=\text{awrt }20.0$ **or** $q=\text{awrt }1.62$	A1
	$p=10^{1.3}$ **and** $q=10^{\frac{3.4-1.3}{10}}$	dM1
	$p=\text{awrt }19.95$ **and** $q=\text{awrt }1.622$	A1
		(4)
(b)	e.g. $"19.95"\times"1.622"^T=700$ $\Rightarrow "1.622"^T=\frac{700}{"19.95"}$ $\Rightarrow T=\log_{"1.622"}\left(\frac{700}{"19.95"}\right)=\text{awrt }7.36$	M1A1
		(2)
		(6 marks)

(a)

M1: A correct expression for p **or** q. May be implied by a correct value so $p=\text{awrt }20.0$ **or** $q=\text{awrt }1.62$ Condone p truncating to 19.9 but not just $p=\text{awrt }20$

May be seen as e.g. $\{N=\}\ 10^{1.3}\times\ldots$ to imply the expression for p or $\{N=\}\ \ldots\times\left(10^{0.21}\right)^t$ to imply the expression for q (but not just $\{N=\}\ \ldots\times10^{0.21t}$).

Note that they may use simultaneous equations i.e. $pq^0=10^{1.3}$ and $pq^{10}=10^{3.4}$ which may lead to $q^{10}=\frac{2511\ldots}{19.95\ldots}=125.8\ldots$ so $q=\sqrt[10]{125.8\ldots}$ which scores the M1.

A1: $p=\text{awrt }20.0$ **or** $q=\text{awrt }1.62$ Condone p truncating to 19.9.

dM1: A correct expression for p **and** a correct expression for q.
May be implied by correct values so $p=\text{awrt }20.0$ **and** $q=\text{awrt }1.62$ Condone p truncating to 19.9 but not just $p=\text{awrt }20$. May also be implied if embedded in their equation.
It is dependent on the previous method mark.

A1: $p=\text{awrt }19.95$ **and** $q=\text{awrt }1.622$ There is no need to see the complete model.
If p and q are not stated explicitly then allow them to be implied by e.g. $\{N=\}\ 19.95\times1.622^t$

SC: Incorrect index work seen e.g. $10^{1.3}+10^{0.21t}$ followed by correct values for p and q scores maximum SC 1100.

(b)

M1: Uses their values for p and q in $pq^T = 700$ or uses $\log 700 = 1.3 + "0.21"T$ and proceeds to a value for $T > 0$ (or t) using the correct order of operations and correct log work.

They cannot proceed directly from $"19.95" \times ("1.622")^T = 700$ to a value for T without at least $("1.622")^T = \dfrac{700}{"19.95"}$ **or** $T = \log_{"1.622"} \dfrac{700}{"19.95"}$ o.e. e.g. $T \ln("1.622") = \ln \dfrac{700}{"19.95"}$

You may need to check their log work on a calculator, in which case 2 sf is sufficient.
Condone slips such as multiplying instead of dividing or miscopying their p or q e.g. 1.62 in place of 1.622.
Note that $\log 700 = pq^T$ is incorrect and scores M0.

A1: awrt 7.36 following award of the M1.

Question Number	Scheme	Mark
5(a)	$f(x) > 5$	M1A1
		(2)
(b)	$\{g^{-1}(x) =\}\ \ln\left(\frac{x-7}{5}\right)+2$ $x > 7$	M1A1 B1
		(3)
(c)	$fg(a) = 8 \Rightarrow \frac{35+25e^{a-2}+2}{7+5e^{a-2}-3} = 8$ **or** $g(a) = f^{-1}(8) \Rightarrow 7+5e^{a-2} = \frac{26}{3}$ $37+25e^{a-2} = 40e^{a-2}+32 \Rightarrow 15e^{a-2} = 5 \Rightarrow a = 2-\ln 3$	M1 dM1A1
		(3)
		(8 marks)

(a)

M1: $f(x)\ldots5$ where … is any inequality or equality.
Ignore another bound e.g. $0 < f(x)$ for this mark.
Condone use of y for $f(x)$ or just f … 5 but not x … 5
May be implied by use of interval notation with or without $f(x) \in$

A1: $f(x) > 5$ or $f > 5$ or $y > 5$ but not e.g. $x > 5$.
$5 < f(x) < \infty$ scores A1 but $5 < f(x) \leqslant \infty$ is A0.
Using interval notation, $(5, \infty)$ scores M1A1 but $(5, \infty]$ or $[5, \infty]$ or $[5, \infty)$ score M1A0 with or without $f(x) \in$.

(b) Use of log in place of ln or $\log_e$ is ambiguous and is condoned only for the M marks.

M1: Complete attempt to find $g^{-1}(x)$. Score for $\{y =\}\ \ln\left(\frac{\pm x \pm 7}{5}\right) \pm 2$ or $\{x =\}\ \ln\left(\frac{\pm y \pm 7}{5}\right) \pm 2$

Do not penalise invisible brackets unless they are clearly incorrect e.g. $\frac{\ln x - 7}{5}$

A1: $\{g^{-1}(x) =\}\ \ln\left(\frac{x-7}{5}\right)+2$ o.e. $\{g^{-1}(x) =\}\ \ln\left|\frac{x-7}{5}\right|+2$ is acceptable.

So accept e.g. $\{g^{-1} =\}\ \ln\left(\frac{x-7}{5}\right)+2\ln e$ or $\ln(x-7)-\ln 5+2$ or $\ln\left(\frac{(x-7)e^2}{5}\right)$

ISW after a correct answer is seen.

There is no need to see a LHS but score A0 if they have e.g. $x = \ln\left(\frac{x-7}{5}\right)+2$

Condone spurious $y =$ e.g. $g^{-1}(x) = y = \ln\left(\frac{x-7}{5}\right)+2$

Condone invisible brackets if they are unambiguous e.g. $\ln\frac{x-7}{5}+2$ but not clearly incorrect $\frac{\ln x - 7}{5}+2$

B1: $x>7$ must be x and not y or g
Using interval notation, $(7,\infty)$ scores B1 but $(7,\infty]$ or $[7,\infty]$ or $[7,\infty)$ score B0.
If using $x\in(7,\infty)$ then it must be x and not y or g

(c) May work in *x* instead of *a* which is acceptable for all marks.
Use of log in place of ln or log$_e$ is ambiguous and is condoned only for the M marks.

M1: Attempts to form the equation $\dfrac{5\left(7+5e^{a-2}\right)+2}{7+5e^{a-2}-3}=8$ **or** attempts to form an equation from $\dfrac{5x+2}{x-3}=8\Rightarrow x=...\left\{\dfrac{26}{3}\right\}$ **and** $g(a)=f^{-1}(8)\Rightarrow 7+5e^{a-2}="\dfrac{26}{3}"$

You may see $\dfrac{5x+2}{x-3}=8\Rightarrow x=...\left\{\dfrac{26}{3}\right\}\Rightarrow\{a=\}\ \ln\left(\dfrac{"\frac{26}{3}"-7}{5}\right)+2$ which scores M1dM1

dM1: Attempts to rearrange the equation to the form $Ae^{a-2}=B$ o.e. with $A\times B>0$
It is dependent on the previous method mark.
Proceeding directly to $\ln\left(\dfrac{"\frac{26}{3}"-7}{5}\right)+2$ scores M1dM1

A1: $\{a=\}\ 2-\ln 3$ or simplified equivalent. Condone $\{a=\}\ 2+\ln\left(\dfrac{1}{3}\right)$ and allow $x=$

Condone $\ln\left(\dfrac{e^2}{3}\right)$ and ISW after a correct simplified equivalent is seen.

Condone also $\{a=\}\ 2+\ln\left|\dfrac{1}{3}\right|$

Question Number	Scheme	Mark
6(a)	$R=5$	B1
	$\tan\alpha=\left(\frac{4}{3}\right)\Rightarrow\alpha=...$	M1
	$5\sin(x+0.927)$	A1
		(3)
(b)(i)	Max = 529	B1ft
(ii)	$2x+"0.927"=\frac{7\pi}{2}\Rightarrow x=...$	M1
	awrt 5.03	A1
		(3)
		(6 marks)

(a)

B1: For $R = 5$. Not scored for just $R = –5$ but condone $R = \pm 5$ for this mark. Condone 5.00 (3sf)

M1: Attempts to find α.

Look for $\tan\alpha=\pm\frac{4}{3}$ or $\tan\alpha=\pm\frac{3}{4}$ leading to $\alpha = \ldots$ (you do not need to check their value of α) and α may come from $\sin\alpha=\pm\frac{4}{5}$ or $\cos\alpha=\pm\frac{3}{5}$

May be implied by $\alpha=$ awrt 0.927 or if working in degrees by awrt 53.1

A1: $5\sin(x+0.927)$ Requires the expression not just correct values for R and α

May be scored if seen at the start of part (b) only if correct i.e. $5\sin(x+0.927)$ seen not $5\sin(2x+0.927)$

Condone 5.00 for R but not ± 5 for R and do not allow invisible brackets.

(b)(i)

B1ft: 529 following $R = 5$ or ft their (a). Look for $\left(4\times(-R)-3\right)^2$

(b)(ii)

M1: Sets $2x+"0.927"=\frac{7\pi}{2}$ and proceeds using a correct order of operations to a value for x

You may need to check their value if their α is incorrect.

May be seen as e.g. $x=\frac{7\pi}{4}-\frac{"0.927"}{2}$ or $x=k\pi+\frac{3\pi}{4}-\frac{"0.927"}{2}$ where $k = 1$

This mark may be withheld if it comes from clearly incorrect work, e.g., $\sin(2x+0.927)=1$ so $2x+"0.927"=\frac{\pi}{2},\ \frac{7\pi}{2}$

A1: awrt 5.03 only following a correct equation. Answer only with no method scores no marks.
If they have more than one answer, then awrt 5.03 must be selected or the others rejected.

Question Number	Scheme	Mark
7(a)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}-2+5\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$	M1A1
	$-2+5\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=0\Rightarrow\frac{5}{2}\sin x=2\Rightarrow x=...$	dM1
	$x=\text{awrt }2.21$	A1
		(4)
(b)	$\int 1-2x+5\sin^2\left(\frac{1}{2}x\right)\mathrm{d}x\Rightarrow\int\frac{7}{2}-2x-\frac{5}{2}\cos x\,\mathrm{d}x$	M1
	$\int\frac{7}{2}-2x-\frac{5}{2}\cos x\,\mathrm{d}x=\frac{7}{2}x-x^2-\frac{5}{2}\sin x\quad(+c)$	A1ft
	$\left(\frac{7}{2}\left(\frac{\pi}{2}\right)-\left(\frac{\pi}{2}\right)^2-\frac{5}{2}\sin\left(\frac{\pi}{2}\right)\right)-\left(\frac{7}{2}\left(\frac{\pi}{6}\right)-\left(\frac{\pi}{6}\right)^2-\frac{5}{2}\sin\left(\frac{\pi}{6}\right)\right)$	dM1
	$=\frac{7\pi}{6}-\frac{2\pi^2}{9}-\frac{5}{4}$	A1
		(4)
		(8 marks)

(a) **Note:** If you just see $-2+\frac{5}{2}\sin x$ with no previous manipulation of the given expression, then send to review.

M1: Attempts to differentiate, achieving $A+B\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$ for A, B non-zero.

Alternatively, uses the identity $\pm\cos x=\pm1\pm2\sin^2\left(\frac{1}{2}x\right)$ to replace $\sin^2\left(\frac{1}{2}x\right)$ and differentiates to the form $\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}$ $\alpha+\beta\sin x$ for α, β non-zero. Note that they may rearrange $\pm\cos x=\pm1\pm2\sin^2\left(\frac{1}{2}x\right)$ incorrectly but allow if an acceptable identity is seen.

A1: $-2+5\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$ o.e. e.g. $-2+\frac{5}{2}\sin x$ There is no need to see a LHS e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}$

May be unsimplified e.g. $-2+\frac{10}{2}\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)$

Condone slips in their constant term for this mark and penalise in the final mark: see SC1.

dM1: Sets $A+B\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=0$, uses $\sin x=2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ and proceeds to solve $A+\frac{B}{2}\sin x=0$ with $\left|\frac{2A}{B}\right|\leqslant1$ **or** solves $\alpha+\beta\sin x=0$ with $\left|\frac{\alpha}{\beta}\right|\leqslant1$ to find a value for x.

There is no need to see e.g. $\sin x=\frac{4}{5}$

Some are incorrectly replacing $2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ with $\sin 2x$ and score dM0.

A1: awrt 2.21 following award of all previous marks. Correct answer only scores no marks.
If there is more than one answer, awrt 2.21 must be clearly selected or the others rejected.
Ignore any attempt to a y coordinate.

SC1: Note that if they use the alternative but make slips on the constant term e.g. a sign error e.g. $y=1-2x-\frac{5}{2}-\frac{5}{2}\cos x$ they may still achieve $\left\{\frac{dy}{dx}=\right\}-2+\frac{5}{2}\sin x$ and go on to find awrt 2.21. In such cases allow M1A1dM1A0 to be scored.

SC2: Similarly, a sign error on the cos$x$ term e.g. $y=1-2x+\frac{5}{2}+\frac{5}{2}\cos x$ followed by a second error when differentiating may fortuitously produce $\left\{\frac{dy}{dx}=\right\}-2+\frac{5}{2}\sin x$ and they may go on to find awrt 2.21. In such cases allow M1A0dM1A0 to be scored.

(b) **Note: if you see work that does not show any algebraic integration but proceeds directly to e.g. $\frac{7\pi}{6}-\frac{2\pi^2}{9}-\frac{5}{4}$ or similar then send to review.**

M1: Attempts to use the identity $\pm\cos x=\pm1\pm2\sin^2\left(\frac{1}{2}x\right)$ to achieve the integral $C+Dx+E\cos x$ where C, D and E are non-zero.
Note that the use of the identity may be seen in part (a) but must be seen again or used in part (b) to score this mark.

A1ft: Integrates to achieve $Cx+\frac{D}{2}x^2+E\sin x$ following through on their values for C, D and E.
This may be unsimplified, i.e., they may not have collected their x terms together or they may have e.g. $-\frac{2}{2}x^2$

dM1: Attempts to substitute in the correct limits and subtracts either way round into a changed expression from their $C+Dx+E\cos x$
It is dependent on the previous method mark.

A1: cao $\frac{7\pi}{6}-\frac{2\pi^2}{9}-\frac{5}{4}$ o.e. e.g. $\frac{42\pi-8\pi^2-45}{36}$ or $\frac{(4\pi-15)(3-2\pi)}{36}$ and apply ISW after a correct answer is seen. Allow equivalent fractions e.g. $\frac{14}{12}\pi$ for $\frac{7\pi}{6}$
All previous marks must be scored but note that the below is acceptable for full marks.

$$\int\frac{7}{2}-2x-\frac{5}{2}\cos x\,\{dx\}=\left[\frac{7}{2}x-x^2-\frac{5}{2}\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\frac{7\pi}{6}-\frac{2\pi^2}{9}-\frac{5}{4}$$

SC3: Note that a sign error on the cos$x$ term followed by a second error when integrating may fortuitously produce the correct integrated expression. Scores maximum 1010 in this case.

Question Number	Scheme	Mark
8(a)	-35	B1
		(1)
(b)	$"-35"+40e^{0.2\times3}=37.88...$ 37 900	M1 A1
		(2)
(c)	$\left\{\frac{dE}{dt}=\right\}\ 40\times0.2\times e^{0.2\times5}=$ awrt 21.7 <u>thousand</u> {cars} <u>per year</u>	M1A1
		(2)
(d)	$150+450e^{-0.2X}="-35"+40e^{0.2X}\Rightarrow 40e^{0.4X}-185e^{0.2X}-450=0$ $40e^{0.4X}-185e^{0.2X}-450=0\Rightarrow e^{0.2X}=6.386522...$ $\Rightarrow 0.2X=\ln(6.3865...)\Rightarrow X=...$ awrt 9.3	M1 dM1 ddM1 A1
		(4)
		(9 marks)

(a)

B1: $\{A=\}\ -35$ only. There is no need to see the complete model.

(b)

M1: Substitutes $t=3$ into the model using their "–35" and proceeds to find a value.
May be implied by their answer correct to 2sf which may need to be checked if A is incorrect.

A1: 37 900 or 37.9 <u>thousand</u>(s) only. Correct answer implies both marks.
37,900 or 37,9 <u>thousand</u> are both also correct.

(c)

M1: Differentiates to the form $pe^{0.2t}$, $p\neq40$, substitutes in $t=5$ and proceeds to find a value.
Allow $8e$ to imply this mark.
Sight of 21.7 or 21700 alone scores M0 with no evidence of differentiation.

A1: awrt 21.7 <u>thousand</u>(s) {cars} <u>per year</u> or awrt 21700 {cars} <u>per year</u> including units.
21,700 {cars} <u>per year</u> or 21,7 <u>thousand</u> {cars} <u>per year</u> are both also correct.
Must follow a correct derivative in some form, so allow it to follow e.g. $40\times0.2\times e^{0.2\times5}$ or $8e$ or $8e^{0.2\times5}$ or $40\times0.2\times e^{0.2t}$
Correct answer only scores no marks.

(d) Note that they may work in a variable other than *X* throughout this part for full marks.

M1: Sets the two expressions equal to each other using their A, multiplies by $e^{\pm 0.2X}$ and proceeds to a 3TQ with terms collected. All terms do not need to be on the same side (so they may not even realise that they have a 3TQ).

Condone incorrect notation when squaring $e^{\pm 0.2X}$ i.e. $e^{(\pm 0.2X)^2}$ provided it is recovered (for all the marks).

dM1: Attempts to solve their 3TQ in $e^{\pm 0.2X}$ using the usual rules but they may use a calculator, in which case you may need to check their values to 2sf.
For reference, if their quadratic is correct, look for $e^{0.2X} = \text{awrt } 6.4$ **or** $\text{awrt } -1.8$
$\left(= \dfrac{37 \pm \sqrt{4249}}{16}\right)$ or $e^{-0.2X} = \text{awrt } 0.16$ **or** $\text{awrt } -0.57$ $\left(= \dfrac{-37 \pm \sqrt{4249}}{180}\right)$
Note that they must achieve $e^{\pm 0.2X} = \ldots$ so if they substitute for $e^{\pm 0.2X}$ then they must revert back but it may be implied by later work.
Dependent on the previous method mark.

ddM1: Uses logs correctly to proceed from $e^{\pm 0.2X} = q$ where $q > 0$ to $X = \ldots$
May be implied by a correct value for X from their $e^{\pm 0.2X} = q$
Dependent on both previous method marks.

A1: awrt 9.3 following award of all three previous marks. Correct answer only scores no marks.

Question Number	Scheme	Mark
9(a)	$\left(\dfrac{k}{2}, 3\right)$	B1B1
		(2)
(b)	$-2x+k+3=\dfrac{1}{2}x+\dfrac{k+7}{2}$ **or** $2x-k+3=\dfrac{1}{2}x+\dfrac{k+7}{2}$	M1
	$x=\dfrac{k-1}{5}$ **or** $x=\dfrac{3k+1}{3}$	A1
	$-2x+k+3=\dfrac{1}{2}x+\dfrac{k+7}{2}$ **and** $2x-k+3=\dfrac{1}{2}x+\dfrac{k+7}{2}$	dM1
	M: $x=\dfrac{k-1}{5}$ **and** N: $x=\dfrac{3k+1}{3}$	A1
		(4)
(c)	y coordinate of R is $k+3$	B1
	$\dfrac{1}{2}\times''(k+3)''\times''\dfrac{(k-1)}{5}''=\dfrac{33}{40}\Rightarrow ...k^2 \pm ...k \pm ...=0$	M1
	$4k^2+8k-45=0 \Rightarrow k=...\{2.5\} \Rightarrow \dfrac{1}{2}\times(''2.5+3'')\times\left(\dfrac{3\times 2.5+1}{3}\right)=\dfrac{187}{24}$	dM1A1
		(4)
		(10 marks)

Mark all parts together.

(a) **Note: May be seen on their diagram if they have drawn one.**

B1: One coordinate correct. Allow e.g. $x=0.5k$ or $y=3$

B1: Both coordinates correct. Allow e.g. $x=\dfrac{1}{2}k$, $y=3$

If they write their answer as a vector $\begin{pmatrix}\dfrac{1}{2}k \\ 3\end{pmatrix}$ then score B1B0

(b) **Ignore any attempt to find y coordinates in this part.**

M1: Attempts to form an equation with modulus brackets removed.
They must deal with the modulus brackets correctly (so $|2x-k|$ must be replaced with either $2x-k$ or $-2x+k$ o.e.) but condone a slip e.g. in multiplying by 2 when they first set up the equation.

So allow e.g. $|2x-k|+3=\dfrac{1}{2}x+\dfrac{k+7}{2} \to 2|2x-k|+3=x+k+7 \to 4x-2k+3=x+k+7$

or e.g. $|2x-k|+3=\dfrac{1}{2}x+\dfrac{k+7}{2} \to |2x-k|+6=x+k+7 \to -(2x-k)+6=x+k+7$

A1: $x=\dfrac{k-1}{5}$ or $x=\dfrac{3k+1}{3}$ o.e. Do not be concerned by labelling.
Ignore incorrect simplification after a correct expression for x in terms of k is seen.

dM1: Attempts to form both equations with modulus brackets removed.
It is dependent on the previous method mark.
They must deal with the modulus brackets correctly (so $|2x-k|$ must be replaced with each of $2x-k$ and $-2x+k$ o.e. in their equations) but condone a slip as above e.g. in multiplying by 2 when they first set up the equations.

A1: x coordinate of M is $x=\dfrac{k-1}{5}$ o.e. **and** x coordinate of N is $x=\dfrac{3k+1}{3}$ o.e.
There is no need to specify which is M or N but withhold this mark if they have been incorrectly paired.
Ignore incorrect simplification after a correct expression for x in terms of k is seen.
If there are other x coordinates given that are clearly answers for part (b) then score final A0.

(c)

B1: y coordinate of R is $k+3$ May be seen elsewhere in their work, but it must clearly be R if being scored in part (a) or (b) (so e.g. $(0, k+3)$ or R: $k+3$ is acceptable but not $\left(\dfrac{k}{2}, k+3\right)$
If they have $(k+3, 0)$ but go on to use $k+3$ as their y coordinate, then allow recovery.

M1: Uses the formula for the area of a triangle using their R and their M, sets equal to $\dfrac{33}{40}$ and rearranges to achieve a 3TQ equation in k. All terms do not need to be on the same side. Condone slips e.g. in expanding their brackets.

dM1: Attempts to solve the 3TQ in k using the usual rules, including by calculator, and uses their value of $k>1$ in the formula for the area of the triangle using their R and their N to find a value. If they have two values for $k>1$ then allow them to use either.

A1: $\dfrac{187}{24}$ or exact equivalent e.g. $7\dfrac{19}{24}$ or $7.791\dot{6}$ but not 7.79166…

Note: There are no marks scored for those that set up an equation e.g.

$$\frac{\text{Area}(ORN)}{\text{Area}(ORM)}=\frac{\frac{1}{2}OR\left("k+\frac{1}{3}"\right)}{\frac{1}{2}OR\left("\frac{k-1}{5}"\right)} \text{ or } \text{Area}(ORN)=\frac{\left("k+\frac{1}{3}"\right)}{\left("\frac{k-1}{5}"\right)}\times\frac{33}{40}$$

without some attempt to set up an equation which can be used to find a value of k.

Note: Any attempts via integration that appear to be worthy of credit should be sent to review.

Question Number	Scheme		Mark
10(a)	**Left to right**	**Right to left**	
	$\cot x - \text{cosec}\,2x \equiv \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin 2x}$	$\cot 2x \equiv \dfrac{\cos 2x}{\sin 2x}$	M1
	$\equiv \dfrac{2\cos^2 x}{2\sin x\cos x} - \dfrac{1}{\sin 2x} \equiv \dfrac{2\cos^2 x - 1}{2\sin x\cos x}$	$\equiv \dfrac{\cos 2x}{2\sin x\cos x}$	M1
	$\equiv \dfrac{\cos 2x}{\sin 2x}$	$\equiv \dfrac{2\cos^2 x}{2\sin x\cos x} - \dfrac{1}{2\sin x\cos x}$	M1
	$\equiv \cot 2x$ *	$\equiv \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin 2x} \equiv \cot x - \text{cosec}\,2x$ *	A1*
			(4)
(b)	$13 + \cot(2\theta + 30°) = \text{cosec}^2(2\theta + 30°)$		B1
	$13 + \cot(2\theta + 30°) = 1 + \cot^2(2\theta + 30°) \Rightarrow \cot^2(2\theta + 30°) - \cot(2\theta + 30°) - 12 = 0$		M1
	$\{\cot(2\theta + 30°) =\}\ 4$ **and** -3		A1
	$\tan(2\theta + 30°) = \dfrac{1}{4} \Rightarrow 2\theta + 30° = 180° + \tan^{-1}\left(\dfrac{1}{4}\right) \Rightarrow \theta = \ldots$ **or** $\tan(2\theta + 30°) = -\dfrac{1}{3} \Rightarrow 2\theta + 30° = 180° + \tan^{-1}\left(-\dfrac{1}{3}\right) \Rightarrow \theta = \ldots$		dM1
	$=$ awrt $65.8°$, awrt $82.0°$		A1
			(5)
			(9 marks)

(a)

M1: Uses one of $\cot x = \dfrac{\cos x}{\sin x}$ or $\text{cosec}\,2x = \dfrac{1}{\sin 2x}$ (or $\cot 2x = \dfrac{\cos 2x}{\sin 2x}$ if working right to left).
The arguments must be correct (may be recovered if missing).

M1: Attempts to use $\sin 2x = 2\sin x\cos x$ (**and** proceeds to a single fraction if working left to right).

M1: Attempts to use $\pm\cos 2x = \pm 2\cos^2 x \pm 1$ (**and** splits the fraction if working right to left).
Alternatively, replaces $\pm 1 = \pm\sin^2 x \pm \cos^2 x$ and divides through by $\pm\cos^2 x$ to create an expression in terms of $\tan x$ only:

$$\frac{2\cos^2 x - 1}{2\sin x\cos x} = \frac{2\cos^2 x - \sin^2 x - \cos^2 x}{2\sin x\cos x} = \frac{2 - \tan^2 x - 1}{2\tan x}\left\{= \frac{1 - \tan^2 x}{2\tan x} = \cot 2x\right\}$$

A1*: Achieves the given answer with no errors. Penalise poor notation e.g. missing arguments or $\cos x^2$ in this mark. Condone the use of $=$ in place of $\equiv$ throughout.
The given answer should be stated at some stage, which may be implied through their working as in the main scheme, or it may be explicitly stated at the start or end of their work.
Note e.g. “LHS =” on its own is not sufficient for this mark.
Condone $\dfrac{\cos 2x}{2\sin x\cos x} = \cot 2x$ at the final step (they have already used $\sin 2x = 2\sin x\cos x$)
They may work from both sides in an attempt to find an expression that is equivalent to both $\cot x - \text{cosec}\,2x$ **and** $\cot 2x$. In such cases there must be some minimal conclusion for A1*.

(b) **Note: the degrees symbol does not need to be seen for any of the marks.**
Note: Some candidates may restart having not successfully completed part (a). In such cases, allow full marks to be scored in part (b).

B1: Uses the result from part (a) correctly to reduce the equation so that the arguments are the same e.g. $13+\cot(2\theta+30°)=\text{cosec}^2(2\theta+30°)$ o.e. e.g. $\dfrac{\cos(2\theta+30°)}{\sin(2\theta+30°)}=\dfrac{1}{\sin^2(2\theta+30°)}-13$

Allow e.g. $13+\cot X=\text{cosec}^2 X$ or $13+\cot 2X=\text{cosec}^2 2X$ even if X is not defined or even if it is not recovered by later work.

M1: Attempts to use $\pm1\pm\cot^2 X=\pm\text{cosec}^2 X$ with their argument and proceeds to a 3TQ. The = 0 may be implied by later work.

A1: Both correct roots from the correct quadratic, i.e., 4 **and** –3, which may come from a calculator or may be implied by going straight to e.g. $\tan X=\dfrac{1}{4}$ **and** $-\dfrac{1}{3}$

Do not be overly concerned with the LHS.

Note that if they multiply by $\tan^2 X$ they should have the roots $\{\tan X=\}\ \dfrac{1}{4}$ **and** $-\dfrac{1}{3}$ coming from $12\tan^2 X+\tan X-1\ \{=0\}$

dM1: Complete and correct order of operations to find a value for θ in the range $0<\theta<90°$
Dependent on the previous method mark **and** on an appropriate method to solve their 3TQ which may be by calculator.
May be implied by one correct angle (2sf) for their roots. Do not allow work in radians.

A1: Both values correct: awrt $65.8°$, awrt $82.0°$ (not just $82°$) and no others in the range $0<\theta<90°$. Ignore any solutions outside the range.