Pearson Edexcel International A Level Mathematics

Pure Mathematics 2

Past Paper Collection (from 2020)

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Last updated: July 1, 2024

Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Tuesday 18 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
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Turn over ▶

Leave blank

Answer all questions. Write your answers in the spaces provided.

1. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 4 - a_n$$
$$a_1 = 3$$

Find the value of

(a) (i) a_2

(ii) a_{107}

(2)

(b) $\sum_{n=1}^{200}(2a_n - 1)$

(2)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The point $P(5, 4)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geqslant 2x - 9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

Leave blank

Question 3 continued

Q3

(Total 4 marks)

Leave blank

4. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

(3)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

(a) According to model *A*, determine Kim's annual salary in year 2.

(3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

(b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10

(3)

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9. (a) Show that the equation

$$\cos\theta - 1 = 4\sin\theta\tan\theta$$

can be written in the form

$$5\cos^2\theta - \cos\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < \frac{\pi}{2}$

$$\cos 2x - 1 = 4\sin 2x\tan 2x$$

giving your answers, where appropriate, to 2 decimal places.

(4)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 8 marks)

Leave blank

10.

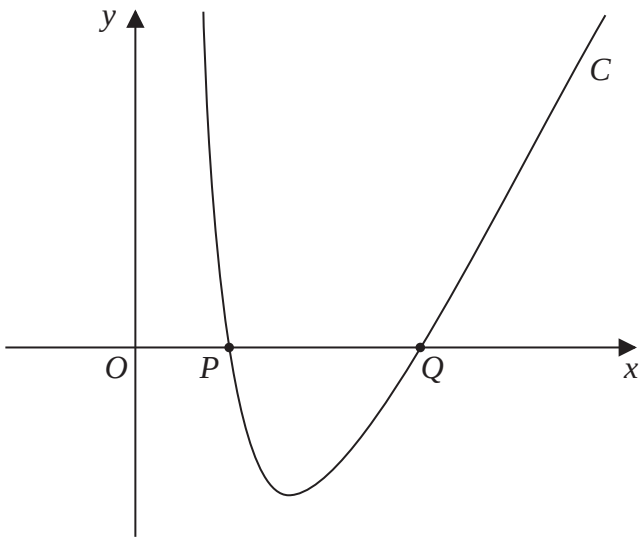

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \qquad x > 0$$

Using calculus,

(a) find the range of values of x for which $f(x)$ is increasing,

(4)

(b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13\right) dx = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C.

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13\right) dx = -8$

(c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13\right) dx$

(ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k\right) dx = 0$

(3)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Wednesday 6 November 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

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Turn over ▶

Leave blank

1. A curve C has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1+\frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{x}{4}\right)^{12}$$

(3)

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. $f(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $f(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $f(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $f(x)$.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(3)

Using your answer to part (b) and making your method clear, estimate

(c) $\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6.

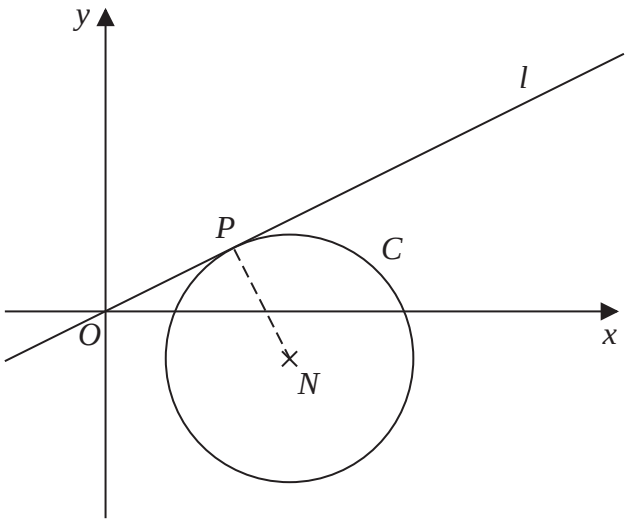

Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Leave blank

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$

(2)

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$

(2)

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$

(3)

Leave blank

Question 7 continued

Q7

(Total 7 marks)

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8. Solutions relying on calculator technology are not acceptable in this question.

(i)

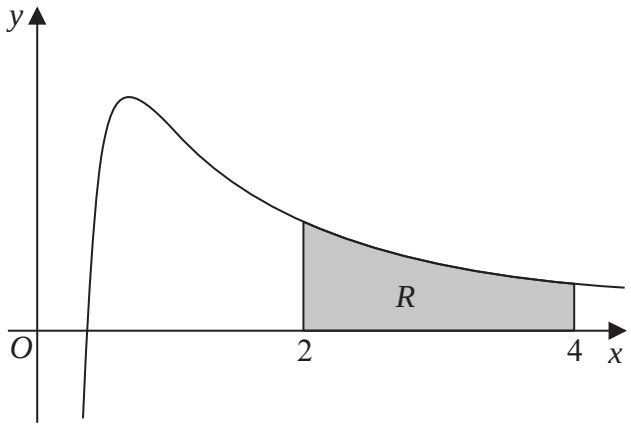

Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) \mathrm{d}x = 55$$

(4)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 12 marks)

Leave blank

10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Wednesday 15 January 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

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Turn over ▶

Leave blank

1. The table below shows corresponding values of x and y for $y = \log_2(2x)$

The values of y are given to 2 decimal places as appropriate.

x	2	5	8	11	14
y	2	3.32	4	4.46	4.81

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for $\int_2^{14} \log_2(2x)\,dx$, giving your answer to one decimal place.

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_2^{14} \frac{\log_2(4x^2)}{5}\,dx$

(ii) $\int_2^{14} \log_2\left(\frac{2}{x}\right)dx$

(4)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. One of the terms in the binomial expansion of $(3 + ax)^6$, where a is a constant, is $540x^4$

(a) Find the possible values of a.

(4)

(b) Hence find the term independent of x in the expansion of

$$\left(\frac{1}{81} + \frac{1}{x^6}\right)(3 + ax)^6$$

(3)

Leave blank

Question 2 continued

(Total 7 marks)

Q2

Leave blank

3. $f(x) = 6x^3 + 17x^2 + 4x - 12$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4.

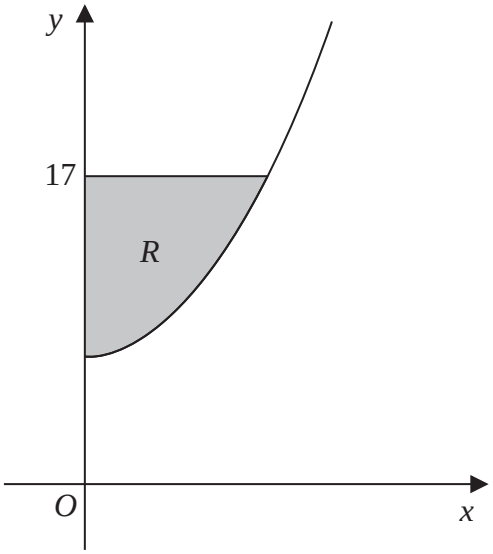

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = 2x^2 + 7 \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the line with equation $y = 17$

Find the exact area of R.

(6)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(5)

Leave blank

Question 5 continued

(Total 8 marks)

Q5

Leave blank

6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. (a) Show that the equation

$$8\tan\theta = 3\cos\theta$$

may be rewritten in the form

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant x \leqslant 90°$, the equation

$$8\tan 2x = 3\cos 2x$$

giving your answers to 2 decimal places.

(4)

Leave blank

Question 7 continued

(Total 7 marks)

Q7

Leave blank

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 7 marks)

Leave blank

9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

(b) Show that the x coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 7 marks)

Leave blank

10. A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C.

Given that P is a stationary point of C,

(a) show that $k = -\frac{3}{2}$

(4)

(b) Determine the nature of the stationary point at P, justifying your answer.

(2)

The curve C has a second stationary point.

(c) Using algebra, find the x coordinate of this second stationary point.

(4)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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International Advanced Level

Centre Number | Candidate Number

Wednesday 21 October 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

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Turn over ▶

Leave blank

1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Hence find the constant term in the series expansion of

$$\left(3 - \frac{1}{x}\right)^2 \left(2 - \frac{x}{4}\right)^{10}$$

(3)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2.

$$y = \frac{2^x}{\sqrt{(5x^2 + 3)}}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	−0.25	0	0.25	0.5	0.75
y	0.462		0.653		0.698

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_{-0.25}^{0.75} \frac{2^x}{\sqrt{(5x^2 + 3)}} \, \mathrm{d}x$$

(3)

Leave blank

Question 2 continued

(Total 4 marks)

Q2

Leave blank

3. $f(x) = ax^3 - x^2 + bx + 4$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

(Total 10 marks)

Q3

Leave blank

4. The points P and Q have coordinates $(-11, 6)$ and $(-3, 12)$ respectively.

Given that PQ is a diameter of the circle C,

(a) (i) find the coordinates of the centre of C,

(ii) find the radius of C.

(4)

(b) Hence find an equation of C.

(2)

(c) Find an equation of the tangent to C at the point Q giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

(Total 9 marks)

Q4

Leave blank

5. Ben is saving for the deposit for a house over a period of 60 months.

Ben saves £100 in the first month and in each subsequent month, he saves £5 more than the previous month, so that he saves £105 in the second month, £110 in the third month, and so on, forming an arithmetic sequence.

(a) Find the amount Ben saves in the 40th month.

(2)

(b) Find the total amount Ben saves over the 60-month period.

(3)

Lina is also saving for a deposit for a house.

Lina saves £600 in the first month and in each subsequent month, she saves £10 less than the previous month, so that she saves £590 in the second month, £580 in the third month, and so on, forming an arithmetic sequence.

Given that, after n months, Lina will have saved exactly £18 200 for her deposit,

(c) form an equation in n and show that it can be written as

$$n^2 - 121n + 3640 = 0$$

(3)

(d) Solve the equation in part (c).

(2)

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible value for n.

(1)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 11 marks)

Q5

Leave blank

6.

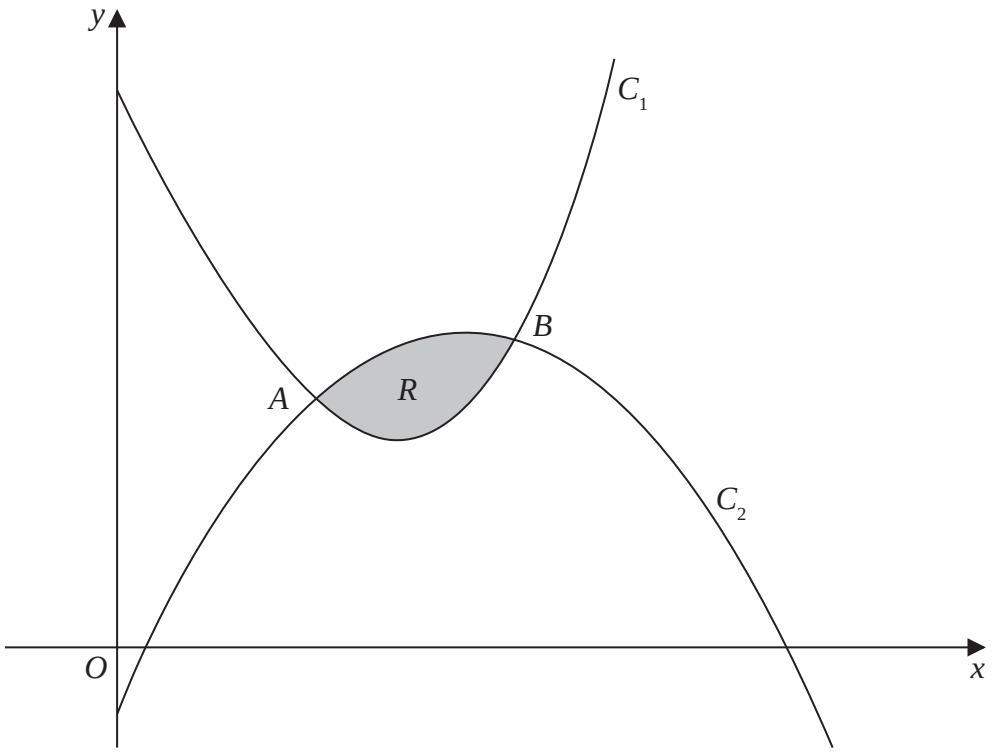

Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1 : y = x^3 - 6x + 9 \qquad x \geqslant 0$$
$$C_2 : y = -2x^2 + 7x - 1 \qquad x \geqslant 0$$

The curves C_1 and C_2 intersect at the points A and B as shown in Figure 1.

The point A has coordinates (1, 4).

Using algebra and showing all steps of your working,

(a) find the coordinates of the point B.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7. (i) Show that

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta} \qquad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(ii) Solve, for $0 \leqslant x < 90°$, the equation

$$3\cos^2(2x + 10°) = 1$$

giving your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Leave blank

Question 7 continued

(Total 7 marks)

Q7

Leave blank

8. A geometric series has first term a and common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(3)

The second term of a geometric series is –320 and the fifth term is $\frac{512}{25}$

(b) Find the value of the common ratio.

(2)

(c) Hence find the sum of the first 13 terms of the series, giving your answer to 2 decimal places.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 8 marks)

Leave blank

9. (i) Find the exact value of x for which

$$\log_3(x + 5) - 4 = \log_3(2x - 1)$$

(4)

(ii) Given that

$$3^{y+3} \times 2^{1-2y} = 108$$

(a) show that

$$0.75^{y} = 2$$

(4)

(b) Hence find the value of y, giving your answer to 3 decimal places.

(2)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Monday 18 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Leave blank

1. $f(x) = x^4 + ax^3 - 3x^2 + bx + 5$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

(b) find the possible values of p.

(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. (i) Use algebra to prove that for all $x \geqslant 0$

$$3x + 1 \geqslant 2\sqrt{3x}$$

(3)

(ii) Show that the following statement is not true.

"The sum of three consecutive prime numbers is always a multiple of 5"

(1)

Leave blank

Question 5 continued

Q5

(Total 4 marks)

Leave blank

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7.

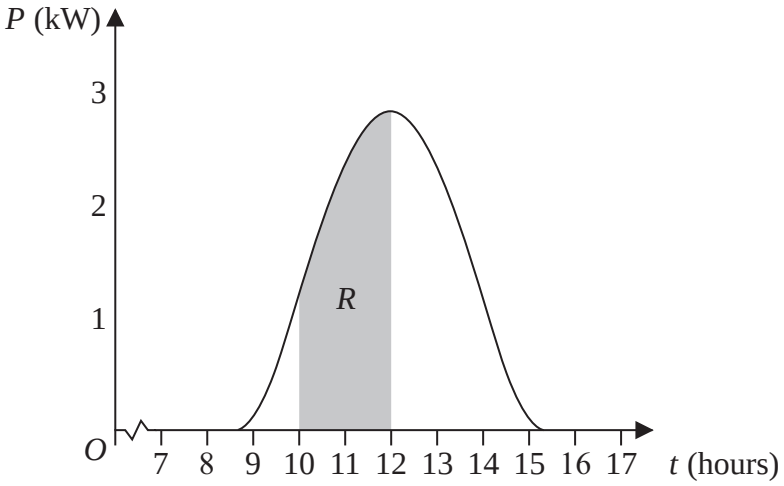

Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

Leave blank

Question 7 continued

Q7

(Total 6 marks)

Leave blank

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 7 marks)

Leave blank

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

(Total 10 marks)

Q9

Leave blank

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan *A*, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan *A*.

(3)

In training plan *B*, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan *B*.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan *A*,

(ii) find the maximum value of x, if he uses training plan *B*.

(5)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

Leave blank

1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

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Question 1 continued

Q1

(Total 7 marks)

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2.

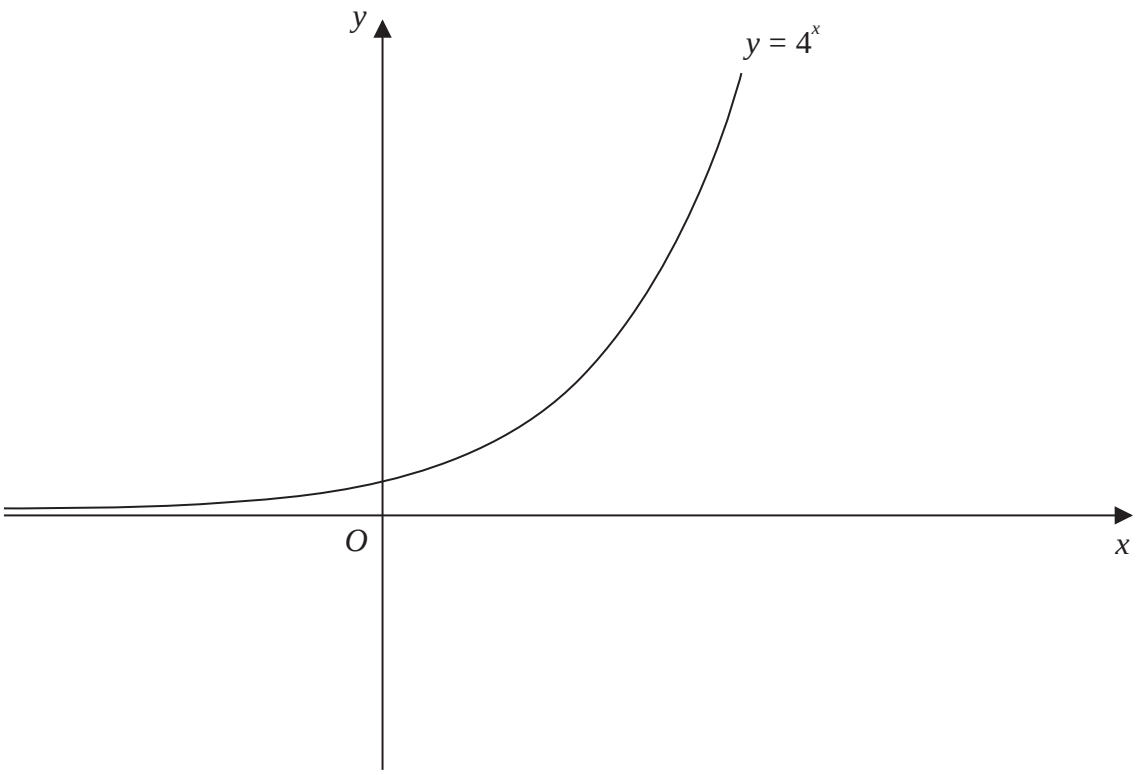

Figure 1

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P.

(b) Using algebra, find the exact coordinates of P.

(4)

Leave blank

Question 2 continued

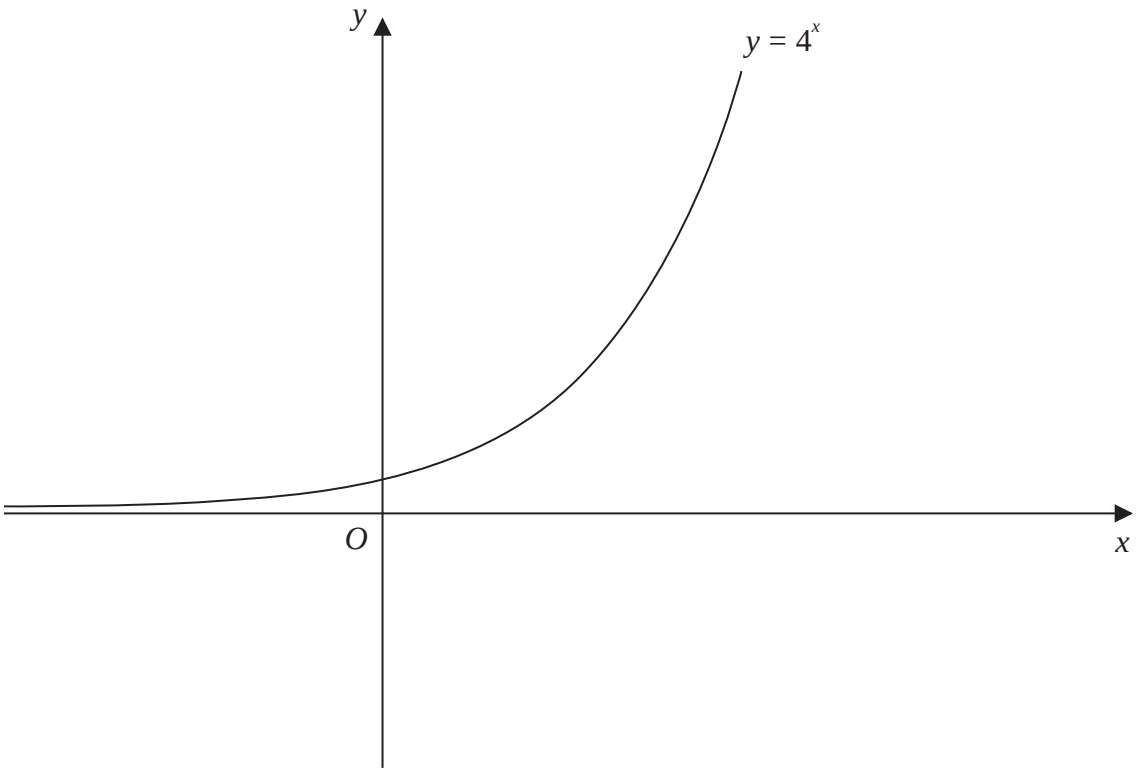

Diagram 1

Q2

(Total 8 marks)

Leave blank

3. (i) Prove that for all single digit prime numbers, p,

$$p^3 + p \text{ is a multiple of } 10$$

(2)

(ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n + 1)^3 - n^3 \text{ is not a multiple of } 3$$

(3)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 5 marks)

Leave blank

4. (a) Find, in ascending powers of x, up to and including the term in x^3, the binomial expansion of

$$\left(2+\frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

(b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

(c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

Leave blank

Question 4 continued

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Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

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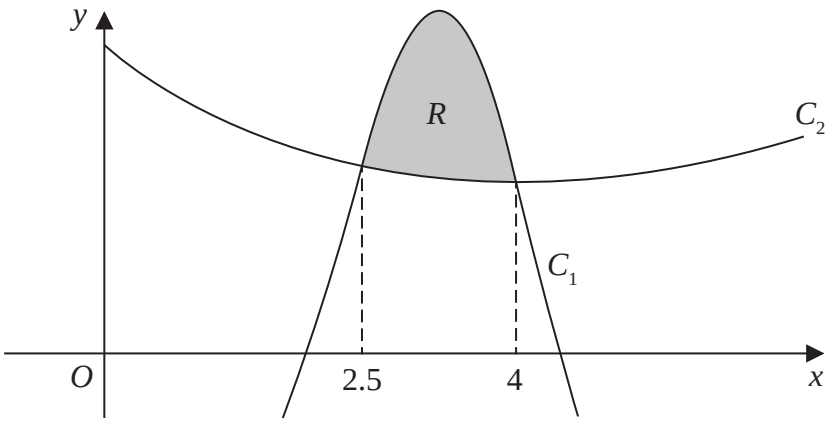

Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\int \left(x^{\frac{3}{2}} - 3x + 9 \right) \mathrm{d}x$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

(Total 10 marks)

Q5

Leave blank

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Leave blank

7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 10 marks)

Leave blank

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

Leave blank

Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 10 marks)

Q8

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9.

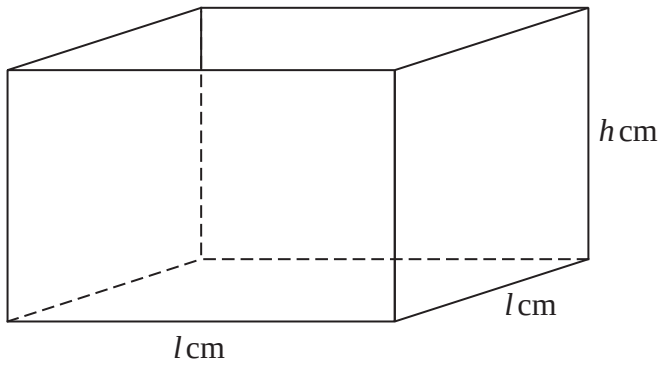

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is $250\,000\,\text{cm}^3$

(a) show that the external surface area, $S\,\text{cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer the questions in the spaces provided
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Leave blank

1. The first three terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^{16}$ are

$$1, \quad -4x \quad \text{and} \quad px^2$$

where k and p are constants.

(a) Find, in simplest form,

(i) the value of k

(ii) the value of p

(3)

$$g(x) = \left(2 + \frac{16}{x}\right)(1 + kx)^{16}$$

Using the value of k found in part (a),

(b) find the term in x^2 in the expansion of $g(x)$.

(3)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where k is a positive constant.

(a) Find, in terms of k, an expression for u_3

(2)

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of k.

(3)

Leave blank

Question 2 continued

Q2

(Total 5 marks)

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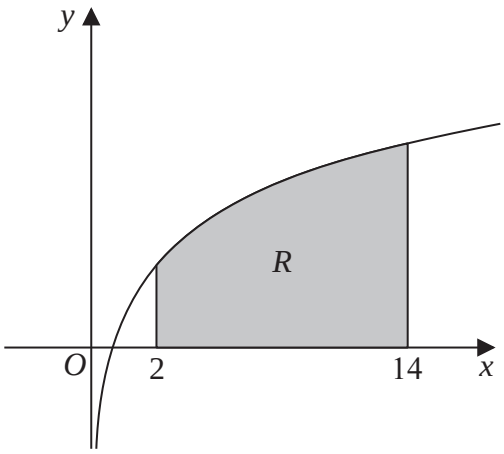

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \log_{10} x$

The region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 14$

Using the trapezium rule with four strips of equal width,

(a) show that the area of R is approximately 10.10

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of R.

(1)

(c) Using the answer to part (a) and making your method clear, estimate the value of

(i) $\displaystyle\int_2^{14} \log_{10} \sqrt{x}\, \mathrm{d}x$

(ii) $\displaystyle\int_2^{14} \log_{10} 100x^3\, \mathrm{d}x$

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. $f(x) = (x^2 - 2)(2x - 3) - 21$

(a) State the value of the remainder when $f(x)$ is divided by $(2x - 3)$

(1)

(b) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(c) Hence,

(i) factorise $f(x)$

(ii) show that the equation $f(x) = 0$ has only one real root.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N,

(b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

(c) Hence, state the value of N.

(1)

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Leave blank

6. (i) The circle C_1 has equation

$$x^2 + y^2 + 10x - 12y = k \qquad \text{where } k \text{ is a constant}$$

(a) Find the coordinates of the centre of C_1

(2)

(b) State the possible range in values for k.

(2)

(ii) The point $P(p, 0)$, the point $Q(-2, 10)$ and the point $R(8, -14)$ lie on a different circle, C_2

Given that

- p is a positive constant
- QR is a diameter of C_2

find the exact value of p.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100}, find the minimum possible value of n.

(3)

(ii) A different geometric sequence has first term a and common ratio r.

Given that

- the second term of the sequence is -6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

(3)

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

(1)

Hence,

(c) state the value of r, giving a reason for your answer,

(1)

(d) find the sum of the first 4 terms of the series.

(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 10 marks)

Q7

Leave blank

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

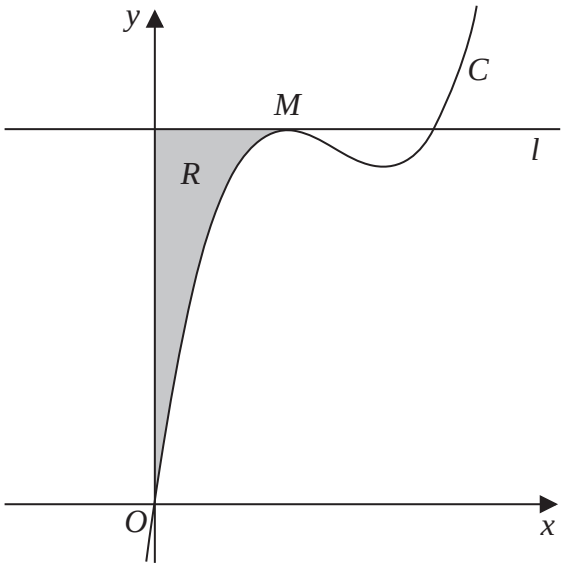

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$

(3)

(b) Determine the range of values of x for which y is increasing.

(2)

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

(c) Find, by algebraic integration, the exact area of R.

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

Leave blank

9. (a) Prove that for all positive values of x and y,

$$\frac{x+y}{2} \geqslant \sqrt{xy}$$

(3)

(b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)

Leave blank

Question 9 continued

Q9

(Total 4 marks)

Leave blank

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^2\left(2x + \frac{\pi}{4}\right) = 3$$

(5)

(ii) Solve, for $0 < \theta < 360°$

$$(2\sin\theta - \cos\theta)^2 = 1$$

giving your answers, as appropriate, to one decimal place.

(5)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Leave blank

1. The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

(i) $\displaystyle\int_5^7 2^{6-\sqrt{x}}\,\mathrm{d}x$

(ii) $\displaystyle\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)\mathrm{d}x$

(4)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(2)

(b) Hence find the coordinates of the stationary point of C.

(4)

(c) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point of C.

(2)

Leave blank

Question 2 continued

(Total 8 marks)

Q2

Leave blank

3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form.

(4)

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

Leave blank

Question 4 continued

Q4

(Total 5 marks)

Leave blank

5. $f(x) = 3x^3 + Ax^2 + Bx - 10$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)g(x)$$

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6.

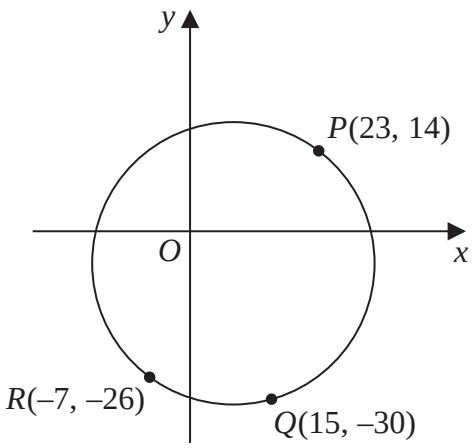

Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Leave blank

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9.

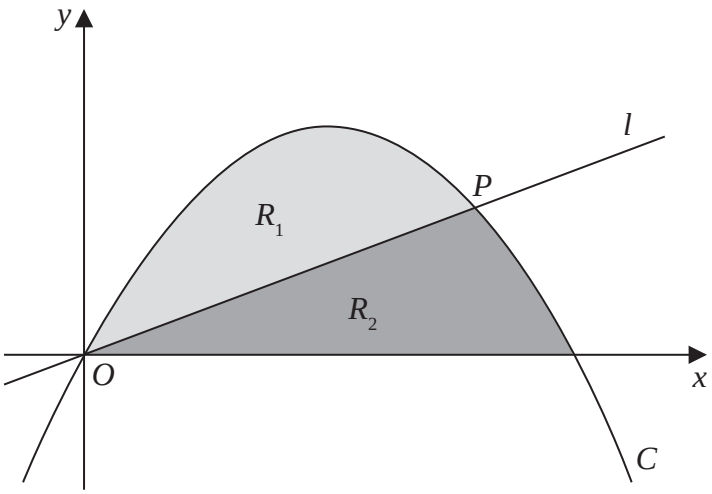

Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

(Total 10 marks)

Q9

Leave blank

10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 5 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
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Information

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- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

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- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Leave blank

1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2.

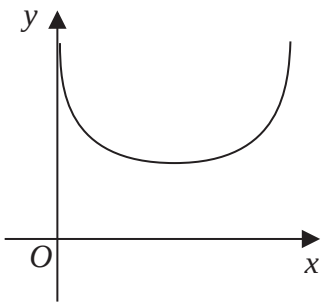

Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319		1.001		1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) \mathrm{d}x$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) \mathrm{d}x$$

(3)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. (i) Show that the following statement is **false**:

"$(n + 1)^3 - n^3$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Leave blank

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. $f(x) = Ax^3 + 6x^2 - 4x + B$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

Leave blank

Question 7 continued

Q7

(Total 7 marks)

Leave blank

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

Leave blank

Question 8 continued

Q8

(Total 8 marks)

Leave blank

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

Leave blank

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

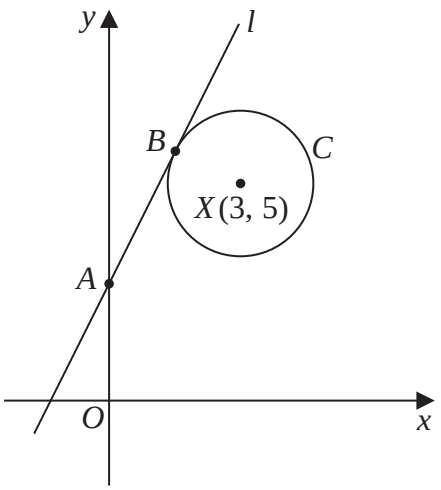

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
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Information

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- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
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Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. Given that a, b and c are integers greater than 0 such that

- $c = b + 2$
- $a + b + c = 10$

Prove, by exhaustion, that the product of a, b and c is always even.

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c
	1	
	2	

Question 1 continued

(Total for Question 1 is 3 marks)

2. A curve C has equation $y = f(x)$ where

$$f(x) = (2 - kx)^5$$

and k is a constant.

Given that when $f(x)$ is divided by $(4x - 5)$ the remainder is $\frac{243}{32}$

(a) show that $k = \frac{2}{5}$

(2)

(b) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{2}{5}x\right)^5$$

giving each term in simplest form.

(3)

Using the solution to part (b) and making your method clear,

(c) find the gradient of C at the point where $x = 0$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) a_1

(ii) a_2

(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n}{3}\right)\right\}$$

You must make your method clear.

(4)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The weight of a baby mammal is monitored over a 16-month period.

The weight of the mammal, w kg, is given by

$$w = \log_a(t + 5) - \log_a 4 \qquad 2 \leqslant t \leqslant 18$$

where t is the age of the mammal in months and a is a constant.

Given that the weight of the mammal was 10 kg when $t = 3$

(a) show that $a = 1.072$ correct to 3 decimal places.

(3)

Using $a = 1.072$

(b) find an equation for t in terms of w

(3)

(c) find the value of t when $w = 15$, giving your answer to 3 significant figures.

(2)

Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$(3\cos\theta - \tan\theta)\cos\theta = 2$$

can be written as

$$3\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

$$(3\cos 2x - \tan 2x)\cos 2x = 2$$

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. The curve C_1 has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 3 decimal places.

(3)

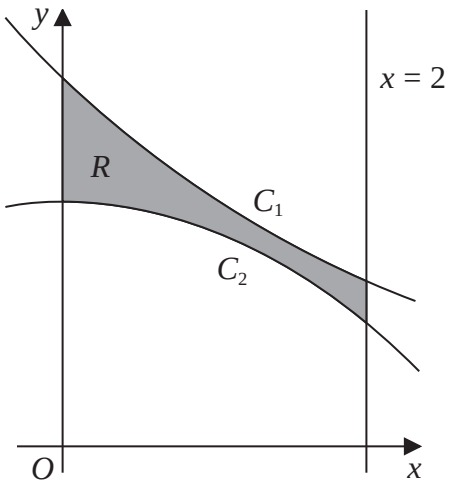


Figure 1

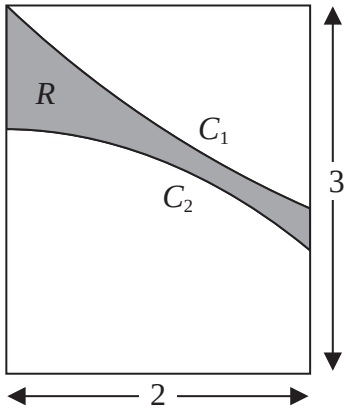


Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \qquad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a, b, c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the k^{th} term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

Question 8 continued

(Total for Question 8 is 7 marks)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

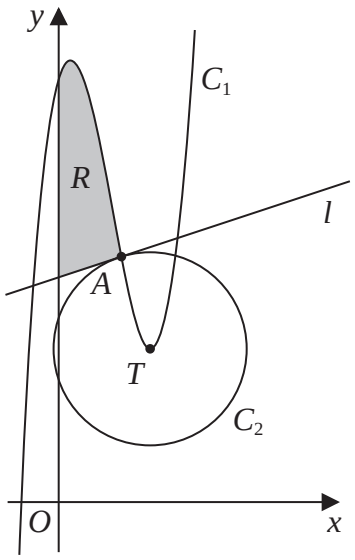


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a,

(a) $\log_2 9$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a.

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

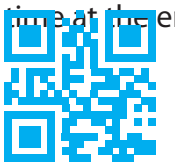
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.



1.

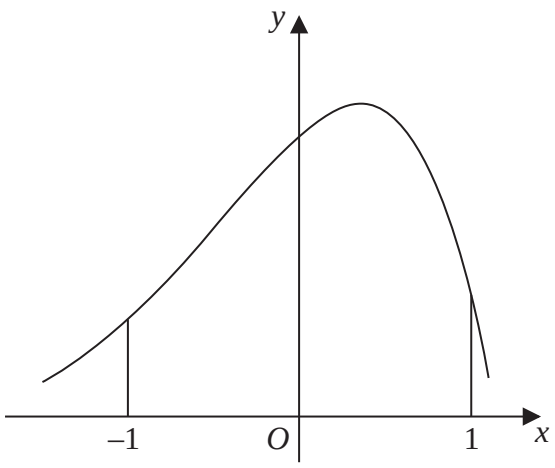


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$

The table below shows some corresponding values of x and y for this curve.

The values of y are given to 3 decimal places.

x	–1	–0.5	0	0.5	1
y	2.287	4.470	6.719	7.291	2.834

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to estimate

(i) $\displaystyle\int_{-1}^{1} \left(\mathrm{f}(x) - 2\right)\mathrm{d}x$

(ii) $\displaystyle\int_{1}^{3} \mathrm{f}(x-2)\,\mathrm{d}x$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

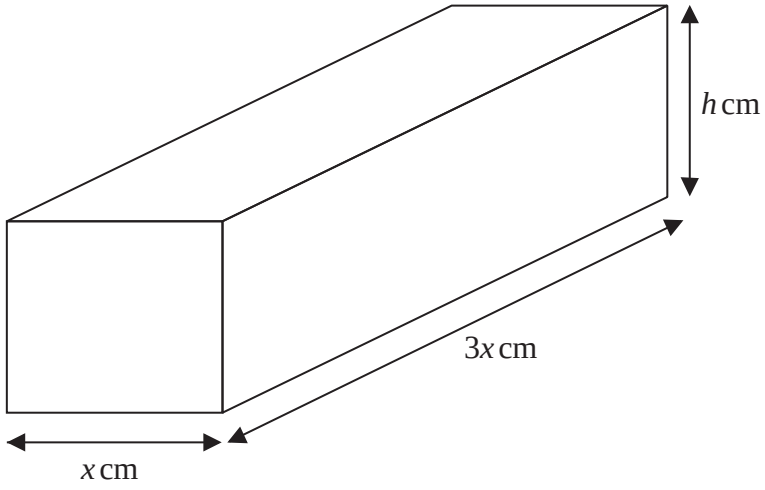


Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is $972\,\text{cm}^3$

(a) Show that the surface area of the brick, $S\,\text{cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\dfrac{\mathrm{d}S}{\mathrm{d}x}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2}$ and hence show that the value of x found in part (c) gives the minimum value of S.

(2)

(e) Hence find the minimum surface area of the brick.

(1)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. $\mathrm{f}(x) = \left(2 + \frac{kx}{8}\right)^7$ where k is a non-zero constant

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $\mathrm{f}(x)$. Give each term in simplest form.

(4)

Given that, in the binomial expansion of $\mathrm{f}(x)$, the coefficients of x, x^2 and x^3 are the first 3 terms of an arithmetic progression,

(b) find, using algebra, the possible values of k.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1, a > 1, y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

Question 4 continued

(Total for Question 4 is 6 marks)

5. $$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51$$

(2)

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0$$

(2)

(c) Hence find the value of p and the value of q.

(3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x)$$

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 7 continued

(Total for Question 7 is 7 marks)

8. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5\sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

(ii) Solve, for $0 < \theta < 360°$, the equation

$$2\tan\theta\sin\theta = 5 + \cos\theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

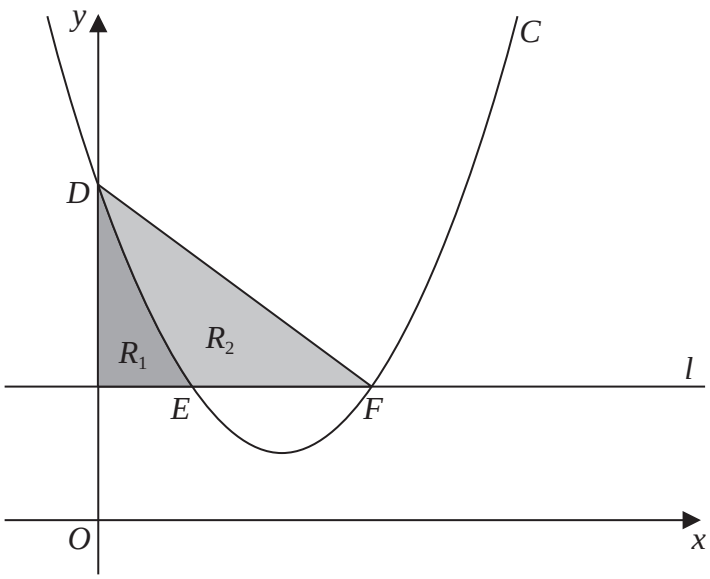


Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$

$$2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$$

which is not divisible by 3

Complete this proof.

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 18 May 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. The continuous curve C has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below.

x	4.0	4.2	4.4	4.6	4.8	5.0
y	9.2	8.4556	3.8512	5.0342	7.8297	8.6

Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_4^5 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -3$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

Question 2 continued

(Total for Question 2 is 6 marks)

3. A circle C has centre (2, 5)

Given that the point $P(8, -3)$ lies on C

(a) (i) find the radius of C

(ii) find an equation for C

(3)

(b) Find the equation of the tangent to C at P giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The binomial expansion, in ascending powers of x, of

$$(3 + px)^5$$

where p is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that

- $B = 18D$
- $p < 0$

(b) find

(i) the value of p

(ii) the value of C

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$

(5)

Question 5 continued

(Total for Question 5 is 5 marks)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

Question 7 continued

(Total for Question 7 is 8 marks)

8. (i) A student writes the following statement:

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10"

Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

Question 8 continued

(Total for Question 8 is 5 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta + 3) = 11 - 5\cos\theta$$

may be written as

$$3\cos^2\theta - 14\cos\theta + 8 = 0$$

(3)

(b) Hence solve, for $0 < x < 360°$

$$3\cos 2x(\tan 2x\sin 2x + 3) = 11 - 5\cos 2x$$

giving your answers to one decimal place.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,\mathrm{d}x = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

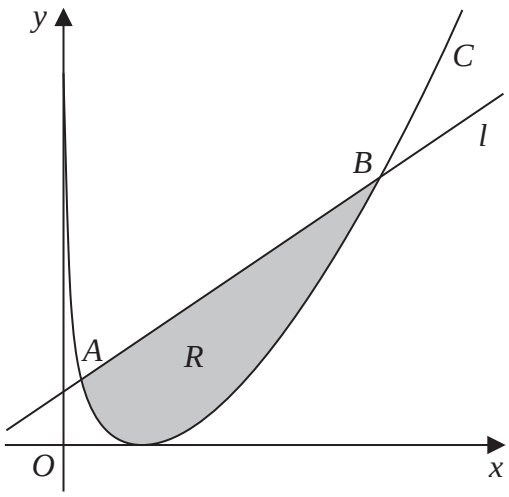

Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where a and b are constants.

(a) Find, in terms of a and b,

(i) u_2

(ii) u_3

(2)

Given

- $\sum_{n=1}^{3} u_n = 153$
- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of u_2

(3)

Question 11 continued

Question 11 continued

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 13 October 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided
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- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. Given that a, b and c are integers greater than 0 such that

- $c = 3a + 1$
- $a + b + c = 15$

prove, by exhaustion, that the product abc is always a multiple of 4

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c	abc

Question 1 continued

(Total for Question 1 is 3 marks)

2. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}$$

(a) Find the value of u_2, the value of u_3 and the value of u_4

(3)

(b) Find the value of

$$\sum_{r=1}^{100} u_r$$

(2)

Question 2 continued

(Total for Question 2 is 5 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Solve, for $0 < \theta \leqslant 360°$ the equation

$$2\tan\theta + 3\sin\theta = 0$$

giving your answers, as appropriate, to one decimal place.

(5)

(b) Hence, or otherwise, find the smallest positive solution of

$$2\tan(2x + 40°) + 3\sin(2x + 40°) = 0$$

giving your answer to one decimal place.

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + ax^2 - 29x + b$$

where a and b are constants.

Given that $(2x + 1)$ is a factor of $f(x)$,

(a) show that

$$a + 4b = -56$$

(2)

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is -25

(b) find a second simplified equation linking a and b.

(2)

(c) Hence, using algebra and showing your working,

(i) find the value of a and the value of b,

(ii) fully factorise $f(x)$.

(5)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$3^a = 70$$

giving the answer to 3 decimal places.

(2)

(ii) Find the exact value of b such that

$$4 + 3\log_3 b = \log_3 5b$$

(4)

Question 5 continued

(Total for Question 5 is 6 marks)

6.

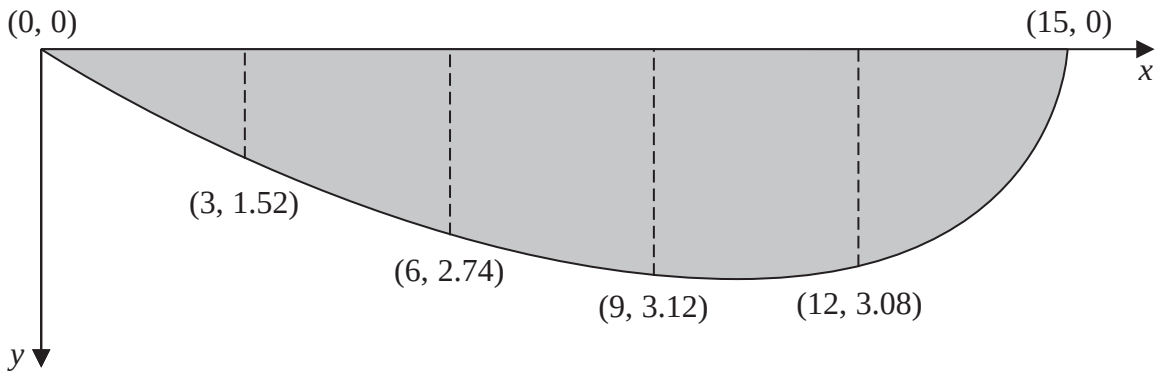

Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m}\,\text{s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

Question 6 continued

(Total for Question 6 is 6 marks)

7.

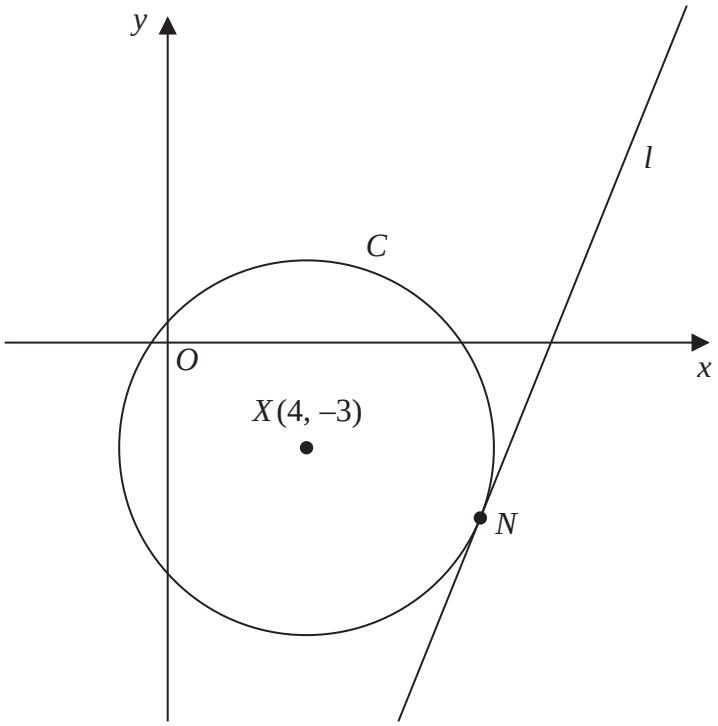

Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9.

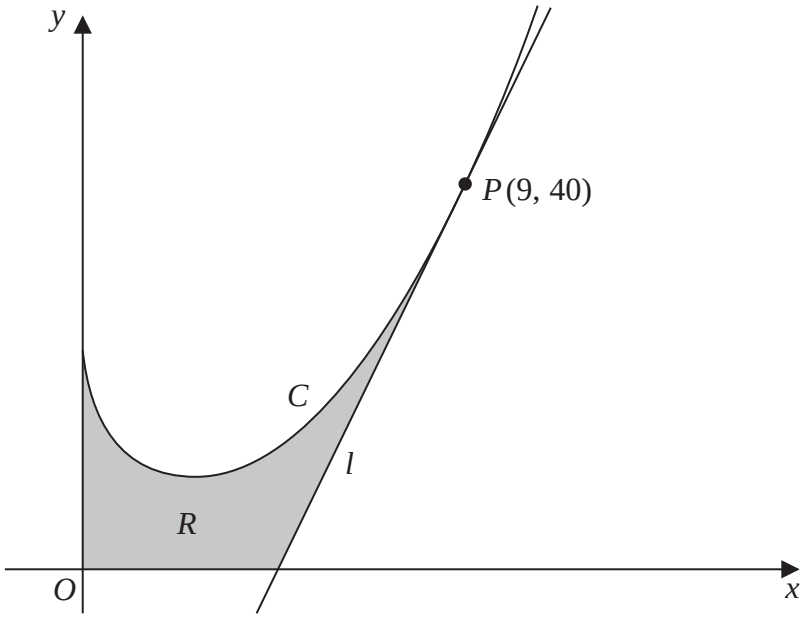

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 16 January 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. $f(x) = ax^3 + 3x^2 - 8x + 2$ where a is a constant

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. Find the coefficient of the term in x^7 of the binomial expansion of

$$\left(\frac{3}{8} + 4x\right)^{12}$$

giving your answer in simplest form.

(3)

Question 2 continued

(Total for Question 2 is 3 marks)

3. The circle C

- has centre $A(3, 5)$
- passes through the point $B(8, -7)$

(a) Find an equation for C.

(3)

The points M and N lie on C such that MN is a chord of C.

Given that MN

- lies above the x-axis
- is parallel to the x-axis
- has length $4\sqrt{22}$

(b) find an equation for the line passing through points M and N.

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Sketch the curve with equation

$$y = a^{-x} + 4$$

where a is a constant and $a > 1$

On your sketch show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote to the curve.

(3)

x	-4	-1.5	1	3.5	6	8.5
y	13	6.280	4.577	4.146	4.037	4.009

The table above shows corresponding values of x and y for $y = 3^{-\frac{1}{2}x} + 4$

The values of y are given to four significant figures, as appropriate.

Using the trapezium rule with all the values of y in the table,

(b) find an approximate value for

$$\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) \mathrm{d}x$$

giving your answer to two significant figures.

(3)

(c) **Using the answer to part (b)**, find an approximate value for

(i) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x}\right) \mathrm{d}x$

(ii) $\displaystyle\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) \mathrm{d}x + \int_{-8.5}^{4} \left(3^{\frac{1}{2}x} + 4\right) \mathrm{d}x$

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. (i) Find the value of

$$\sum_{r=1}^{\infty} 6 \times (0.25)^r$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$
$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

(2)

(b) State the order of this sequence.

(1)

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4000 tonnes of wheat.

Model *A* assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model *A*, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model *B* assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model *B*, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4"

(4)

Question 8 continued

(Total for Question 8 is 6 marks)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

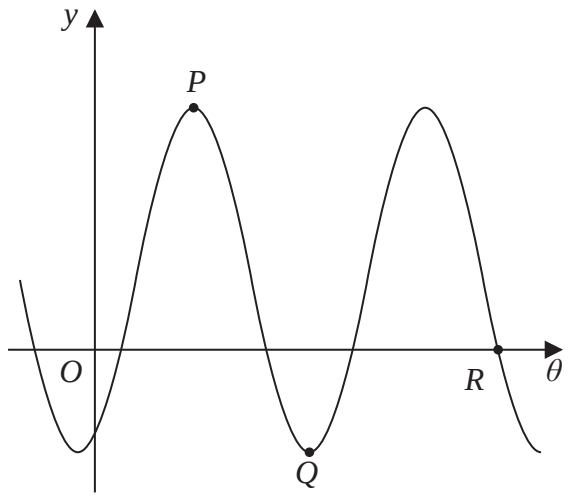

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 14 marks)

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

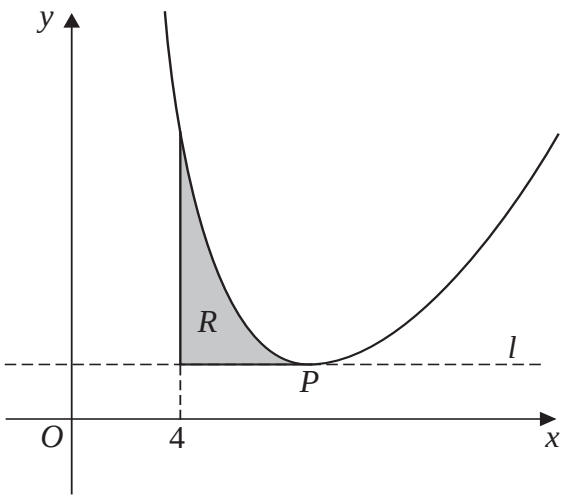

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 14 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{1}{6}x\right)^9$$

giving each term in simplest form.

(3)

(b) Hence find the coefficient of x^3 in the expansion of

$$(10x+3)\left(1-\frac{1}{6}x\right)^9$$

giving the answer in simplest form.

(2)

Question 1 continued

(Total for Question 1 is 5 marks)

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.

(4)

(b) Hence find the smallest value of n for which

$$S_n > 8000$$

(3)

Question 2 continued

(Total for Question 2 is 7 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10)$$

(5)

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where a is a positive constant greater than 1

(1)

Question 3 continued

(Total for Question 3 is 6 marks)

4. $f(x) = (x - 2)(2x^2 + 5x + k) + 21$

where k is a constant.

(a) State the remainder when $f(x)$ is divided by $(x - 2)$

(1)

Given that $(2x - 1)$ is a factor of $f(x)$

(b) show that $k = 11$

(2)

(c) Hence

(i) fully factorise $f(x)$,

(ii) find the number of real solutions of the equation

$$f(x) = 0$$

giving a reason for your answer.

(5)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show detailed reasoning.**

(a) Given that x and y are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

(4)

(b) Using a counter example, show that the result in part (a) is not true for all real numbers.

(2)

Question 5 continued

(Total for Question 5 is 6 marks)

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x - 5)^2 + (y + 8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 6 marks)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, H°C, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

9.

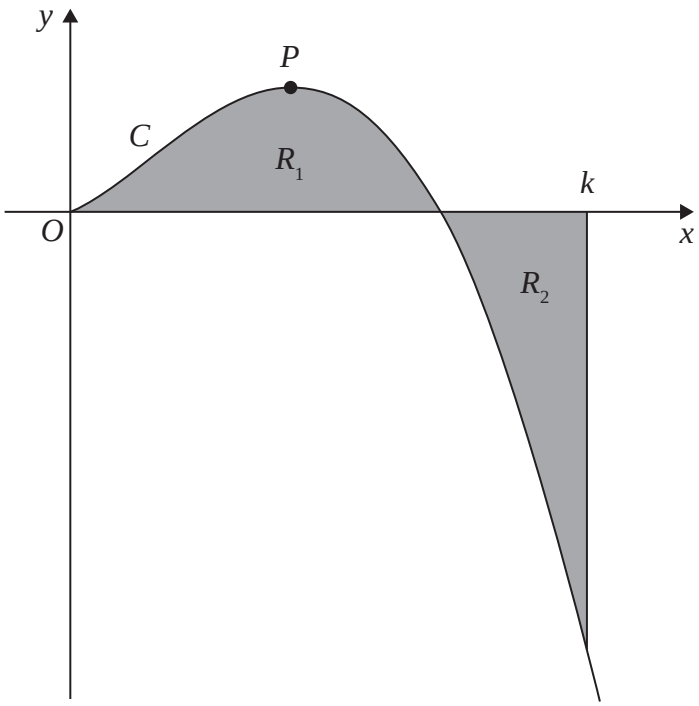

Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that
- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS