Question Number	Scheme	Marks
1. (a)	$(3+kx)^7 = \underline{3^7} + {}^7C_1 3^6 (kx)^1 + {}^7C_2 3^5 (kx)^2 + {}^7C_3 3^4 (kx)^3 + \ldots$	<u>B1</u>, M1
	$= 2187 + 5103kx + 5103k^2x^2 + 2835k^3x^3 + \ldots$	A1, A1
		(4)
(b)	Sets $5103k^2 = 3\times 2835k^3 \Rightarrow k = \ldots$	M1
	$k = \frac{3}{5}$	A1
		(2) **(6 marks)**

(a)

B1: For a first term (constant term) of 3^7 or 2187 or even ${}^7C_0 \times 3^7$

M1: For an attempt at the binomial expansion. Score for a correct attempt at one of terms 2, 3 or 4

Accept sight of ${}^7C_1 3^6 (kx)^1$ or ${}^7C_2 3^5 (kx)^2$ or ${}^7C_3 3^4 (kx)^3$ condoning the Omission of brackets.

Accept any **correct** coefficient appearing from Pascal's triangle.

A1: For any two simplified terms of $2187 + 5103kx + 5103k^2x^2 + 2835k^3x^3$

A1: For $2187 + 5103kx + 5103k^2x^2 + 2835k^3x^3$ ignoring terms with greater powers. Allow as a list and allow in descending order.

ALT For the first two marks

B1: Takes 3^7 out as a common factor to obtain $3^7\left(1+\frac{kx}{3}\right)^7$

M1: For an attempt at the binomial expansion Score for a correct attempt at one of terms 2, 3 or 4 inside the main bracket.

Accept $\ldots\left(1+(7\times)\left(\frac{kx}{3}\right),\ \left(\frac{7\times 6}{2!}\right)\times\left(\frac{kx}{3}\right)^2,\ \left(\frac{7\times 6\times 5}{3!}\right)\times\left(\frac{kx}{3}\right)^3\right)$ condoning the omission of brackets

(b)

M1: For setting $'5103'k^2 = 3\times '2835'k^3$ leading to $k = \ldots$following **correct** processing M0 if they include x's, but allow if recovered

A1: For $k = \frac{3}{5}$ or exact equivalent. No other values stated.

Question Number	Scheme	Marks
2 (a)	$h = 3$	B1
	Area $\approx \frac{3}{2}\{1.3195 + 0.4665 + 2\times(1.0718 + 0.8706 + 0.7071 + 0.5743)\}$	M1
	$= 12.35$	A1
		(3)
(b)	States either 'use more trapezia', or other such as 'make the strip widths narrower'	B1
		(1)
(c)	$\int_{-4}^{11} 2^{3-0.1x}\,\mathrm{d}x = 2^3 \times \int_{-4}^{11} 2^{-0.1x}\,\mathrm{d}x = 8\times '12.35' = 98.8$	M1, A1
		(2)
		(6 marks)

(a)

B1 For $h = 3$ seen anywhere even if it is not later used correctly.

This is implied by sight of $\frac{3}{2}$ in front of the bracket

M1 For a correct content of the bracket within Trapezium Rule, condoning slips.
E.g., missing end bracket(s) or MR on no more than two values

FYI: Area A $\approx \left(\frac{3}{2}\right)(1.786 + 2[3.2238]) \Rightarrow \left(\frac{3}{2}\right)(8.2336) = 12.3504$

A1 awrt 12.35 following the award of M1
Note that the calculator answer for this integral is 12.31

(b)

B1: States any valid method that states or implies thinner strips, more strips, more trapezia, reduce the size of *h* etc.,

We will also condone for the award of this mark;

Use more *y* values
Use more *x* values
Use more *x* and *y* values

(c)

M1: Attempts to use the rule $2^{3-0.1x} = 2^3 \times 2^{-0.1x}$ seen anywhere.

A1: awrt 99 following 8×12.35
Note that the calculator answer is 98.4

Question Number	Scheme	Marks
3.(i)	Sets $\text{f}(-2)=0 \Rightarrow 4(-2)^3+6(-2)+k=0 \Rightarrow k=44$	M1, A1
	Attempts $\text{f}(5)=4x^3+6x+'44'=4\times5^3+6\times5+'44'=574$	dM1, A1
		(4)
(ii)	Sets $6x^3-15x^2-21x+8=\left(Ax^2+Bx+C\right)(2x+3)+R$	
	And attempts to find values of A, B, C and R via inspection/ substitution etc	M1
	Two correct of A, B, C and R	A1
	$Q(x)=3x^2-12x+\dfrac{15}{2} \qquad R=-\dfrac{29}{2}$	A1
		(3)
		(7 marks)

(i) M1: Sets $\text{f}(\pm2)=0 \rightarrow$ leading to a value for k

Uses Polynomial Division:

$$x+2\,\overline{)\,4x^3+0x^2+6x+k} \quad \text{quotient } 4x^2-8x+22 \qquad \text{Rem}=k-44=0\Rightarrow k=44$$

Allow for $4x^2+\alpha x+\beta$ with a remainder $k\pm\phi=0$

A1: $k=44$ seen or implied

dM1: Attempts $\text{f}(\pm5)$ with their value for k leading to a value for the remainder.

Uses Polynomial division

$$x-5\,\overline{)\,4x^3+\left(0x^2\right)+6x+'44'} \quad \text{quotient } 4x^2+20x+106 \qquad \text{Rem}=574 \text{ or Rem}=530+k$$

Allow for $4x^2+\mu x+\nu$ with a constant remainder

A1: 574

(ii) M1: Full attempt to find both the quotient and remainder.

See main scheme but valid via division. Look for a 3 term quadratic quotient and a constant remainder

Alt - uses polynomial division

Look for a 3-term quadratic quotient and a constant remainder

$$2x+3\,\overline{)\,6x^3-15x^2-21x+8} \quad \text{quotient } 3x^2-12x+\frac{15}{2} \qquad r=-\frac{29}{2}$$

A1: 2 correct terms of the 4 terms in $Q(x)=3x^2-12x+\dfrac{15}{2}$ and $R=-\dfrac{29}{2}$

A1: $\text{Q}(x)=3x^2-12x+\dfrac{15}{2}$ and $\text{R}=-\dfrac{29}{2}$ correctly labelled and identified. Allow Quotient (or even Q) in place of Q(x)

Question Number	Scheme	Marks
4.	$\int \left(\frac{12}{x^2}+4\right)\mathrm{d}x = -\frac{12}{x}+4x$	M1
	Sets $\int_k^{2k}\left(\frac{12}{x^2}+4\right)\mathrm{d}x = 14 \Rightarrow \left[-\frac{12}{x}+4x\right]_k^{2k} = 14$	
	$\left(-\frac{6}{k}+8k\right)-\left(-\frac{12}{k}+4k\right)=14 \Rightarrow 2k^2-7k+3=0$ o.e.	dM1, A1
	$(2k-1)(k-3)=0 \Rightarrow k=\frac{1}{2}, 3$	ddM1, A1
		(5 marks)

M1: Attempts to integrate with one index correct

dM1: Applies the limits either way around with a subtraction, **and** sets = 14

A1: Forms a correct 3TQ. Terms do not need to be on the same side of the $=$ sign.

Look for $4k^2+6=14k$ o.e

ddM1: Uses any method of solving a 3TQ including a calculator.

A1: $k=\frac{1}{2}, 3$ following all previous marks.

Question Number	Scheme	Marks
5 (a)	$x^2+y^2-6x+5y-41=0$	
	Attempts $(x-3)^2+\left(y+\frac{5}{2}\right)^2=\ldots$	M1
	Correct centre $\left(3,-\frac{5}{2}\right)$	A1
	Exact radius $\frac{15}{2}$	A1
		(3)
(b)	Sets up appropriate equation $(k+'3')^2+\left('\frac{5}{2}'\right)^2=\left(5+'\frac{15}{2}'\right)^2$	M1
	$(k+3)^2=150\Rightarrow k+3=\sqrt{150}$	dM1
	$\Rightarrow k=5\sqrt{6}-3$	A1
		(3)
		(6 marks)

(a)

M1: Attempts to complete the square for both variables. Look for $(x\pm3)^2, \left(y\pm\frac{5}{2}\right)^2=\ldots$

This mark can be implied from the coordinates of the centre $\left[\left(3,-\frac{5}{2}\right)\right]$ just written down.

A1: $\left(3,-\frac{5}{2}\right)$

A1: $\frac{15}{2}$ condone $\sqrt{\frac{225}{4}}$ o.e which may be scored following $(x-3)^2+\left(y+\frac{5}{2}\right)^2=\frac{225}{4}$

(b)

M1: Uses Pythagoras' theorem to set up an equation in k

Look for $(k\pm'3')^2+\left('\frac{5}{2}'\right)^2=\left(5+'\frac{15}{2}'\right)^2$ following through on their $\left(3,-\frac{5}{2}\right)$ and $\frac{15}{2}$

Also look for $k^2+6k-141=0$ o.e.

dM1: Uses an appropriate method to proceed to a value for k

A1: $k=5\sqrt{6}-3$ o.e.

Question Number	Scheme	Marks
6 (i)	Can write p and/or q in an appropriate form. E.g $p = 2n+3,\ q = 2n+1$ $n \in \mathbb{N}$	B1
	$p^2 - q^2 = (2n+1)^2 - (2n-1)^2$	M1
	$= 4n^2 + 4n + 1 - (4n^2 - 4n + 1)$	A1
	$= 8n$ which is a multiple of 8, hence proven *	A1*
		(4)
(ii)	$y = x^3 + 12x^2 + 49x + 2 \Rightarrow \frac{dy}{dx} = 3x^2 + 24x + 49$	M1,
	$\Rightarrow \frac{dy}{dx} = 3(x+4)^2 + 1$	dM1, A1
	Gradient is always …1 so cannot ever be 0, so no stationary points on C *	A1*
		(4)
		(8 marks)

(i)

B1: Can write p and/or q in an appropriate form. Condone omission of $n \in \mathbb{N}$
Allow any letter to be used in place of n
Allow $(p =)\ 2n+3, (q =)\ 2n+1$ p and q to be implied

M1: Starts the proof by writing $[p^2 - q^2 =]\ (2n+1)^2 - (2n-1)^2$ o.e

OR, such as $p^2 - q^2 = (2n+3)^2 - (2n+1)^2$

NB Must be in one variable only and *p* and *q* must be consecutive.

A1: Correct multiplication to an intermediate or fully simplified answer,
Allow $4n^2 + 4n + 1 - (4n^2 - 4n + 1)$

A1*: Reaches a correct answer, E.g. $8n$, gives a valid statement ‘which is a multiple of 8’ or a minimal conclusion. E.g., ‘QED’, ‘Shown’, □ , #, etc.,
Note with $p^2 - q^2 = (2n+3)^2 - (2n+1)^2$ you will get $8n + 8$

SC Uses *p* < *q* Award marks as above, but score B1 M1 A1 A0 for an otherwise fully correct proof.

(ii)

M1: Attempts to differentiate and achieves a 3TQ

An attempt to differentiate is defined in General Guidance.

dM1: And then uses a valid method of showing that $\frac{dy}{dx} \neq 0$

Valid methods could be

1. Attempting to complete the square
2. Attempting the discriminant $(\text{Disc} = -12)$
3. Attempting to solve, with working, $'3x^2 + 24x + 49' = 0$
   Accept 'not real roots' following some working but not 'Math Error'
   $\left(\text{Roots are } \frac{-12 \pm \sqrt{3}\text{i}}{3}\right)$

A1: Correct calculations [As above]

A1*: Valid conclusion. E.g.

1. $\frac{dy}{dx}$ …1 hence is never 0 **so no stationary points**

2. The Discriminant is negative **so no (real) roots** and **hence no stationary points**

3. The roots are complex so there are **no (real) solutions** and **hence no stationary points.**

Accept reference to turning points.

Question Number	Scheme	Marks
7 (i)	$8^{2x-5} = 20$	
	Take $\log_2$ of <u>both</u> sides and uses power rule $(2x-5)\log_2 8 = \log_2 20$	M1
	Uses $\log_2 8 = 3$ or $\log_2 20 = \log_2 4 + \log_2 5$	dM1
	$3(2x-5) = 2 + \log_2 5$	A1
	$x = \frac{17}{6} + \frac{1}{6}\log_2 5$	A1
		(4)
(ii)	$\log_3(13+2y) + 3 = 2\log_3(4-y)$	
	One correct log law $2\log_3(4-y) \rightarrow \log_3(4-y)^2$ or $3 \rightarrow \log_3 27$	B1
	Correct attempt to combine two terms	M1
	Correct equation (not involving logs) E.g. $(4-y)^2 = 27(13+2y)$	A1
	$y^2 - 62y - 335 = 0$	
	$(y+5)(y-67) = 0$	dM1
	$y = -5$ only	A1
		(5) **(9 marks)**

(i)

M1: Take $\log_2$ of <u>both</u> sides and uses power rule $(2x-5)\log_2 8 = \log_2 20$

Allow this mark for log work with any or no base: E.g. $(2x-5)\log 8 = \log 20$

dM1: Uses $\log_2 8 = 3$ or $\log_2 20 = \log_2 4 + \log_2 5$ **correctly** to obtain an equation in x involving just $\log_2 5$

A1: A **correct** intermediate equation in x involving just $\log_2 5$

A1: $x = \frac{17}{6} + \frac{1}{6}\log_2 5$

There are several ways of solving part (i). These are the General Principles for marking this. Allow numerical slips only, index and log work must be correct.

M1: Deal with the index and take logs of both sides

dM1: Simplify logs/use the addition law **correctly** to obtain an equation involving x, $\log_2 5$ and constant(s) only. If candidates change from base 8 to base 2 it must be correct throughout.

A1: For a **correct** equation involving x and $\log_2 5$ (no other logs) and constant(s)

A1: For $x = \frac{17}{6} + \frac{1}{6}\log_2 5$

(ii)

B1: One correct log law $2\log_3(4-y) \to \log_3(4-y)^2$ or $3 \to \log_3 27$

M1: Correct attempt to **combine** two terms

$$\log_3(13+2y)+3 = \log_3 27(13+2y) \text{ or}$$

$$2\log_3(4-y) - \log_3(13+2y) = \log_3 \frac{(4-y)^2}{(13+2y)}$$

Do not award this mark for an equation from incorrect log work when **combining** the terms, for example:

$$2\log_3(4-y) - \log_3(13+2y) = \frac{\log_3(4-y)^2}{\log_3(13+2y)} \Rightarrow \log_3 \frac{(4-y)^2}{(13+2y)}$$

A1: Correct equation (not involving logs) E.g. $(4-y)^2 = 27(13+2y)$

dM1: Correct method of solving 3TQ in y. Allow for either 67 or – 5 seen.
Dependent upon previous M

A1: A1: $y = -5$ **only** If $y = 67$ is not rejected then award A0

Question Number	Scheme	Marks
8 (i)	$4\tan^2(2\theta-30)°+1=49$	
	$\tan(2\theta-30)°=(\pm)\sqrt{12}$	M1
	$(2\theta-30)=$ any of awrt 74, 106, 254, 286	A1
	Correct method to find θ E.g. $\theta=\text{awrt }\dfrac{74+30}{2}$	dM1
	Two of $\theta=$ awrt 51.9, 68.1, 141.9, 158.1	A1
	All four of $\theta=$ awrt 51.9, 68.1, 141.9, 158.1	A1
		(5)
(ii)	Uses $\tan x=\dfrac{\sin x}{\cos x}\Rightarrow 2\times\dfrac{\sin x}{\cos x}\sin x+3=0$	M1
	$2\sin^2 x+3\cos x=0\Rightarrow 2\left(1-\cos^2 x\right)+3\cos x=0$	dM1
	$\Rightarrow 2\cos^2 x-3\cos x-2=0$	A1
	$\Rightarrow(2\cos x+1)(\cos x-2)=0\Rightarrow\cos x=-\dfrac{1}{2}$	ddM1
	$\Rightarrow x=\dfrac{2}{3}\pi,\dfrac{4}{3}\pi$	A1
		(5) **(10 marks)**

(i)

M1: **Either:** Attempts to make $\tan(2\theta-30)°$ the subject. Allow $\tan\alpha°=3.46$

An attempt is defined as a numerical slip in finding '12'

Allow the use of a substitution here e.g., let $2\theta-30=x\Rightarrow\tan x°=(\pm)\sqrt{12}$

OR: Uses the sin/cos and Pythagorean identities as follows:

$$\tan^2\alpha=12\Rightarrow\frac{\sin^2\alpha}{\cos^2\alpha}=12\Rightarrow 1-\cos^2\alpha=12\cos^2\alpha$$

$$\cos^2\alpha=\frac{1}{13}\Rightarrow\cos\alpha=(\pm)\sqrt{\frac{1}{13}}\quad\text{o.e.}$$

OR $$\sin\alpha=(\pm)\sqrt{\frac{12}{13}}\quad\text{o.e.}$$

Award this mark for obtaining a value for either sin or cos allowing for numerical slips only.

A1: Achieves $(2\theta-30)=$ any of one of awrt 74, 106, 254, 286

Allow a substitution, so achieves any one of $x=$ awrt 74, 106………..

dM1: Dependent upon the previous M mark. It is for attempting to find θ. Score for $\frac{\arctan k \pm 30}{2}$

A1: For two of $\theta =$ awrt $51.9,\ 68.1, 141.9, 158.1$

A1: All four of $\theta =$ awrt $51.9,\ 68.1, 141.9, 158.1$ and no other values in the range
Ignore values outside of range.

(ii)

M1: Uses $\tan x = \frac{\sin x}{\cos x} \Rightarrow 2 \times \frac{\sin x}{\cos x} \sin x + 3 = 0$

dM1: Uses $\sin^2 x = 1 - \cos^2 x$ and multiplies by $\cos x$ and attempts to form a 3TQ in $\cos x$

A1: Correct simplified 3 term quadratic $2\cos^2 x - 3\cos x - 2 = 0$ in any form.

ddM1: Solves a 3TQ using any valid method [including calculators] in $\cos x$ leading to at least one value for $\cos x$

A1: $x = \frac{2}{3}\pi, \frac{4}{3}\pi$ and no other values in the range Ignore values outside of range

Question Number	Scheme	Marks
9.(a)	Attempts to use $7\,500 = 12\,400 + 14d$ to find 'd'	M1
	Finds $d = (-350)$ and uses this in $12\,400 + 4d$	M1
	Lithium mined in Year 5 is 11 000	A1
		(3)
(b)	Attempts $\frac{15}{2}(12400 + 7500)$ or $\frac{15}{2}\{2\times 12400 + 14\times -350\} = 149\,250$	M1, A1
		(2)
(c)	Attempts to use $7\,500 = 12\,400 r^{14}$ to find 'r'	M1
	Finds $r = (0.9647)$ and uses this in $12\,400 \times r^{9}$	M1
	Lithium mined in Year 10 $8580 \leq L \leq 9000$	A1
		(3)
(d)	Limit $= \dfrac{12400}{1 - '0.9647'} = 351\,300$ Accept $310\,000 \leq S_\infty \leq 351\,600$	M1, A1
		(2)
		(10 marks)

]

(a)

M1: Attempts to use the AP formula in an attempt to find 'd'
Accept an attempt at $7\,500 = 12\,400 + 14d$ resulting in a value for d.
Accept the calculation $\dfrac{7500 - 12400}{14}$ condoning slips on the 7500 and 12400

M1: A correct attempt, using $12\,400 + 4d$, to find the mass of lithium mined in year 5.
You may award this following an "incorrect" AP formula with $15d$ being used instead of $14d$
Eg $7\,500 = 12\,400 + 15d$ or more likely $\dfrac{7500 - 12400}{15}$ usually leading to an answer of awrt 11093

A1: Lithium mined in Year 5 is 11 000

(b)

M1: Any correct method to find the sum of the AP .
Look for $\frac{15}{2}\{2\times 12400 + 14\times -350\} = \ldots$ following through on their 'd'
OR $\frac{15}{2}(12400 + 7500) = \ldots$ using first + last formula

A1: 149 250

(c)

M1: Attempts to use the GP *n*th term formula in an attempt to find '*r*'

Accept an attempt at $7\,500 = 12\,400r^{14} \Rightarrow r^{14} = \dfrac{7500}{12400} \Rightarrow r = \ldots$ condoning numerical slips on the values.

Accept the calculation $\sqrt[14]{\dfrac{7500}{12400}}$

M1: A correct attempt, using $12\,400 \times r^{9}$, to find the 10th term ft their positive *r*

You may award this following an "incorrect" GP formula with 15 being used instead of 14

Eg following $7\,500 = 12\,400r^{15}$. or $r = \sqrt[15]{\dfrac{7500}{12400}}$.

A1: Accept awrt in the range $8580 \leqslant \ldots \leqslant 9000$

(d)

M1: A correct method for the sum to infinity of a GP.

Look for an attempt at $\dfrac{12400}{1-'0.9647'}$ following through on their 0.9647

Allow this mark only if their $|r| < 1$

A1: Accept awrt in the range $310\,000 \leqslant S_\infty \leqslant 355\,000$

Question Number	Scheme	Marks
10.(a)	$y=\dfrac{\sqrt{x}\left(100-x^2\right)}{40}=\dfrac{5}{2}x^{\frac{1}{2}}-\dfrac{1}{40}x^{\frac{5}{2}}$	
	$\dfrac{dy}{dx}=\dfrac{5}{4}x^{-\frac{1}{2}}-\dfrac{1}{16}x^{\frac{3}{2}}$	M1, A1
	Stationary point $\dfrac{5}{4}x^{-\frac{1}{2}}-\dfrac{1}{16}x^{\frac{3}{2}}=0\Rightarrow x^2=20$	dM1
	$x=2\sqrt{5}$	A1
		(4)
(b)	$\displaystyle\int \frac{5}{2}x^{\frac{1}{2}}-\frac{1}{40}x^{\frac{5}{2}}\,dx=\frac{5}{3}x^{\frac{3}{2}}-\frac{1}{140}x^{\frac{7}{2}}$	M1, A1
	$\dfrac{5}{3}k^{\frac{3}{2}}-\dfrac{1}{140}k^{\frac{7}{2}}=0\Rightarrow k^2=\dfrac{140\times 5}{3}=\dfrac{700}{3}$	dM1
	$\Rightarrow k=\sqrt{\dfrac{700}{3}}$ or $\dfrac{10}{3}\sqrt{21}$	A1
		(4)
		(8 marks)

(a)

M1: An attempt to differentiate.
Writes as a sum of two terms and reduces the power of at least one term by 1

A1: $\dfrac{dy}{dx}=\dfrac{5}{4}x^{-\frac{1}{2}}-\dfrac{1}{16}x^{\frac{3}{2}}$ which may be left unsimplified

dM1: Sets $\dfrac{dy}{dx}=0$ **and** proceeds to find a value for x^2 or kx^2

A1: $x=2\sqrt{5}$ or exact equivalent

(b)

M1: An attempt to integrate.
Writes as a sum of two terms and increases the power of at least one term by 1

A1: $\dfrac{5}{3}x^{\frac{3}{2}}-\dfrac{1}{140}x^{\frac{7}{2}}$ which may be left unsimplified. Ignore $+C$

dM1: Substitutes $x=k$ in their $\dfrac{5}{3}x^{\frac{3}{2}}-\dfrac{1}{140}x^{\frac{7}{2}}=0$ with only two terms in k [and any constant terms eliminated by cancelling] **and** solves for k^2

A1: $k=\dfrac{10}{3}\sqrt{21}$ or exact equivalent