Question Number	Scheme	Marks
1 (a)	$54=\frac{9}{2}\left(2a+8d\right)$	M1
	$\Rightarrow a+4d=6$ *	A1*
		(2)
(b)	Correct equation $a+7d=\frac{1}{2}\left(a+6d\right)$	B1 (M1 on EPEN)
	Substitutes or eliminate terms $\Rightarrow 6-4d+7d=\frac{1}{2}\left(6-4d+6d\right)$ $\Rightarrow d=\ldots$ (or $a=\ldots$)	M1
	$d=-1.5$ (o.e.) , $a=12$	A1, A1
		(4)
		(6 marks)

(a)

M1: Attempts to use e.g. $S_9=\frac{9}{2}\left(2a+8d\right)$ or $S_9=\frac{9}{2}\left(2a+\left(9-1\right)d\right)$ with $S_9=54$

Alternatively uses $S_9=\frac{9}{2}\left(a+l\right)$ with $S_9=54$ and $l=a+8d$

or lists terms $a+a+d+\ldots+a+8d=54\Rightarrow 9a+36d=54$ scored when a correct equation is achieved (i.e. without …).

There is no need to see the notation S_9

A1*: Proceeds to the given answer via a correct equation as in the main scheme.

There is no need to see any rearranging between this and the given answer.

Just $9a+36d=54\Rightarrow a+4d=6$ scores M0A0*: a valid formula or listing of terms in a sum must be seen.

(b)

B1: Correct equation e.g., $a+7d=\frac{1}{2}\left(a+6d\right)$ or $2\left(a+\left(8-1\right)d\right)=a+\left(7-1\right)d$

There is no need to see any notation e.g. u_8

Note that the simplified equation is $a+8d=0$

M1: Uses both equations, condoning a slip such as misplacing the $\frac{1}{2}$, e.g., $\frac{1}{2}\left(a+7d\right)=a+6d$, **and** solves simultaneously to find a value for *a* **or** *d*.

This mark depends on the use of acceptable expressions for u_7 and u_8.

Don't be concerned with the process of solving their simultaneous equations provided they have two acceptable equations and arrive at a solution for *a* **or** *d*.

Question Number	Scheme	Marks
2 (a)	$u_2 = 2 - \frac{4}{3} = \frac{2}{3}$, $u_3 = 2 - \frac{4}{"\frac{2}{3}"} = -4$, $u_4 = 2 - \frac{4}{"-4"} = 3$	M1 A1 A1
		(3)
(b)	$\{u_{61} =\}\ 3$	B1
		(1)
(c)	$\left\{\sum_{i=1}^{99} u_i =\right\} \frac{99}{3} \times \left(3 + "\frac{2}{3}" + "-4"\right)$	M1
	$= -11$	A1
		(2)
		(6 marks)

(a)

M1: Attempts to use the formula correctly using **values** at least once which may be implied by $u_2 = \frac{2}{3}$

or

e.g. by $u_2 = 5, u_3 = \frac{6}{5}$ where their u_3 follows from their u_2 Not scored for e.g. just $u_2 = 2 - \frac{4}{u_1}$

A1: 2 correct answers. Do not allow e.g. 0.66 or 0.66… for $\frac{2}{3}$ but allow $0.\dot{6}$

A1: All 3 correct. If the terms are not labelled, then the order must be clear to score this mark.

(b)

B1: cao. Must be clearly identified as their answer to part (b) or labelled as u_{61}

They do not need to show that they have used a period of 3 in their answer, but if they have **clearly** used a period of e.g. 4 in part (b), withhold the mark.

(c)

M1: Uses a valid method. Note that $3 + \frac{2}{3} - 4 = -\frac{1}{3}$ and you might see $99 + 22 - 132 = -11$

A1: cao. Answer only scores both marks.

Question Number	Scheme	Marks
3 (a)	"Strip width" or $h = 2$	B1
	Area $\approx \frac{"h"}{2}\left(\{0+\}\ 2.8284+2\times(1.4142+2+2.4495)\right)$	M1
	$=$ awrt 14.556	A1
		(3)
(b)	$\left\{\int_{-2}^{6}\left(2x+\sqrt{4x+8}\right)\mathrm{d}x=\right\}\ \left[x^2\right]_{-2}^{6}+2\times"14.556"$	M1
	$=6^2-\left(-2\right)^2+2\times"14.556"$	dM1
	$=$ awrt 61.1S	A1
		(3)
		(6 marks)

(a) **Note: Algebraic integration in part (a) scores 000. They can go on to score 110 in part (b).**

B1: Uses a strip width of 2 units. This may be embedded in the (correct) trapezium rule or seen as $h = 2$.

M1: Uses the correct form of the trapezium rule.

Look for $\frac{\cdots}{2}(2.8284+2\times(1.4142+2+2.4495))$ condoning transcription errors but they must be using the y values in the correct places.

Just $2.8284+2\times(1.4142+2+2.4495)$ scores B1M1 (and the A1 is available if they get awrt 14.556)

Note that $\frac{2}{2}(0+2.8284)+2\times(1.4142+2+2.4495)$ is acceptable unless it follows an incorrect formula e.g. $\frac{h}{2}(y_0+y_4)+2\times(y_1+y_2+y_3)$

A1: awrt 14.556. Note the unrounded value is 14.5558 and scores full marks.
Calculator integration achieves 15.085 (to 3 d.p.) and scores no marks unless a valid attempt is seen.
Ignore any units or e.g. u^2 if seen.

(b)

M1: For an attempt to split up the integral and achieve an expression of one of the below forms:
Look for e.g. $\left[x^2\right]_{-2}^{6}+k\times"14.556"$ **or** $32+k\times"14.556"$ **or** $\left[\alpha x^n\right]_{-2}^{6}+2\times"14.556"$ with $n\neq 1$ but allow $k=1$ for this mark. They may attempt the two parts separately but must combine for M1.
The $\left[x^2\right]_{-2}^{6}$ might be seen as e.g. $\frac{1}{2}\times6\times12-\frac{1}{2}\times2\times4$ or $\frac{1}{2}\times6\times12+\frac{1}{2}\times2\times\left(-4\right)$ if they treat it as the area of two triangles or they might apply the trapezium rule to $2x$ only (which should give the correct 32).
Condone slips in the base and/or height of the triangles if the other aspect $2\times"14.556"$ is correct.

dM1: Full and correct **method** using their answer to part (a) to find a value.

Question Number	Scheme	Marks
4 (a)	$\text{f}(2)=0 \Rightarrow 24+4a+2b-10=0$	M1
	$\Rightarrow 2a+b=-7$ *	A1*
		(2)
(b)	$\text{f}(-1)=-36 \Rightarrow -3+a-b-10=-36 \quad \{\Rightarrow a-b=-23\}$	M1, A1
	Solve simultaneously $\Rightarrow a=-10, b=13$	dM1, A1
		(4)
(c)	$\text{f}(x)=3x^3-10x^2+13x-10$	
	Full method to find both quotient and remainder	
	e.g. $3x^3-10x^2+13x-10 \equiv (x-5)(3x^2+5x+38)+180$	M1, A1
	States quotient is $(3x^2+5x+38)$ and remainder is 180	A1
		(3)
		(9 marks)

(a) **Note that attempts using algebraic division in part (a) score no marks.**

M1: Attempts to set $\text{f}(2)=0$ to form an equation in a and b, e.g., $3(2)^3+a(2)^2+2b-10=0$
The = 0 may be implied for this mark only, by e.g., $\text{f}(2)=24+4a+2b-10 \Rightarrow -14=4a+2b$
but not by proceeding directly to the given answer e.g. $\text{f}(2)=24+4a+2b-10 \Rightarrow -7=2a+b$

A1*: Shows that $2a+b=-7$ via an intermediate line.
$\text{f}(2)=0$ must be present but may be seen as e.g. f (2) = … = 0
Allow e.g. when $x = 2$, f(x) = … = 0
Some examples:
$\text{f}(2)=3(2)^3+a(2)^2+2b-10 \Rightarrow 2a+b=-7$ scores M0A0* (no equation prior to given answer)
$3(2)^3+a(2)^2+2b-10=24+4a+2b-10=0 \Rightarrow 2a+b=-7$ scores M1A0* (no $\text{f}(2)=0$)
$\text{f}(2)=24+4a+2b-10=0 \Rightarrow 2a+b=-7$ scores M1A1*
$\text{f}(2)=24+4a+2b-10 \Rightarrow 14+4a+2b=0 \Rightarrow 2a+b=-7$ scores M1A1*

(b)

M1: Sets $\text{f}(-1)=-36$ to form an equation in a and b, e.g., $3(-1)^3+a(-1)^2-b-10=-36$
Alternatively, attempts to divide f(x) algebraically by (x + 1) leading to a quotient of the form $(3x^2+Px+S)$ with remainder T where P, S and T are functions of a and/or b, followed by setting their remainder T equal to –36. For reference $P = a – 3$, $S = b + 3 – a$ and $T = a – b – 13$

A1: Correct unsimplified equation in a and b.
Any indices must be correctly evaluated but terms do not need to be collected.
e.g., $-3+a-b-10=-36$ or $a-b=-23$ o.e.

dM1: Solves simultaneously to find values for a **and** b. Dependent upon the previous M mark.
Don't be concerned with the process of solving their simultaneous equations provided they are using the given answer to (a) (or a miscopy) and arrive at a solution for a **and** b.

A1: $a=-10, b=13$ following award of all previous marks in part (b).

Question Number	Scheme	Marks
5	$4x^{-\frac{1}{2}} \rightarrow ...x^{\frac{1}{2}}$ **or** $k \rightarrow ...x$	M1
	$\left\{\int \left(\frac{4}{\sqrt{x}}+k\right)\mathrm{d}x=\right\} \int \left(4x^{-\frac{1}{2}}+k\right)\mathrm{d}x = \frac{4x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+kx \ \{+c\}$	A1
	$\left[\frac{4x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+kx\right]_2^4 = 30 \Rightarrow \left(8\sqrt{4}+4k\right)-\left(8\sqrt{2}+2k\right)=30$	dM1
	$\Rightarrow 2k+16-8\sqrt{2}=30$	
	$\Rightarrow k=7+4\sqrt{2}$	ddM1A1
		(5)
		(5 marks)

Note: Condone spurious inclusion of $\int$ throughout.

M1: Increases the power by one on a correct term. Score for either $4x^{-\frac{1}{2}} \rightarrow ...x^{\frac{1}{2}}$ or $k \rightarrow ...x$ where … is a constant and may be unsimplified.

The indices must be evaluated, so do not score for $4x^{-\frac{1}{2}} \rightarrow ...x^{-\frac{1}{2}+1}$

Candidates that write e.g. $\frac{4}{\sqrt{x}}=4x^{\frac{1}{2}} \rightarrow \frac{4}{\left(\frac{3}{2}\right)}x^{\frac{3}{2}}$ or e.g. $\frac{4}{\sqrt{x}}=4x^{-2} \rightarrow \frac{4}{(-1)}x^{-1}$ will not score this mark (unless they integrate k to … x).

A1: Correct integration (may be unsimplified) $\frac{4x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+kx \ \{+c\}$ ignoring limits.

The indices must be evaluated, so do not score for $4x^{-\frac{1}{2}} \rightarrow ...x^{-\frac{1}{2}+1}$
There is no need for $+c$ in the whole question.

dM1: Substitutes both $x=4$ and $x=2$ into an integrated expression (at least one term must have been integrated), subtracts the correct way round and sets equal to 30.
Condone missing brackets which lead to a sign error, e.g., $8\sqrt{4}+4k-8\sqrt{2}+2k=30$
It is dependent upon the previous method mark i.e. there must be some clear algebraic integration.
If there is no algebraic integration but you think it may be implied then send to review.

ddM1: Dependent upon both previous method marks **and** having correctly integrated k to kx
It is for an attempt to solve the resulting equation in k leading to a value which may be inexact.
Condone slips when solving their equation: do not be concerned about their processing provided they proceed from a linear equation in k to a value for k.

Question Number	Scheme	Marks
6.(a)	{Mid-point of XY =} (3, 7)	B1
	gradient $XY = \frac{11-3}{6-0}$ $\left\{=\frac{4}{3}\right\}$	M1
	$y-"7"="-\frac{3}{4}"(x-"3")$	dM1
	$3x + 4y - 37 = 0$	A1
		(4)
(b)	Substitute $y = 10$ into their line equation to give $x = \ldots$	M1
	$x=-1$	A1
		(2)
(c)	e.g. $r^2=("-1"-0)^2+(10-3)^2$ or $r=\sqrt{("-1"-6)^2+(10-11)^2}$	M1
	$(x+1)^2+(y-10)^2=50$	dM1, A1
		(3)
(d)	Full method to find W or e.g. $(\ldots, 9)$ **or** $("-8", \ldots)$	M1 (B1 on EPEN)
	$(-8, 9)$	A1 (B1 on EPEN)
		(2)
		(11 marks)

Note: Throughout the question be generous with parts that are mislabelled or not labelled at all, provided it is clear what their intention is.

(a)

B1: Correct coordinates for $\{M =\}$ (3, 7)

M1: Correct method for gradient of $XY = \frac{11-3}{6-0}$ $\left\{=\frac{4}{3}\right\}$

An alternative is to set up and solve an equation e.g. $11=6m+3$ leading to $m = \ldots$

dM1: Correct method for the equation of the line through Z and their M (which must not be X or Y).

Look for use of the negative reciprocal of their $\frac{4}{3}$ with their (3, 7), e.g., $y-"7"="-\frac{3}{4}"(x-"3")$

If using $y = mx + c$ they must proceed as far as $c = \ldots$ Dependent on the previous method mark.

A1: $3x+4y-37=0$ but allow any integer multiple of this and any order of terms.

The = 0 must be present. $3x+4y=37$ is A0 without also giving an alternative in the required form.

(b)

M1: Substitutes $y = 10$ into their $3x + 4y - 37 = 0$ (or earlier form of this equation) and solves to find a value for x

or Alt uses the fact that $XZ = YZ$ to give $x^2+7^2=(x-6)^2+1^2$ and solves by expanding and then cancelling the x^2 terms to find a value for x

A1: cao $x=-1$ Note: if they use an equation from part (a) it must be correct.

If using **Alt** then you do not need to check their equation in part (a).

(c)

M1: Valid method to find the radius or radius^2, usually using Pythagoras with $("-1", 10)$ and either X or Y. Alternatively, uses $MZ^2 + MY^2 = r^2$ or $MZ^2 + MX^2 = r^2$

dM1: Correct method for the equation of the circle $(x\pm"-1")^2+(y\pm10)^2="\sqrt{50}"^2$ using their radius found using a valid method and condoning sign errors. Dependent on the first method mark.

A1: cao $(x+1)^2+(y-10)^2=50$ must be simplified. Do not allow e.g. $(x+1)^2+(y-10)^2=\sqrt{50}^2$
This mark is dependent on scoring the A1 in part (b) because we require that $x=-1$ comes from correct working.
ISW after a correct answer is seen e.g. if they go on to attempt to expand the brackets.
Those who fortuitously achieve a correct equation using x = 1 will score A0.

(d)

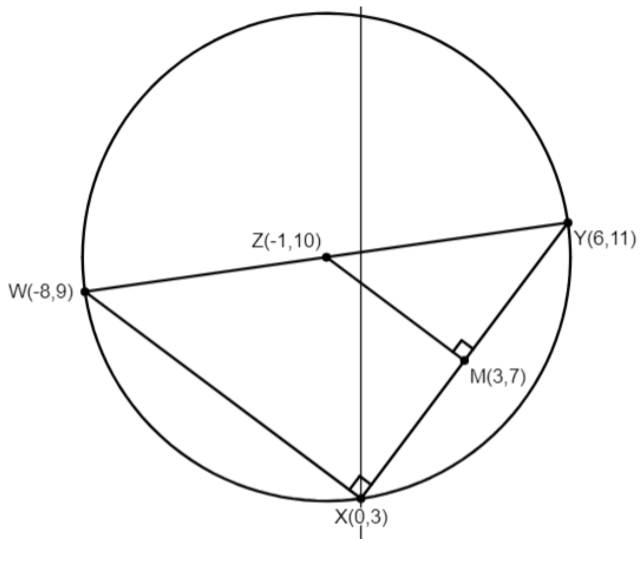
M1: For a full **method** as far as a value for x **or** y which *would* lead to a correct value for either coordinate of W following from their answers to previous parts of the question.
May be implied by one correct coordinate following through on their $x=-1$ using the main scheme, so either $(...,9)$ or $("-8",...)$. Follow through on $6-2\times(6-("-1"))$ for the x coordinate.
Allow this mark if they slip and write (9, –8).

Note that there are many ways to find the coordinates of W.
In any of these approaches, condone slips in copying or arithmetic mistakes but the core method must be valid, so any quadratic equation created **must** be solved using an appropriate strategy which may be by calculator. Do not be concerned by how they label any lines e.g. mislabel XW as YW.
Some examples are:

- Using $\overrightarrow{OY}+2\overrightarrow{YZ}$ (as in the main scheme).
- Finding the equation of XW $\left(y-3="-\frac{3}{4}"x\right)$ and solving simultaneously with the equation of their circle.
- Finding the equation of YW $\left(y-11="\frac{1}{7}"(x-6)\text{ or } y-10="\frac{1}{7}"(x-"-1")\right)$ and solving simultaneously with the equation of their circle.
- Finding the equation of XW and the equation of YW (as above) and solving simultaneously.
- Finding $\overrightarrow{OX}+2\overrightarrow{MZ}$
- Using Pythagoras $XY^2+XW^2=(2r)^2$ with $W(a, b)$ lying on any of XW, YW or the circle.

Any of these approaches may let $W(a, b)$ first: e.g. in bullet 2, let $W(a, b)$ then gradient of WX is $\frac{b-3}{a}=-"\frac{3}{4}"$ leading to $a=4-\frac{4}{3}b$ which can then be substituted into the equation of the circle or their YW.

A1: cao $(-8,9)$ and no others seen, i.e., if two possible pairs of coordinates are given then A0.
This mark is dependent on scoring the A1 in part (b) because we require that $x=-1$ comes from correct working.



Question Number	Scheme	Marks
7 (a)	$100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$	M1
	$y = \frac{100}{x} - \frac{\pi x}{8}$	A1
	$P = 2y + x + \frac{1}{2}\pi x = \frac{200}{x} - \frac{\pi x}{4} + x + \frac{1}{2}\pi x$	dM1
	$P = \frac{1}{4}x(4+\pi) + \frac{200}{x}$ *	A1*
		(4)
(b)	$\left\{\frac{dP}{dx} =\right\} 1 + \frac{\pi}{4} - \frac{200}{x^2}$	B1
	$1 + \frac{\pi}{4} - \frac{200}{x^2} = 0 \Rightarrow x^2 = \frac{800}{4+\pi}$	M1
	$\Rightarrow x = \sqrt{\frac{800}{4+\pi}}$	A1
		(3)
(c)	$\frac{d^2P}{dx^2} = \frac{\lambda}{x^3} = \frac{\lambda}{"10.6"^3} = ...$ or makes reference to the sign of $\frac{d^2P}{dx^2} = \frac{\lambda}{x^3}$	M1
	e.g. $\frac{d^2P}{dx^2} = \frac{400}{x^3}$ so (since $x > 0$) $\frac{d^2P}{dx^2} > 0 \Rightarrow$ minimum	A1
		(2)
(d)	Substitutes $x = \sqrt{\frac{800}{4+\pi}} \approx 10.6$ into $P = \frac{200}{x} + \frac{1}{4}x(4+\pi) = ...$	M1
	$=$ awrt 37.8 {m}	A1
		(2)
		(11 marks)

(a)

M1: Sets $100 = xy + kx^2$ where k is a non-zero constant e.g. $100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ (may be implied by later work)

A1: Achieves $y = \frac{100}{x} - \frac{\pi x}{8}$ or equivalent correct y in terms of x e.g. $y = 100x^{-1} - \frac{\pi x}{8}$ or $y = \frac{800 - \pi x^2}{8x}$

dM1: Substitutes their y in terms of x into their expression for the perimeter. Don't be concerned about the mechanics of rearranging the subject for this mark.
Dependent on the previous method mark.

A1*: Full proof showing all relevant lines with no errors or omissions.
There is no need to collect terms in x prior to factorisation but there must be evidence of correct substitution of a correct y in terms of x into a correct P in terms of y and x before the given answer.
The $P =$ (allow perimeter = or even perimeter:) must be present at some stage but accept if it is first seen in the answer line.
Condone $200x^{-1}$ in place of $\frac{200}{x}$ and allow the terms in any order.

(b) Note: Full marks in (b) can be scored from use of $\frac{dy}{dx}$ or $f'(x)$ in place of $\frac{dP}{dx}$ (or P')

B1: Fully correct derivative $\left\{\frac{dP}{dx}=\right\}1+\frac{\pi}{4}-\frac{200}{x^2}$ o.e e.g. $\left\{\frac{dP}{dx}=\right\}\frac{4+\pi}{4}-200x^{-2}$

The indices must be processed, i.e., not $\left\{\frac{dP}{dx}=\right\}1+\frac{\pi}{4}-200x^{-1-1}$

M1: Sets their $\frac{dP}{dx}=0$ which must be of the form $A-Bx^{-2},\ A,B>0$ **and** proceeds at least to $x^{\pm 2}=C,\ C>0$ which may be implied by their solution of e.g. $D=Ex^{-2},\ D\times E>0$
Alternatively, they may use the quadratic formula. Condone the use of an inequality for this mark.

A1: cao $x=\sqrt{\frac{800}{4+\pi}}$ o.e. such as $\frac{20\sqrt{2}}{\sqrt{4+\pi}}$ or $\frac{20\sqrt{8+2\pi}}{4+\pi}$ and ISW after an acceptable answer.
It must be exact and in a simple form and **not** a decimal 10.58… or embedded fractions e.g.
$\sqrt{\frac{200}{1+\frac{\pi}{4}}}$ or $\frac{10\sqrt{2}}{\sqrt{1+\frac{\pi}{4}}}$ are A0 $\pm\sqrt{\frac{800}{4+\pi}}$ is also A0. Must now be an equality and not e.g.
$x>\sqrt{\frac{800}{4+\pi}}$

(c) **Notes: All marks in (c) and (d) can be scored if x = 10.6 or better.**
Mark part (c) and part (d) together.

M1: For an attempt to

- Either substitute their answer to part (b) into $\frac{d^2P}{dx^2}=\frac{\lambda}{x^3}=\ldots$ and find a value (you may need to check if this is implied)
- Or make reference to the sign of $\frac{d^2P}{dx^2}=\frac{\lambda}{x^3}$
- Alternatively, they may substitute x values either side of their value found in part (b) into their $\frac{dP}{dx}$ to find two values for $\frac{dP}{dx}$

A1: cso Requires a correct $\frac{d^2P}{dx^2}=\frac{400}{x^3}$, a value of 10.6 or better in part (b), a statement and a reason, e.g. $\frac{d^2P}{dx^2}=\frac{400}{x^3}>0\Rightarrow$ minimum
If they calculate the value of the 2nd derivative it must be 0.3 (1sf) or better and they must state > 0
If using the 2nd derivative they must use the notation $\frac{d^2P}{dx^2}$ or P'' for this mark (condone $\frac{dP^2}{d^2x}$)
In the alternative, the two values substituted in must be either side of $\sqrt{\frac{800}{4+\pi}}$, the values of $\frac{dP}{dx}$ must be correct to 1sf, they must use the notation $\frac{dP}{dx}$ (or P') and they must conclude that the gradient changes from negative to positive implying a minimum point (or equivalent).

Question Number	Scheme	Marks
8(a)	$\dfrac{\cos\theta}{2\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$	M1
	$\cos^2\theta = 2\sin\theta(1+\sin\theta) \Rightarrow 1-\sin^2\theta = 2\sin\theta(1+\sin\theta)$	dM1
	$\Rightarrow 1-\sin^2\theta = 2\sin\theta + 2\sin^2\theta \Rightarrow 3\sin^2\theta + 2\sin\theta - 1 = 0$ *	A1*
		(3)
(b)	$3\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{3},\ \{-1\}$	B1
	Uses $\cos^2\theta \equiv 1-\sin^2\theta \Rightarrow \cos^2\theta = 1-\left(\text{"}\frac{1}{3}\text{"}\right)^2$	M1
	$\Rightarrow \{\cos\theta =\}\ \dfrac{2\sqrt{2}}{3}$	A1
		(3)
(c)	Attempts common ratio $= \dfrac{\cos\theta}{2\sin\theta} = \dfrac{\left(\text{"}\frac{2\sqrt{2}}{3}\text{"}\right)}{\left(\text{"}\frac{2}{3}\text{"}\right)} = \ldots \{\sqrt{2}\}$	M1
	Finds $ar^{10} = \text{"}\frac{2}{3}\text{"} \times \left(\text{"}\sqrt{2}\text{"}\right)^{10} = \ldots$	dM1
	$\dfrac{64}{3}$	A1
		(3)
		(9 marks)

(a)

M1: Uses common ratios and forms a correct equation in terms of θ, e.g.,

$u_1u_3 = u_2^2 \Rightarrow \cos^2\theta = 2\sin\theta(1+\sin\theta)$ or e.g. $1+\sin\theta = 2\sin\theta\left(\dfrac{\cos\theta}{2\sin\theta}\right)^2$

dM1: Cross multiplies (if necessary) and uses $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to form a quadratic equation in $\sin\theta$
Dependent on the previous method mark.

A1*: Expands the brackets and proceeds correctly to $3\sin^2\theta + 2\sin\theta - 1 = 0$ with no errors.
They must have a correct intermediate line i.e. expanded any brackets before the given answer.
Condone the occasional slip in missing or incorrect arguments or use of e.g. $\sin\theta^2$ but penalise consistent errors in the main body of their work (not their side working). Requires the = 0.

(b)

B1: Solves the given equation find $\sin\theta = \dfrac{1}{3}$ Allow the use of a calculator to solve the 3TQ here.
May be scored in part (a). Ignore any reference to $\sin\theta = -1$ for this mark only.
Not scored for e.g. $x = \sin\theta$ followed by $x = \dfrac{1}{3}$ unless used later in e.g. $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$

M1: Uses $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to achieve an exact value for $\cos^2\theta$, i.e., $\cos^2\theta = 1 - \sin^2\theta = 1 - \left("\frac{1}{3}"\right)^2$ or an exact value for $\cos\theta$ which may come directly from a right-angled triangle with sides that are multiples of 1, $2\sqrt{2}$ (or $\sqrt{8}$) and 3 (or otherwise for their $\sin\theta = ...$ provided $\sin\theta \neq \pm 1$ or 0)

Condone if they return to $\frac{\cos\theta}{2\sin\theta} = \frac{1+\sin\theta}{\cos\theta} \Rightarrow \frac{\cos\theta}{2\left(\frac{1}{3}\right)} = \frac{1+\left(\frac{1}{3}\right)}{\cos\theta} \Rightarrow \cos^2\theta = ...$ for this mark.

Alternatively, condone if they find r through e.g. $u_3 = u_1 r^2 \Rightarrow r^2 = \frac{1+"\frac{1}{3}"}{2\left("\frac{1}{3}"\right)} = ...\{2\}$ **and** go on to find $u_2 = u_1 r = 2\left("\frac{1}{3}"\right)"\sqrt{2}" = ...$ Note that this approach will also score the first M1 in part (c).

Attempting $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ $\{= 0.9428...\}$ is not an acceptable method.

A1: $\{\cos\theta =\}\ \frac{2\sqrt{2}}{3}$ only (o.e e.g. $\{\cos\theta =\}\ \frac{\sqrt{8}}{3}$) and ISW after a correct answer.

(c) **No marks can be scored in part (c) from use of $\sin\theta = \pm 1$ as these trivialise the solutions. Similarly, no marks can be scored if their $\sin\theta$ or $\cos\theta$ are inexact.**

M1: Attempts common ratio using either $\frac{\cos\theta}{2\sin\theta} = \frac{"\frac{2\sqrt{2}}{3}"}{2\left("\frac{1}{3}"\right)} = ...\left\{\sqrt{2}\right\}$ or $\frac{1+\sin\theta}{\cos\theta} = \frac{1+"\frac{1}{3}"}{"\frac{2\sqrt{2}}{3}"} = ...\left\{\frac{2}{\sqrt{2}}\right\}$ as far as a value but may be implied. Note this may be seen in (b).

May come from equivalent trigonometric expressions e.g. $\frac{1}{2}\cot\theta = \frac{2\sqrt{2}}{2}$ $\left\{= \sqrt{2}\right\}$

Alternatively, attempts r^2 through e.g. $u_3 = u_1 r^2 \Rightarrow r^2 = \frac{1+"\frac{1}{3}"}{2\left("\frac{1}{3}"\right)} = ...\{2\}$ which may be seen in (b).

dM1: Finds $ar^{10} = "\frac{2}{3}" \times \left("\sqrt{2}"\right)^{10} = ...$ where a is their $2\sin\theta$

Dependent on the previous method mark but note that the marks may be scored simultaneously.

You may see $ar^{10} = "\frac{2}{3}" \times \left("2"\right)^5 = ...$ which is also acceptable, as are other correct alternatives.

A1: $\frac{64}{3}$ o.e. e.g. $21\frac{1}{3}$ Do not allow e.g. 21.33 or 21.33… but allow $21.\dot{3}$

Must follow a correct answer in part (b) unless they only use the **alternative** method which does not require a value for cos*θ*.

but do **not** allow this mark if they clearly use incorrect e.g. $\cos\theta = -\frac{2\sqrt{2}}{3}$ or $\cos\theta = \text{awrt } 0.943$

Note you may see $ar^{10} = 2\sin\theta \times \left(\frac{\cos\theta}{2\sin\theta}\right)^{10} = \frac{64}{3}$ which will score full marks provided it follows a correct exact answer in part (b).

Do not ISW if they e.g. go on to find the sum of the first 11 terms.

Question Number	Scheme	Marks
9.	**Way 1** $\log_4 \frac{a}{b} = 3$ **or** $\log_4 a = \log_4 64b$ $\frac{a}{b} = 64$ Solves $ab = 25$ with $\frac{a}{b} = k$ and proceeds to $a = \ldots$ **or** $b = \ldots$ e.g. $a^2 = 1600 \Rightarrow a = \ldots$ $a = 40$ **and** $b = \frac{5}{8}$	M1 A1 dM1 A1 **(4)** **(4 marks)**

Note: Condone the absence of base 4 throughout.

If candidates write $\log_4 a - \log_4 b = 3 \rightarrow \frac{\log_4 a}{\log_4 b} = 3 \rightarrow \frac{a}{b} = 4^3$ **allow maximum 1010.**

M1: **Correct** use of addition or subtraction law for logs e.g. $\log_4 a - \log_4 b \rightarrow \log_4 \frac{a}{b}$

or $\log_4 a = 3 + \log_4 b = \log_4 p + \log_4 b = \log_4 pb$ (for some constant $p > 0$ with $p \neq 3$)

or (**Way 2**) $\log_4 ab \rightarrow \log_4 a + \log_4 b$

Alternatively, via **Way 3**, achieves a correct $a = 4^{3+\log_4 b}$ or $b = 4^{\log_4 a - 3}$

In **Way 4**, attempts to make a or b the subject of the first equation and substitutes into the second equation.

Not scored for just e.g. $3 = \log_4 64$ or for e.g. $\log_4 a - \log_4 b = 3$ going directly to $\frac{a}{b} = 3$

A1: **Way 1**: Achieves a second correct equation in terms of a and b with logs removed i.e. $\frac{a}{b} = 4^3$ o.e.

Way 2: Achieves a correct equation only in a **or** only in b which may still have logs.
e.g. $2\log_4 a = 3 + \log_4 25$ **or** $2\log_4 b = \log_4 25 - 3$

Way 3: Achieves a correct equation only in a **or** only in b which may still have logs.
e.g. $b \times 4^{3+\log_4 b} = 25$ or $a \times 4^{\log_4 a - 3} = 25$

Way 4: Writes e.g. $a = \frac{25}{b}$ and substitutes into the second equation to achieve a correct equation only in a or only in b e.g. $\log_4 \frac{25}{b} - \log_4 b = 3$ (proceed to mark as in **Way 2**).

Note that in all approaches they may have already replaced 3 with $\log_4 64$

dM1: Dependent on first M mark.

Way 1: Solves to find a value for a or b using simultaneous equations. Expect to see a method for solving the simultaneous equations but allow slips in rearranging or e.g. transcription errors.
If they just write down values then they should be correct for their simultaneous equations.

Ways 2 & 4: Replaces 3 with $\log_4 p$ ($p > 0$ with $p \neq 3$) and otherwise uses logs **correctly** to find a or b.

Way 3: Replaces 4^3 with p ($p > 0$ with $p \neq 3$) and otherwise uses logs **correctly** to find a or b.

There must be a clear method to remove the logs: they cannot proceed directly from e.g $\log_4 \frac{25}{b^2} = 3$ to $b = \ldots$

Some examples:

- **Way 2:** $\log_4 a^2 = \log_4 p + \log_4 25 \Rightarrow \log_4 a^2 = \log_4 (25p) \Rightarrow a^2 = 25p \Rightarrow a = ...$
- **Way 2:** $\log_4 b^2 = \log_4 25 - \log_4 p \Rightarrow \log_4 b^2 = \log_4 \left(\frac{25}{p}\right) \Rightarrow b^2 = \frac{25}{p} \Rightarrow b = ...$
- **Way 3:** $a = 4^{3+\log_4 b} \Rightarrow b \times 4^{3+\log_4 b} = 25 \Rightarrow b^2 \times 64 = 25 \Rightarrow b = ...$
- **Way 3:** $b = 4^{\log_4 a\ -3} \Rightarrow a \times 4^{\log_4 a\ -3} = 25 \Rightarrow \frac{a^2}{64} = 25 \Rightarrow a = ...$
- **Way 4:** $a = \frac{25}{b} \Rightarrow \log_4 \frac{25}{b} - \log_4 b = 3 \Rightarrow \log_4 \frac{25}{b^2} = 3 \Rightarrow \frac{25}{b^2} = 64 \Rightarrow b = ...$
- **Way 4:** $b = \frac{25}{a} \Rightarrow \log_4 a - \log_4 \frac{25}{a} = 3 \Rightarrow \log_4 \frac{a^2}{25} = 3 \Rightarrow \log_4 \frac{a^2}{25} = \log_4 64 \Rightarrow a = ...$

A1: Both a and b correct and no other solutions. Allow $b = 0.625$ but not a rounded or truncated decimal.

If e.g. $b = -\frac{5}{8}$ is found it must be rejected.

Question Number	Scheme	Marks
10 (a)	Either $kn = 36$ **or** $\frac{n(n-1)}{2}k^2 = 567$	M1
	Both $kn = 36$ **and** $\frac{n(n-1)}{2}k^2 = 567$	A1
	$\Rightarrow \frac{n(n-1)}{2}\left(\frac{36}{n}\right)^2 = 567 \Rightarrow n = ...$ **or** $\Rightarrow \frac{36}{k}\left(\frac{36}{k}-1\right)\times\frac{k^2}{2} = 567 \Rightarrow k = ...$	dM1
	e.g. $\Rightarrow \frac{648(n-1)}{n} = 567 \Rightarrow 648n - 648 = 567n \Rightarrow n = 8$ **or** $\Rightarrow 648 - 18k = 567 \Rightarrow k = \frac{9}{2}$ o.e.	A1
	$k(\text{"8"}) = 36 \Rightarrow k = ...$ **or** $\left(\text{"}\frac{9}{2}\text{"}\right)n = 36 \Rightarrow n = ...$	ddM1
	$n = 8$ **and** $k = \frac{9}{2}$ o.e.	A1
		(6)
(b)	$\{p =\}\ \frac{n(n-1)(n-2)}{3!}k^3 = \frac{\text{"8"}\times\text{"7"}\times\text{"6"}}{6}\times\left(\text{"}\frac{9}{2}\text{"}\right)^3 = ...$	M1
	$= 5103$	A1
		(2)
		(8 marks)

Note: **Misreading 567 as 576 should lead to *n* = 9 and *k* = 4 and may score maximum (a) M1A1dM1A0ddM1A0 (b) M1A1 Other misreads of 567 should be treated similarly.**

(a)

M1: States either $kn = 36$ **or** $\frac{n(n-1)}{2}k^2 = 567$ accepting equivalents but see note below.

Condone an incorrect $\frac{n(n-1)}{2}k = 567$ (usually coming from a bracketing error) or use of e.g. 576.

This mark cannot be awarded from e.g. $\binom{n}{1}k = 36$ or ${}^nC_2(kx)^2 = 567x^2$ or with factorials

e.g. $\frac{n!}{(n-2)!}k^2 = 567$ or versions including x terms such as $knx = 36x$ unless followed by a correct $kn = 36$ or $\frac{n(n-1)}{2}k^2 = 567$

A1: Both $kn = 36$ **and** $\frac{n(n-1)}{2}k^2 = 567$ correct.

dM1: Forms an equation in one variable and solves leading to a value for k or n.