Question Number	Scheme	Marks
1(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) 6\times3(3x-2)^5 \quad (=18(3x-2)^5)$	M1A1
		(2)
(b)	$"18"(3\times\tfrac{1}{3}-2)^{"5"}=-18$	M1
	$-18\rightarrow\frac{1}{18}$	M1
	$y-1="\frac{1}{18}"\left(x-\frac{1}{3}\right)$	dM1
	$3x-54y+53=0$	A1
		(4)
		(6 marks)

(a)

M1 Applies the chain rule to achieve a form of $A(3x-2)^5$ which may be unsimplified. The index does not need to be processed for this mark.

Alternatively, multiplies out the brackets to achieve an expression of the form $Ax^6+Bx^5+Cx^4+Dx^3+Ex^2+Fx+G$ (where all coefficients do not need to be simplified) and reduces the power by 1 on at least one of the terms. You will need to check this carefully to make sure it is the relevant term where the power has decreased

eg $Bx^5\rightarrow ...\times Bx^4$

Condone slips in their expansion.

(The actual expansion is $64-576x+2160x^2-4320x^3+4860x^4-2916x^5+729x^6$)

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)18(3x-2)^5$ or unsimplified equivalent eg $6\times3(3x-2)^5$ but the index must be processed. If they have multiplied out then accept $-576+4320x-12960x^2+19440x^3-14580x^4+4374x^5$.

Accept x^1 for x

(b)

M1 Substitutes $x = \frac{1}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be of the form $A(3x-2)^5$ (or accept a polynomial of degree 5) and finds a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Finds the negative reciprocal of their gradient. May be implied by the equation of their line. It cannot be scored for setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, solving to find x and then taking the negative reciprocal of this.

dM1 Attempts to find the equation of the normal. Look for $y - 1 = "\frac{1}{18}"\left(x - \frac{1}{3}\right)$ (must be a changed gradient from their tangent gradient). It is dependent on the first method mark. If they use $y = mx + c$ they must proceed as far as $c = \ldots$ $\left(= "\frac{53}{54}"\right)$

A1 $3x - 54y + 53 = 0$ or any integer multiple of this with all terms on one side of the equation eg $-6x + 108y - 106 = 0$ You must see the $= 0$
isw after a correct answer is seen. Eg if they proceed to state values for a, b and c which contradict their equation then take their equation as their final answer.

Question Number	Scheme	Marks
2(a)	$\text{fg}(5) = \text{f}(3) = \dfrac{5 - "3"}{3 \times "3" + 2}, = \dfrac{2}{11}$	M1, A1
		(2)
(b)	$\text{f}^{-1}(x) = \dfrac{5 - 2x}{3x + 1}$	M1A1
	$x \in \mathbb{R},\ x \neq -\dfrac{1}{3}$	B1
		(3)
(c)	$\dfrac{5 - \frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7$	B1ft
	$\dfrac{5 - \frac{1}{a}}{3 \times \frac{1}{a} + 2} = 2(a+3) - 7 \Rightarrow 4a^2 - a - 2 = 0$	M1A1
	$(a =)\ \dfrac{1 \pm \sqrt{33}}{8}$	A1
		(4)
		(9 marks)

(a)

M1 Correct order of operations applying g before f on 5

Allow an attempt to substitute $x = 5$ into $\text{fg}(x) = \dfrac{5 - (2x - 7)}{3(2x - 7) + 2}$ condoning slips. They must proceed to find a value.

This may be completed in two steps so allow for an attempt at $\text{g}(5) = A \rightarrow \text{f}(A) = ...$

Condoning arithmetical slips and bracket errors/omissions.

A1 $\dfrac{2}{11}$ or exact equivalent. Accept $0.\dot{1}\dot{8}$ but not eg 0.18 or 0.18….

(b)

M1 Changes the subject of $y = \dfrac{5 - x}{3x + 2} \rightarrow x = \dfrac{... \pm 2y}{\pm 3y \pm ...}$ or $x = \dfrac{5 - y}{3y + 2} \rightarrow y = \dfrac{... \pm 2x}{\pm 3x \pm ...}$

A1 $\quad \mathrm{f}^{-1}(x)=\frac{5-2x}{3x+1}$ or $\mathrm{f}^{-1}(x)=\frac{2x-5}{-3x-1}$ or exact equivalent expressions such as $\mathrm{f}^{-1}(x)=\frac{5-2x}{3\left(x+\frac{1}{3}\right)}$

.

Accept $\mathrm{f}^{-1}:x\to\frac{5-2x}{3x+1}$ Condone $\mathrm{f}^{-1}=...$ (or $\mathrm{f}^{-1}=y=...$) but **do not allow just** $y=...$ or $\mathrm{f}^{-1}:y=...$

B1 $\quad x\neq-\frac{1}{3}$. The omission of $x\in\mathbb{R}$ is condoned.

Note: It is also acceptable to define f^{-1} in any variable eg as $\mathrm{f}^{-1}(t)=\frac{5-2t}{3t+1}\quad t\neq-\frac{1}{3}$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than *x* it must be fully defined eg $\mathrm{f}^{-1}(t)=...$ not just $\mathrm{f}^{-1}=...$

(c)

B1ft Sets $\frac{5-\frac{1}{a}}{3\times\frac{1}{a}+2}=2(a+3)-7$ or their $\frac{1}{a}=\mathrm{f}^{-1}(2a-1)\Rightarrow\frac{1}{a}=\frac{"5"-"2"(2a-1)}{"3"(2a-1)+"1"}$

(may be in terms of *x*). Follow through on their $\mathrm{f}^{-1}(x)$.
May be implied by later work if they have invisible brackets.

M1 They must start with an equation of one of these forms (which may be in *x*):

- $\frac{A\pm\frac{1}{a}}{B\times\frac{1}{a}\pm C}=Da+E$ or equivalent so typically look for $\frac{5-\frac{1}{a}}{3\times\frac{1}{a}+2}=2(a+3)-7$

  (may be manipulated to $\frac{Aa\pm1}{B\pm Ca}=Da+E$)

- $\frac{1}{a}=\frac{Aa\pm B}{Ca\pm D}$ or equivalent

Rearranges their equation of an allowable form to a 3TQ. They must be attempting to solve $\mathrm{f}\left(\frac{1}{a}\right)=\mathrm{g}(a+3)$ so it cannot be awarded for an attempt to solve $\mathrm{f}(a)=\mathrm{g}(a)$ or $\mathrm{f}\left(\frac{1}{a}\right)=\mathrm{g}\left(\frac{1}{a}\right)$

Do not be concerned with the mechanics of their arrangement and condone invisible brackets.

A1 $\quad 4a^2-a-2=0$ or equivalent equation involving 3 terms. Eg $4a^2-a=2$ (may be in terms of *x*). May be implied by awrt ±0.843 and the $=0$ may be implied by their attempt to solve their quadratic equation.

A1 $\quad (a=)\ \frac{1\pm\sqrt{33}}{8}$ or exact equivalent. Do not accept rounded decimals. Condone $x=\frac{1\pm\sqrt{33}}{8}$

Question Number	Scheme	Marks
3(a)	$\int \frac{9x}{3x^2+k}\,dx = \frac{3}{2}\ln\left(3x^2+k\right)+C$	M1A1
		(2)
(b)	$\frac{3}{2}\ln(75+k)-\frac{3}{2}\ln(12+k)=\ln 8$	M1
	$\frac{3}{2}\ln\left(\frac{75+k}{12+k}\right)=\ln 8$ oe	dM1
	$\frac{75+k}{12+k}=4 \Rightarrow k=...$	ddM1
	$(k=)\ 9$	A1
		(4)
		(6 marks)

(a)

M1 Integrates to $A\ln(3x^2+k)$ with or without the + C. Allow A = 1 for this mark. Condone invisible brackets.

A1 $\frac{3}{2}\ln\left(3x^2+k\right)+C$ or $\frac{3}{2}\ln B\left(3x^2+k\right)$ oe Must include the constant of integration that is different from k. Must have brackets around $3x^2+k$

Also allow eg $\ln\left[B\left(3x^2+k\right)\right]^{\frac{3}{2}}$

(b) *Be aware they can solve equations on the calculator which is not acceptable*

M1 Attempts to substitute in 5 and 2 into their changed expression which must include x and k, subtracting either way round and sets equal to $\ln 8$. The values embedded in an equation is sufficient to score this mark. Condone slips if they attempted to evaluate when substituting in. Ignore the $+C$.

dM1 Attempts to apply the subtraction (or addition) law of logarithms once to an equation of the form $\pm A\ln(B+k)\pm A\ln(C+k)=\ln D$ or equivalent (allow $A = 1$).

Typically look for $A\ln(B+k)-A\ln(C+k)=\ln D \Rightarrow A\ln\left(\frac{B+k}{C+k}\right)=\ln D$ or equivalent

Do not be concerned with arithmetical errors and condone slips dealing with their A

Condone invisible brackets.

ddM1 Solves their equation of the form $\ln\left(\frac{B+k}{C+k}\right)=\ln D^{\frac{1}{A}}$ or $\ln\left(\frac{B+k}{C+k}\right)^A=\ln D$

where $A\neq 1$, and the A is correctly dealt with by
- removing lns correctly
- rearranging to form a linear equation in k
- proceeding to find a value for k.

eg $"\ln\left(\frac{"75"+k}{"12"+k}\right)=\ln 8^{"\frac{2}{3}"} \Rightarrow "75"+k="48"+"4k" \Rightarrow k=\ldots$

It is dependent on both of the previous method marks.

Condone arithmetical slips in their rearrangement and condone invisible brackets as long as the intention is clear. Also condone this mark to be scored if k is negative.

A1 9 cao (with intermediate working seen)

They cannot proceed from $\frac{3}{2}\ln(75+k)-\frac{3}{2}\ln(12+k)=\ln 8$ **to** $(k=)\ 9$ **without some intermediate working seen (eg an equation with the lns removed)**

Note an answer of $\ln(3x^2+k)\ (+C)$ in (a) will score a maximum of M1dM1ddM0A0 in (b)

Question Number	Scheme	Marks
4(a)	Gradient $=\dfrac{3.85-3.08}{5-0}$ (=0.154 or $\dfrac{77}{500}$)	M1
	$\log_{10} N = 3.08 + 0.154t$ oe	A1
		(2)
(b)	$a = 10^{3.08} =$ awrt 1200	B1
	$\log_{10} b = \text{"}0.154\text{"}$ or $b = 10^{\text{"}0.154\text{"}}$	M1
	$b =$ awrt 1.43	A1
		(3)
(c)	$\log_{10} 500\,000 = 3.08 + 0.154T \Rightarrow T = ...$ or $T = \log_{\text{"}1.43\text{"}}\left(\dfrac{500\,000}{\text{"}1200\text{"}}\right) = ...$	M1
	$(T =)$ awrt 17	A1
		(2)
		(7 marks)

Mark (a) and (b) together

(a)

M1 A correct attempt to find the gradient between the given coordinates. This may be implied by $10^{0.154}$ or awrt 1.43 or $\dfrac{77}{500}$. The values embedded in the expression $\dfrac{3.85-3.08}{5-0}$ or $\dfrac{3.08-3.85}{-5}$ is sufficient. Check Figure 1 and may be seen in (b)

A1 $\log_{10} N = 3.08 + 0.154t$ Must be in terms of $\log_{10} N$ and t (may be seen in (b))

Condone $\log N$ or $\lg N$ for $\log_{10} N$ but do not accept $\ln N$

Accept equivalent equations such as $\log_{10} N = \dfrac{77}{25} + \dfrac{77}{500}t$ or $500\log_{10} N = 1540 + 77t$ or $t = \dfrac{500}{77}\log_{10} N - \dfrac{1540}{77}$

Do not accept $N = 1200 \times 1.43^t$

(b)

B1 awrt 1200

M1 A correct equation to find b using their gradient in (a). Can be awarded for $\log_{10} b = "0.154"$ or $b = 10^{"0.154"}$ (may be implied by awrt 1.43). Allow log or lg but not ln.

A1 awrt 1.43

(c)

M1 Substitutes in N = 500 000 into their $N = ab^t$ using their values for a and b and uses a valid method to find a value or expression for T (can be left as a logarithm)
This can be either by:

- using correct inverse laws
- using a calculator and proceeding directly to a value for t

Typically, $"1200" \times "1.43"^T = 500\,000 \Rightarrow "1.43"^T = \dfrac{500\,000}{"1200"} \Rightarrow T = \dfrac{\log_{10} "\frac{1250}{3}"}{\log_{10} "1.43"} = \ldots$

$$\Rightarrow "1.43"^T = \frac{500\,000}{"1200"} \Rightarrow T = \log_{"1.43"}\left(\frac{500\,000}{"1200"}\right) = \ldots$$

Alternatively uses their $\log_{10} N = "3.08" + "0.154"t$ and rearranges to find a value for T allowing sign slips only:

$$\log_{10} 500\,000 = "3.08" + "0.154"T \Rightarrow T = \frac{\log_{10} 500\,000 - "3.08"}{"0.154"} = \ldots$$

May be implied by a correct answer.

A1 $(T =)$ awrt 17 isw after a correct answer is seen (cannot be left as a logarithm)
This mark can only be scored provided EITHER their equation in (a) is correct OR their values for *a* and *b* are correct in (b).

Question Number	Scheme	Marks
5(a) **(i)**	7	B1
(ii)	$\dfrac{9}{k}$	B1
		(2)
(b)	$kx-9-2=0$ and $-kx+9-2=0 \Rightarrow \dfrac{\dots}{k}, \dfrac{\dots}{k}$	M1
	CVs $\dfrac{7}{k}, \dfrac{11}{k}$	A1
	$\dfrac{7}{k} < x < \dfrac{11}{k}$	A1
		(3)
(c)	$"-2" = -2 \times "\dfrac{9}{k}" + 3 \Rightarrow k = \dots$	M1
	$(k=)\ 3.6$	A1
	$k > 3.6$	A1
		(3)
		(8 marks)
Alt(b)	$-2 < kx - 9 < 2$	
	$\dfrac{\dots}{k} < x < \dfrac{\dots}{k}$	M1
	$\dfrac{7}{k} < x < \dfrac{11}{k}$	A1A1

(a) Check Figure 2 and by the questions

(i)

B1 7 Condone $(0, 7)$ but not $(7, 0)$

(ii)

B1 $\frac{9}{k}$ oe Condone $\left(\frac{9}{k}, -2\right)$ but not $\left(-2, \frac{9}{k}\right)$

(b)

M1 A correct attempt to find the two critical values. Eg

- Forms two correct equations (or linear inequalities) with the modulus signs removed and solves to find the values of x where the graph crosses the x-axis (see main scheme) or the values of x such that $|kx-9|-2<0$
- Solves the given inequality to isolate x in the middle (Alt(b)) by:
  $|kx-9|-2<0 \Rightarrow |kx-9|<2 \Rightarrow -2<kx-9<2 \Rightarrow ...<x<...$
- Solves the given inequality by squaring both sides and solving:
  $(kx-9)^2<4 \Rightarrow k^2x^2-18kx+77<0 \Rightarrow x^2-\frac{18}{k}x+\frac{77}{k^2}<0 \Rightarrow \left(x-\frac{9}{k}\right)^2<\frac{4}{k^2} \Rightarrow ...<x<...$

Do not be too concerned by their use of $=, \geqslant, \leqslant, >, <$ throughout their working.

A1 Correct critical values $\frac{7}{k}, \frac{11}{k}$ cao which may be seen in their final answer.

A1 $\frac{7}{k}<x<\frac{11}{k}$ or simplified equivalent eg "$x>\frac{7}{k} \cap x<\frac{11}{k}$" or "$\left\{x \in \mathbb{R} : \frac{7}{k}<x<\frac{11}{k}\right\}$" or $x \in \left(\frac{7}{k}, \frac{11}{k}\right)$ Must be in terms of x

There must be some attempt to bring the two inequalities together if separate. The mark cannot be scored for $x>\frac{7}{k}$ $x<\frac{11}{k}$ DO NOT ACCEPT eg "$x>\frac{7}{k}$ or $x<\frac{11}{k}$" $x \in \left[\frac{7}{k}, \frac{11}{k}\right]$

(c)

M1 Uses a correct method to find a value for k. For example:

- Substitutes in $\left("\frac{9}{k}", "-2"\right)$ into $y=3-2x$ and attempts to find a value for k. They may solve
  $"-2"=3-2x \Rightarrow x="2.5" \Rightarrow "\frac{9}{k}"<"2.5" \Rightarrow k>...$
- Finds when $y=3-2x$ intersects with AB ($y=-kx+7$) and then when $y=3-2x$ intersects with B and forms the inequality $\frac{4}{k-2}<"\frac{9}{k}"$ or similar.
- Finds the value of x when $y=3-2x$ intersects with AB, $y=-kx+7$, $\left(=\frac{4}{k-2}\right)$ and then when $y=3-2x$ intersects $y=kx-11$ $\left(=\frac{14}{k+2}\right)$ and sets equal to each other to find the value for k (or solves simultaneously to find $\Rightarrow x="2.5" \Rightarrow "\frac{9}{k}"<"2.5" \Rightarrow k>..$)
- Finds the gradient between $(0,3)$ and B and deduces that $-2>\frac{"-2"-3}{"\frac{9}{k}"-0}$ oe leading $k=...$
- Sets $f(x)=3-2x$ and attempts the discriminant leading to a value for k:
  $3-2x=|kx-9|-2 \Rightarrow (5-2x)^2=(kx-9)^2 \Rightarrow (4-k^2)x^2+(18k-20)x-56=0$

$b^2 - 4ac = 0 \Rightarrow (18k-20)^2 - 4(4-k^2)\times(-56) = 0 \Rightarrow 100k^2 - 720k + 1296 = 0 \Rightarrow k = ...$
(allow two values for k from a quadratic)
In each method do not be too concerned by their use of $=, \geqslant, \leqslant, >, <$ throughout their working.
Condone arithmetical slips.

A1 $(k =)\ 3.6$ oe

Question Number	Scheme	Marks
6(a)	$(\text{f}'(x)=)\ 5(x^2-2)\times\frac{1}{2}\times 4\times(4x+9)^{-\frac{1}{2}} + 10x(4x+9)^{\frac{1}{2}}$	M1A1
	$(\text{f}'(x)=)\ \dfrac{10(x^2-2)+10x(4x+9)}{(4x+9)^{\frac{1}{2}}} = \dfrac{50x^2+90x-20}{(4x+9)^{\frac{1}{2}}}$	dM1
	$(\text{f}'(x)=)\ \dfrac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$	A1
		(4)
(b)	Sets $\text{f}'(x)=0$ to give $x=-2, \frac{1}{5}$	B1
		(1)
(c)	$y = 5\left(("-2")^2-2\right)\left(4\times("-2")+9\right)^{\frac{1}{2}} = 10$	M1
	$(-2, 10)$	A1cao
		(2)
(d)	$\text{f}(0) = -30$	B1
	Upper bound $= 2\times"10"+4 = ...$ or Lower bound $= 2\times"-30"+4 = ...$	M1
	$-56 \leqslant \text{g}(x) \leqslant 24$	A1
		(3)
		(10 marks)

A1 $k > 3.6$ or eg $k > \frac{18}{5}$ but not $k \geqslant \frac{18}{5}$ If $k < -2$ is found then it must be rejected

(a)

M1 Attempts the product rule. Award for $\pm A(5x^2-10)(4x+9)^{-\frac{1}{2}} \pm Bx(4x+9)^{\frac{1}{2}}$ or equivalent where $A, B \neq 0$ The indices do not need to be processed for this mark.

If they quote the product rule eg $uv' + u'v$ and state the expressions for u, u', v and v' then allow a miscopy when substituted. They must have eg $u=5x^2-10 \rightarrow u'=Ax$ and $v=(4x+9)^{\frac{1}{2}} \rightarrow v'=B(4x+9)^{-\frac{1}{2}}$

May also attempt the product rule on $5x^2(4x+9)^{\frac{1}{2}}-10(4x+9)^{\frac{1}{2}}$ so award for

$\pm Ax(4x+9)^{\frac{1}{2}} \pm Bx^2(4x+9)^{-\frac{1}{2}} \pm C(4x+9)^{-\frac{1}{2}}$ $A, B, C \neq 0$

A1 Correct unsimplified expression for $f'(x)$ eg $2(5x^2-10)(4x+9)^{-\frac{1}{2}}+10x(4x+9)^{\frac{1}{2}}$ oe

dM1 Proceeds to a single fraction, multiplies out the brackets on the numerator and proceeds to the form $\frac{3\text{TQ}}{(4x+9)^{\frac{1}{2}}}$ or $3\text{TQ}\,(4x+9)^{-\frac{1}{2}}$. They cannot proceed straight to the given 3TQ or a multiple of it without some working seen to score this mark.

Alternatively, you may only see manipulation of the numerator and then $\frac{3\text{TQ}}{(4x+9)^{\frac{1}{2}}}$ which is acceptable: $10(x^2-2)+10x(4x+9)=10(x^2-2+4x^2+9x)=10(5x^2+9x-2) \Rightarrow \frac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$

Condone arithmetical slips in their working but their method to proceeding to a single fraction must be correct.

It is dependent on the previous method mark.

A1 $\left(f'(x)=\right)\frac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$ with no errors seen in the main body of their solution.

Accept $\frac{10(5x^2+9x-2)}{\sqrt{4x+9}}$

Condone invisible brackets as long as they are recovered, or their subsequent working makes their intention clear. They do not need to explicitly state $k = 10$ but if there is a contradiction between their expression and their stated value then the expression takes precedence. Do not be concerned with what they have before the =

(b) Mark (b) and (c) together

B1 Sets $f'(x)=0$ to give $-2, \frac{1}{5}$ Withhold if one is rejected. Condone $(-2,0)$ $\left(\frac{1}{5},0\right)$. May be seen in (c)

(c)

M1 Substitutes their smaller root from solving $5x^2+9x-2=0$ into $y=5(x^2-2)(4x+9)^{\frac{1}{2}}$ and proceeds to find a value for y. Condone bracket omissions, miscopying or arithmetical slips as long as it is clear that they are attempting to substitute their smaller root into $f(x)$

A1cao $(-2, 10)$ only (Allow $x=-2,\ y=10$)

(d)

B1 $(\mathrm{f}(0)=)-30$ seen (which may be implied by sight of -56)

M1 A correct attempt at the lower or upper bound.

- Either attempts $2\times"10"+4$ and identifies this as the upper bound. Follow through on their part (c) y coordinate. $<"24"$ or $\leqslant"24"$ is acceptable
- Or attempts $2\times"-30"+4$ and identifies this as their lower bound. $>"-56"$ or $\geqslant"-56"$ is acceptable for this mark. It cannot be scored for using $2\times\mathrm{f}\left(\frac{1}{5}\right)+4$ as their lower bound.

Note that $g>-56$ or $g\geqslant-56$ will score B1M1.
Allow use of max or min for upper and lower bound for this mark.

A1 $-56\leqslant \mathrm{g}(x)\leqslant 24$ oe eg "$\{\mathrm{g}(x)\in\mathbb{R}:-56\leqslant \mathrm{g}(x)\cap \mathrm{g}(x)\leqslant 24\}$" or "$-56\leqslant \mathrm{g}(x)$ AND $\mathrm{g}(x)\leqslant 24$" or $\mathrm{g}\in[-56, 24]$ or other variations similar to these. Condone g for g(x) and $-56\leqslant y\leqslant 24$ or $-56\leqslant 2\mathrm{f}(x)+4\leqslant 24$ but not $-56\leqslant \mathrm{f}\leqslant 24$

DO NOT ACCEPT "$-56\leqslant \mathrm{g}(x)\cup \mathrm{g}(x)\leqslant 24$" or "$-56\leqslant \mathrm{g}(x)$ OR $\mathrm{g}(x)\leqslant 24$"

Question Number	Scheme	Marks
7(a)	$3\cot^2 2\theta - 7 = \dfrac{13}{2\sin\theta\cos\theta}$ **or** $2\sin\theta\left(3(\text{cosec}^2 2\theta - 1) - 7\right) = 13\sec\theta$ **oe**	M1
	$3(\text{cosec}^2 2\theta - 1) - 7 = \dfrac{13}{2\sin\theta\cos\theta}$	dM1
	$\Rightarrow 3\text{cosec}^2 2\theta - 10 = 13\text{cosec}\, 2\theta \Rightarrow 3\text{cosec}^2 2\theta - 13\text{cosec}\, 2\theta - 10 = 0$ *	ddM1A1*
		(4)
(b)	$3\text{cosec}^2 2\theta - 13\text{cosec}\, 2\theta - 10 = 0$	
	$(3\text{cosec}\, 2\theta + 2)(\text{cosec}\, 2\theta - 5) = 0 \Rightarrow (\text{cosec}\, 2\theta) = 5 \Rightarrow (\sin 2\theta) = \dfrac{1}{5}$	M1A1
	$\theta = \dfrac{\sin^{-1}\left("\frac{1}{5}"\right)}{2} = \ldots$	dM1
	$(\theta =)$ awrt 0.101, 1.47	A1
		(4)
		(8 marks)
Alt(a)	$2\sin\theta\left(\dfrac{3(1-\sin^2 2\theta)}{\sin^2 2\theta} - 7\right) = 13\sec\theta$	M1
	$\dfrac{3(1-\sin^2 2\theta)}{\sin^2 2\theta} - 7 = \dfrac{13}{2\sin\theta\cos\theta}$	dM1
	$\dfrac{3}{\sin^2 2\theta} - 10 = \dfrac{13}{\sin 2\theta}$	ddM1
	$3\text{cosec}^2 2\theta - 13\text{cosec}\, 2\theta - 10 = 0$ *	A1*

(a)

M1 Attempts **EITHER**

- to divide both sides by $(2)\sin\theta$ and use $\sec\theta = \dfrac{1}{\cos\theta}$ (or $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$) to achieve a term of the form $\dfrac{A}{B\sin\theta\cos\theta}$ oe (implied by $\dfrac{\ldots}{\ldots\sin 2\theta}$ or $\ldots\text{cosec}\,\theta\sec\theta$).
  Note $\sin 2\theta(3\cot^2 2\theta - 7) = 13$ does not score this mark. They have not divided by $(2)\sin\theta$

- to use $\pm 1 \pm \cot^2 2\theta = \pm\text{cosec}^2 2\theta$ to replace the $\cot^2 2\theta$ term in the equation or in the alternative method it is for using $\cot^2 2\theta = \dfrac{\cos^2 2\theta}{\sin^2 2\theta}$ and using $\pm 1 \pm \sin^2 2\theta = \pm\cos^2 2\theta$ to replace the $\cos^2 2\theta$

Condone arithmetical slips and invisible brackets.

dM1 Attempts **BOTH**

- to divide both sides by $(2)\sin\theta$ and use $\sec\theta = \dfrac{1}{\cos\theta}$ (or $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$) to achieve a term of the form $\dfrac{A}{B\sin\theta\cos\theta}$ oe (implied by $\dfrac{...}{...\sin 2\theta}$ or $...\operatorname{cosec}\theta\sec\theta$). May be implied
- to use $\pm 1 \pm \cot^2 2\theta = \pm\operatorname{cosec}^2 2\theta$ to replace the $\cot^2 2\theta$ term in the equation or in the alternative method it is for using $\cot^2 2\theta = \dfrac{\cos^2 2\theta}{\sin^2 2\theta}$ and using $\pm 1 \pm \sin^2 2\theta = \pm\cos^2 2\theta$ to replace the $\cos^2 2\theta$

Condone arithmetical slips and invisible brackets. It is dependent on the first mark.

ddM1 A full attempt to achieve a 3TQ in $\operatorname{cosec} 2\theta$ only. It is dependent on both of the previous method marks. They must

- use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ (or $\operatorname{cosec} 2\theta = \dfrac{1}{2}\operatorname{cosec}\theta\sec\theta$) and proceed to a $\operatorname{cosec} 2\theta$ term
- multiply out the brackets and collect terms on one side to form a 3TQ in $\operatorname{cosec} 2\theta$ only.

Do not be concerned with the mechanics of their arrangement and they may proceed to the printed answer at this stage. Condone arithmetical slips and invisible brackets.

A1* Achieves $3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$ with no arithmetical errors in the main body of their solution. You must see either $\sin\theta\cos\theta$ or $\sin 2\theta$ within their working.
Allow recovery of invisible brackets in their solution.
Withhold the final mark for poor notation such as $\operatorname{cosec}^2 2\theta$ being written as $\operatorname{cosec} 2\theta^2$ or if the 2 is missing from 2θ
They may work in a different variable, but the final answer must be in terms of 2θ

(b) *Be aware that answers with no intermediate working seen is not acceptable*

M1 Attempts to solve the quadratic and finds the reciprocal of one of their roots (usual rules apply for solving a quadratic). Do not be concerned by their labelling.

Look for eg $.... = 5 \to \dfrac{1}{5}$ or eg $.... = -\dfrac{3}{2} \to -\dfrac{2}{3}$

A1 $(\sin 2\theta =)\ \dfrac{1}{5}$ (Ignore any reference to $-\dfrac{3}{2}$) $\dfrac{1}{5}$ on its own is sufficient. Condone eg $\sin^2 2\theta = \dfrac{1}{5}$ or $\sin X = \dfrac{1}{5}$ or $\sin\theta = \dfrac{1}{5}$

dM1 Correct method to find a value for θ by finding arcsin(their $\dfrac{1}{5}$) and dividing by 2.

May be implied by awrt 0.10 or awrt 1.5 (or in degrees awrt 5.8 or awrt 84).
It is dependent on seeing either the quadratic solved or their root from the quadratic (either $\geqslant 1$ or $\leqslant -1$) or the reciprocal of the root being stated first.
It cannot be scored for solving eg $\sin^2 2\theta = ...$

A1 $(\theta =)$ awrt 0.101, awrt 1.47 and no other angles within the given interval.
Some intermediate working must be shown. Examples of minimum working shown in (b):

$\sin 2\theta = \frac{1}{5} \Rightarrow (\theta =)\ 0.101, 1.47$

$\text{cosec } 2\theta = 5 \Rightarrow (\theta =)\ 0.101, 1.47$

Question Number	Scheme	Marks
8(a)	$\frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t} = 0 \Rightarrow 9 = e^{1.75t-10}$ or $9e^{-0.75t} = e^{t-10}$	M1A1
	$9 = e^{1.75t-10} \Rightarrow t = \frac{10+\ln 9}{1.75}$ or $\ln 9 - 0.75t = t - 10 \Rightarrow t = \frac{10+\ln 9}{1.75}$	M1
	$t =$ awrt 6.97	A1
	awrt 11.9 (ms^{-1})	A1
		(5)
(b)	$\int 12 - 12e^{-0.75t} - e^{(t-10)}\, dt = 12t - e^{(t-10)} + 16e^{-0.75t}\ (+C)$	M1A1
	$\left[12t - e^{(t-10)} + 16e^{-0.75t}\right]_0^T = 100 \Rightarrow 12T = ...$	M1
	$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$ *	A1*
		(4)
(c) (i)	$T_2 = \frac{1}{12}\left(116 - 16e^{-0.75\times 10} + e^{10-10} - e^{-10}\right)$	M1
	$T_2 =$ awrt 9.7493	A1
(ii)	9.7293 (seconds)	A1
		(3)
		(12 marks)

(a) *** Be aware this can be solved directly on a calculator. Calculus must be seen***

M1 Differentiates to $\pm Ae^{t-10} \pm Be^{-0.75t}$ and **sets equal to 0** Must be seen in (a) Do not accept differentiating to eg $...te^{t-10}$. Condone an expression of $12 \pm Ae^{t-10} \pm Be^{-0.75t}$ only

A1 $9 = e^{1.75t-10}$ or equivalent eg $9e^{-0.75t} - e^{t-10} = 0$

M1 Proceeds from their equation of the form $Ae^{Ct\pm D} - Be^{Et\pm F} = 0$ or equivalent leading to a value or an expression (which may be unsimplified) for t. If log work is seen it must be correct.

Eg $\frac{40+4\ln 9}{7}$ or $\frac{10+\ln 9}{1.75}$ will score this mark

$9e^{-0.75t} - e^{t-10} = 0 \Rightarrow$ awrt 6.97 can also score this mark

This cannot be scored from an unsolvable equation.

A1 awrt 6.97 or awrt 6.96 including exact unsimplified expressions such as $\frac{40+4\ln 9}{7}$ or $\frac{10+\ln 9}{1.75}$.

A1 awrt 11.9 (ms^{-1}) with all previous marks scored. Condone lack of units

Note: awrt 6.97 and awrt 11.9 with no calculus scores 0 marks

(b)

M1 Attempts to integrate the expression for v. Award for $12t \pm Ae^{(t-10)} \pm Be^{-0.75t}$.

Do not accept eg $12t \pm Ae^{(t-10)+1} \pm Be^{-0.75t+1}$

Allow if their attempt to integrate appears in part (a) as long as it is a clear attempt at integration. Limits may have been substituted in.

A1 $12t - e^{(t-10)} + 16e^{-0.75t}$ (with or without $+C$) and allow unsimplified eg $12t - e^{(t-10)} - \frac{12}{-0.75}e^{-0.75t}$

Do not withhold this mark if they write $12x$ but subsequently replace it with T later on.

M1 Substitutes T and 0 into their changed expression with three different terms in t, sets equal to 100 and attempts to make $12T$ or T the subject.

Eg $\left(12T - e^{T-10} + 16e^{-0.75T}\right) - \left(-e^{-10} + 16\right) = 100 \Rightarrow 12T = ...$

Condone arithmetical slips in their rearrangement.

A1* $T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$ with no errors including bracket errors/omissions.

Must have $T = \ldots$

(c)

(i)

M1 Substitutes 10 into the iterative formula. The expression with 10 embedded is sufficient to score this mark.

May be implied by awrt 9.7492 or awrt 9.7493. May also be implied by 9.7306

A1 awrt 9.7493

(ii) ***Be aware that this can be found directly on a calculator which is not acceptable***

A1 9.7293 (seconds) (cannot be awarded without the method mark)

SC If they over round in (i) and (ii) and achieve 9.749 and 9.729 then award M1A0A1

Question Number	Scheme	Marks
9(a)	$\frac{dy}{dx}=\frac{(1+\sin x)(-2\sin x)-(1+2\cos x)\cos x}{(1+\sin x)^2}$	M1A1
	$(1+\sin x)(-2\sin x)-(1+2\cos x)\cos x=0$ $\Rightarrow -2\sin x-\cos x-2=0$	dM1
	$2\sin x+\cos x=-2$ *	A1*
		(4)
(b)		
	Possible solutions from $R\sin(x+\alpha)$ or $R\cos(x-\alpha)$: $2\sin x+\cos x=-2\Rightarrow\sqrt{5}\sin(x+0.464)=-2$ or $\sqrt{5}\cos(x-1.107)=-2$ or eg solutions from squaring $(2\sin x+\cos x)^2=4\Rightarrow\cos x(3\cos x-4\sin x)=0$	M1M1A1
	-- $\sqrt{5}\sin(x+0.464)=-2$ or $\sqrt{5}\cos(x-1.107)=-2\Rightarrow x=...$ or $\tan x=\frac{3}{4}\Rightarrow x=...$	dM1
	$x=$ awrt 3.79	A1
		(5)
		(9 marks)

(a)

M1 Attempts to differentiate by the product rule or quotient rule and proceeds to:

Quotient rule: $\frac{\pm A(1+\sin x)(\sin x)\pm(1+2\cos x)\cos x}{....}$ (Scored for the numerator only)

Product rule: $\pm\cos x(1+2\cos x)(1+\sin x)^{-2}\pm A\sin x(1+\sin x)^{-1}$

Alternatively, they express $\frac{1+2\cos x}{1+\sin x}=\frac{1}{1+\sin x}+\frac{2\cos x}{1+\sin x}$ and proceed to either

Quotient rule: $\pm\frac{\cos x}{(1+\sin x)^2}\pm\frac{A\sin x(1+\sin x)\pm\cos x(2\cos x)}{(1+\sin x)^2}$

Product rule: $\pm\cos x(1+\sin x)^{-2}\pm A\cos^2 x(1+\sin x)^{-2}\pm B\sin x(1+\sin x)^{-1}$

Condone arithmetical slips and invisible brackets.

A1 $\left(\frac{dy}{dx}=\right)\frac{(1+\sin x)(-2\sin x)-(1+2\cos x)\cos x}{(1+\sin x)^2}$ or equivalent (which may be unsimplified)

If is is via the product rule then it may appear as
$\left(\frac{dy}{dx}=\right)-\cos x(1+2\cos x)(1+\sin x)^{-2}-2\sin x(1+\sin x)^{-1}$ or equivalent (which may be unsimplified).

dM1 Sets the numerator of their quotient equal to zero, multiplies out the brackets and attempts to use the identity $\pm\sin^2 x\pm\cos^2 x=\pm1$ (which may be implied by their working) leading to an equation in $\sin x$ and $\cos x$ only. There must be no squared terms.
If it is via the product rule they will need to either combine their terms into a single fraction and then set the numerator equal to zero (or multiply both sides by $(1+\sin x)^2$), and then attempt to use the identity $\pm\sin^2 x\pm\cos^2 x=\pm1$ leading to an equation in $\sin x$ and $\cos x$ only. There must be no squared terms.
Condone slips and invisible brackets. It is dependent on the first method mark.

A1* Achieves $2\sin x+\cos x=-2$ with no errors or missing brackets around $1+2\cos x$ or $1+\sin x$ when needed in the main body of their solution.
(Do not penalise the fraction line not being quite long enough for the numerator.)
It is acceptable to proceed to a correct $\frac{dy}{dx}$, multiply out the brackets and proceed directly from $-2\sin x-2\sin^2 x-\cos x-2\cos^2 x=0\Rightarrow 2\sin x+\cos x=-2$ to score full marks. You do not need to explicitly see $\sin^2 x+\cos^2 x=1$ stated or substituted in.

Note if they only ever use the numerator of the quotient formula: $vu'-uv'$ from the beginning
Maximum score is M1A0dM1A0*

(b) *Be aware this can be solved entirely using a calculator which is not acceptable*
Solution by expressing $2\sin x+\cos x$ in the form $R\sin(x\pm\alpha)$ or $R\cos(x\pm\alpha)$

M1 $R=\sqrt{2^2+1^2}=\sqrt{5}$

M1 $(\alpha=)\tan^{-1}\left(\pm\frac{1}{2}\right)$ or $(\alpha=)\tan^{-1}(\pm2)$ (or $\cos^{-1}\left(\pm\frac{2}{\sqrt{5}}\right)$ $\sin^{-1}\left(\pm\frac{1}{\sqrt{5}}\right)$)

A1 $"\sqrt{5}"\sin(x+"0.464")=-2$ or $"\sqrt{5}"\cos(x-"1.107")=-2$ oe (or in another variable)

dM1 Attempts to solve their equation and proceeds to find a value for x.
eg $x=\left(\pi-\sin^{-1}\left(\frac{-2}{"\sqrt{5}"}\right)\right)\pm"0.464"$ or $x=\sin^{-1}\left(\frac{-2}{"\sqrt{5}"}\right)\pm"0.464"$
You may need to check their angle on your calculator. Condone slips in their rearrangement.
It is dependent on both of the previous method marks.

A1 awrt 3.79 only

Solutions squaring both sides and using the identity $\sin^2 x+\cos^2 x\equiv1$
M1 Attempts to square both sides and multiplies out the brackets. Condone sign slips only on the brackets they are multiplying out. Do not accept just squaring all the individual terms.

M1 Attempts to use $\pm\sin^2 x\pm\cos^2 x=\pm1$ and proceeds to a quadratic equation in either $\sin x$ or $\cos x$ or an equation of the form $\pm A\cos^2 x\pm B\cos x\sin x=0$.
The use of the identity may be done before squaring both sides. Eg look for
$2\sin x+2=\cos x\Rightarrow 2\sin x+2=\sqrt{1-\sin^2 x}\Rightarrow\ldots$

Condone sign slips only.

A1 $3\cos^2 x - 4\sin x\cos x = 0$ oe **or** $5\sin^2 x + 8\sin x + 3 = 0$ oe **or** $5\cos^2 x + 4\cos x = 0$ oe
The = 0 may be implied by later work and terms do not need to be collected on one side.

dM1 Attempts to solve their quadratic equation in either $\sin x$ or $\cos x$ or an equation of the form $\pm A\cos^2 x \pm B\cos x\sin x = 0$ and proceeds to find a value for *x*. Do not allow for solving $\cos x = 0$
Usual rules apply for solving a quadratic. They may even just state the roots from their calculator.
It is dependent on both of the previous method marks.

A1 awrt 3.79 only
The table below may help with following through a particular approach:

	$(2\sin x + \cos x)^2 = 4$	$(2\sin x + 2)^2 = \cos^2 x$	$(\cos x + 2)^2 = 4\sin^2 x$
M1	$4\sin^2 x + 4\sin x\cos x + \cos^2 x = 4$	$4\sin^2 x + 8\sin x + 4 = \cos^2 x$	$\cos^2 x + 4\cos x + 4 = 4\sin^2 x$
M1	Attempts to use $\pm\sin^2 x \pm \cos^2 x = \pm 1$		
A1	$3\cos^2 x - 4\sin x\cos x = 0$ oe	$5\sin^2 x + 8\sin x + 3 = 0$ oe	$5\cos^2 x + 4\cos x = 0$ oe
dM1	Eg $\cos x(3\cos x - 4\sin x) = 0 \Rightarrow x = ...$	Eg $(5\sin x + 3)(\sin x + 1) = 0 \Rightarrow x = ...$	Eg $\cos x(5\cos x + 4) = 0 \Rightarrow x = ...$
A1	awrt 3.79		

It is possible that they may divide the equation by either $\sin x$ or $\cos x$ and use $1 + \tan^2 x = \sec^2 x$ or $1 + \cot^2 x = \text{cosec}^2 x$ resulting in the following equations (or equivalent simplified rearrangements of these) to solve:

- $3\cot^2 x = 4\cot x$
- $\cot x(5\cot x + 4\cot x\,\text{cosec}\,x) = 0$
- $3\,\text{cosec}^2 x + 8\,\text{cosec}\,x + 5 = 0$
- $4\tan x = 3$
- $4\sec x + 5 = 0$

The mark scheme should be applied in the same way but looking for use of $\pm 1 \pm \tan^2 x = \pm\sec^2 x$ or $\pm 1 \pm \cot^2 x = \pm\text{cosec}^2 x$ and proceeding to equations involving cot *x*, cosec *x*, tan *x* or sec *x*
Send to review if unsure.

Solutions using the *t* method substitution

Using $t = \tan\left(\frac{x}{2}\right) \Rightarrow \sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

M1 Attempts to express $2\sin x + \cos x = -2$ in terms of *t*. Attempts to use $\sin x = \frac{\pm 2t}{\pm 1 \pm t^2}$ and $\cos x = \frac{\pm 1 \pm t^2}{\pm 1 \pm t^2}$.

M1 Proceeds from an equation of the form $2\left(\frac{2t}{1+t^2}\right) + \frac{1-t^2}{1+t^2} = -2$ to a 3TQ

$$\frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2} = -2 \Rightarrow 4t + 1 - t^2 = -2(1+t^2) \Rightarrow ...$$

A1 $t^2 + 4t + 3 = 0$ or equivalent. The = 0 may be implied by later work and the three terms do not need to be on the same side of the equation.

dM1 Correctly rearranges their equation to find a value for x
$t^2 + 4t + 3 = 0 \Rightarrow t = -3 \Rightarrow x = 2\left(\arctan("-3") + \pi\right)$ Ignore any reference to $t = -1$
It is dependent on both of the previous method marks.

A1 awrt 3.79

There may be other variants of these methods, but the mark scheme should still be able to be applied. If in doubt send to review.

Answer only scores 0 marks