Question	Scheme	Marks
1(a)	Remainder is $\mathrm{f}(2)=2\times 2^3-7\times 2^2+5\times 2+4=..$	M1
	$=2$	A1
		(2)
(b)	$\begin{array}{r} 2x^2\,\text{"}-3x-1\text{"} \\ x-2\,\big)\,2x^3-7x^2+5x+4 \\ 2x^3-4x^2 \\ \text{"}-3x^2\text{"}+5x+4 \\ -3x^2+6x \\ -x+4 \\ -x+2 \\ 2 \end{array}$	M1
	(Quotient is) $2x^2-3x-1$	A1
		(2)
		(4 marks)

(a) **Note that part (a) demands that the remainder theorem must be used.**

M1: Attempts f (2). Score for an attempt to substitute $x=2$ into the expression and proceed to a value.

See the main mark scheme. Condone slips.

E.g. $\mathrm{f}(2)=2\times 2^3\pm 7\times 2^2+5\times 2+4=..$ or $\mathrm{f}(2)=64-28+10+4=..$

Allow as evidence $\mathrm{f}(2)=2$ for both marks but $\mathrm{f}(2)=6$ for example, without sight of any intermediate working would be M0. Attempting to get the remainder via division alone is M0

A1: Correct answer following M1.

It must be the answer to part (a), their first answer or identified as $R=$, f (2) = o.e.

(b) Work done in part (a) can be scored in part (b).

M1: Attempts long division of f(x) by $x-2$

Look for a full attempt at division (including synthetic division) reaching at least a 3-term quadratic of the form $2x^2+ax+b$ and a constant remainder

The remainder may be 0 and may be implied.

For a full attempt at division look for the terms in x^3, x^2 and x to be fully 'cancelled out' as seen below and is acceptable even though the remainder has not been reached

$$
\begin{array}{r l}
 & \;2x^2\;"-3x-1" \\
x-2 & \big)\;2x^3-7x^2+5x+4 \\
 & \;\;2x^3-4x^2 \\
 & \qquad\;\;"-3x^2"+5x+4 \\
 & \qquad\;\;-3x^2+6x \\
 & \qquad\qquad\;\;-x+4 \\
 & \qquad\qquad\;\;-x+2
\end{array}
$$

A1: Correct quotient which **must** be seen separately to the long division sum.

It must be the answer to part (b) or identified as $Q=$ o.e.

Alternatives in (b)

M1: Subtracts their r from f(x) and attempts to factor out $(x-2)$

$$\frac{\mathrm{f}(x)}{x-2}=\mathrm{q}(x)+\frac{r}{x-2}\Rightarrow \mathrm{q}(x)(x-2)=\mathrm{f}(x)-r=2x^3-7x^2+5x+2$$

$$=(x-2)\left(2x^2+\ldots\pm1\right)$$

Alternatively sets $2x^3-7x^2+5x+4=(x-2)\left(2x^2+Px+Q\right)+'2'$ and attempts to find P and Q

A1: Correct quotient.

This **must** be separately stated, not as part of a factorised cubic unless clearly identified

Question	Scheme	Marks
2(i)	$1+\sin^2 x+\cos x=0 \Rightarrow 1+1-\cos^2 x+\cos x=0$	B1
	$\Rightarrow \cos^2 x-\cos x-2=0 \Rightarrow (\cos x-2)(\cos x+1)=0 \Rightarrow \cos x=...$	M1
	$\Rightarrow \cos x=-1 \Rightarrow x=\pi$	A1
		(3)
(ii)	$\sin(\theta+60^o)=4\cos(\theta+60^o) \Rightarrow \tan(\theta+60^o)=4$	B1
	$\Rightarrow \theta=\arctan('4')-60^\circ$	M1
	$=16.0^\circ$	A1
		(3)
		(6 marks)

(i) Condone omission of the variable *x* at any point in the solution

B1: Correct application of $\sin^2 x=1-\cos^2 x$ to the equation. E.g. sight of $1+1-\cos^2 x+\cos x=0$

M1: Forms and solves a 3-term quadratic in $\cos x$

- allow this mark to be awarded if sign errors are made e.g. $\sin^2 x=\pm 1\pm\cos^2 x$
- allow a solution of a 3TQ in $\cos x$ via any means including via a calculator leading to $\cos x=...$ If a calculator is used the roots (values of $\cos x$) must be correct for their quadratic
- do NOT allow a solution without the intermediate step. E.g. $\cos^2 x-\cos x-2=0 \Rightarrow \pi(\text{or } 180°)$

A1: Correct answer only. It must be exact and **must** follow B1, M1

If they correctly find $x=\pi$ and then follow this with 3.14 then allow. An answer of 180° is A0

Do not accept any other answers within the range $0<x\leqslant 2\pi$

(ii) Condone omission of degrees symbol at any point in the solution. Missing brackets can only be condoned if their subsequent work implies their intended presence.

B1: Correct application of '$\tan(\theta+60^\circ)=\sin(\theta+60^\circ)/\cos(\theta+60^\circ)$' to reach $\tan(\theta+60^\circ)=4$

You may see $x=\theta+60°$ in which case it would be for $\tan x=4$

M1: Correct process from $\tan(\theta+60^\circ)=k,\quad k\neq 0$ to reach a value for θ, allow $\theta=\arctan(k)\pm 60^\circ$

So $\tan(\theta+60^\circ)=\frac{1}{4} \Rightarrow \theta+60^\circ=14° \Rightarrow \theta^\circ=-46°$ or indeed 74° would score M1

Note that $\tan\left(\theta+60^{\circ}\right)=4 \Rightarrow \theta+60^{\circ}=1.326 \Rightarrow \theta^{\circ}=\ldots$ would score M0 due to mixed units

It is possible to use $\dfrac{\tan\theta+\tan 60}{1-\tan\theta\tan 60}=k \Rightarrow \tan\theta=p \Rightarrow \theta=\arctan p$

A1: Correct answer only. Accept awrt 16.0 but not 15.9 or 16 unless you previously see an allowable answer. It **must** follow B1, M1.

Condone extra solutions **only if** they are outside the range

Longer methods via squaring may be seen in (ii)

E.g. I

B1: Applies tan $\theta=\sin\theta/\cos\theta$ correctly in their solution to reach $\tan\theta=\dfrac{4-\sqrt{3}}{1+4\sqrt{3}}$

M1: Attempts both compound angle formulae, rearranges to tan $\theta=\ldots$ and applies arctan

$$\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta=4\left(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right)\Rightarrow\tan\theta=\tan\theta=\frac{4-\sqrt{3}}{1+4\sqrt{3}}\Rightarrow\theta=\tan^{-1}(\ldots)=\ldots$$

Condone sign slips in their compound angle identities and processing errors as allowed in the main method

A1: Correct answer following B1, M1

E.g. II

$$\sin^2\left(\theta+60^{o}\right)=16\cos^2\left(\theta+60^{o}\right)\Rightarrow\sin^2\left(\theta+60^{o}\right)=\frac{16}{17}\text{ or }\Rightarrow\cos^2\left(\theta+60^{o}\right)=\frac{1}{17}$$

B1: Correct equation. Either $\sin^2\left(\theta+60^{o}\right)=\dfrac{16}{17}$ or $\cos^2\left(\theta+60^{o}\right)=\dfrac{1}{17}$

M1: Correct process from, for example, $\sin^2\left(\theta+60^{o}\right)=p \qquad 0<p<1$

$$\sin^2\left(\theta+60^{o}\right)=p\Rightarrow\sin\left(\theta+60^{o}\right)=\sqrt{p}\Rightarrow\theta=\arcsin\left(\sqrt{p}\right)\pm 60^{o}$$

A1: Correct answer following B1, M1

Question	Scheme	Marks
3(a) (i)	$d = u_2 - u_1 = -6 - 3 = -9$	M1
	Also, $u_4 - u_2 = -24 - (-6) = -18 = 2\times -9$ Difference is consistent so could be arithmetic	A1
	$u_3 = -15$	B1
(ii)	$r = \dfrac{u_2}{u_1} = \dfrac{-6}{3} = -2$	M1
	Also, $\dfrac{u_4}{u_2} = \dfrac{-24}{-6} = 4 = (-2)^2$ Ratio is consistent so could be geometric	A1
	$u_3 = 12$	B1
		(6)
(b)	$u_1 = u_5 = u_9 = u_{13}$ etc $\Rightarrow \sum_{i=1}^{50} u_i = 12\times(3-6+u_3-24)+3-6$	M1
	$\Rightarrow 12(u_3-27)-3=141 \Rightarrow u_3 = \ldots$	dM1
	$u_3 = 39$	A1
		(3)
		(9 marks)

(a)(i)

M1: Attempts to find the common difference of the first two terms.

Implied by sight of ± 9 or $(-6)-(3)$ either way around

A1: Must be using a correct common difference of -9.

It requires a correct reason and minimal conclusion.

Acceptable reasons must include the first, second and fourth terms.

Examples of allowable reasons and conclusions are;

- if arithmetic $d=-9,\ u_4 = a+3d = 3+3\times -9 = -24$ ✓ (Calculation seen)
- $3, \quad -6, \quad -15, \quad -24$ terms **all** decrease by 9 so (could be) arithmetic (explained in words)

B1: $u_3 = -15$

A minimal possible solution scoring 3 marks is;

$3, \xrightarrow{-9} -6, \xrightarrow{-9} u_3, \xrightarrow{-9} -24$ scores M1,

followed by $u_3 = -15$ scores B1 (which may be seen in the list), followed by appropriate wording

‘Differences are constant, so (could be) arithmetic’ which scores the A1

Or ‘Common difference (in words, not just $d =$) is -9, so ✓ , which scores A1’

(a)(ii)

M1: Attempts to find the common ratio of two terms. Implied by sight of -2, $\frac{-6}{3}$ $-\frac{1}{2}$ or $\frac{3}{-6}$

A1: Must be using a correct common ratio of -2.

It requires a correct reason and minimal conclusion.

Acceptable reasons must include the first, second and fourth terms.

Examples of allowable reasons and conclusions are;

- if geometric $r=-2,\ u_4 = ar^3 = 3\times -2^3 = -24$ ✓ (Calculation seen)
- $3,\quad -6,\quad 12,\quad -24$ you multiply.by -2 **each time**, so geometric (explained in words)

B1: $u_3 = 12$

A minimal possible solution scoring 3 marks is;

$\times -2\ \ \times -2\ \ \times -2$
$3,\quad -6,\quad 12,\quad -24$ scores M1, B1

followed 'by multiplies by -2 each time, so (could be) geometric scores A1'

or 'common ratio (in words, not just $r=$) is -2 so ✓ scores A1'

(b)

M1: Attempts to write the summation as 12 sets of the sum of four consecutive terms plus the first two terms. Allow if there are sign slips in the two extra terms

Alternatively writes as a summation of 13 sets of the sum of four consecutive terms minus the last two terms. Allow if there are sign slips in the two extra terms.

E.g. Allow for $12\times(3-6+u_3-24)+3+6$

or $13\times(3-6+u_3-24)-24-u_3$ (both showing common sign slips)

dM1: Sets their sum equal to 141 (condoning slips e.g. 114) and solves to find u_3

Look for all of

- the award of the previous M1
- correct order of solving, condoning arithmetical slips only E.g.
  $12\times(3-6+u_3-24)+3+6=141$
  $12(u_3-21)+9=141 \Rightarrow 12(u_3-21)=132 \Rightarrow u_3-21=11 \Rightarrow u_3=32$

A1: Correct value.

Question	Scheme	Marks
4(i)	E.g. try [f (2) = −8, f (3) = 8, f (4) = 64] f (5) = ...	M1
	So, a counterexample is f (5) = 175	A1
	Since $5^3 = 125 < 175 < 216 = 6^3$ so f (5) cannot be a perfect cube.	A1
		(3)
(ii)	$(x+1)^2 - 6x + 4 = x^2 + 2x + 1 - 6x + 4$	M1
	$x^2 - 4x + 5 = (x-2)^2 - 4 + 5$	M1
	$(x-2)^2 + 1 > 0$ since $(x-2)^2 \geqslant 0$ for all x, hence $(x+1)^2 > 6x - 4$ *	A1*
		(3)
		(6 marks)

(i)

n	f(n)	f(n)^(1/3)
2.00	-8.00	-2.00
3.00	8.00	2.00
4.00	64.00	4.00
5.00	175.00	5.59
6.00	356.00	7.09
7.00	622.00	8.54
8.00	988.00	9.96
9.00	1469.00	11.37
10.00	2080.00	12.77
11.00	2836.00	14.15

M1: Shows evidence of searching for a counter example, e.g. attempts to use a positive integer greater than 1 in the expression. Note n = 1,2,3,4 all give perfect cubes, but this mark may be scored for the evidence of trying n = 2, 3 or 4 even if they think one is not a cube. Alternatively, attempting any greater integer is fine. If their answer is correct e.g. f (3) = 8, this is sufficient, but if it is incorrect you will need evidence such as sight of the 3 being substituted into f(n) condoning slips

A1: For the correct result for a suitable counterexample.

A1: Demonstrates/shows that their example fails. They are likely to do this by taking the cube root of their value and showing it is not an integer. All work should be correct, values to at least 1 d.p. with a minimal conclusion. A brief solution for 3 marks would be f (6) = 356 and $\sqrt[3]{356} \approx 7.1$ so statement is not true

(ii)

M1: Gathers terms on one side and expands the brackets. May be done as part of steps with inequalities (using equivalences or not).

M1: Attempts to complete the square on the gathered quadratic. (See general principles)

A1*: Completes the proof with correct work, reason and conclusion given.

This requires all of

- $x^2 - 4x + 5 \equiv (x-2)^2 + 1$
- A reason for $(x-2)^2 + 1 > 0$ such as $(x-2)^2 \geqslant 0$
- A conclusion that refers back to the given statement

Example I: Scores M1, M1, A0 as it lacks both a reason and a reference back to the given statement

$(x+1)^2 > 6x - 4 \Rightarrow x^2 + 2x + 1 - 6x + 4 > 0 \Rightarrow x^2 - 4x + 5 > 0 \Rightarrow (x-2)^2 + 1 > 0$ which is true.

Example II: Scores M1, M1, A0 as the reason is incorrect. It does refer back due to use of $\Leftrightarrow$

$(x+1)^2 > 6x - 4 \Leftrightarrow x^2 + 2x + 1 - 6x + 4 > 0 \Leftrightarrow x^2 - 4x + 5 > 0 \Leftrightarrow (x-2)^2 + 1 > 0$ which is true as $(x-2)^2 > 0$.

Example III: Scores M1, M1, A1

$(x-2)^2+1>0 \Rightarrow x^2-4x+5>0 \Rightarrow x^2+2x+1-6x+4>0 \Rightarrow x^2+2x+1>6x-4 \Rightarrow (x+1)^2>6x-4$

Other methods are possible once the candidates reach the gathered quadratic.

Alternative I

Using their x^2-4x+5 which must be a 3TQ

M1: Attempts BOTH $b^2-4ac=16-20=...$ AND shape of graph 'U shaped (may be a sketch)'

Alternatively, they could attempt $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{4\pm\sqrt{-4}}{2}$ AND shape of graph 'U shaped

A1: Concludes that

- $b^2-4ac=16-20=-4<0$ there are no roots
- curve $y=x^2-4x+5$ is always **above** the x-axis (which may be from a sketch)
- $x^2-4x+5>0 \Rightarrow (x+1)^2>6x-4$

Alternative II

Using their x^2-4x+5 which must be a 3TQ

M1: Attempts BOTH $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x-4=0$ AND shape of graph 'U shaped (may be a sketch)'

A1: Concludes that

- there is a minimum at $(2,1)$
- curve $y=x^2-4x+5$ is always above the x-axis
- $x^2-4x+5>0 \Rightarrow (x+1)^2>6x-4$

Alternative III

M1: Candidates can attempt this part using **BOTH** $x=2n$ and $x=2n+1$ but must make progress

with both aspects as far as the main scheme with the terms on one side of the 'equation'

E.g. with $x=2n$ a correct intermediate stage would be $4n^2-8n+5=4(n-1)^2+1$

with $x=2n+1$ a correct intermediate stage would be $4n^2-4n+2=4\left(n-\frac{1}{2}\right)^2+1$

If you see candidates who adopt this approach, make sufficient progress and you are not sure how to award then please send to review

Question	Scheme	Marks
5(a)	$h = 0.6$	B1
	Area $\approx \frac{0.6}{2}\left\{1.5+2\left(0.48+0.30+0.26+0.26+0.30+0.48\right)+1.5\right\}$	M1
	$0.3\times 7.16\left(\frac{0.3\times 179}{25}\right)=2.15\left(\text{m}^2\right)$	A1
		(3)
(b)	Total side area = $0.9\times 1.5\times 2+'2.148'=\left(4.848..\right)$	M1
	Volume $\approx 2.5\times '4.848'$	dM1
	$12.1\left(\text{m}^3\right)$	A1
		(3)
(c)	Overestimate with valid reason, e.g. area of trapezia are bigger than cross sectional area.	B1
		(1)
		(7 marks)

(a)

B1: Correct value for h stated or used. The mark may be implied by 0.3 × {} or $\frac{0.6}{2}\{....\}$

M1: Correct attempt at the trapezium rule with their value for h.

Look for $\frac{h}{2}\left\{1.5+1.5+2\left(0.48+0.30+0.26+0.26+0.30+0.48\right)\right\}$ o.e.

Allow the M1 if one of the middle values is missing or miscopied e.g. 0.84 for 0.48 but the overall shape of the formula must be correct. Condone a missing trailing bracket

If they write $\frac{0.6}{2}(1.5+1.5)+2\times\left(0.48+0.30+0.26+0.26+0.30+0.48\right)$ it is M0 unless followed by the correct answer.

May see $\frac{"0.6"}{2}\left\{3+4\left(0.48+0.30+0.26\right)\right\}$ if they realise the symmetry, or calculations involving individual trapezia.

A1: Correct answer, accept awrt 2.15. Units are not required. You may see $\frac{537}{250}$ which is fine

(b)

M1: Attempts to find the total area of cross section. Look for $0.9\times1.5\times2+'2.148'=$

Condone attempts where only one end is used. Look for $0.9\times1.5+'2.148'=$

This may be implied by a correct attempt at a volume calculation. See the dM1

So $(0.9\times1.5\times2+'2.148')\times2.5=\ (2.7+'2.148')\times2.5=$ which scores M1, dM1,

And $(0.9\times1.5+'2.148')\times2.5=(1.35+'2.148')\times2.5$ which scores M1, dM0

Longer methods exist. E.g. Cross-sectional area = rectangle - empty curved region;

Region above C is 4.2×1.5 – '2.148' = 4.152. So Cross-sectional area = $6\times1.5-4.152$

dM1: Correct attempt at a volume of the ramp. It is dependent upon the previous M mark

Look for $(0.9\times1.5\times2+'2.148')\times2.5=\ (2.7+'2.148')\times2.5=..$

or equivalent such as $0.9\times1.5\times2.5+0.9\times1.5\times2.5+'2.148'\times2.5=...$

Note that longer methods may be seen

A1: Correct answer, awrt 12.1 (m^3). Ignore any reference to units.

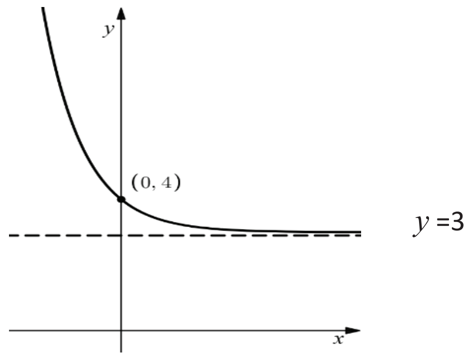
SPECIAL CASE 1, 0, 0: Some candidates mis-interpret the question and believe that the concrete is only used on the curved section

Calculates the concrete for the curved section or the curved section with one 'end'.

So, for M1, dM0, A0 the calculation $'2.148'\times2.5=$ with no additional work

(c)

B1: States **overestimate** AND gives a valid reason.
The reason, in almost all cases, should explicitly compare the area of the trapezia with the shaded area.
All reasons must make some reference to 'trapezia' or 'area of a trapezium'
The Figure could be used to explain their reasoning so look carefully at what the candidate states
Condone 'the trapezia are bigger than the cross section'
'the top lines of the trapezia are above the curve'
'the curve is **under** the lines of the trapezia'
'part of the trapezium is **above** the ramp'
Do NOT accept any reference to convex or concave without further clarification or trapezium lying outside the curve without a diagram to explain that it creates more area

Question	Scheme		Marks
6(i)(a)		Correct shape or correct y intercept	M1
		Correct shape, position and y intercept	A1
		Correct equation of the asymptote	B1
			(3)
(b)	$\log\frac{1}{5^x}=\log 20 \Rightarrow x=-\frac{\log 20}{\log 5}$ or e.g. $x=\log_{\frac{1}{5}} 20$		M1
	$=-1.86$		A1
			(2)
(ii)	(a) Addition law $\log_4 64x=\log_4 64+\log_4 x$		M1
	$3+a$		A1
			(2)
	(b) $\log_4\frac{2}{x^2}=\log_4 2-\log_4 x^2$		M1
	$\log_4\frac{2}{x^2}=\log_4 2-2\log_4 x$		dM1
	$\frac{1}{2}-2a$		A1
			(3)
			(10 marks)

(i)(a)

M1: For **either** of

- an exponential decay type shape, in any position. Be tolerant with "pen flicks" at each end
- the correct y intercept either stated as (0, 4) or labelled on the graph at 4. The graph must cross (not touch) the y - axis at this point. Condone (4, 0) as long as it is in the correct place

A1: Fully correct shape and position. Requires

- curve in quadrants 1 and 2 only
- Allowable gradients. Look at Practice and the Exemplar sheet
- y intercept labelled on graph as 4, (0, 4) or (4, 0) which is condoned

B1: For stating that $y=3$ is the asymptote to their graph. The curve must be asymptotic at $y=3$

It must be an equation, not just the line labelled at 3. It must be the only asymptote and the graph must appear to level out below the intercept

(b)

M1: Look for the following

- first of all, attempts to make $\frac{1}{5^x}$ or 5^x the subject
- then takes logs (to any base) leading to a value or expression that equates to $\pm x$.

For example, $\frac{1}{5^x}+3=23\Rightarrow\frac{1}{5^x}=20\Rightarrow(\pm)x=\frac{\log 20}{\log 5}$ or $\frac{\log 20}{\log\frac{1}{5}}$

$$\frac{1}{5^x}+3=23\Rightarrow\frac{1}{5^x}=20\Rightarrow 5^x=\frac{1}{20}\Rightarrow x=\log_5\frac{1}{20}$$

$$\frac{1}{5^x}+3=23\Rightarrow\frac{1}{5^x}=20\Rightarrow 5^x=\frac{1}{20}\Rightarrow x\log 5=\log\frac{1}{20}\Rightarrow x=-1.86$$

A1cao: Correct answer and must be to 3 s.f and must follow M1.

(ii) (a)

M1: Evidence of the addition law of logs on the given expression. Condone missing bases

E.g. writes $\log_4 64x=\log_4 64+\log_4 x$

A1: Correct answer in terms of *a*. Allow this to be written down for both marks. Do not ISW here

Withold this mark if they go on to solve an equation = 0 to find the value of *a*

(ii) (b)

M1: Correct application of subtraction law of logs on $\log_4\frac{2}{x^2}$. E.g. $\log_4\frac{2}{x^2}=\log_4 2-\log_4 x^2$ o.e.

They may use the addition law on $\log_4 2x^{-2}$ E.g. $\log_4 2x^{-2}=\log_4 2+\log_4 x^{-2}$

Also allow for correct use of power law of logs. E.g. for $2\log_4\frac{\sqrt{2}}{x}$. Condone missing bases

dM1: Uses both power and subtraction/addition laws **correctly** to reach either of the following

Look for $\log_4\frac{2}{x^2}=\log_4 2-2\log_4 x$ or $\log_4\frac{2}{x^2}=2\log_4\sqrt{2}-2\log_4 x$. Bases must be present

A1: Correct answer, as in scheme. The answer may be written down for all 3 marks. Do not ISW here

Withold this mark if they go on to solve an equation = 0 to find the value of *a*

Question	Scheme	Marks
7(a)	$\left(1+\frac{5}{16}x\right)^{10}=1+10\left(\frac{5}{16}x\right)+\frac{10\times9}{2}\left(\frac{5}{16}x\right)^2+\ldots$	M1 A1
	$\frac{25}{8}x$ or $\frac{1125}{256}x^2$	A1
	$=1+\frac{25}{8}x+\frac{1125}{256}x^2$	A1
		(4)
(b)	$x=\frac{1}{10}\Rightarrow\left(1+\frac{5}{16}x\right)^{10}=\left(1+\frac{5}{16}\times\frac{1}{10}\right)^{10}=\left(\frac{33}{32}\right)^{10}$	B1
	So $\left(\frac{33}{32}\right)^{10}\approx1+\frac{25}{80}+\frac{1125}{25600}$	M1
	$=\frac{1389}{1024}$	A1
		(3)
(c)	$\left(\frac{33}{32}\right)^{10}\approx\frac{1389}{1024}\Rightarrow\sqrt[10]{1389}\approx\frac{33}{32}\times\sqrt[10]{1024}$	M1
	$=\frac{33}{16}$	A1
		(2)
		(9 marks)

(a)

M1: Correct binomial coefficient linked with correct power of $\frac{5}{16}x$ for the second or third term of their expansion. Accept alternative notation, such as ${}^{10}C_1$ or just 10 etc, for the coefficients. (The power 1 need not be seen.) Allow if the brackets are omitted. Condone a misread on $\frac{5}{16}x$, e.g. $\frac{5}{6}x$

Look for either of the following

- a second term of $10\times\frac{5}{16}x$ or unsimplified version such as $\binom{10}{1}\times\frac{5}{16}x$
- a third term of $\frac{10\times9}{2}\times\left(\frac{5}{16}x\right)^2$ condoning unsimplified versions with bracketing slips such as ${}^{10}C_2\times\frac{5}{16}x^2$

A1: A correct unsimplified expansion with correct bracketing and no combination notation. See main scheme. Condone a list for this mark.

A1: Correct simplified second or third term.

A1: Fully correct and **simplified expansion**. A list is insufficient but award this mark if the correct expansion is seen or implied in part (b). Ignore any extra terms in x^3 etc even if they are incorrect. If they go on to multiply by 256 withhold this mark.

It may be awarded without sight of working, and/or with the **EXACT** decimal equivalents for $\frac{25}{8}$ and $\frac{1125}{256}$

Note: M1A0A1A0 is possible (e.g. if the 5/16 is not squared after a correct expression).

(b) Score (b) and (c) together

B1: Substitutes $x=\frac{1}{10}$ into $\left(1+\frac{5}{16}x\right)^{10}$ **AND SHOW** that it gives $\left(\frac{33}{32}\right)^{10}$.

Some may achieve $x=\frac{1}{10}$ by solving $1+\frac{5}{16}x=\frac{33}{32}$ which is fine as long as it is followed by the $\left(\frac{33}{32}\right)^{10}$ which may be seen in (c)

Look for $\left(1+\frac{5}{16}\times\frac{1}{10}\right)^{10}$ or $\left(1+\frac{5}{16}\times\frac{1}{10}\right)=\frac{33}{32}$ followed by $\left(\frac{33}{32}\right)^{10}$ which may be seen in (c)

M1: Substitutes $x=\frac{1}{10}$ into their $1+\frac{25}{8}x+\frac{1125}{256}x^2$ which may contain more terms

Score for sight of $1+\frac{25}{8}\times\frac{1}{10}+\frac{1125}{256}\times\left(\frac{1}{10}\right)^2$

A1: Simplifies to the correct fraction, $\frac{1389}{1024}$. This is dependent upon the M only

So $\left(\frac{33}{32}\right)^{10}=1+\frac{25}{8}\times\frac{1}{10}+\frac{1125}{256}\times\left(\frac{1}{10}\right)^2=\frac{1389}{1024}$ scores B0, M1, A1

(c) **Marks in part (c) cannot be scored unless the fraction is of the form 1389/*k***

M1: Takes the 10th root of both sides and multiplies by the denominator.

Award for $\left(\frac{33}{32}\right)^{10}=\frac{1389}{'k'}\Rightarrow\sqrt[10]{1389}=\frac{33}{32}\times\sqrt[10]{k}$.

A1: Correct answer $\frac{33}{16}$ but $\frac{66}{32}$ is acceptable.

If part (b) is correct we can award the correct answer both marks in part (c)

Question	Scheme	Marks
8(a)	$(x\pm4)^2\,..(y\pm6)^2=..$	M1
	Centre is $(4,-6)$	A1
	$r=\sqrt{16+36-27}=5$	A1
		(3)
(b)	Deduces that the x coordinate of B is 8	B1ft
	$\frac{dy}{dx}=0$ at $x='8'\Rightarrow 2ax-\frac{3}{2}=0\Rightarrow 16a-\frac{3}{2}=0$	M1
	$a=\frac{3}{32}$ *	A1*
		(3)
(c)	$\int \frac{3}{32}x^2-\frac{3}{2}x-3\,dx=..x^3+..x^2+..x$	M1
	$\int_0^{'8'}\frac{3}{32}x^2-\frac{3}{2}x-3\,dx=\left[\frac{1}{32}x^3-\frac{3}{4}x^2-3x\right]_0^{'8'}=\left[\frac{8^3}{32}-\frac{3}{4}\times8^2-3\times8\right]-(0)$	dM1
	$=-56$	A1
	$X=(8,0)$, Area trapezium $OABX=\frac{1}{2}(3+'9')\times'8'=...$	M1
	Area semi-circle $=\frac{1}{2}\pi\times'5'^2=\frac{'25'\pi}{2}$	B1ft
	Area $R=$ $'56'-'48'+'\frac{25\pi}{2}'$	dM1
	$=8+\frac{25\pi}{2}$	A1
		(7)
		(13 marks)

(a)

M1: Attempt to complete the square witnessed by sight of $(x\pm4)^2\,..(y\pm6)^2=...$

Implied by a centre given as $(\pm4,\pm6)$

A1: Centre is $(4,-6)$ o.e. Condone brackets not being present

A1: Radius is 5, not $\sqrt{25}$. M1 must be awarded but may be scored following an incorrect centre $(\pm4,\pm6)$

(b)

B1ft: States or implies the correct x coordinate for point B.

Follow through on twice their x coordinate for their centre. It may be awarded from work in (c) or on the graph. It is implied by their top limit in their integral

M1: Differentiates, sets equal to 0 and substitutes their x at B to set up an equation in a.

Alternatives exist:

Alt I: Completes the square and sets x coordinate of vertex equal to their x coordinate of B and solves for a. Alternatively they find the line of symmetry and set equal to their 8

E.g. $y = a\left(\left(x - \frac{3}{4a}\right)^2 - ...\right) - 3 \Rightarrow \frac{3}{4a} = '8'$

Alt II: Works out the coordinates of B and substitutes into the parabola equation to find a

E.g. $A = (0, -3) \to \text{Centre} = (4, -6) \to B = (8, -9)$ so B is $(8, -9) \Rightarrow -9 = 64a - 12 - 3$.

Any point that lies on the parabola apart from $(0, -3)$ would be suitable. So, via symmetry

$A = (0, -3) \to B = (8, -9) \to (16, -3)$ lies on curve, $\Rightarrow -3 = 256a - 24 - 3$

A1*: Fully correct. Note that this is a printed answer and you must see an appropriate method

(c)

M1: Integrates the quadratic to the form $..x^3 + ..x^2 + ..x$. There is no requirement for a constant of integration or it to be simplified. This may be awarded from anywhere in their solution, even (a)

Condone slips on the coefficients of the given quadratic before integrating, allow even with 'ax^2'

dM1: Substitutes their 8 (2 × the x coordinate of their centre of C) and 0 into the integral and subtracts either way round. It is dependent upon having integrated to the form $..x^3 + ..x^2 + ..x$

The substitution of 0 may be implied which is fine. This can be implied by a correct value for their $\left[..x^3 + ..x^2 + ..x\right]_0^8$ but $\int_0^8 \frac{3}{32}x^2 - \frac{3}{2}x - 3\,dx = \pm 56$ is M0, dM0 as there is no algebraic integration

A1: Correct result of integrating and substituting, accept either 56 or −56 as they may realise areas must be positive. May be implied if not seen directly (allow if integration was correct and a correct attempt to substitute was made and used in working).

M1: Attempts the area of trapezium $OABX$ where X is their (8,0). It is dependent upon

- the '8' used must be 2 × the x coordinate of their centre of C
- the '9' used must be the y coordinate of the curve or circle at x = '8'

An alternative could be to integrate the equation of line AB between 0 and 8. Condone slips

B1ft: Correct semi-circle area following through their radius from (a). May be left unsimplified

dM1: Correct overall strategy for finding the required area.

Look for an attempt at the area of integral minus trapezium plus semi circle

It is dependent upon a correct method for the area of ALL shapes which must be combined correctly

A1: Correct answer.

Alt for first 4 marks: Candidate attempts to combine line *AB* and curve

M1: Attempts integral of (quadratic − line AB) either way around. Condone slips on equation of AB

Look for $...x^2 + ...x \to ...x^3 + ...x^2$ but condone $...x^2 + ...x + ... \to ...x^3 + ...x^2 + ...x$

dM1: Applies limits 0 and their 8 and subtracts either way round. See main method for conditions

A1: Correct answer $= \pm 8$.

M1: For subtracting the equation of AB from the equation of the curve (or vice versa), integrating and applying limits. For example,

$$\pm\int_0^8 \left\{\left(\frac{3}{32}x^2-\frac{3}{2}x-3\right)-\left(-\frac{3}{4}x-3\right)\right\}\mathrm{d}x=\pm\left[\frac{3}{96}x^3-\frac{3}{8}x^2\right]_0^8$$

The method of finding the equation of line AB must be sound, but condone slips.

dM1: In this method it would be for adding together the area between the line and curve and the semi- circle

You may see alternatives where translations have been used, e.g. up 9 units where the trapezium becomes a triangle

Question	Scheme	Marks
9(a)	**States** that $x=2r+10$ o.e. **and** attempts $k\pi r^2=k\pi\left(\frac{x-10}{2}\right)^2$ where $k=1,\frac{1}{2},\frac{1}{4}$	M1
	Area = $xy+2\times\frac{\pi}{4}\times\left(\frac{x-10}{2}\right)^2 \Rightarrow A=xy+\frac{\pi}{8}(x-10)^2$ *	A1*
		(2)
(b)	$A=10000\Rightarrow y=\frac{10000}{x}-\frac{\pi}{8x}(x-10)^2$ $\Rightarrow P=2\left(\frac{10000}{x}-\frac{\pi}{8x}(x-10)^2\right)+\left(2+\frac{\pi}{2}\right)x-5\pi$	M1 A1
	$=\frac{20000}{x}-\frac{\pi}{4x}\left(x^2-20x+100\right)+\left(2+\frac{\pi}{2}\right)x-5\pi$ $=\frac{20000}{x}-\frac{\pi x}{4}+5\pi-\frac{25\pi}{x}+\left(2+\frac{\pi}{2}\right)x-5\pi=\frac{\cdots}{x}+(....)x+...$	M1
	$\Rightarrow P=\frac{20000-25\pi}{x}+\left(2+\frac{\pi}{4}\right)x$ *	A1*
		(4)
(c)	$\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)=-\frac{20000-25\pi}{x^2}+2+\frac{\pi}{4}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}x}=0\Rightarrow\frac{20000-25\pi}{x^2}=2+\frac{\pi}{4}\Rightarrow x^2=\frac{20000-25\pi}{2+\pi/4}\Rightarrow x=\ldots$	dM1
	$x=84.6$	A1
		(3)
(d)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{40000-50\pi}{x^3}>0$ for $x>0$	M1
	As $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}>0$ so x gives a minimum value for the perimeter.	A1
		(2)

(11 marks)

(a) This is a 'show that' and the answer is given, but condone $r = R$

M1: Look for a statement of $x = 2r + 10$ o.e, AND an attempt at the area of a full circle, semi-circle or quarter circle. Score for $\pi\left(\frac{x}{2}-5\right)^2$, $\frac{1}{2}\times\pi\left(\frac{x-10}{2}\right)^2$ or $\frac{1}{4}\times\pi\left(\frac{x-10}{2}\right)^2$ seen anywhere in (a)

A1*: Completes correctly to the given answer, no errors seen and suitable intermediate work. Look for

- A or area = being used at some point in the proof
- An intermediate line with the correct rectangle area xy being added to two quarter circles (or semi-circle) $\frac{1}{2}\times\pi\left(\frac{x-10}{2}\right)^2$ or unsimplified equivalent
- The final given result (Note: A = does not necessarily need to be on the final line)

(b) Note that this was originally marked M1, dM1, ddM1, A1. Now it is M1, A1, M1, A1

You may see candidates going back through their proof amending the number of 0's. Please look carefully at each line and award according to the scheme. The A marks cannot be awarded without the appropriate M mark.

M1: Finds y in terms of x using A = 10000 cm^2 and uses the result to find P in terms of x.

Look for $y = \frac{10000}{x} - \frac{\pi}{8x}(x-10)^2 \Rightarrow P = 2\left(\frac{10000}{x} - \frac{\pi}{8x}(x-10)^2\right) + \left(2+\frac{\pi}{2}\right)x - 5\pi$ o.e

Condone slips such as

- the number of 0's used in their 10000 but candidates **must not** be using just 1
- algebraic/bracketing/coefficient slips
- a lack of $P =$. Look for an expression in x with an appropriate form

A1: A fully correct (intermediate) expression (for P) in terms of x

M1: A valid attempt to expand the brackets and reach the required expression. Look for

- $(x-10)^2$ expanded to $x^2 \pm 20x \pm 100$ followed by an attempt to divide by kx
- finally an attempt to collect like terms

Condone an attempt with A = 1 but other aspects of the previous M1 should have been achieved.

A1*: Completes correctly to the given answer, no errors seen and suitable intermediate work. The $P =$ must be seen on at least one line but it does not need to appear on the last line

(c) Allow $\frac{dP}{dx}$ appearing as $\frac{dy}{dx}$, P' or y'. You may not even see the LHS as it is not required

M1: Attempts the derivative of the given expression (allowing for miscopies) and proceeds to the form $\pm\frac{A}{x^2} + B, \quad A, B > 0$

dM1: Sets derivative of the form $-\frac{A}{x^2} + B, \quad A, B > 0$ equal to 0 (which could be implied) and solves to find x. Don't be concerned with the particular method of solving the equation, but it should involve a $\sqrt{}$ It can be implied by a correct value for their equation (which must be in the correct form,)

A1: Correct value, awrt 84.6 (ignore reference to units).

You may see attempts via decimal equivalents which is acceptable if values are given to 3sf+. E.g. $P = \dfrac{19921}{x} + 2.785x \Rightarrow \dfrac{\mathrm{d}P}{\mathrm{d}x} = -\dfrac{19921}{x^2} + 2.785$

(d)

M1: Attempts the second derivative to $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} = \pm\dfrac{C}{x^3}$ and considers its sign or value.

Allow $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2}$ appearing as $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$, P'' or y''

So, allow a calculation of their $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} = \pm\dfrac{C}{x^3}$ at the value of x found in part (c) condoning slips

Alternatively score for example for $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} = \dfrac{40000}{x^3}$ with statement that $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} > 0$

A1: Requires

- Correct second derivative using suitable notation $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2}$ condoning use of P''
- Correct value of x **(to 2sf)** and reason e.g. $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} = 0.07 > 0$ or as $x > 0$, $\dfrac{\mathrm{d}^2 P}{\mathrm{d}x^2} > 0$
- Correct conclusion 'hence minimum'