Question Number	Scheme	Marks
	$\mathrm{g}(x)=3x-5+5\sin\left(x^2\right)$	
1 (a)	$\mathrm{g}(0.7)=-0.55,\quad \mathrm{g}(0.8)=0.39$	M1
	Change of sign and continuous function, hence root in $[0.7,0.8]$	A1
		(2)
(b)	$0=3x-5+5\sin\left(x^2\right)\Rightarrow x=\left[\arcsin(1-0.6x)\right]^{\frac{1}{2}}$	B1
		(1)
(ci)	$x_2=\left[\arcsin(1-\text{"}0.6\text{"}\times 0.7)\right]^{\frac{1}{2}}=$ awrt 0.7866	M1, A1
(cii)	$\alpha=0.7589$	A1
		(3)
		(6 marks)

(a)

M1 Attempts the value of g at 0.7 and 0.8 with one correct (to 1 s.f. rounded or truncated). Use of a narrower range is possible, providing the lower x value gives a negative function value and the higher one gives a positive function value. You may need to check these function values.

A1 Requires

- Both values correct to 1 s.f., rounded or truncated.
- Reason: sign change (or $\mathrm{g}(0.7)<0, \mathrm{g}(0.8)>0$ or $\mathrm{g}(0.7)\mathrm{g}(0.8)<0$, etc) and "continuous function" (or "continuous graph/curve", or just "continuity" etc.)
- Minimal conclusion, e.g. "root", "root in interval", "root in [0.7,0.8]", "α lies in interval", etc). Do not condone the incorrect statement "Change of sign hence continuous function and root in interval" etc.

(b)

B1 Correct value of p stated or implied. May be embedded in formula. If embedded, allow e.g. $-\frac{3x}{5}$ in place of $-0.6x$. Condone slips in the formula if the correct value of p is clearly seen. If an incorrect p value is seen in (b) it is B0, regardless of what the candidate uses in (c). If there is no obvious attempt at (b), then you can mark any formula used in (c). Apply isw if, for example, a correct formula is given, followed by $p=-0.6$.

(ci)

M1 Attempts to substitute $x_1=0.7$ into the correct formula with their $\pm p$ to reach a value. Do not be concerned about the labelling of this second approximation, e.g. it may be labelled x_1 or x_3. This mark is available for candidates working in degrees, in which case, if a correct formula is used, they will get $x_2=5.954$. For an incorrect formula and no working, you may have to check their answer, whether working in radians or degrees.
Can be implied by 0.786... following $p=0.6$

A1 awrt 0.7866. Condone incorrect labelling.

(cii)

A1 cao $\alpha=0.7589$. Accept $x=0.7589$ or e.g. $x_{11}=0.7589$ if it is clear this is their final answer.

Question Number	Scheme	Marks
2 (i)	$y=x^2\ln 2x\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2x\ln 2x+x$ o.e.	M1 A1
(ii)	$\left(\mathrm{f}'(x)=\right)\frac{2(4x+3)^2\mathrm{e}^{2x}-8(4x+3)\mathrm{e}^{2x}}{(4x+3)^4}$ or $2\mathrm{e}^{2x}(4x+3)^{-2}-8\mathrm{e}^{2x}(4x+3)^{-3}$	M1 A1
	$=\frac{\cancel{(4x+3)}\mathrm{e}^{2x}\left[2(4x+3)-8\right]}{(4x+3)^{\cancel{4}3}}$ or $\mathrm{e}^{2x}(4x+3)^{-3}\left[2(4x+3)-8\right]$	dM1
	$=\frac{2\mathrm{e}^{2x}(4x-1)}{(4x+3)^3}$	A1
		(6 marks)

(i)

M1 Attempts the product rule to achieve $Ax\ln 2x+Bx$ o.e. with $A,B\neq 0$ which may be unsimplified.

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2x\ln 2x+x$. Condone $\ln 2x(2x)+x$. isw after a correct answer.

(ii)

M1 Attempts the quotient rule to achieve $\frac{P(4x+3)^2\mathrm{e}^{2x}-Q(4x+3)\mathrm{e}^{2x}}{(4x+3)^4}$ with $P,Q>0$. Allow $-Q(4x+3)\mathrm{e}^{2x}$ to be expanded, i.e. if correct $-(32x+24)\mathrm{e}^{2x}$ or $-32x\mathrm{e}^{2x}-24\mathrm{e}^{2x}$.

Alternatively, attempts the product rule. Look for $P\mathrm{e}^{2x}(4x+3)^{-2}-Q\mathrm{e}^{2x}(4x+3)^{-3}$ with $P,Q>0$.

In either case, Q may be written as the product of two integers.

Condone miscopying slips on $(4x+3)$, e.g. $(3x+4)$

A1 Using quotient rule: $\left(\mathrm{f}'(x)=\right)\frac{2(4x+3)^2\mathrm{e}^{2x}-8(4x+3)\mathrm{e}^{2x}}{(4x+3)^4}$ o.e.

Using product rule: $\left(\mathrm{f}'(x)=\right)2\mathrm{e}^{2x}(4x+3)^{-2}-8\mathrm{e}^{2x}(4x+3)^{-3}$ o.e.

In either case, allow 8 to be written as (2)(4).

dM1 Dependent on previous M1. Correctly takes out a common factor of $(4x+3)\mathrm{e}^{2x}$ from their numerator and cancels $(4x+3)$. If using product rule, must take out a common factor of $(4x+3)^{-3}\mathrm{e}^{2x}$. They may also take out a factor of 2 at this stage, but this is not required for dM1.

A1 Achieves $\frac{2\mathrm{e}^{2x}(4x-1)}{(4x+3)^3}$. Do not allow $2\mathrm{e}^{2x}(4x-1)(4x+3)^{-3}$. Allow recovery of invisible brackets at any stage.

Question Number	Scheme	Marks
3 (i)	$\int 2(3x+5)^7\,\mathrm{d}x = \frac{(3x+5)^8}{12}(+c)$	M1 A1
		(2)
(ii) (a)	By division $\begin{array}{r} 4x-3 \\ x^2+x-2\overline{\smash{\big)}4x^3+x^2-7x+14} \end{array}$	M1 A1
	Remainder $=4x+8$	M1
	$\frac{4x+8}{x^2+x-2}=\frac{4\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$	
	$\frac{4x^3+x^2-7x+14}{(x-1)(x+2)}\equiv 4x-3+\frac{4}{x-1}$	A1
		(4)
(b)	$\int_3^9 \mathrm{f}(x)\,\mathrm{d}x=\left[2x^2-3x+4\ln(x-1)\right]_3^9$	M1 A1ft
	$=126+4\ln 8-4\ln 2=126+8\ln 2$	dM1 A1
		(4)
		(10 marks)

(i)

M1 Integrates and achieves $k(3x+5)^8$ where k is a constant. Any attempt that involves expanding $(3x+5)^7$ is unlikely to reach an answer in its simplest form, so probably scores M0A0, but if you think there is a meaningful attempt to factorise the answer into the required form, send to review. If a substitution is used, they must revert to x and reach $k(3x+5)^8$

A1 $\frac{(3x+5)^8}{12}$ with or without + c

(ii) (a)

By division

M1 Divides the cubic by x^2+x-2 and achieves a linear quotient $4x\pm\ldots$. Watch out for different ways to present the division, e.g. grid method. They may divide first by $(x+2)$ and then by $(x-1)$ or vice versa. See exemplification below. Do not allow misreads for the quadratic divisor; or if dividing by the two linear factors separately, these must be correct.

A1 Achieves the linear quotient $4x-3$. This mark can be given even if slips in the division lead to an incorrect remainder.

M1 See one of the following 3 cases below for award of this mark. All 3 cases depend on a valid attempt at division and $Q(x)$ is their linear quotient after division.

1) If the candidate has divided the cubic by x^2+x-2, achieving a remainder $px+q$, this mark can be awarded for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

2) If they have divided by $(x+2)$ and then $(x-1)$, they should reach an expression of the form $Q(x)+\frac{c}{x-1}$, where *c* is an integer. If there are slips in the division by $(x+2)$ leading to a non-zero remainder, award for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

3) If they have divided by $(x-1)$ and then $(x+2)$, this mark can be awarded for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

In Cases (2) and (3) there must be a correct method for combining the two remainders into a single fraction.

A1 $4x-3+\frac{4}{x-1}$ which may be seen in (ii) (b)

Division by the two linear factors separately – exemplification

$\frac{4x^3+x^2-7x+14}{x+2}=4x^2-7x+7$; followed by $\frac{4x^2-7x+7}{x-1}=4x-3+\frac{4}{x-1}$ (M1A1M1A1)

Or $\frac{4x^3+x^2-7x+14}{x-1}=4x^2+5x-2+\frac{12}{x-1}$; followed by $\frac{4x^2+5x-2}{x+2}=4x-3+\frac{4}{x+2}$ (M1A1 here for correct quotient). Then $\frac{12}{(x-1)(x+2)}+\frac{4}{x+2}=\frac{4}{x-1}\Rightarrow \mathrm{f}(x)=4x-3+\frac{4}{x-1}$ (M1A1)

Alt: By comparing terms / using an identity. This must be of the correct form but condone slips (say in signs).
$4x^3+x^2-7x+14\equiv(ax+b)(x+2)(x-1)+c(x+2)$

M1 Correct method to find two constants, e.g. $x=1\Rightarrow 12=3c\Rightarrow c=4$, compare terms in $x^3\Rightarrow 4=a$
A1 Two correct constants
M1 Correct method to find all three constants
A1 $4x-3+\frac{4}{x-1}$ which may be seen in (ii) (b)

(ii) (b)

M1 Achieves a correct $c\ln(x-1)$ AND one of the other two terms: $\frac{1}{2}ax^2$ or *bx* for their *a*, *b* and *c*.
Condone invisible brackets in the log term as long as there is a recovery when substituting limits.

A1ft $\int ax+b+\frac{c}{x-1}\,\mathrm{d}x=\frac{1}{2}ax^2+bx+c\ln(x-1)$. Follow through on their a,b and c. Condone a spurious integration symbol and d*x* after integration.

dM1 Dependent on the first M1. Substitutes in both limits and subtracts. Condone missing brackets around the expression involving the lower limit.

A1 $126+4\ln 4$ or $126+8\ln 2$ or any equivalent involving a single log term. Withhold this mark if a spurious integration symbol or d*x* features in their final answer. Isw after a correct expression.

Question Number	Scheme	Marks
4	$\log_{10} h = 2.4 - 0.25\log_{10} m$	
(a)	$\log_{10} h = 2.4 - 0.25\log_{10} 3 \Rightarrow h = 10^{2.4-0.25\log_{10} 3}$	M1
	$\Rightarrow h = 191$	A1
		(2)
(b)	Example: One correct law $\log_{10} h = 2.4 + \log_{10} m^{-0.25}$	M1
	Full method in forming an equation linking *h* and *m*, e.g. $h = 10^{2.4} \times m^{-0.25}$	dM1
	$\Rightarrow h = \dfrac{251}{m^{0.25}}$	A1
		(3)
(c)	The resting heart rate (in beats per minute) for (a mammal of mass) 1 kg	B1
		(1) **(6 marks)**

(a)

M1 Substitutes $m = 3$ into the given equation and uses the correct order of operations to find *h*. Can be implied by a correct value for *h*.

A1 awrt 191

(b)

M1 Makes a first step towards achieving an answer. Sight of a correct rule or law used. Will usually be awarded for one of:

- Application of a power rule e.g. $-0.25\log_{10} m \to \log_{10} m^{-0.25}$
- An attempt to make *h* the subject. E.g. $\log_{10} h = 2.4 - 0.25\log_{10} m \to h = 10^{2.4-0.25\log_{10} m}$
- Both of the above applied at once, e.g. $\log_{10} h = 2.4 - 0.25\log_{10} m \to h = \dfrac{10^{2.4}}{m^{0.25}}$ or $10^{2.4} \times m^{-0.25}$

In this case, M1 dM1 are scored together.

dM1 Dependent on the first M1. Full and complete method to make *h* the subject, with logs being correctly removed, e.g. $\log_{10} h = 2.4 + \log_{10} m^{-0.25} \Rightarrow h = 10^{2.4} \times m^{-0.25}$

A1 Achieves $h = \dfrac{251}{m^{0.25}}$ with at least one intermediate step (see M1) between the given equation and the final answer. Accept ¼ or 0.250 in place of 0.25 and allow awrt 251. Do not accept $p = 251, q = 0.25$ for this mark without seeing the full equation.

Note: It is possible to score 000 in (b) despite a numerator of 251 if an incorrect method is used.

Note: Allow full marks for part (b) if full working is shown correctly in (a).

SC: Award SC110 for use of $m = 3$ in part (a), and making *h* the subject, reaching $h = \dfrac{10^{2.4}}{3^{1/4}}$ and then writing down $p = 251, q = 0.25$ in part (b).

(c)

B1 "The (resting) heart rate (in beats per minute) for (a mammal of mass) 1 kg". Also accept "The beats per minute for (a mammal of mass) 1 kg."

Question Number	Scheme	Marks
5 (a)	$\mathrm{ff}(x)=\dfrac{2\times\left(\dfrac{2x+16}{x-4}\right)+16}{\left(\dfrac{2x+16}{x-4}\right)-4}$, $\dfrac{2\times(2x+16)+16(x-4)}{(2x+16)-4(x-4)}$,$=\dfrac{10x-16}{-x+16}$	M1, dM1, A1 **(3)**
(b)	$\dfrac{2x+16}{x-4}=12\Rightarrow 10x=64\Rightarrow x=6.4$	M1 A1 **(2)**
(c)	$\dfrac{2\ln a+16}{\ln a-4}=\ln a\Rightarrow(\ln a)^2-6\ln a-16=0$	M1
	$\Rightarrow(\ln a-8)(\ln a+2)=0\Rightarrow\ln a=8,-2$	dM1
	$\Rightarrow a=\mathrm{e}^{-2},\mathrm{e}^{8}$	A1 **(3)** **8 marks**

(a)

M1 Attempts to fully substitute f(x) for x in $\dfrac{2x+16}{x-4}$. Condone **one** slip as long as the intention is clear and there are two substitutions of f(x) for x.

dM1 Dependent on the first M1. Correct attempt to form a single fraction (see scheme) by multiplying two terms in the numerator and two terms in the denominator by $(x-4)$

A1 $\dfrac{10x-16}{-x+16}$ or exact simplified equivalent

(b)

M1 Attempts to solve $\mathrm{f}(x)=12$ using correct order of operations (multiplying by $x-4$ and grouping terms in x).

Alternatively, reaches $\mathrm{f}^{-1}(x)=\dfrac{ax+b}{cx+d}$ using correct order of operations and attempts to substitute $x=12$. Note: $\mathrm{f}^{-1}(x)=\dfrac{4x+16}{x-2}$

A1 6.4 o.e.

(c) **Note:** If correct answers appear without a 3TQ, send to review.

M1 Sets up a 3TQ in $\ln a$. Must have three terms, but not necessarily on one side of the equation. Condone a four term quadratic with all terms on one side if it is followed by a correct attempt to solve. Condone missing “=0”.

dM1 Dependent on the first M1. Full attempt to solve a 3TQ in $\ln a$, using any valid method (including calculator), reaching two values, which do not necessarily have to be labelled $\ln a$. Allow a substitution, e.g. $x=\ln a$ or even $a=\ln a$.

A1 $a=\mathrm{e}^{8},\mathrm{e}^{-2}$ (or $\dfrac{1}{\mathrm{e}^{2}}$) and no others. Accept answers labelled as x or unlabelled. Apply isw (e.g. if exact answers are followed by decimals). Must follow M1 dM1.

Question Number	Scheme	Marks
6 (a)	$2\cos(\theta-60°)=3\sin\theta \Rightarrow 2\cos\theta\cos 60°+2\sin\theta\sin 60°=3\sin\theta$	M1, A1
	$\Rightarrow \cos\theta=\left(3-\sqrt{3}\right)\sin\theta$ or $\Rightarrow 1+\sqrt{3}\tan\theta=3\tan\theta$	dM1
	$\Rightarrow \tan\theta=\dfrac{1}{3-\sqrt{3}}$	
	$\Rightarrow \tan\theta=\dfrac{1}{6}\left(3+\sqrt{3}\right)$ *	A1*
		(4)
(b)	Deduces that $\theta=2x-10°$	B1
	$\tan(2x\pm\alpha)=\dfrac{1}{6}\left(3+\sqrt{3}\right)\Rightarrow 2x\pm\alpha=\text{awrt}\,38$ or $218\Rightarrow x=...$	M1
	$x=\text{awrt}\,24.1°$, awrt $114.1°$.	A1
		(3) **(7 marks)**

(a)

M1 Attempts to use $\cos(\theta-60°)=\cos\theta\cos 60°\pm\sin\theta\sin 60°$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 3 of $3\sin\theta$

A1 Correct equation in $\sin\theta$ and $\cos\theta$

dM1 Dependent on the first M1. Uses $\cos 60°=\dfrac{1}{2}, \sin 60°=\dfrac{\sqrt{3}}{2}$ AND attempts one of:

- Dividing all 3 terms by $\cos\theta$ or $\sin\theta$ to set up an equation in just $\tan\theta$; or
- Collecting terms in $\sin\theta$ to form an equation of the form $P\sin\theta\pm Q\cos\theta=0$ o.e.

A1* Correctly proceeds to given answer showing all necessary steps: **see the steps set out in the main scheme above**. It is not necessary to see the rationalising of the denominator. Withhold this mark for invisible brackets or for use of the wrong variable, e.g. *x*, or a missing variable, at any stage within the body of the solution.

(b) **Notes**

- **Correct answers from no working score B0 M0 A0**
- **This is a "hence" question, so starting from scratch also scores B0 M0 A0**

B1 Deduces that $\theta=2x-10°$

M1 Attempts correct method from $\tan(2x\pm\alpha)=\dfrac{1}{6}\left(3+\sqrt{3}\right)\Rightarrow x=....$

Look for the correct order of operations, i.e. arctan, then add or subtract *α*, then divide by 2.

As a minimum look for $\tan(2x\pm\alpha)=\dfrac{1}{6}\left(3+\sqrt{3}\right)\Rightarrow 2x\pm\alpha=\text{awrt}\,38$ or $218\Rightarrow x=...$

If $\tan(2x\pm\alpha)$ is expanded to $\dfrac{\tan 2x\pm\tan\alpha}{1\mp\tan 2x\tan\alpha}$ then $\tan 2x$ must be made the subject using the correct order of operations before arctan and division by 2.

Calculations must be in degrees.

A1 $x=\text{awrt}\,24.1°$, awrt $114.1°$ and no others in the range.

Question Number	Scheme	Marks
7 (a)	$\left(\frac{11}{3}, -4\right)$	B1 B1
		(2)
(b)	Attempts $3x-11-4=8$ and $-3x+11-4=8$	M1, dM1
	$x=-\frac{1}{3}, \frac{23}{3}$	A1
		(3)
(c)	$m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$	M1, A1, A1
		(3)
(d)	$a=-4,\ b=\frac{4}{3}$	B1ft, B1ft
		(2)
		(10 marks)

Note: Check working near to the question text or written on the diagram.

(a)

B1 For one correct coordinate

B1 For $\left(\frac{11}{3}, -4\right)$. Allow missing brackets, or $x=\frac{11}{3}, y=-4$. Accept equivalent fractions for $\frac{11}{3}$

(b)

M1 For a full attempt at either $3x-11-4=8$ or $-3x+11-4=8$. Must lead to a value for x.

dM1 Dependent on the first M1. For a full attempt at both $3x-11-4=8$ and $-3x+11-4=8$ leading to values for x.

A1 $x=-\frac{1}{3}, \frac{23}{3}$ only. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.

Alt

M1 Reaches $|3x-11|=12$, squares both sides and solves a quadratic equation. Must lead to a value for x.

dM1 Reaches two values for x following a valid method for solving their quadratic equation.

A1 $x=-\frac{1}{3}, \frac{23}{3}$. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.

(c)

M1 Finds any of the three critical values for m. This need not be presented as an inequality.

A1 Two of $m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$. Do not give this mark if there are contradictions, e.g. $m=-\frac{12}{11}, m>-\frac{12}{11}$ unless the correct answer is clearly indicated. Accept set notation.

A1 All three of $m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$ with no contradictions (see above). Accept set notation.

(d) **Note: If *a*=… and *b*=… are not stated, they could be embedded in $y=a\mathrm{f}\left(x-b\right)$. The stated values take precedence.**

Question Number	Scheme	Marks
8	$x = e^{2\tan y} \Rightarrow \frac{dx}{dy} = e^{2\tan y} \times 2\sec^2 y$	B1
	$\Rightarrow \frac{dx}{dy} = 2x\left(1+\tan^2 y\right) = 2x\left(1+\left(\frac{\ln x}{2}\right)^2\right)$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{4}{2x\left(4+\left(\ln x\right)^2\right)}$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{2}{x\left(4+\left(\ln x\right)^2\right)}$	A1
		(4) **(4 marks)**

Note: Condone incorrect notations for $\left(\ln x\right)^2$ such as $\ln x^2$ and $\ln^2 x$ in working, but not in the final answer.

B1 Correct expression for $\frac{dx}{dy}$ as in scheme or in an equivalent form

The following two M marks require their $\frac{dx}{dy}$ to be in the form $k e^{2\tan y} \sec^2 y$.

M1 Either:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; OR
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\frac{dx}{dy}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; AND
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\frac{dy}{dx}$ in terms of x.

A1 $\frac{dy}{dx} = \frac{2}{x\left(4+\left(\ln x\right)^2\right)}$. Condone missing $\frac{dy}{dx}$ on LHS as long as it appears somewhere in working and it is clear that it refers to their final answer.

Alt for the two M marks using cos

The following two M marks require their $\frac{dx}{dy}$ to be in the form $\frac{k e^{2\tan y}}{\cos^2 y}$.

M1 Either:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; OR
- Forms an expression for $\cos^2 y$ using a valid method and gets $\frac{dx}{dy}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ to form an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$; AND
- Forms an expression for $\cos^2 y$ using a valid method and gets $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

Alt using arctan. Note that B1 M1 M0 A0 is not possible via this route

B1 $x=\mathrm{e}^{2\tan y} \Rightarrow y=\arctan\left(\frac{1}{2}\ln x\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{1}{2}\ln x\right)^2}\times\ldots$ where … could be 1.

M2 $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\left(\frac{1}{2}\ln x\right)^2}\times\frac{1}{2x}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{x\left(4+\left(\ln x\right)^2\right)}$

Alt using implicit differentiation

B1 $x=\mathrm{e}^{2\tan y} \Rightarrow \ln x=2\tan y \Rightarrow \frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}y}=2\sec^2 y$ OR $x=\mathrm{e}^{2\tan y} \Rightarrow \ln x=2\tan y \Rightarrow \frac{1}{x}=2\sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}$

Other variations are possible. Then as main method

Question Number	Scheme	Marks
	$\mathrm{h}(x)=8+3\sin x\left(\cos x-2\sin x\right)$	
9 (a)	$\mathrm{h}(x)=8+3\sin x\cos x-6\sin^2 x$	
	$\mathrm{h}(x)=8+\frac{3}{2}\sin 2x+3\cos 2x-3$ o.e.	M1, dM1
	$R\cos\alpha=\frac{3}{2},\ R\sin\alpha=3$	A1
	Let $\frac{3}{2}\sin 2x+3\cos 2x=R\sin\left(2x+\alpha\right)$	
	$R^2=\left("\frac{3}{2}"\right)^2+"3"^2\Rightarrow R=...$	ddM1
	$\tan\alpha=\dfrac{"3"}{\left("\frac{3}{2}"\right)}\Rightarrow\alpha=...$	ddM1
	$\left(\mathrm{h}(x)=\right)5+\frac{3}{2}\sqrt{5}\sin\left(2x+1.107\right)$	A1
		(6)
(b)	$5-\frac{3}{2}\sqrt{5}\leqslant\mathrm{h}\left(x\right)\leqslant 5+\frac{3}{2}\sqrt{5}$	B1ft
		(1)
(c)	$2x+"1.107"=\frac{7\pi}{2}\Rightarrow x=...$	M1
	$x=$ awrt 4.94	A1
		(2) **(9 marks)**

(a)

M1 Multiplies out bracket and attempts to use **either** of
- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

dM1 Dependent on the first M1. Multiplies out bracket and attempts to use **both** of
- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

Expands $R\sin\left(2x+\alpha\right)$ and compares coefficients to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

Alt for the first two marks using the given form of the answer, expanding and equating coefficients.

M1 Expands $R\sin(2x+\alpha)$ to $R\sin 2x\cos\alpha+R\cos 2x\sin\alpha$ and uses **either** $\sin 2x=2\sin x\cos x$ or $\cos 2x=1-2\sin^2 x$

dM1 Dependent on the first M1. Expands using **both** $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$ to reach $2R\sin x\cos x\cos\alpha+R\sin\alpha-2R\sin^2 x\sin\alpha$

And compares coefficients of $\sin x \cos x$ and $\sin^2 x$ to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

A1 $R\cos\alpha = \frac{3}{2}$ and $R\sin\alpha = 3$ or equivalent equations. Note that a value of R may be embedded if it has already been calculated by inspection of coefficients or construction of a triangle. In some cases, the two equations may not be explicitly stated, but implied by further work.

The following two M marks are both dependent on the first two M marks, i.e. we require a correct method leading to $R\cos\alpha = "\frac{3}{2}"$ and $R\sin\alpha = "3"$.

ddM1 $R^2 = \left("\frac{3}{2}"\right)^2 + "3"^2 \Rightarrow R = ...$ May be implied by their R value, even if R is inexact.

ddM1 $\tan\alpha = \frac{"3"}{\left("\frac{3}{2}"\right)} \Rightarrow \alpha = ...$ but condone $\tan\alpha = \frac{\left("\frac{3}{2}"\right)}{"3"} \Rightarrow \alpha = ...$ Alternatively α can be found using $\sin\alpha = \frac{"3"}{R}$ or $\cos\alpha = \frac{"3/2"}{R}$ with their R.

A1 $(\mathrm{h}(x) =) 5 + \frac{3}{2}\sqrt{5}\sin(2x + 1.107)$. A full expression must be given for h(x), not just values of P, R and α. This expression must be seen in part (a).

(b) **Note: this mark can be scored following incorrect working in part (a).**

B1ft $5 - \frac{3}{2}\sqrt{5} \leqslant \mathrm{h}(x) \leqslant 5 + \frac{3}{2}\sqrt{5}$ o.e. allowing follow through on their P and R

$\left[5 - \frac{3\sqrt{5}}{2}, 5 + \frac{3\sqrt{5}}{2}\right]$ and $\left\{\mathrm{h}(x) : 5 - \frac{3\sqrt{5}}{2} \leqslant \mathrm{h}(x) \leqslant 5 + \frac{3\sqrt{5}}{2}\right\}$ are suitable alternatives.

Condone y or h in place of h(x) but not f(x) or x.

(c) **Note: full marks can be scored in part (c) following incorrect working in part (a).**

M1 Attempts to solve $2x + "1.107" = \frac{7\pi}{2}$ using the correct order of operations, leading to a value for x.

Alternatively solves $\sin(2x + "1.107") = -1$ using the correct order of operations, and selects the second positive value of x.

Or differentiates to reach $2\cos(2x + "1.107") = 0$ and solves to find the fourth positive solution.

A1 $x =$ awrt 4.94

Question Number	Scheme	Marks
10 (a)	$\left(\frac{\mathrm{d}N_A}{\mathrm{d}t}\right) = 900\times 0.2\mathrm{e}^{0.2\times 5} = \text{awrt } 489$ or $180\mathrm{e}$	M1, A1
		(2)
(b)	States or implies that $P = 2\,000$	B1
	$11570 = 8000 + 2000\mathrm{e}^{k\times 4} \Rightarrow 2000\mathrm{e}^{4k} = 3570$	M1
	$\Rightarrow k = \text{awrt } 0.145$	dM1, A1
		(4)
(c)	$8000 + 900\mathrm{e}^{0.2T} = 8000 + "2000"\mathrm{e}^{0.145T}$	
	$\mathrm{e}^{(0.2-"0.145")T} = \frac{"2000"}{900}$ or $\ln\left(900\mathrm{e}^{0.2T}\right) = \ln\left("2000"\mathrm{e}^{0.145T}\right)$ $\ln 900 + 0.2T = \ln"2000" + 0.145T$	M1
	$T = \frac{1}{(0.2-"0.145")}\ln\left(\frac{"2000"}{900}\right)$	dM1
	$T = 14.5$	A1
		(3) **(9 marks)**

(a) **Note: if a correct answer is seen without any working, please send to review.**

M1 $\frac{\mathrm{d}N_A}{\mathrm{d}t} = C\mathrm{e}^{0.2\times 5} = \ldots$ where $C \neq 900$. Withhold this mark if they substitute $t = 5$ before differentiation.

A1 awrt 489 or exact 180e. isw

(b)

B1 States or implies that $P = 2\,000$

M1 Proceeds to $A\mathrm{e}^{4k} = B$ where $A \times B > 0$

dM1 Dependent on previous M1. Correct attempt to solve using ln, following the correct order of operations. Proceeding from $A\mathrm{e}^{4k} = B$ to a value for *k* with no ln work is dM0 A0.

A1 $k = \text{awrt } 0.145$. Exact answer only is A0.

(c) **Condone use of *t* in place of *T* throughout this part.**

M1 Cancels 8000 on each side, then **either**:

- uses correct index work to achieve an equation in $\mathrm{e}^{(0.2-"0.145")T}$ that could be rearranged to the form $\mathrm{e}^{(0.2-"0.145")T} = K$ where $K > 0$; **or**
- takes ln of both sides and uses the log addition law on both sides, giving an equation involving two terms in *T* and at least one constant term.

dM1 Dependent on previous M1. Correct ln work leading to a positive value or positive exact expression for *T*. Proceeding from $\mathrm{e}^{(0.2-"0.145")T} = K$ to a value for *T* with no ln work is dM0 A0.

A1 *T* = awrt 14.5. Exact answer only is A0.