Question Number	Scheme	Marks
	$f(x) = 2\sec x + 6x - 3$	
1. (a)	$f(0.1) = -0.39 \qquad f(0.2) = 0.24$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	Sets $f(x) = 0$, uses $\sec x = \dfrac{1}{\cos x}$ and makes x of $6x$ the subject	
	$\Rightarrow 6x = 3 - \dfrac{2}{\cos x} \Rightarrow x = \dfrac{1}{2} - \dfrac{1}{3\cos x}$ *	B1*
		(1)
(c)	(i) $x_2 = \dfrac{1}{2} - \dfrac{1}{3\cos 0.15} = 0.16..$	M1
	$x_2 =$ awrt 0.1629	A1
	(ii) $\alpha = 0.1622$	A1
		(3)
		(6 marks)

Note: If the student uses degrees mode they will get values -0.39999 and 0.200001 in (a) and 0.1666… in (c). An answer in (c) of 0.16666… would be an indicator of the mode used for part (a) if rounded values are given see notes below.

(a)

M1: Attempts the value of $f(x)$ at 0.1 and 0.2 with one **correct** to at least 1 sf rounded or truncated. Note that degrees mode gives -0.39999 and 0.200001 and these can be accepted for the M mark.

A1: Both values correct to at least 1sf (but see warning), rounded or truncated with reason (Sign change **and** continuous function) and minimal conclusion (root). For "sign change" accept $f(0.1) < 0$, $f(0.2) > 0$ shown, or $f(0.1)f(0.2) < 0$

Warning: Degrees mode answers should score A0, so if the values -0.39999 and 0.200001 are used score A0. However, if just $f(0.1) = -0.4$ and $f(0.2) = 0.2$ are used with no contrary evidence – see note above – the A1 will be scored if reason and conclusion are given.

Note: Use of a narrower interval is possible. You may need to check values, but score as per the main scheme with their end points as long as their interval contains the root.

(b)

B1*: Shows all necessary steps to show given result. Allow if α is used in place of x.

Sets $f(x) = 0$, uses $\sec x = \dfrac{1}{\cos x}$ and makes the x of the $6x$ the subject. The "=0" must be seen at some stage.

(c) (i)

M1: Uses iterative formula and $x_1 = 0.15$ to find $x_2 = 0.16..$ There must be some evidence of use of the iterative formula, either substitution seen or awrt 0.16 **other than** just 0.1622(315…), which may be from a calculator solve. If all that is seen is 0.1622(315…) with no other evidence then M0A0A0 will be scored.

A1: $(x_2 =)$ awrt 0.1629 Allow if the x_2 is omitted, so just awrt 0.1629 can score M1A1

(c)(ii) A1: $(\alpha =)\, 0.1622$ following the award of the M mark (see note).

An answer of 0.1666… may score the first M in (c) as long as there is evidence of the iterative formula used but the value only will be M0 (calculator solve gives this value).

Question Number	Scheme	Marks
2 (a)	10 m^2	B1
		(1)
(b)	$\log_{10} 25 = 1 + 0.03T \Rightarrow T = ...$	M1
	$T = 13.26$	A1cso
		(2)
		(3 marks)

(a)

B1: 10 m^2. Requires the units

(b)

M1: Substitutes $A = 25$ and finds value or exact expression for T or t. Condone poor algebra in the log work for this mark. Look for the process of substituting A and reaching a value/expression for T or t.

You may see attempts at making T the subject first. Look for substituting A=25 into their expression, condoning any poor algebra in the rearrangement and proceeding to a value/expression for T or t.

A1cso: Awrt 13.26 but condone awrt 13.26 weeks and isw rounding errors after a correct answer. Must have come from fully correct log work.

Do not accept expressions, they must evaluate to give a decimal correct to at least 2d.p.

Question Number	Scheme	Marks
3.	$\dfrac{dy}{dx}=\dfrac{4(x+3)^2-2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2-2(4x+1)(x+3)=0$ $\Rightarrow(x+3)(10-4x)=0\Rightarrow x=...$	M1
	Critical value of $\dfrac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\dfrac{dy}{dx}>0\Rightarrow -3<x<\dfrac{5}{2}$	A1
		(6 marks)

Note this appears as M1A1M1A1M1A1 on epen but is being marked as M1A1M1A1B1A1

M1: Attempts the quotient rule and achieves

$$\frac{dy}{dx}=\frac{a(x+3)^2-b(4x+1)(x+3)}{(x+3)^4}\quad a,b>0\quad \text{condoning slips}$$

They may attempt to multiply out the $(x+3)^2$ first which is fine as long as they reach a quadratic

Product rule may be attempted, look for $\dfrac{dy}{dx}=A(x+3)^{-2}-B(4x+1)(x+3)^{-3}$, $A,B>0$

Note: Only the numerator is required for this problem, and this mark can be awarded if a student attempts just $u'v-uv'=0$. If the denominator is considered it must be treated correctly.

A1: For a correct (unsimplified) $\dfrac{dy}{dx}$ or $u'v-uv'$ (if this is all that is considered). By the Product rule $\dfrac{dy}{dx}=4(x+3)^{-2}-2(4x+1)(x+3)^{-3}$

M1: Solves their numerator/their $\dfrac{dy}{dx}=0$ (or with any inequality) to obtain at least one critical value from a non-calculator method. If the numerator is expanded to a quadratic a method of solution must be shown (e.g. factorisation or completing the square or formula seen applied) They may be working with equalities or inequalities for this mark. Note this means students who cancel the $(x+3)$ and lose the second critical value can still score M1.

A1: Critical value of $\dfrac{5}{2}$ (allow equivalent fractions) found provided the previous M has been scored.

B1: Finds or identifies -3 as the other critical value (no need for method shown).

A1: Depends on first M. For $-3<x<\dfrac{5}{2}$ or $-3<x\leqslant\dfrac{5}{2}$ or equivalent set notation, e.g. $x\in\left(-3,\dfrac{5}{2}\right]$ Condone $-3\leqslant x\leqslant\dfrac{5}{2}$ or $-3\leqslant x<\dfrac{5}{2}$. Must be simplified fraction. Allow split inequalities $-3<x$ and $x<\dfrac{5}{2}$, condoning "or" or a comma between but if set notation used it must be intersection not union.

Note: If the denominator of $\dfrac{dy}{dx}$ is incorrectly treated but the numerator is correct, a correct answer may be achieved. In such cases award SC M0 A0 M1 A1 B1 A1 if the subsequent marks are earned.

Note: answers relying on calculator technology where roots appear from an unsimplified derivative can score maximum M1A1M0A0B1A1

Question Number	Scheme	Marks
4 (a)	$$\frac{4x^3+2x^2+3x+8}{x^2+4} \equiv Ax+B+\frac{Cx+D}{x^2+4}$$	
(i)	Any correct value of $A=4,\ B=2$ or $C=-13$	B1
	Full method to find values for A, B and C E.g. $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$ Or $x^2+0x+4\overline{)4x^3+2x^2+3x+8}$ with quotient $4x+2$ … $-13x$	M1
	$A=4,\ B=2$ and $C=-13$	A1
(ii)	E.g. Using $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$	
	For example $x^2 \Rightarrow B=2 \quad x=0 \Rightarrow 4\times2+D=8 \quad D=0$ *	B1*
		(4)
(b)	$\int\left(Ax+B+\frac{Cx}{x^2+4}\right)\mathrm{d}x=\frac{Ax^2}{2}+Bx,+\frac{1}{2}C\ln\left(k\left(x^2+4\right)\right)$ k any constant	B1 ft, M1 A1ft
	$\int_1^4 \frac{4x^3+2x^2+3x-8}{x^2+4}\,\mathrm{d}x=\left[2x^2+2x-\frac{13}{2}\ln\left(k\left(x^2+4\right)\right)\right]_1^4$	
	$=\left(32+8-\frac{13}{2}\ln 20\right)-\left(2+2-\frac{13}{2}\ln 5\right)=36+\frac{13}{2}\ln\frac{1}{4}=36-13\ln 2$	dM1 A1
		(5)
		(9 marks)

(a)(i) Note – mark part (a) as a whole.

B1: Any correct value of A, B or C. Must be correctly identified to its letter by label or position in the identity, not just a value.

M1: A full method to find values for A, B and C.

If they attempt $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$ the expression must be correct.

If they attempt by division then they must proceed to a linear quotient and a linear remainder. Alternative notations of long division may be seen. Correct values for A, B and C will imply the method mark.

A1: Correct A, B, C or correct expression.

Note: attempts at long division but mixing up the constants to give $-13x+\frac{4x+2}{x^2+4}$ can score the method mark only.

(a)(ii)

B1*: Fully shows that $D = 0$ from clear and correct work.

Using $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ they would need to set up (at least) two correct equations and solve, with appropriate substitutions seen, to show that $D = 0$

Using division the working must be fully correct with the remainder shown as just $-13x$

FYI

$$\begin{array}{r|l} & 4x + 2 \\ \hline x^2 + 0x + 4 & 4x^3 + 2x^2 + 3x + 8 \\ & 4x^3 + 0x^2 + 16x \\ & \quad 2x^2 - 13x + 8 \\ & \quad 2x^2 + \ 0x + 8 \\ \hline & \qquad\qquad -13x \end{array}$$

Make sure the remainder is −13x and not just −13

Using verification is also acceptable, e.g. after finding values, expanding both sides with $D = 0$ in place, and checking expressions agree.

(b)

B1ft: Integrates their $Ax + B$ to $\frac{Ax^2}{2} + Bx$ $(A, B \neq 0)$

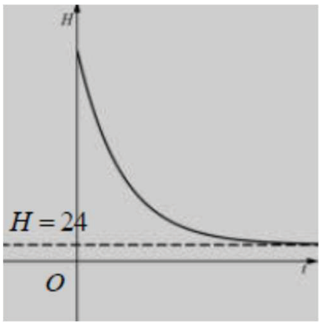
M1: Integrates their $\frac{Cx}{x^2 + 4}$ to $\ldots\ln\left(k\left(x^2 + 4\right)\right)$ where k is any constant (most likely $k = 1$). Condone invisible brackets, $\ln x^2 + 4$. Condone the answer coming from $\frac{Cx}{2x}\ln(x^2 + 4)$ as long as the spurious x's are cancelled.

They should be using $D = 0$ to access this mark, $\frac{Cx + D}{x^2 + 4}$ with $D \neq 0$ to $\ldots\ln\left(k\left(x^2 + 4\right)\right)$ is M0. If they split the fraction a valid attempt must be made at both parts in order to score the M – send to review in such instances.

A1ft: Correct integration for their $\frac{Cx}{x^2 + 4}$ Condone invisible brackets, $\ln x^2 + 4$ (oe)

dM1: Substitutes 1 and 4 into an expression of the form $Px^2 + Qx + R\ln\left(k\left(x^2 + 4\right)\right)$, subtracts either way around and attempts to collect terms (condone slips combining ln terms) reaching single constant and ln terms. “Invisible brackets” on a $\ln x^2 + 4$ term must have been recovered to score this mark.

A1: $36 - 13\ln 2$

Question Number	Scheme	Marks
5 (a)	$304(°C)$	B1
		(1)
(b)	 Shape or Asymptote	M1
	Fully correct Shape and Asymptote	A1
		(2)
(c)	$144 = 280\mathrm{e}^{-0.05t} + 24 \Rightarrow 280\mathrm{e}^{-0.05t} = 120$	M1
	Correct use of lns $\Rightarrow -0.05t = \ln\frac{120}{280} \Rightarrow t = 16.95$	dM1 A1
		(3)
(d)	$\left(\frac{\mathrm{d}H}{\mathrm{d}t} = \right) - 0.05\times 280\mathrm{e}^{-0.05t}$	M1
	$\frac{\mathrm{d}H}{\mathrm{d}t} = -0.05\times(H-24) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = 1.2 - 0.05H$	M1, A1cso
		(3)
		(9 marks)

(a)

B1: 304

(b)

M1: Correct shape or correct asymptote. For the shape the curve must be at least in the upper half plane with negative gradient which levels out to 0, but which may be asymptotic to the x-axis. Be tolerant with curves that begin to turn back on themselves for this mark if the intent is clear.
For the asymptote it must be an asymptote for their curve (not just a dashed line with no curve approaching it), and may just be labelled 24 on the y-axis for this mark.

A1: Correct shape and asymptote. For the shape the curve must be in quadrant 1 only, starting on the H axis with negative gradient which levels to 0 at the asymptote which must be above the x-axis. It must not cross the asymptote or clearly turn back on itself, but condone touching the asymptote and slight "pen flicks" at the end.
The asymptote must be an equation, $H = 24$ but condone $y = 24$. Must be an equation, not just 24 labelled as the intercept of the asymptote.

(c)

M1: Substitutes $H = 144$ into the equation and proceeds at least as far as $A\mathrm{e}^{\pm 0.05t} = B$ condoning slips.

dM1 Uses correct ln work to find t.

Method 1: $A\mathrm{e}^{-0.05t} = B \to \mathrm{e}^{\pm 0.05t} = k \to \pm 0.05t = \ln k \to t = ...$

Method 2: $A\mathrm{e}^{-0.05t} = B \to \ln A - 0.05t = \ln B \to t = ...$

May be implied by a correct f.t. value following reaching an equation $A\mathrm{e}^{\pm 0.05t} = B$

A1 Awrt 16.95. Condone awrt 16.95 minutes.

(d)

M1: Differentiates to a form $k\mathrm{e}^{-0.05t}$ where $k \neq 1$ or 280. May be called $\frac{\mathrm{d}y}{\mathrm{d}x}$ or unlabelled as long as it is clearly an attempt at the derivative.

M1: Rearranges the equation for H to find $\mathrm{e}^{-0.05t}$ or a multiple of $\mathrm{e}^{-0.05t}$ which is then substituted into their $k\mathrm{e}^{-0.05t}$ (not dependent, so their k may be 280 for this mark). May be implied as long as a derivative has been attempted.

A1: cso $\frac{\mathrm{d}H}{\mathrm{d}t} = 1.2 - 0.05H$ or with fractions e.g. $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{6}{5} - \frac{H}{20}$, which must follow correct work and notation. The $\frac{\mathrm{d}H}{\mathrm{d}t}$ must have been seen to score this mark.

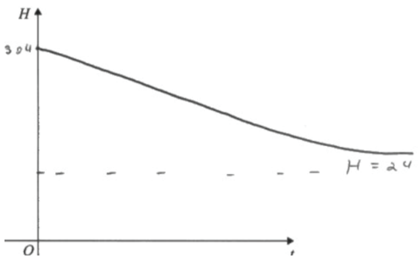
Alt:

M1: Makes t the subject and differentiates: $H = 280\mathrm{e}^{-0.05t} + 24 \Rightarrow t = -20\ln\frac{H-24}{280} \Rightarrow \left(\frac{\mathrm{d}t}{\mathrm{d}H} = \right)\frac{-20}{H-24}$

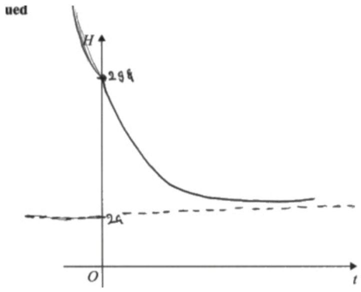
M1: Applies $\frac{\mathrm{d}H}{\mathrm{d}t} = 1 \div \frac{\mathrm{d}t}{\mathrm{d}H}$ with their attempt at $\frac{\mathrm{d}t}{\mathrm{d}H}$

A1: cso per main scheme.

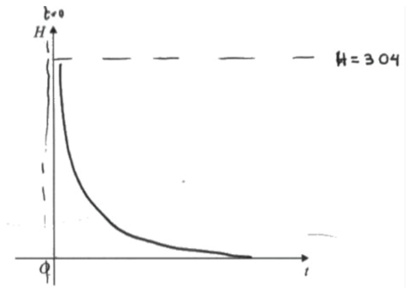
Some examples of scores for graphs:



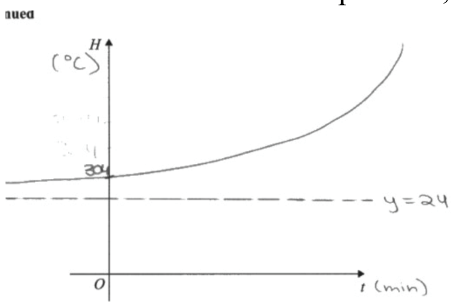
M1A0 Asymptote correct and labelled, shape incorrect



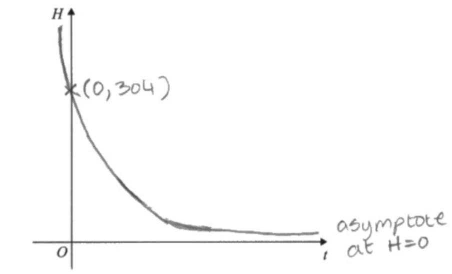
M1A0 Asymptote correct labelled but graph extends into second quadrant, and no equation.



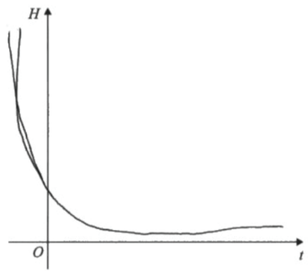
M1A0 Shape acceptable, but asymptote incorrect and does not start on H axis



M1A0 Asymptote correct and labelled (and is an asymptote to their graph), shape incorrect



M1A0 Shape acceptable, asymptote incorrect (and curve extends to Q2)



M0A0 Shape and asymptote both incorrect. Gradient increases above zero.

Question Number	Scheme	Marks
6 (a)	$y=\dfrac{4x+3}{x-2}\Rightarrow yx-2y=4x+3 \Rightarrow yx-4x=2y+3$ | $y=4+\dfrac{11}{x-2}\Rightarrow y-4=\dfrac{11}{x-2}$, $x-2=\dfrac{11}{y-4}$	M1
	$\Rightarrow x=\dfrac{2y+3}{y-4}\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{2x+3}{x-4}$ | $\Rightarrow x=2+\dfrac{11}{y-4}\Rightarrow \mathrm{f}^{-1}(x)=2+\dfrac{11}{x-4}$	A1
	$x\neq 4$	B1
		(3)
(b)	$\mathrm{ff}(x)=\dfrac{4\times\dfrac{4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}-2}$ or $\mathrm{ff}(x)=4+\dfrac{11}{4+\dfrac{11}{x-2}-2}$	M1
	$=\dfrac{4\times(4x+3)+3(x-2)}{4x+3-2(x-2)}=\dfrac{19x+6}{2x+7}$	dM1 A1
		(3)
(c)	Either $x=1$ or $y=38$	M1
	$(1, 38)$	A1
		(2)
		(8 marks)

(a)

M1: Attempts to change the subject. May interchange x and y at the start or work towards $x=\ldots$ Must proceed as far as getting the two x terms on one side of the equation and the terms not in x on the other or equivalent if x and y are swapped first. Alternatively may divide through first, gather terms and proceed as far as cross multiplying.

A1: $\mathrm{f}^{-1}(x)=\dfrac{2x+3}{x-4}$ (oe) but accept $y=\dfrac{2x+3}{x-4}$ and even $\mathrm{f}^{-1}=\dfrac{2x+3}{x-4}$ or e.g. $\mathrm{f}(x)^{-1}=\dfrac{-2x-3}{4-x}$ Accept equivalent simplified forms.

B1: States correct domain $x\neq 4$ $(x\in\mathbb{R})$ Ignore reference to $y\neq 2$ but extra x exclusions is B0.

(b)

M1: Attempts to replace both the x's in $\dfrac{4x+3}{x-2}$ by $\dfrac{4x+3}{x-2}$. Condone a slip such as missing the "+3" as long as the attempt to replace both x's is seen. Alternatively if dividing through look for the attempt to replace the x by $4+\dfrac{11}{x-2}$ condoning a slip missing the extra "-2"

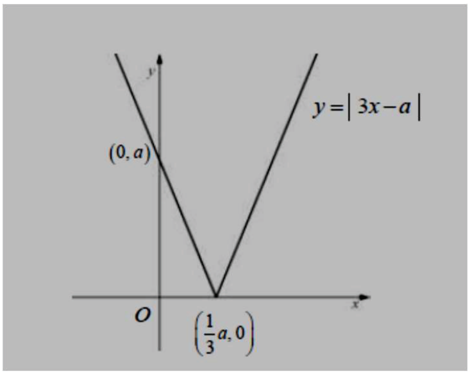
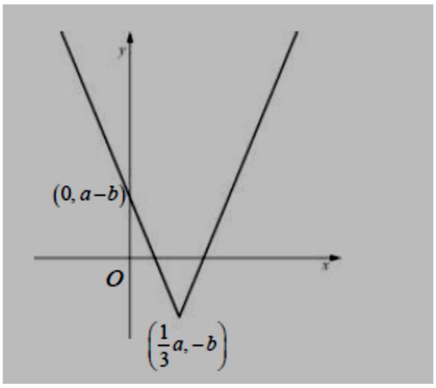
dM1: Full attempt to form a fraction of the required form by multiplying all terms in the numerator and denominator by $(x-2)$ If dividing through they must combine to a single fraction.
An equally valid alternative is to put both numerator and denominator over common factors of $(x-2)$

A1: $\dfrac{19x+6}{2x+7}$ but allow equivalent fractions of this form.

(c) Mark only the final attempt here as the candidate may "build up" their answer in stages.
Also watch out for the answer stated next to the question, which can be taken as their final answer.

M1: Either $x=1$ or $y=38$

A1: (1, 38) but allow to be written out separately, or with the brackets omitted.

Question Number	Scheme		Marks
7 (a)(i)		Shape	B1
		Vertex $\left(\frac{1}{3}a, 0\right)$	B1
		y intercept $(0, a)$	B1
(a)(ii)		Shape	B1ft
		Vertex $\left(\frac{1}{3}a, -b\right)$	B1ft
		y intercept $(0, a-b)$	B1ft
			(6)
(b)	Attempts to solve $-3x+a-b=5x \Rightarrow x=...$		M1
	$\Rightarrow x=\dfrac{a-b}{8}$ ONLY		A1
			(2)
			(8 marks)

Note: accept both diagrams on the same sketch as long as the information for each is clear.

If two graphs are given unlabelled, treat their first as part (a)(i) and their second as part (a)(ii).

(a)(i)

B1: A V shaped curve sitting on the positive x-axis and appearing in at least quadrant 1. Don't be too concerned about the graph being non symmetrical unless it appears wildly incorrect

B1: The vertex is on the positive x- axis and is marked $\left(\frac{1}{3}a, 0\right)$ but condone $(x=)\frac{1}{3}a$ which may be seen as the intercept on the graph. May be scored from an incorrect shape for the graph, but must have a single vertex on the x-axis.

B1: The graph passes through the positive y-axis at $(0, a)$ but condone $(y=)a$ which may be seen as the intercept on the graph.

SC If a "Y" shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but all B0B1B1 if vertex and positive intercept are correct.

(a)(ii)

B1ft: A V shaped curve with vertex in quadrant 4 or follow through a V shape that is a translation down from their answer in (a)(i)

B1ft: The vertex is in quadrant 4 and is marked $\left("\frac{1}{3}a", -b\right)$, which may be indicated by appropriate labelling on the graph. Follow through on their x intercept of the vertex in (i) (which must have been on the x-axis)

B1ft: The graph passes through the positive y-axis at $(0, "a"-b)$, following through on their y intercept from (i), but condone $(y=)\,a-b$ which may be seen as the intercept on the graph.

SC If a "Y" shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but all B0B1B1 if vertex and positive intercept are correct.

(b)

M1: Attempts to solve $-3x+a-b=5x \Rightarrow x=...$ There may be other equations attempted

A1: $x=\frac{a-b}{8}$ or exact equivalent only. If multiple values are found the incorrect ones must be rejected or this correct answer clearly indicated as their final answer (e.g. by underlining). Ignore any reasoning given, mark the value.

Question Number	Scheme	Marks
8 (i)	States or uses $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\text{cosec}\,\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\dfrac{3}{8} \Rightarrow \tan\left(2x - 70°\right) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right) + 70°}{2}$	dM1
	awrt $24.7°$, awrt $114.7°$	A1
		(4)
		(9 marks)

(i)

B1: States or uses $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

M1: Uses both $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to set up an equation in $\sin 2\theta$

A1: $\sin 2\theta = \dfrac{3}{4}$

M1: Correct method of solving an equation of the form $\sin 2\theta = k$, $|k| < 1$ to find at least one value for θ. May be solving in degrees for this mark ($24.30°, 65.70°$)

A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range (isw for incorrect rounding).

(ii) Main scheme

M1: Uses the compound angle identities to reach $\tan\left(2x \pm 70°\right) = -\dfrac{3}{8}$

A1: $\tan\left(2x - 70°\right) = -\dfrac{3}{8}$

dM1: Correct order of operations to find a value for x. Condone if radians mode used, e.g.

$2x - 70 = -0.3587... \Rightarrow x = \dfrac{70 - 0.3587...}{2} = 34.82...$ May be scored from

$2x + 70 = ... \rightarrow x = \dfrac{-70 - ....}{2}$ Condone errors in finding the second solution such at $180 - PV$.

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 1

M1: Cross multiplies and makes $\tan 2x$ the subject. Condone $\tan 70°$ being replaced by 2.75

A1: $\tan 2x = \dfrac{8\tan 70° - 3}{8 + 3\tan 70°} = \text{awrt } 1.17$

dM1: Correct order of operations to find one value for x. See note on main scheme.
Implied by a correct value for their $\tan 2x = \text{awrt } 1.17$ provided the previous M has been scored

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in $\tan x$ (condoning slips and not necessarily gathered)

$$\frac{\frac{2\tan x}{1-\tan^2 x}-\tan 70°}{1+\frac{2\tan x}{1-\tan^2 x}\tan 70°}=-\frac{3}{8}\Rightarrow 16\tan x-8\tan 70°\left(1-\tan^2 x\right)=-3\left(1-\tan^2 x+2\tan x\tan 70°\right)$$

$$\left(\Rightarrow t^2\left(8\tan 70°-3\right)+t\left(16+6\tan 70°\right)+3-8\tan 70°=0\ \left(18.98t^2+32.48t-18.98=0\right)\right)$$

A1: A correct value for $\tan x$, $\tan x=$ awrt 0.4604 or awrt −2.172

dM1: Applies arctan to achieve at least one value for x.

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ (or equivalent identity)	B1
	$3\operatorname{cosec}\theta=8\cos\theta\Rightarrow 9\operatorname{cosec}^2\theta=64\cos^2\theta$ and uses both $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sin^2\theta+\cos^2\theta=1$ to get equation in one trig term	M1
	$\Rightarrow 9=64\sin^2\theta\left(1-\sin^2\theta\right)\Rightarrow \sin^2\theta=\frac{4\pm\sqrt{7}}{8}$	A1
	$\Rightarrow \theta=\arcsin\left(\sqrt{\frac{4\pm\sqrt{7}}{8}}\right)=$ awrt 0.424, awrt 1.15	M1 A1
		(5)

B1: States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot\theta=\frac{\cos\theta}{\sin\theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2\theta=\frac{4\pm\sqrt{7}}{8}$ note it is the same roots for $\cos^2\theta$

M1: Correct method of solving an equation of the form $\sin^2\theta=k$ or $\cos^2\theta=k$, $0<k<1$ to find at least one value for θ. May be solving in degrees for this mark ($24.30°, 65.70°$)

A1: $\theta=$ awrt 0.424, awrt 1.15 and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}=0\Rightarrow\ln(4x)=-2$	dM1
	$Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2\times y$ co-ordinate of Q	M1
	Range $g(x)\leqslant\frac{12}{e}$	A1ft
		(2)
		(8 marks)

(a)

B1: $\frac{1}{4}$ or exact equivalent. May be seen as the x coordinate in a coordinate pair.

(b)

M1: Attempts to differentiate using the product rule and achieves $f'(x)=\frac{a}{\sqrt{x}}\ln(4x)+b\sqrt{x}\times\frac{1}{x}$

A1: $f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$. Note the $\frac{1}{x}$ may be seen as $\frac{4}{4x}$

dM1: Sets $f'(x)=0$ and proceeds to $\ln(4x)=k$. The "=0" may be implied by the attempt to solve. Condone attempts where multiplying through by $\sqrt{x}$ occurs before setting equal to 0.

A1: $x=\frac{1}{4e^2}$ oe but allow awrt 0.034 following correct equation.

A1: $Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$ Both coordinates correct and simplified, but may be listed separately, $x=\ldots$, $y=\ldots$ and allow if y is found in (c). Accept negative powers of e, but must be simplified coefficients and simplified powers.

(c)

M1: Attempts $-2\times$ their y co-ordinate of Q. Also allow following decimal answer

A1ft: Range $g(x)\leqslant\frac{12}{e}$ following through on their negative y value for Q. Accept with y instead of g(x) and wtih negative powers or expression in e following their (b), e.g. $y\leqslant 12e^{-1}$ Accept decimals following a decimal answer to (b).

Question Number	Scheme	Marks
10.(a)	$x = 3\cos 2y \Rightarrow \left(\frac{dx}{dy} =\right) -6\sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their } \frac{dx}{dy}}$	M1
	$\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$	
	$x^2 = 5 \Rightarrow a = \sqrt{5}$	M1 A1
	$\sqrt{5} = 3\cos 2y \Rightarrow b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6\sin 2y} = -\frac{1}{4}$	
	$\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2}\arcsin\left(\frac{2}{3}\right)$	M1A1
	$x = 3\cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	dM1 A1
		(4)
		(9 marks)

(a)

M1: $\left(\frac{dx}{dy} =\right) k\sin 2y$ Alt: $x = 3(2\cos^2 y - 1) \Rightarrow \left(\frac{dx}{dy} =\right) k\cos y \sin y$ (oe with sine identity etc)

A1: $\left(\frac{dx}{dy} =\right) -6\sin 2y$ Alt: $\left(\frac{dx}{dy} =\right) -12\cos y \sin y$

(b)

M1: Either

- Uses $\sin^2 2y + \cos^2 2y = 1$ with $x = 3\cos 2y$ to find $\sin 2y$ in terms of x (condone slips with the 3 when rearranging)
- Or uses $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$

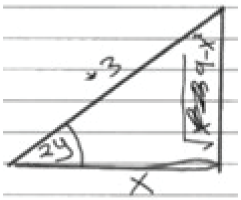
dM1: Uses both of the above to find $\frac{dy}{dx}$ in terms of x.

A1: $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}}$ including the $\frac{dy}{dx}$ seen at some stage associate with the answer, may have the −1/2 in the numerator seen and isw attempts to take the 2 into the numerator.

Alt for first bullet point: Longer ways may be possible, but look for correct formulae used, allowing algebraic slips simplifying, and must reach an expression for sin 2y in terms of x. E.g.

$$\sin 2y = 2\sin y\cos y = 2\sqrt{\frac{1}{2}\left(1-\frac{x}{3}\right)}\sqrt{\frac{1}{2}\left(1+\frac{x}{3}\right)}$$

Another alternative for finding sin 2y in terms of x is use of appropriate triangles, e.g.



To score the M they must proceed as far as identifying sin 2y in terms of x, here $\sin 2y = \frac{\sqrt{9-x^2}}{3}$. The trig ratios and application of Pythagoras theorem must be correct.

(c)

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $x^2 = p, p > 0$. Condone slips in rearranging.

A1: $a = \sqrt{5}$ Accept $x = \ldots$

dM1: Substitutes the value of x in $x = 3\cos 2y$ and uses arccos to find y (allowing decimal values – you may need to check).

A1: $b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$ Accept $y = \ldots$ Accept alternative notation such as $\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$ or even acos.

Alt method

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $\sin 2y = p, |p| < 1$. Condone slips in rearranging.

A1: $b = \frac{1}{2}\arcsin\frac{2}{3}$ Accept $y = \ldots$ Accept alternative notation such as $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$ or even asin

dM1: Substitutes the value of y in $x = 3\cos 2y$ and uses Pythagorean identity to find exact value of x.

A1: $a = \sqrt{5}$ Accept $x = \ldots$