

Question	Scheme	Marks
1(a)	$f(\pm 2) = a \times (\pm 2)^3 + 7(\pm 2)^2 \pm 46 \times 2 + 24$	M1
	[Remainder is] $8a - 40$	A1
		(2)
(b)	$8a - 40 = 0 \Rightarrow a = 5^*$	B1*
		(1)
(c)	$5x^3 + 7x^2 - 46x + 24 = (x - 2)(5x^2 + 17x - 12)$	M1
		A1
	$5x^2 + 17x - 12 = (5x - 3)(x + 4)$	dM1
	$[f(x)] = (x - 2)(5x - 3)(x + 4)$	A1
		(4)
(7 marks)		

Notes:**(a)****M1:** Attempts $f(\pm 2)$ **OR Polynomial Division**

$$\begin{array}{r}
 ax^2 + (7 + 2a)x + 4a - 32 \\
 x - 2 \overline{) \quad \quad \quad} \\
 \underline{ax^3 - 2ax^2} \\
 (7 + 2a)x^2 - 46x + 24 \\
 \underline{(7 + 2a)x^2 - 2(7 + 2a)x} \\
 (4a - 32)x + 24 \\
 \underline{(4a - 32)x - 2(4a - 32)} \\
 8a - 40
 \end{array}$$

M1 for a complete method to obtain a quotient of $ax^2 + f(a)x + g(a)$ where $f(a)$ and $g(a)$ are linear in a and then obtains a linear expression in a for the remainder.

OR Tabular method

	ax^2	$(7 + 2a)x$	$4a - 32$
x	ax^3	$(7 + 2a)x^2$	$(4a - 32)x$
-2	$-2ax^2$	$-2(7 + 2a)x$	$64 - 8a$

So remainder is $24 - (64 - 8a)$

M1: A complete method as above and reaches a linear expression in a in the bottom right hand box and subtracts it from 24

A1: Correct expression in a [Accept $f(2) = 8a - 40$]

Following polynomial division the remainder must be extracted or identified as the remainder.

If their remainder has **not** been clearly identified or extracted, but then used in part (b) then award A1 here.

(b)

B1*: Sets $8a - 40 = 0$ and correctly shows $a = 5$. Allow a restart, but the work must be correct for this mark, there is no ft.

(c)

M1: Attempts to factor out, or divide through, by the $(x - 2)$.

Look for $(x - 2)(5x^2 + \alpha x + \beta)$ OR $(5x^2 + \alpha x + \beta)$ as the quotient from long division where $\alpha, \beta \neq 0$

A1: Correct quadratic factor found.

dM1: Factorises their quadratic factor (usual rules).

A1: Fully factorised cubic all together on one line. Accept $[f(x)] = 5(x - 2)\left(x - \frac{3}{5}\right)(x + 4)$ but not forms with common factors in the linear terms.

Note: Question specifies all stages of working must be shown, so answers without first finding the quadratic factor score no marks. Accept alternative forms of long division.

Question	Scheme	Marks
2(a)	$\int kx^{-3} + 4x^{\frac{3}{2}} dx = k \frac{x^{-2}}{-2} + 4 \frac{x^{\frac{5}{2}}}{5/2} (+c)$	M1 A1
	$\int_1^4 kx^{-3} + 4x^{\frac{3}{2}} dx = \left[-\frac{k}{2x^2} + \frac{8x^{\frac{5}{2}}}{5} \right]_1^4 = \left(-\frac{k}{2 \times 16} + \frac{8(4)^{\frac{5}{2}}}{5} \right) - \left(-\frac{k}{2} + \frac{8}{5} \right)$	M1
	$= \frac{15k}{32} + \frac{248}{5}$	A1
		(4)
(b)	$\frac{15k}{32} + \frac{248}{5} = 50 \Rightarrow k = \dots$	M1
	$k = \frac{64}{75} \text{ o.e.}$	A1
		(2)
(6 marks)		
Notes:		
(a) M1: Raises the power by 1 in both terms. Need not be simplified Accept for example, $\alpha x^{-3+1} + \beta x^{\frac{3}{2}+1}$ for this mark. A1: Correct integral with or without +c. Need not be simplified although indices must be simplified for this mark. Look for $k \frac{x^{-2}}{-2} + 4 \frac{x^{\frac{5}{2}}}{5/2}$ as a minimum. M1: Substitutes and subtracts either way around into an expression with changed indices. A1: Correct simplified answer. Accept e.g., $\frac{75k + 7936}{160}$ condone e.g., $0.46875k + 49.6$ with no rounded decimals. (b) M1: Sets the result of their changed expression from (a) [provided it is a linear expression of least 2 terms in terms of k] equal to 50, and solves to find k A1: Correct answer from a correct equation. Accept equivalent fractions and also accept $0.85\dot{3}$		

Question	Scheme	Marks
3(a)	$u_2 = \frac{3}{2}(5) - \frac{1}{2}(5)^2 + 4 = \dots (= -1)$	M1
	$u_3 = \frac{3}{2}("1") - \frac{1}{2}("1")^2 + 4 = (2), u_4 = \frac{3}{2}("2") - \frac{1}{2}("2")^2 + 4 = (5) [= u_1]$	M1
	<u>e.g.</u> , $(u_4 = u_1)$ (so sequence is periodic) with period 3.	A1
		(3)
(b)	$u_{200} = -1$	B1
		(1)
(c)	$\sum_{r=1}^{200} u_r = (u_1 + u_2 + u_3) + \dots + (u_{196} + u_{197} + u_{198}) + [u_{199} + u_{200}]$	M1
	$= 66 \times (5 - 1 + 2) + 5 - 1 \quad \text{OR} \quad = 67 \times (5 - 1 + 2) - 2$	M1
	$= 400$	A1
		(3)
(7 marks)		
Notes:		
(a) M1: Substitutes 5 into the equation to find u_2 M1: Continue by using their u_2 to find the next term(s) until a repeat is found. A1: All terms correct, (concludes periodic) and states period is 3 Accept either $u_1 = u_4$ or $(u_1 = 5 \text{ and } u_4 = 5)$ seen in the working with a conclusion (b) B1: Correct value coming from a correct sequence. (c) M1: Attempts to group the sum into multiples of the three term sum. Look for any attempt at using a multiple of the sum of their terms from part (a) e.g $\alpha(5 - 1 + 2)$ where α is an integer. M1: Splitting the sum correctly to get 66 sets of 3 and the extra terms, or 67 sets of 3 minus one term A1: For 400		

Question	Scheme	Marks
4(a)	$\frac{dy}{dx} = ax^2 + b + \frac{c}{x^2}$	M1
	$\frac{dy}{dx} = 6x^2 + 7 - \frac{3}{x^2}$	A1
	$6x^2 + 7 - \frac{3}{x^2} = 0 \Rightarrow 6x^4 + 7x^2 - 3 = 0 \Rightarrow (3x^2 - 1)(2x^2 + 3) = 0 \Rightarrow x^2 = \dots$	M1
	$\left[x^2 = -\frac{3}{2} \text{ impossible} \right] x^2 = \frac{1}{3} \Rightarrow (x =) \pm \frac{1}{\sqrt{3}}$	A1
		(4)
(b)	$6x^2 + 7 - \frac{3}{x^2} \geq 0 \Rightarrow 6x^4 + 7x^2 - 3 \geq 0 \text{ (as } x^2 \geq 0)$	
	So need " $3x^2 - 1$ " $\geq 0 \Rightarrow x \leq -\frac{1}{\sqrt{3}}$, $x \geq \frac{1}{\sqrt{3}}$	M1
	$x \leq -\frac{1}{\sqrt{3}}, x \geq \frac{1}{\sqrt{3}}$ or $x < -\frac{1}{\sqrt{3}}, x > \frac{1}{\sqrt{3}}$	A1
		(2)
(6 marks)		
Notes:		
(a) M1: Differentiates with power decreased by 1 in at least two of the three terms A1: Correct derivative. Coefficients need not be simplified, but indices must be simplified. M1: Sets equal to 0, multiplies through and attempts to solve the quadratic in x^2 via a non-calculator method reaching at least one value for x^2. They must show us their method for solving the 3TQ in x^2 explicitly. (Allow any letter to substitute for x^2) A1: Correct values only with no other values given. (b) M1: Selects the outside region for their values. A1: Correct range, accept with loose or strict inequalities. Accept any valid notation. For example: $\left\{ x : x \leq -\frac{1}{\sqrt{3}} \right\} \cup \left\{ x : x \geq \frac{1}{\sqrt{3}} \right\} \text{ or } \left\{ x : x < -\frac{1}{\sqrt{3}} \right\} \cup \left\{ x : x > \frac{1}{\sqrt{3}} \right\}$ $x \leq -\frac{1}{\sqrt{3}}$ OR $x \geq \frac{1}{\sqrt{3}}$ Do not accept $x \leq -\frac{1}{\sqrt{3}}$ AND $x \geq \frac{1}{\sqrt{3}}$ for the A mark		

Question	Scheme	Marks
5(a)	$\frac{1}{2} \times 0.25 (5.389 + 6.469 + 2(6.231 + 6.815 + 7.028 + 6.901 + 6.732))$	B1 M1
	$\left(= \frac{1}{8} \times (11.858 + 2(33.707)) = \frac{1}{8} \times 79.272 \right)$	
	= awrt 9.91	A1
		(3)
(b)	$\int \sqrt{x} \left(3 + \frac{1}{x^2} \right) dx = \int \left(3x^{\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx = \dots x^{\frac{3}{2}} \pm \dots x^{-\frac{1}{2}}$	M1
	$= 2x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} (+c)$	A1A1
		(3)
(c)	$\int_3^{4.5} \sqrt{x} \left(3 + \frac{1}{x^2} \right) dx = \left(2(4.5)^{\frac{3}{2}} - 2(4.5)^{-\frac{1}{2}} \right) - \left(2(3)^{\frac{3}{2}} - 2(3)^{-\frac{1}{2}} \right) = \dots (8.911\dots)$	M1
	$\Rightarrow \text{Area } R = "9.909" - "8.911" = \dots$	
	= awrt 1.0	A1
		(2)

(8 marks)**Notes:****(a)**

B1: For using $h = 0.25$ or equivalent for the width in an attempt at the trapezium rule. Must be used as part of the attempt, not just stated separately.

M1: Correct bracket structure for the trapezium rule.

$$\dots\dots\dots (5.389 + 6.469 + 2(6.231 + 6.815 + 7.028 + 6.901 + 6.732))$$

Look for first + last y values + 2(attempt at sum of remaining values). If one value is missing, allow the M as a slip, but M0 if 1st or last value repeated. Must be y values used.

A1: awrt 9.91

(b)

M1: Expands the brackets and attempts to integrate, look for power increasing by 1 at least once in either a term with power $\frac{1}{2}$ or $-\frac{3}{2}$

A1: One of the two terms correct. Need not be simplified for this mark.

A1: Both terms correct (no need for c). Must be simplified coefficients. Accept equivalents, e.g.

$$2 \left(\sqrt{x^3} - \frac{1}{\sqrt{x}} \right) \text{ or } 2 \left(\frac{x^2 - 1}{\sqrt{x}} \right)$$

(c)

M1: Applies limits 3 and 4.5 to their answer to (b) and attempts to subtract the result from the answer to (a).

A1: awrt 1.0 isw Note: Allow an answer of $9.91 - 8.91 = 1$ following correct work.

Question	Scheme	Marks
6(i)	E.g. $7^{x+3} = 5 \Rightarrow (x+3)\log 7 = \log 5 \Rightarrow x = \frac{\log 5}{\log 7} - 3$ or $x+3 = \log_7 5 \Rightarrow x = \dots$ Alt: $7^{x+3} = 5 \Rightarrow 7^x = \frac{5}{7^3} \Rightarrow x = \log_7 \frac{5}{343}$	M1
	$= -2.17$	A1
		(2)
(ii)(a)	$\log_8 64y^3 = \log_8 64 + \log_8 y^3$ or $\log_8 y^3 = 3\log_8 y$ or $\log_8 64 = 2$ Or $\log_8 (64y^3) \Rightarrow \log_8 (4y)^3 \Rightarrow 3\log_8 (4y)$ or $\log_8 4 = \frac{2}{3}$	M1
	$18(\log_8 y)^2 + 5\log_8 (64y^3) = 73 \Rightarrow 18(\log_8 y)^2 + 5\log_8 64 + 5\log_8 y^3 = 73$ $\Rightarrow 18(\log_8 y)^2 + 5 \times 2 + 15\log_8 y = 73$	dM1
	$\Rightarrow 18(\log_8 y)^2 + 15\log_8 y - 63 = 0 \Rightarrow 6(\log_8 y)^2 + 5\log_8 y - 21 = 0^*$	A1*
		(3)
(b)	$(2\log_8 y - 3)(3\log_8 y + 7) = 0 \Rightarrow \log_8 y = \frac{3}{2}, -\frac{7}{3} \Rightarrow y = 8^k$	M1
	e.g. $y = 8^{\frac{3}{2}}$ or $8^{-\frac{7}{3}}$ oe	A1
	e.g. $16\sqrt{2}$ or $\sqrt{512}$ and $\frac{1}{128}$ oe	A1
		(3)
(8 marks)		

Notes:**(i)****M1:** Correct method for solving to find x , e.g. takes logs and rearranges, as shown in scheme. May divide through by 7^3 first or take log to base 7. There must be evidence of correct use of logs.**A1:** Correct answer, accept awrt -2.17 **(ii)(a)****M1:** Attempt the sum or power law of logs on the $\log_8 64y^3$ or applies $\log_8 64 = 2$ in the proof orNote this cannot be scored for an attempt at $(\log_8 y)^2 = 2\log_8 y$ **dM1:** Correct use of both the sum and the power law of logs seen or implied on the $\log_8 64y^3$ term and simplifies the $\log_8 64$ to achieve a quadratic in any form.**A1*:** Correctly rearranges to the printed equation **with no errors seen** and use of the log laws clear.**(b)****M1:** Solves the **given** quadratic and uses a correct method to obtain at least one solution for y from an equation of form $\log_8 y = k$ i.e., $\log_8 y = k \Rightarrow y = 8^k$. Accept a calculator method to solve the 3TQ.**A1:** At least one correct **exact** answer in either exponential or numerical form.**A1:** Both correct, accepting any equivalent **exact** forms. i.e., exponential or numerical

Question	Scheme	Marks
7(a)	(3,6)	B1
		(1)
(b)	Gradient = $\frac{6-0}{3-0}$ (=2)	M1
	Hence equation of l is $y = 2x$	A1
		(2)
(c)	Distance from O to centre is $\sqrt{3^2 + 6^2} = \dots (= 3\sqrt{5})$	M1
	$\Rightarrow r = 3\sqrt{5} \pm \sqrt{5}$	M1
	For radius either $2\sqrt{5}$ or $4\sqrt{5}$ or one correct value for k	A1
	For both 20 and 80	A1
		(4)
(7 marks)		
Notes:		
(a) B1: Correct centre. Allow $x = 3, y = 6$ (b) M1: Attempts the gradient from O to the centre. May be implied by $\pm \frac{6}{3}$ ft their centre. OR from the equation $y = 2x$ seen. A1: Correct equation from correct centre. (c) M1: Attempts the distance from the origin to the centre of the circle. M1: Finds the radius as the sum or difference of their distance and $\sqrt{5}$ A1: For one of the two possible radii correct or one of the two possible values for k correct. A1: Both values correct.		
Alt 1(c)	A lies on the line $y = 2x \Rightarrow x^2 + (2x)^2 = (\sqrt{5})^2 \Rightarrow x = \dots$	M1
	$\Rightarrow k = (" \pm 1 " - 3)^2 + (" \pm 2 " - 6)^2 = \dots$	M1
	One of 20 or 80 ; both 20 and 80	A1;A1
		(4)
M1: Uses the equation to obtain a general point on the line and applies distance $OA = \sqrt{5}$ to find at least one x coordinate. M1: Uses their coordinates of A to find at least one value for k using the circle equation. A1: One of the two possible values for k correct. A1: Both values correct.		

Alt 2 (c)	Deduces that as A lies on $y = 2x$ then EITHER $\sqrt{5} = \sqrt{1^2 + 2^2}$ OR $\sqrt{5} = \sqrt{(-1)^2 + (-2)^2}$	M1
	$\Rightarrow k = (" \pm 1 " - 3)^2 + (" \pm 2 " - 6)^2 = \dots$	M1
	One of 20 or 80 ; both 20 and 80	A1;A1
		(4)

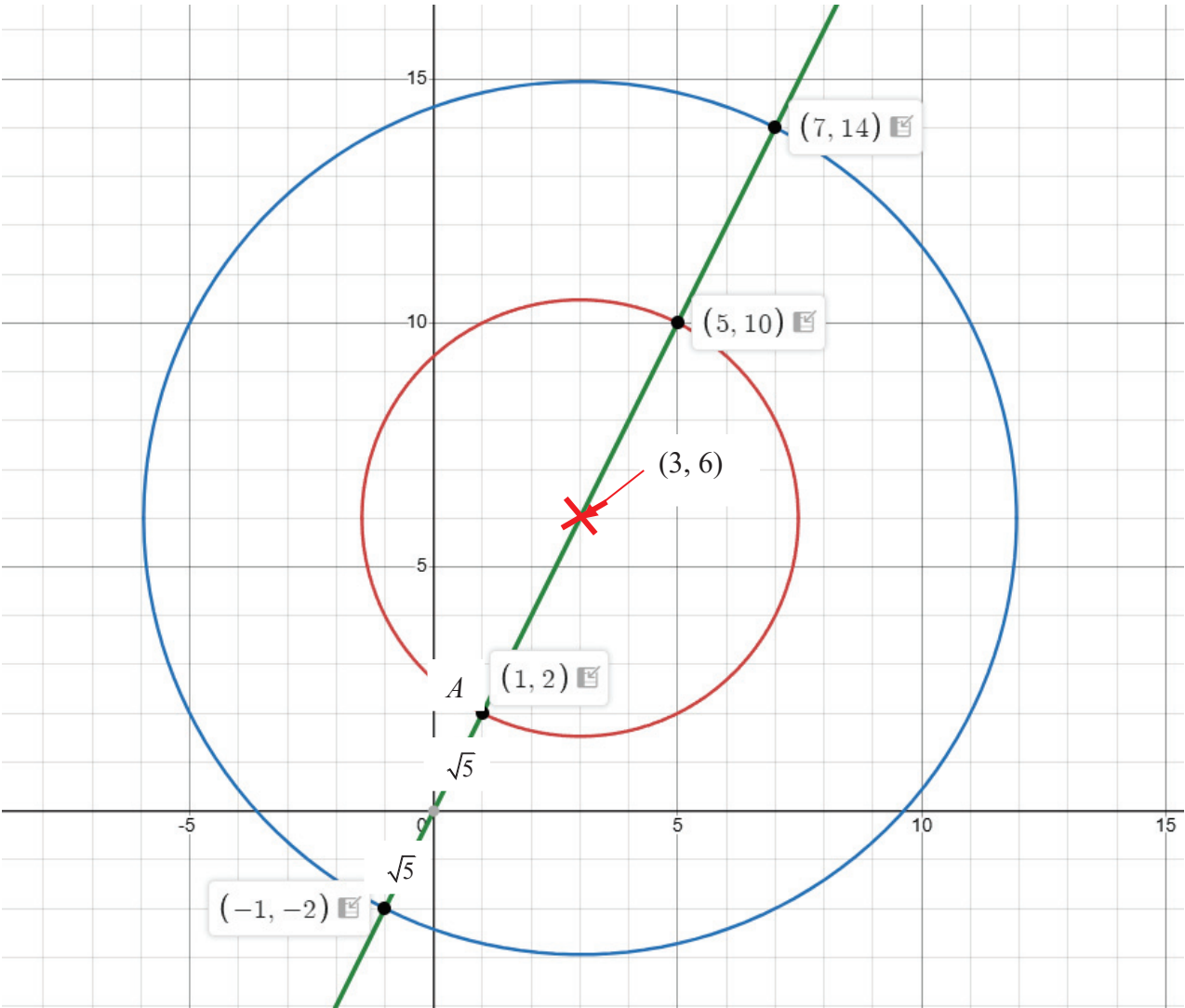
M1: Deduces that the coordinates of A must be either $(1, 2)$ or $(-1, -2)$

M1: Uses their coordinates of A to find at least one value for k using the circle equation.

A1: One of the two possible values for k correct.

A1: Both values correct.

USEFUL SKETCH



Question	Scheme	Marks
8(a)	$(k + 4x)^{12} = k^{12} + \dots$	B1
	$+ \binom{12}{1} k^{11} (4x)^1 + \binom{12}{2} k^{10} (4x)^2 + \binom{12}{3} k^9 (4x)^3 + \dots$	M1
	$(k + 4x)^{12} = k^{12} + 48k^{11}x + 1056k^{10}x^2 + 14080k^9x^3 + \dots$	A1A1
		(4)
(b)	$14080k^9 = \frac{55}{2} \Rightarrow k^9 = \frac{1}{512} \Rightarrow k = \dots$	M1
	$k = \frac{1}{2}$	A1
		(2)
(c)	" $48\left(\frac{1}{2}\right)^{11}$ " or " $1056\left(\frac{1}{2}\right)^{10} \times 2$ "	M1
	$48\left(\frac{1}{2}\right)^{11} + 1056\left(\frac{1}{2}\right)^{10} \times 2 = \dots = \frac{267}{128}$	dM1A1
		(3)

(9 marks)**Notes:****(a)**

B1: For the correct constant term of k^{12} in their expression. They may have initially factored out the k , but must achieve the correct constant in the final expression for this mark.

M1: **Correct** binomial coefficients linked with **correct** powers of x in their expansion. Accept alternative notation, such as ${}^{12}C_1$ or just 12, 66, 220, for the coefficients. (The power 1 need not be seen.) The powers of 4 and k may be missing or incorrect.

ALT for the M mark

$$k^{12} \left(1 + \frac{4x}{k} \right)^{12} = \left[k^{12} \right] \left(1 + \underline{12} \times \left(\frac{4x}{k} \right) + \left[\frac{12 \times 11}{2!} \right] \times \left(\frac{4x}{k} \right)^2 + \left[\frac{12 \times 11 \times 10}{3!} \right] \times \left(\frac{4x}{k} \right)^3 + \dots \right)$$

With **correct** coefficients (with the correct denominators [2 and 6 oe]) and with the **correct** power of x

A1: Any two of the final three terms correct and simplified

A1: All three of the final terms correct and simplified.

Note: Students may attempt to take out a common factor of k^{12} first. In such cases the **B** mark is for correct constant term in their final expansion, **M** follows as above, correct coefficients linked to

correct powers of x . The first **A** can be awarded as above or SC $k^{12} \left(1 + \frac{48}{k}x + \frac{1056}{k^2}x^2 + \frac{14080}{k^3}x^3 \right)$

(oe), final **A** as scheme

(b)

M1: Sets their x^3 coefficient equal to $\frac{55}{2}$ and proceeds to a value for k . They must have had a k in their coefficient.

A1: Correct answer.

(c)

M1: Correct attempt for at least one of the two terms in y^3 in the expansion

- Using their coefficients from (a) (which may not have involved k).
- Realises the y 's are squared.
- May be part of an attempt at a full expansion. Allow with k or their value of k .

dM1: Correct attempt at finding both terms and adding, must have both involved k (which may be left as k), but otherwise follow through their attempt at (a). May be part of an attempt at a full expansion.

A1: Correct answer, accept $\frac{267}{128}y^3$, but must have been extracted from a full expansion and identified as the answer.

Question	Scheme	Marks
9(a)	At 12 noon $t = (12 - 8) = 4$ so $\theta = 17 + 5 \sin\left(\frac{4\pi}{12}\right)$	M1
	$\theta = 17 + 5 \sin\left(\frac{4\pi}{12}\right) = (17 + 4.33...) \approx 21.3(^{\circ}\text{C})$	A1
		(2)
	Alt: Working backwards from the given answer	
	$21.3 = 17 + 5 \sin\left(\frac{\pi t}{12}\right) \Rightarrow \sin\left(\frac{\pi t}{12}\right) = \frac{4.3}{5} \Rightarrow t = \frac{12}{\pi} \arcsin\left(\frac{4.3}{5}\right) = \dots$	M1
	$t \approx 4$ with a conclusion; 8am + 4 = 12 noon	A1
		(2)
(b)	$17 + 5 \sin\left(\frac{\pi T}{12}\right) = 16 + 6 \sin\left(\frac{\pi T}{12}\right) + 3 \cos^2\left(\frac{\pi T}{12}\right)$ Use of: $\cos^2\left(\frac{\pi T}{12}\right) = 1 - \sin^2\left(\frac{\pi T}{12}\right)$	M1 B1
	$3 \sin^2\left(\frac{\pi T}{12}\right) - \sin\left(\frac{\pi T}{12}\right) - 2 = 0 *$	A1*
		(3)
(c)	$\Rightarrow \left(3 \sin\left(\frac{\pi T}{12}\right) + 2\right) \left(\sin\left(\frac{\pi T}{12}\right) - 1\right) = 0 \Rightarrow \sin\left(\frac{\pi T}{12}\right) = -\frac{2}{3}, 1$	B1
	$\Rightarrow T = \frac{12}{\pi} \left[(\pi -) \arcsin\left(-\frac{2}{3}\right) \right]$ or $\frac{12}{\pi} \left[\arcsin\left(-\frac{2}{3}\right) + 2\pi \right]$ or $T = \frac{12}{\pi} \arcsin(1)$	M1
	$T = 6, \text{awrt } 14.8, \text{awrt } 21.2$	A1A1 A1
		(5)
(10 marks)		

Notes:**(a)****M1:** Substitutes $t = 4$ into the equation.**A1:** Correct expression **seen** leading to awrt 21.3Note: This is a show question and using $t = 8$ leading to a value of $\theta = 21.3$ is M0A0**Alt****M1:** Substitutes 21.3 into the equation and works backwards to find a value for T

Allow working in degrees for the M mark

A1: Correct value for T with a conclusion that 8 am + 4 hrs = 12 noon.

(b)**M1:** Sets the two equations equal.**B1 [M1 in Epen]:** For use of **correct** Pythagorean identity to form an expression in just sine. This mark is specifically for the **correct** use of $\cos^2\left(\frac{\pi T}{12}\right) = 1 - \sin^2\left(\frac{\pi T}{12}\right)$ **A1*:** Correct simplified quadratic with no errors seen. Allow with t or T .**(c)****B1:** Correct roots of quadratic seen – **both required**. Allow these value from a substitution for $\sin\left(\frac{\pi T}{12}\right)$, for example $x = -\frac{2}{3}$, 1 seen, but do **not** isw if one of these values is ‘rejected’.**M1:** Uses arcsin with any of their solutions and rearranges to find a value for T (or t). Can be scored from any equation $\sin\left(\frac{\pi T}{12}\right) = k$ where $|k| \leq 1$, Allow for any appropriate solution (including negatives). E.g. if $\sin\left(\frac{\pi T}{12}\right) = -\frac{2}{3}$ or $\sin\left(\frac{\pi T}{12}\right) = 1$ has been found then any of -2.787 , 14.787 , 21.213 or 6 can imply this mark (if method not explicitly shown).**Note:** If they work in degrees they must convert $\frac{\pi}{12} = 15^\circ$ otherwise it is M0**A1:** At least one correct positive value for T or t (awrt 3 s.f. where appropriate). See scheme for answers**A1:** Two correct values**A1:** All three values correct to 3.s.f. as appropriate.

Ignore any extra values outside of the range – withhold this mark for extra values within range.

Question	Scheme	Marks														
10(i)	Two of : $f(3) = 504 - (119 + 216) = \dots, f(4) = 672 - (119 + 384) = \dots,$ $f(5) = 840 - (119 + 600) = \dots, f(6) = 1008 - (119 + 864) = \dots$	M1														
	All of the above, with at least two correct values.	M1														
	So $f(1) = 25 = 5^2 = f(6), f(2) = 121 = 11^2 = f(5), f(3) = 169 = 13^2 = f(4)$ hence all are perfect squares.	A1														
	<table><tr><th>Value of n</th><th>Value of $f(n)$</th></tr><tr><td>1</td><td>$25 = 5^2$</td></tr><tr><td>2</td><td>$121 = 11^2$</td></tr><tr><td>3</td><td>$169 = 13^2$</td></tr><tr><td>4</td><td>$169 = 13^2$</td></tr><tr><td>5</td><td>$121 = 11^2$</td></tr><tr><td>6</td><td>$25 = 5^2$</td></tr></table>	Value of n	Value of $f(n)$	1	$25 = 5^2$	2	$121 = 11^2$	3	$169 = 13^2$	4	$169 = 13^2$	5	$121 = 11^2$	6	$25 = 5^2$	
	Value of n	Value of $f(n)$														
1	$25 = 5^2$															
2	$121 = 11^2$															
3	$169 = 13^2$															
4	$169 = 13^2$															
5	$121 = 11^2$															
6	$25 = 5^2$															
	(3)															
(ii)	${}^nC_r + {}^nC_{r+1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-(r+1))!}$ $\left[(n-r)! = (n-r)(n-r-1)! \Rightarrow (n-(r+1))! = \frac{(n-r)!}{(n-r)} \right]$	M1														
	$= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$ OR $\frac{n!(n-r-1)!(r+1)! + n!r!(n-r)!}{r!(n-r)!(r+1)!(n-r-1)!}$	dM1														
	$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$ OR $\frac{n![(n-r-1)!(r+1)! + r!(n-r)!]}{r!(n-r)!(r+1)!(n-r-1)!}$	ddM1														
	$= \frac{n!(n+1)}{(r+1)!(n-r)!} = \frac{(n+1)!}{(r+1)!((n+1)-(r+1))!} = {}^{n+1}C_{r+1}^*$	A1*														
		(4)														
(7 marks)																
Notes:																
(i) M1: Attempts $f(n)$ for at least two values between 3 and 6 inclusive (need not be correct values), but if the values are incorrect, working must be seen with values obtained.																

M1: Attempts all 4 remaining values, with at least two correct values.

A1: All remaining values for $f(n)$ for $n = 3$ to 6 attempted with correct values found, shown to be squares and minimal conclusion.

Note: If all valid values are correct, but $f(0)$ and/or $f(7)$ are also included, do not award the A mark in (i)

(ii)

M1: Attempts the sum of the binomial coefficients using the formula.

Allow a slip in $(n-r+1)!$ instead of $(n-r-1)!$

dm1: Puts the terms over a common denominator which need not be simplified in either one of the two forms shown.

ddM1: Factorises the numerator which need not be simplified in either of one of the two forms shown.

A1*: Achieves the correct form for the new coefficient from correct work and deduces equal to the required coefficient. Must achieve the correct expression with $(n+1) - (r+1)$ shown.

Alt (ii)	${}^{n+1}C_{r+1} - {}^nC_{r+1} = \frac{(n+1)!}{(r+1)!(n+1-r-1)!} - \frac{n!}{(r+1)!(n-(r+1))!}$	M1
	$= \frac{(n+1) \times n!}{(r+1)!(n-r) \times (n-r-1)!} - \frac{n!}{(r+1)!(n-r-1)!} = \frac{n!}{(r+1)!(n-r-1)!} \times \left(\frac{n+1}{n-r} - 1 \right)$	dm1

	$= \frac{n!}{(r+1)!(n-r-1)!} \times \left(\frac{r+1}{n-r} \right)$	ddM1
	$= \frac{n!}{r!(n-r)!} = {}^nC_r \Rightarrow {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1} *$	A1*

		(4)
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Notes:

Alt (ii)

M1: Attempts the shown difference of binomial coefficients using the formula. Allow if slips are made in the signs, e.g. $(n-r+1)!$ instead of $(n-r-1)!$

dm1: Splits the factorials and takes out the common factor. Alt per main scheme is putting over a common denominator but with the difference instead of sum. The denominator need not be simplified.

ddM1: Simplifies the factored term to a single fraction (oe if putting over common denominator).

A1*: Deduces equal to the required coefficient and concludes the requisite result.