Question	Scheme	Marks
1(a)	$2x^3-4x-15=(Ax+B)(x^2+3x+4)+C(2x+3)\Rightarrow A=\ldots$ or $x^2+3x+4\overline{\big)2x^3+0x-4x-15}$ with quotient $2x\ldots$, $2x^3+\ldots$	M1
	$A=2$	A1
	$B=-6, C=3$	A1A1
		(4)
(b)	$\int \mathrm{f}(x)\,\mathrm{d}x="A"\frac{x^2}{2}+"B"x+"C"\ln\left(x^2+3x+4\right)$	M1 A1ft
	$\int \mathrm{f}(x)\,\mathrm{d}x=x^2-6x+3\ln\left(x^2+3x+4\right)$	
	$\int_3^5 \mathrm{f}(x)\,\mathrm{d}x=\left(5^2-6\times5+3\ln\left(5^2+3\times5+4\right)\right)-\left(3^2-6\times3+3\ln\left(3^2+3\times3+4\right)\right)=..$	dM1
	$=-5+3\ln 44+9-3\ln 22=4+3\ln\left(\frac{44}{22}\right)$	M1
	$=4+\ln 8$	A1
		(5)
		(9 marks)

Notes:

(a)

M1: A correct method leading to at least one of the constants. May attempt long division getting the term A, or multiply through and compare coefficients or substitute values. Note that achieving $A = 2$ implies this mark.

A1: $A=2$ May be seen in the long division or in the expression.

A1: Either $B=-6$ or $C=3$ Stated or as part of the expression

A1: Both $B=-6$ and $C=3$ Stated or as part of the expression

(b) Note: use of letters *A, B* and *C* can score max 3 marks M1A1ftdM0M1A0.

M1: Attempts to integrate their answer to (a), look for $x^n\to x^{n+1}$ at least once and $\frac{2x+3}{x^2+3x+4}\to k\ln(x^2+3x+4)$ Allow with letters used if values have not been found.

A1ft: Correct integration following through their *A, B* and *C* (or with letters).

dM1: Applies the limits to their integral and subtracts. *Must have numerical values for this mark..*

M1: Uses correct log laws to combine the log terms into a single log term. (Need not have used the power law at this stage, unless earlier error leads to mismatch of coefficients.) Allow if letters used, so 3*C*ln(44/22) can score this mark.

A1cao: Correct answer in the form specified.

Question	Scheme	Marks
2(a)	Either $\mathrm{f}(x)<5$ or $\mathrm{f}(x)\geqslant 3$	M1
	$3\leqslant \mathrm{f}(x)<5$	A1
		(2)
(b)	(i) $y=5-\dfrac{4}{3x+2}\Rightarrow \dfrac{4}{3x+2}=5-y\Rightarrow \dfrac{4}{5-y}=3x+2\Rightarrow x=...$	M1
	$\left(\mathrm{f}^{-1}(x)\right)=\dfrac{1}{3}\left(\dfrac{4}{5-x}-2\right)$ oe such as $\dfrac{4}{15-3x}-\dfrac{2}{3}$ or $\dfrac{2x-6}{15-3x}$	A1
	(ii) Domain is $3\leqslant x<5$	B1ft
		(3)
(c)	$\mathrm{fg}(-\pi)=\mathrm{f}\left(\left\lvert 4\sin\left(-\dfrac{\pi}{3}+\dfrac{\pi}{6}\right)\right\rvert\right)=\mathrm{f}\left(\left\lvert 4\sin\left(-\dfrac{\pi}{6}\right)\right\rvert\right)=\mathrm{f}(2)=...$	M1
	$=5-\dfrac{4}{6+2}=\dfrac{9}{2}$	A1
		(2)
		(7 marks)

Notes:

(a)
M1: One correct end of range, though allow with $\leqslant$ or $<$ and $\geqslant$ or $>$ (in correct direction) at the respective ends, and accept with y instead of f(x), or even with x for the M1.
A1: Correct range, allow with y instead of f(x) or with other correct set notation (but use of x is A0 unless in formal set notation). Accept as two separate inequalities.

(b)(i)
M1: Attempts to make x or a swapped y the subject of the equation. Allow sign slips, but there should be a correct order of operations.
A1: Correct rule, must be in terms of x, accept equivalents and isw after a correct answer is seen. Do not be concerned with the lhs (accept y = … and even condone f(x) =...)

(b)(ii)
B1ft: Follow through on their answer to (a). Accept intervals or set notation answers. Do not accept e.g. $3\leqslant \mathrm{f}^{-1}(x)<5$ or with y

(c)
M1: Attempts to evaluate g at $-\pi$ and substitutes into f. (Allow even if a negative value is found.) Evaluation of $\mathrm{g}(-\pi)$ must be attempted, not just substitution into the expression. They may find the expression for fg(x) first and substituted, in which case the trigonometric expression must be evaluated before the mark is awarded.
A1: Correct answer. Accept as decimal. Dot not isw if they try to change to degrees.

Question	Scheme	Marks
3(a)	$f'(x)=2(x-2)\times e^{3x}+(x-2)^2\times 3e^{3x}$	M1 A1
	$f'(x)=0\Rightarrow e^{3x}(x-2)\left(2+3(x-2)\right)=0\Rightarrow x=\ldots$	M1
	$\Rightarrow x=\dfrac{4}{3}$	A1
	$y=\dfrac{4}{9}e^4$ $\quad\left(\text{so } A \text{ is } \left(\dfrac{4}{3},\dfrac{4}{9}e^4\right)\right)$	A1
		(5)
(b)	$k>0$ or $k\leqslant$ their $\dfrac{4}{9}e^4$ or with … or < respectively.	M1
	$0<k\leqslant$ their $\dfrac{4}{9}e^4$	A1ft
		(2)
		(7 marks)

Notes:

(a)

M1: Differentiates using the product rule reaching the form $\alpha(x-2)\times e^{3x}+(x-2)^2\times\beta e^{3x}$

A1: Correct derivative, need not be simplified.

M1: Sets their derivative equal to zero and attempts to cancel or factorise out the exponential term and the $(x-2)$ term (or multiple thereof) and solve for x. Alternatively, they may expand to a 3 term quadratic $(3x^2-10x+8)$ and attempt to solve this via correct method. Allow recovery on slips in the exponent for this mark (and the A's if a correct quadratic expression is solved).

A1: Correct x value.

A1: Correct simplified y value. Allow if seen in part (b). Isw, after a correct answer, but the follow through in (b) will not apply on an incorrectly simplified answer if this mark has been awarded.

(b)

M1: Identifies one correct boundary for k (the direction of inequality must be correct). Accept in terms of y or f(x) for this mark.

A1ft: Correct range for k following their **positive** y coordinate in (a). Must have correct inequalities. Accept awrt 24.3 for the upper bound. Allow as separate inequalities, or set or interval notation.

Question	Scheme	Marks
4(a)	$y=\log_{10}(2x+1)\Rightarrow 10^y=2x+1\Rightarrow x=...$	M1
	$\Rightarrow x=\dfrac{10^y-1}{2}$	A1
		(2)
(b)	$\left(\dfrac{dx}{dy}=\right)\dfrac{1}{2}10^y\ln 10$	M1
	$\dfrac{dy}{dx}=1\bigg/\dfrac{dx}{dy}=\dfrac{1}{\frac{1}{2}10^y\ln 10}$	M1
	$\dfrac{dy}{dx}=\dfrac{2}{(2x+1)\ln 10}$	A1
		(3)
		(5 marks)

Notes:

(a)

M1: Correctly undoes the logarithm and attempts to rearrange to make x the subject.

A1: Correct expression for x. Apply isw after a correct answer.

(b)

M1: Attempts to differentiate 10^y. Accept $\alpha 10^y \to \beta 10^y \ln 10$ (ignore any extra terms). Do not be concerned about the lhs for this mark.

M1: Applies reciprocal $\dfrac{dy}{dx}=1\bigg/\dfrac{dx}{dy}$. Variables must be consistent.

A1: Correct answer. Accept equivalents in terms of x, e.g. with $10^{\log(2x+1)}$ isw after a correct answer.

Alt for (b): Allow use of change of base formula.

M1: $y=\dfrac{\ln(2x+1)}{\ln 10}$ (effectively a B mark via this method)

M1: $\dfrac{dy}{dx}=\left(\dfrac{1}{\ln 10}\times\right)\dfrac{1}{2x+1}\times...$ Attempts to differentiate using the chain rule.

A1: Correct answer.

Note: Candidates who write $\dfrac{dy}{dx}=\dfrac{1}{2x+1}\times...\left(\text{or } \dfrac{dy}{dx}=\dfrac{1}{10^y}\times...\right)$ with no ln 10 term can score M0M1A0 under the Alt.

Alt 2: Implicit Differentiation

M1: As main scheme, attempts to differentiate as part of their work, so $\alpha 10^y \to \beta 10^y \ln 10\dfrac{dy}{dx}$

M1: Makes $\dfrac{dy}{dx}$ the subject. **A1:** Correct answer.

Question	Scheme	Marks
5(a)	$P(1)=\frac{4-1}{10}+\frac{3}{4}\ln\left(\frac{2}{3^2}\right) \quad (=-0.828...)$	M1
	$P(1)=-0.828...$ (which is negative so a loss of £0.828 million,) so approximately £830 000 loss.	A1*
		(2)
(b)	$P(6)=-0.08799\ldots$ and $\text{P}(7)=0.1975\ldots$	M1
	There is a sign change and hence as *P* is continuous on [6,7], so the root for *t* lies in [6,7].	A1
		(2)
(c)	$P=0\Rightarrow\frac{4t-1}{10}=-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right)\Rightarrow t=...$	M1
	$\Rightarrow t=\dfrac{10\left(-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right)\right)+1}{4}=\frac{1}{4}+\frac{30}{16}\ln\left(\frac{t+1}{(2t+1)^2}\right)^{-1}$ $\Rightarrow t=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{(2t+1)^2}{t+1}\right)$ *	A1*
		(2)
(d)	$t_2=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{13^2}{7}\right)=...(=6.219978...)$	M1
	$t_2=\text{awrt }6.220$	A1
	$t_6=6.314$	A1
		(3)
(e)	$"6.3..."\times12=...\text{months}$, or repeated iteration to root 6.31487 gives $75.7785\ldots$ or allow $"0.314..."\times12="3.768..."$	M1
	So it will take 76 months. (Accept 75 or awrt 76 months)	A1
		(2)
		(11 marks)

Notes:

(a)

M1: Attempts to substitute $t=1$ into the given formula. Allow if there is a slip but an attempt at substitution is seen, or allow for sight of awrt $-0.828...$

A1*: Correct value for $P(1)$ seen to at least 2.s.f. if substitution has been shown, or at least 3 s.f. if no substitution was shown, followed by suitable conclusion that it is a loss of approximately £830 000, though accept awrt £830 000. Must mention "loss" and include units (£ or pounds). Negative value given is A0.

(b)

M1: Attempts both $P(6)$ and $P(7)$ with at least one correct to 1 s.f. rounded or truncated. A tighter interval could be used but must contain the root (6.31487)

A1: Both correct to1 s.f. rounded or truncated with suitable conclusion made. Must mention sign change and continuity as well as conclusion about root in the interval.

(c)

M1: Attempts to isolate αt from the $\frac{4t-1}{10}$ after setting equal to zero.

A1*: Correct work to reach the given answer with no incorrect work seen and at least one intermediate step with either $\alpha t = \ldots$ reached or with the power law applied on the ln term (need not see the power explicitly used, but must have been applied correctly).

(d)

M1: Attempts to use the formula with $t_1 = 6$. Accept with 6 embedded in formula followed by a value, or awrt 6.2 (or even 6.3) as implying the attempt.

A1: awrt 6.220 Accept 6.22.

A1: Correct and given to 3 d.p.

(e)

M1: Multiplies their final root from (d) by 12, or uses repeated iteration to narrow further then multiplies by 12. Allow the method if they multiply the fractional part only by 12 and get, e.g., 6 years 3 months.

A1: Accept 75 or 76 months, or anything that rounds to 76 months.

Question	Scheme	Marks
6	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x+\sin x)=-\sin x+\cos x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x+\sin x)(3\cos x)-(-\sin x+\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$	M1 A1
	$=\frac{3\cos^2 x+\cancel{3\sin x\cos x}+2\sin x+3\sin^2 x-2\cos x-\cancel{3\sin x\cos x}}{\cos^2 x+2\sin x\cos x+\sin^2 x}$ $=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}$	M1
	$=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}\times\frac{\sec x}{\sec x}=\frac{3\sec x+2\sec x\sin x-2}{\sec x+2\sin x}$	M1
	$=\frac{2\tan x+3\sec x-2}{\sec x+2\sin x}$	A1
		(6)
		(6 marks)

Notes:

B1: Correct differentiation of $\cos x+\sin x\rightarrow-\sin x+\cos x$ seen somewhere in the proof. This can be scored if seen in workings for quotient rule, or even in the denominator of an incorrect attempt at u'/v'.

M1: Differentiates using the quotient rule or product rule. For quotient rule look for

$$\frac{(\cos x+\sin x)(\pm a\cos x)-(\pm\sin x\pm\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$$

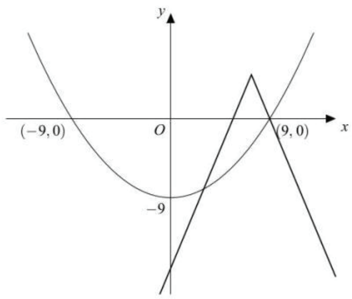
For product rule look for $(\cos x+\sin x)^{-1}(\pm a\cos x)\pm(\cos x+\sin x)^{-2}(\pm\cos x\pm\sin x)(2+3\sin x)$

A1: Fully correct derivative.

M1: Expands numerator or denominator and applies $\sin^2 x+\cos^2 x=1$ or other appropriate correct Pythagorean identity at least once in the proof.

M1: Attempts to multiply through by sec x in numerator and denominator (to achieve the 2sin x term). Not dependent and may be scored before the previous M.

A1: Correct answer. Terms may be in different order. Allow minor slips in notation (e.g. a single missing x) and recovery from missing brackets if the intent is clear, but A0 for persistent incorrect notation throughout (e.g no x's in the trig terms).

Question	Scheme		Marks
7(a)	(i) $\left(\frac{22}{3},5\right)$		B1
	(ii) $(0,-17)$		B1
			(2)
(b)	$5-(3x-22)=0 \Rightarrow x=...$ **and** $5+(3x-22)=0 \Rightarrow x=...$		M1
	$x=9$ and $x=\frac{17}{3}$		A1
			(2)
(c)		Correct U shape symmetric about y-axis with vertex on negative y-axis.	B1
		Graphs meet $(9,0)$ with $(-9,0)$ also shown.	B1
		Intercept at $(0,-9)$ stated or labelled.	B1
			(3)
(d)	Intersect at $(9,0)$		B1
	Or when $5+3x-22=\frac{1}{9}x^2-9$		M1
	$\Rightarrow (x-3)(x-24)=0 \Rightarrow x=...$		dM1
	Need smaller root $x=3 \Rightarrow y=...$		dM1
	$(3,-8)$		A1
			(5)
			(12 marks)

Notes:

(a)

Mark (a) and (b) as a whole

(i) B1: Correct coordinates. May be listed as separate coordinates, $x = \ldots, y = \ldots$

(ii) B1: Correct y intercept (and no other). Accept as coordinates, or stated as $y = \ldots$ (allow without the $x = 0$) or seen on graph as the y intercept. However just -17 on its own is B0.

(b)

M1: Attempts to solve **both** equations, though allow sign slips when expanding the brackets.

A1: Both values correct.

(c)

B1: Correct U shape symmetric about y-axis with vertex on negative y-axis. Allow a little tolerance with shape, but the curve must not clearly bend back on itself, and the minimum should be clearly intended as on the y-axis.

B1: Graphs meet at $(9,0)$ both x intercepts labelled or clearly stated (and no others). Shape need not be correct for this mark e.g. inverted parabola may be shown). Must pass through the same point as the modulus graph at (9,0).

B1: States or labels the y intercept at $(0,-9)$ and intercept must be on the negative y-axis.

(d)

B1: Deduces one point of intersection is (9,0). Must be seen in (d), or clearly stated as the answer if parts are not labelled, do not accept just this marked on the diagram.

M1: Sets up equation for the intersection of the quadratic with the positive gradient line segment, accept $5+3x\pm 22=\frac{1}{9}x^2-9$

dM1: Solves their equation, any valid means. After the equation is seen you may see just the correct root, or 24, which implies the M. Alternatively, expanding and solving the quadratic by usual rules.

dM1: Depends on first M mark. Selects the correct (smaller) root of their quadratic and attempts to find the y value (accept any y value appearing after choosing the correct root as an attempt). The larger root must be rejected. Following a correct equation, if no working is shown for finding the root (see above) they must have achieved $x=3$ and go on to find y for this mark. From an incorrect equation/incorrect method to solve, if only one root is given it must correspond to the smaller root of their equation/method.

A1: $(3,-8)$ only

Alternative by squaring.

B1: As main scheme.

M1: Rearranges to $|3x-22|=a+bx^2\left(=14-\frac{1}{9}x^2\right)$ and squares both sides to reach a quartic equation. If correct it is $\frac{1}{81}x^4-\frac{109}{9}x^2+132x-288=0$

dM1: Solves the resulting quartic (any means). If no method is shown, a correct value from the correct equation will imply the mark.

dM1: Depends on first M mark Selects correct root (second of the four in ascending order) and finds the y coordinate. If no method shown, but the quartic was correct, they must be using $x=3$.

A1: As scheme.

Question	Scheme	Marks
8(a)	$R = 17$	B1
	$\tan\alpha = \frac{15}{8}$	M1
	$\alpha = 1.081$	A1
		(3)
(b)	(i) Min f(x) $= \frac{15}{41+2\times"17"}$	M1
	$= \frac{1}{5}$	A1
	(ii) Occurs when $\sin(x-1.081)=1 \Rightarrow x-1.081=\frac{\pi}{2} \Rightarrow x = ...$	M1
	$x =$ awrt 2.65	A1
		(4)
(c)	$-\frac{23}{5}$ (or -4.6)	B1ft
		(1)
(d)	Awrt 1.33	B1ft
		(1)
		(9 marks)

Notes:

(a)

B1: For 17 only.

M1: Attempts an equation in α. Accept $\tan\alpha = \pm\frac{15}{8}$ or $\tan\alpha = \pm\frac{8}{15}$. If using R accept $\cos\alpha = \pm\frac{8}{"17"}$ or $\sin\alpha = \pm\frac{15}{"17"}$. Implied by a correct value for α

A1: Awrt 1.081. Must be in radians.

(b)(i)

M1: Attempts to apply the result from (a) to find the minimum. Allow for $\frac{15}{41\pm 2\times"\text{their } R"}$

A1: cao.

(ii)

M1: Attempts to solve $x \pm "\alpha" = \frac{\pi}{2}$

A1: cao

(c)

B1ft: Correct answer or follow through $2\times\left(\text{their}\frac{1}{5}\right)-5$ and no other solutions.

(d)

B1ft: For awrt 1.33 or follow through $0.5\times\text{their }2.65$ and no other solutions.

Question	Scheme	Marks
9(a)	$\dfrac{\cos^2\theta}{\cos 2\theta-\sin 3\theta}=\dfrac{\cos^2\theta}{\cos 2\theta-(\sin 2\theta\cos\theta+\cos 2\theta\sin\theta)}$	M1
	$=\dfrac{\cos^2\theta}{1-2\sin^2\theta-2\sin\theta\cos^2\theta-\sin\theta(1-2\sin^2\theta)}$	M1
	$=\dfrac{1-\sin^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta}=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1-2\sin\theta-4\sin^2\theta)}$	M1
	$=\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}$ *	A1*
		(4)
(b)	$\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}=2\operatorname{cosec}\theta\Rightarrow(1+\sin\theta)\sin\theta=2(1-2\sin\theta-4\sin^2\theta)$	M1
	$\Rightarrow 9\sin^2\theta+5\sin\theta-2=0$	A1
	$\Rightarrow\sin\theta=\dfrac{-5\pm\sqrt{25-4\times9\times-2}}{18}=...\Rightarrow\theta=\sin^{-1}...$	M1
	Two of $\theta=15.6°,164.4°,235.6°,304.4°$	A1
	All of $\theta=15.6°,164.4°,235.6°,304.4°$	A1
		(5)
		(9 marks)

Notes:

(a)

M1: Applies the compound angle formula to $\sin 3\theta$. Accept $\pm\sin\theta\cos 2\theta\pm\sin 2\theta\cos\theta$ Allow for students who apply $\sin 3\theta=3\sin\theta-4\sin^3\theta$ directly. May be seen anywhere (e.g. in separate working).

M1: Uses correct double angle formula for $\cos 2\theta$ **and** $\sin 2\theta$ (or assumed used correctly if correct $\sin 3\theta$ formula was used) to achieve all terms in single angle. (May be scored after the third M.)

M1: Uses Pythagorean identity to identify the factor of $1-\sin\theta$ in numerator. They need not have achieved single angle arguments for all terms for this mark. (May be scored before the second M.) There must be an intermediate step before the given answer with $1-\sin\theta$ cancelled if the factor is not explicitly seen in the numerator.

A1: Cancels the $1-\sin\theta$ from numerator and denominator and simplifies to the given result with no incorrect steps or consistently incorrect notation.

(b)

M1: Substitutes the result from (a) and cross multiplies to get an equation in $\sin\theta$ only.

A1: Correct simplified quadratic in $\sin\theta$

M1: Solves the resulting quadratic by formula, completing square or calculator (allow by factorisation if their quadratic factorises) and applies inverse sine to at least one root. May be implied by any correct arcsin of one of their solutions, such as $-55.6°$. My be implied by a radians answer (e.g. 0.272, 2.869…)

A1: Any two correct solutions in the range, accept awrt. Answers in radians scores A0.

A1: Awrt all four solutions correct and no others in the range.

Question	Scheme	Marks
9(a)	$\dfrac{\cos^2\theta}{\cos 2\theta-\sin 3\theta}=\dfrac{\cos^2\theta}{\cos 2\theta-(\sin 2\theta\cos\theta+\cos 2\theta\sin\theta)}$	M1

Example Alt	$=\dfrac{1-\sin^2\theta}{\cos 2\theta(1-\sin\theta)-2\sin\theta(1-\sin^2\theta)}$ $=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)\cos 2\theta-2\sin\theta(1+\sin\theta)(1-\sin\theta)}$	**3rd M1**
	$=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1-2\sin^2\theta-2\sin\theta-2\sin^2\theta)}$	**2nd M1**
	$=\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}$ *	**A1***
		(4)

Notes

The above approach can be marked via the main scheme but scores the 3rd M before the 2nd M.
Alternative approaches will hopefully fit the scheme similarly. Other approaches can be similarly applied as each of the three method marks will generally be required. E.g. see below (allowing first M for students who apply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ directly)
Note in Alt 2 below it is more likely the 1st M will be gained by aside work to expand sin3θ with substitution later seen into the equation.

9(a) Alt2 In reverse	$\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta}=\dfrac{1-\sin^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta}$ $=\dfrac{\cos^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta}$	**3rd M1**
	$=\dfrac{\cos^2\theta}{1-2\sin^2\theta-3\sin\theta+4\sin^3\theta}=\dfrac{\cos^2\theta}{\cos 2\theta-(3\sin\theta-4\sin^3\theta)}$	**2nd M1**
	$=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(3-4\sin^2\theta)}=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(1+2(1-\sin^2\theta)-2\sin^2\theta)}$ $=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(1+2\cos^2\theta-(1-\cos 2\theta))}$ $=\dfrac{\cos^2\theta}{\cos 2\theta-(2\sin\theta\cos^2\theta+\sin\theta\cos 2\theta)}$ $=\dfrac{\cos^2\theta}{\cos 2\theta-(\sin 2\theta\cos\theta+\cos 2\theta\sin\theta)}$	**1st M1**
	$=\dfrac{\cos^2\theta}{\cos 2\theta-\sin 3\theta}$ *	**A1***
		(4)

9(a) **Alt3** **By cross multiplying**	$\dfrac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = \dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta} \Leftrightarrow$ $\cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)=\left(1+\sin\theta\right)\left(\cos 2\theta-\sin 3\theta\right)\Leftrightarrow$ $\cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)=\left(1+\sin\theta\right)\left(\cos 2\theta-\underline{\sin 2\theta\cos\theta-\cos 2\theta\sin\theta}\right)$	**M1**
	$\Leftrightarrow \cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)$ $=\left(1+\sin\theta\right)\left(\underline{1-2\sin^2\theta-2\sin\theta\cos^2\theta-\left(1-2\sin^2\theta\right)\sin\theta}\right)$	**M1**
	$\Leftrightarrow \cos^2\theta-\cancel{2\sin\theta\cos^2\theta}-4\sin^2\theta\cos^2\theta=1-2\sin^2\theta-\cancel{2\sin\theta\cos^2\theta}$ $-\cancel{\sin\theta}+\cancel{2\sin^3\theta}+\cancel{\sin\theta}\cancel{-2\sin^3\theta}-2\sin^2\theta\cos^2\theta-\sin^2\theta+2\sin^4\theta$ $\Leftrightarrow \cos^2\theta-2\sin^2\theta\cos^2\theta=1-3\sin^2\theta+2\sin^2\theta\underline{\left(1-\cos^2\theta\right)}$	**M1**
	$\Leftrightarrow \cos^2\theta-\cancel{2\sin^2\theta\cos^2\theta}=1-3\sin^2\theta+2\sin^2\theta-\cancel{2\sin^2\theta\cos^2\theta}$ $\Leftrightarrow \cos^2\theta=1-\sin^2\theta$ which is true, hence given result is true.*	**A1***
		(4)

Notes:

(a)
M1: Applies the compound angle formula to $\sin 3\theta$ at some stage in the working. Accept $\pm\sin\theta\cos 2\theta\pm\sin 2\theta\cos\theta$ Allow for students who apply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ directly. May be seen anywhere (e.g. in separate working).
M1: Uses correct double angle formula for $\cos 2\theta$ **and** $\sin 2\theta$ (or $\sin 3\theta$) to achieve all terms in single angle. (May be before or after cross multiplying.)
M1: Cross multiplies and uses Pythagorean identity somewhere in the working in an attempt to achieve a true statement by reaching the same expression on both sides, or reducing to a known true trigonometric identity (as shown in scheme). Award at the stage the identity is applied to an appropriate term in an expression where the angles have been reduced to just θ.
A1: Reaches a correct true statement with no errors seen **and** makes conclusion that the original result is true.