Question Number	Scheme	Marks
1 (a)	$78+17\times-3=27$	M1, A1
		(2)
(b)	$\frac{100}{2}(2\times78+99\times-3)$	M1
	$=-7050$	A1
		(2)
		(4 marks)

(a)

M1: Attempts to find the value of $a+(n-1)d$ with *a* = 78, *n* = 18 and *d* = –3. E.g.

Award for $78+17\times-3=...$

Condone slips, so for example 78 written as 87...

BUT writing 78 as 18 (which is the value of *n*) or -3 as 3 are not regarded as slips

Writing down 27 scores both marks

Listing is acceptable only if they write down 27 as the answer

A1: 27

(b)

M1: Attempts the correct sum formula for 100 terms. So *n* = 100 must be used (twice).

Condone with $d=\pm3$ and slips on 78 .... BUT writing 78 as 18 is not regarded as a slip

Award for an attempt at $\frac{100}{2}(2\times78+99\times-3)$ or $\frac{100}{2}(2\times78+99\times3)$ o.e.

Writing down -7050 scores both marks

You may see the alternative formula used, usually seen with an attempt to find u_{100} first.

Look for $\frac{100}{2}(78+u_{100})$ condoning the same slips on 78 and u_{100} found via $78+99\times\pm3$

A1: -7050 but this is not isw. Withhold this mark if they write -7050 and follow this with 7050

Guidance on applications of formulae for this question:

If a candidate writes out a fully correct formula including bracketing, followed by the correct embedded values they can be awarded M1.
E.g. $n^{\text{th}}\text{ term} = a + (n-1)d \Rightarrow 18^{\text{th}}\text{ term} = 78 + (18-1) - 3 = 92$ should be awarded M1, A0 as the correct formula and embedded values are seen

However, $18^{\text{th}}\text{ term} = 78 + (18-1) - 3 = 92$ should be awarded M0 as the correct formula cannot be implied

Similarly writing $18^{\text{th}}\text{ term} = 78 + 19 \times -3 = \ldots$ should be awarded M0, A0 as it implies the incorrect formula $a + (n+1)d$

Question Number	Scheme	Marks
2	$(1+px)^{10}=1+10px+\frac{10\times9}{2}p^2x^2+\frac{10\times9\times8}{6}p^3x^3+\ldots$	
	Sets $10p=15\Rightarrow p=1.5$ o.e.	M1 A1
	Finds the value of $45p^2 \Rightarrow q=101.25$ o.e.	M1 A1
	Finds the value of $120p^3 \Rightarrow r=405$	M1 A1
		(6) **(6 marks)**

M1: Sets $10p=15\Rightarrow p=\ldots$ or $10px=15x\Rightarrow p=\ldots$

Alternatively score for a correct simplified expansion of
$(1+px)^{10}=1+10px+45p^2x^2+120p^3x^3+\ldots$

A1: $p=1.5$ o.e. Candidates who simply state that $p=1.5$ can be awarded M1, A1

M1: Finds a value for $45p^2$ or $\frac{10\times9}{2}p^2x^2$ or equivalent using their value of p which may or may not have been found via a correct method.

A1: States a correct value for $q=101.25$ o.e following the award of M1

M1: Finds the value for $120p^3$ or $\frac{10\times9\times8}{3!}p^3x^3$ or equivalent using their value of p which may or may not have been found via a correct method.

A1: States a correct value for $r=405$ following the award of M1

Special Case I: Candidates who leave the x^2 AND x^3 in their answer

Candidates who show a correct method but state $q = 101.25x^2$ **AND** $r = 405x^3$ following M1, A1 can score SC 1,1,1,1,1,0

Special Case II: Candidates who find the value of *p* and then expand $\left(1+\frac{3}{2}x\right)^{10}$

Candidates who show a correct method to find *p* then expand $\left(1+\frac{3}{2}x\right)^{10} = 1+15x+101.25x^2+405x^3$ without explicitly stating *q* and *r* can be awarded all of the marks

Answers with limited or no working;

If the candidate gives a correct expansion of $(1+px)^{10}$, whether simplified or not, then all marks are available.

E.g $(1+px)^{10} = 1+10px+45(px)^2+120(px)^3 \Rightarrow p=\frac{3}{2}, q=\frac{405}{4}, r=405$ scores 6 marks

If the candidate just writes down values without any valid equations/working then we will apply the following.

- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

So just stating $p=1.5$, $q=101.25$ and $r=405$ without the necessary equations or expansion is SC: 1,1,1,0,0,0

Question Number	Scheme	Marks
3.(a)	$300\,000 = 3\times 2^{-k} \Rightarrow 2^{-k} = 100000 \Rightarrow -k = \dfrac{\log 100000}{\log 2}$	M1
	$(k) = -16.61$	A1
		(2)
(b) (i)	Strip width = 1.5	B1
	$\frac{1.5}{2}\{4.243+0.023+2(1.5+0.530+0.188+0.066)\}$ = 6.63	M1, A1
		(3)
(ii)	$\int_{-0.5}^{7} 2^{-x}\,dx + \int_{-7}^{0.5} 2^{x}\,dx = \frac{1}{3}\times\text{"6.63"}+\frac{1}{3}\times\text{"6.63"} = 4.42$	M1, A1
		(2)
		(7 marks)

(a)

M1: Correct **use of logs** leading to an expression or value for x or $-x$.

Alternatively applies the correct **use of logs** leading to an expression for k or $-k$

There are lots of variations, examples of which include

- $(\pm k) = \dfrac{\log 100000}{\log 2}$, $(\pm x) = \log_2 \dfrac{1}{100000}$ and $(\pm x) = \dfrac{\ln 100000}{\ln 2}$
- $300\,000 = 3\times 2^{-k} \Rightarrow \log 300000 = \log 3 - k\log 2 \Rightarrow k = \ldots$

Condone a slip on the number of zeros (4 or 6 zeros can still score this method mark)

It cannot be scored if they incorrectly combine the 3 and the 2 and work from $6^{-k} = 300\,000$

A1: AWRT –16.61 following M1. ISW after a correct answer

Note that the answer only is M0, A0. The demand is that the candidates use logs (b)(i)

B1: Correct strip width. E.g. States or uses $h = 1.5$. Condone it being unprocessed. E.g. $\frac{7-(-0.5)}{5}$ o.e.

Can be implied by a correct h value within an attempted trapezium rule which does not need to be perfect. A correct application would be

$\frac{1.5}{2}\{4.243+0.023+2(1.5+0.530+0.188+0.066)\}$ or

$0.75\{4.243+0.023+2(1.5+0.530+0.188+0.066)\}$

M1: Correct application of the trapezium rule

$\frac{"h"}{2}\{4.243+0.023+2(1.5+0.530+0.188+0.066)\}$

Condone slips on values but there must be the correct number of them and they must be in the correct place within the formula.

Condone a missing trailing bracket, but hard to decipher bracketing can be awarded this mark only if followed by awrt 6.6. Answers only score M0

A1: awrt 6.63 (Answer in full is 6.6255). ISW after a correct answer

Note that the calculator answer for this integral is 6.087

(b)(ii) Attempts to (b)(ii) must use their answer to (b)(i) but could use an additional attempt at the trapezium rule on $\int_{-7}^{0.5} 2^x \, dx$ with $h = 1.5$ or smaller

M1: Attempts to find $\frac{2}{3}\times"6.63"$ or alternatively $\frac{1}{3}\times"6.63"+\frac{1}{3}\times"6.63"$ following through on their 6.63

Note that $\frac{1}{3}\times"6.63"+2.03=4.24$ is M0 as 2.03 is the calculator answer for $\int_{-7}^{0.5} 2^x \, dx$

A1: 4.42. Allow 4.417 or awrt 4.42 if the full answer is used.

Note that the calculator answer for the sum of the two integrals is 4.058...

Question Number	Scheme	Marks
4 (a)	$\mathrm{f}(x)=4x^3+13x^2-10x+8$ $\begin{array}{r l} & 4x^2+21x+32 \\ x-2 \,)\!\! & 4x^3+13x^2-10x+8 \\ & \underline{4x^3-8x^2} \\ & \quad 21x^2-10x \\ & \quad \underline{21x^2-42x} \\ & \qquad 32x+8 \\ & \qquad \underline{32x-64} \\ & \qquad 72 \end{array}$ Synthetic Division $\begin{array}{c\|c c c \| c} 2 & 4 & 13 & -10 & 8 \\ & & 8 & 42 & 64 \\ \hline & 4 & 21 & 32 & 72 \end{array}$	
(i)	$Q(x)=4x^2+21x+32$	M1, A1
(ii)	$R=72$	M1, A1
		(4)
(b)(i)	Attempts $\mathrm{f}(-4)=4\times-64+13\times16-10\times-4+8$ $=-256+208+40+8=0$ Hence $(x+4)$ is a factor *	M1 A1*
(ii)	$\mathrm{f}(x)=4x^3+13x^2-10x+8=(x+4)(4x^2-3x+2)$	M1
	For their $4x^2-3x+2$ attempts "b^2-4ac" $=9-32$, 'b^2-4ac' <0 so $4x^2-3x+2$ has no real roots and $\mathrm{f}(x)=0$ has one at $x=-4$	M1 A1*
		(5)
(c)	$(\mathrm{f}'(x))=12x^2+26x-10=2(3x-1)(2x+5)$ $-\frac{5}{2}<x<\frac{1}{3}$	M1 dM1, A1
		(3) **(12 marks)**

(a) (i)

M1: Attempts to divide $\mathrm{f}(x)$ by $(x-2)$ and achieves a 3TQ quotient of the form $4x^2+bx+c$ $\quad b\neq 0, c\neq 0$ **AND** a remainder that is a constant. See equivalent form using synthetic division

A1: Achieves a Quotient or $Q(x)=4x^2+21x+32$. It is implied if they write (i) $4x^2+21x+32$

It cannot just be awarded from the expression within the division sum

(a) (ii)

M1: Attempts to divide $f(x)$ by $(x-2)$ and achieves a 3TQ quotient **AND** a remainder that is a constant.
Alternatively attempts to find the value of $f(2)$. Allow $f(2)=72$ but if it is incorrect you must see some evidence for the award such as showing an intermediate line of $4\times 2^3+13\times 2^2-10\times 2+8$
or else $32+52-20+8$ (or equivalent) condoning slips. See equivalent form using synthetic division

A1: States that Remainder or $R = 72$. It is implied if they write (ii) 72
It cannot just be awarded from sight of 72 within the division sum BUT if a candidate gets the division sum completely correct and does not state $Q(x)=4x^2+21x+32$ and $R = 72$ (o.e.) they should be awarded SC 1,1,1,0

Alt (a) via an identity BUT other acceptable methods exist such as by inspection which is a less formal version of this.

Sets up a correct identity $f(x)=4x^3+13x^2-10x+8\equiv(x-2)\left(Ax^2+Bx+C\right)+R$ **AND then**

M1: Compares terms and achieves $A=4$ and values for B and C

A1: States that Quotient, (i) or $Q(x)=4x^2+21x+32$

M1: Finds R via f(2) or solving $8=R\pm 2C$ with their value for C.

A1: States that Remainder, (ii) or $R = 72$

(b)(i)

M1: Attempts to find the value of $f(-4)$. For this to be awarded you must see some values. Look for, for example,

- $f(-4)=4\times(-4)^3+13\times(-4)^2-10\times(-4)+8$ with or without the brackets
- $f(-4)=-256+208+40+8$ condoning slips

You must be convinced that they have attempted $f(-4)$ and not $f(+4)$

Note that dividing $f(x)=4x^3+13x^2-10x+8$ by $(x+4)$ is M0 as it doesn't satisfy the demand of the question.

Note that simply stating $f(-4)=0$ without sight of evidence (e.g. intermediate values) is M0

A1*: Requires

- The award of M1
- Sight of correct values or correct expression, e.g. $f(-4) = -256 + 208 + 40 + 8$ or $f(-4) = 4\times(-4)^3 + 13\times(-4)^2 - 10\times(-4) + 8$ (must be correct bracketing) followed by = 0
- and a minimal conclusion which may be QED, hence true, ✓

(b)(ii). Mark part (b) as one complete section. Work in part (i) can count in part (ii)

Although the question states hence, these marks can be awarded independently of (b)(i) so 00111 is possible

M1: Divides $f(x)$ by $(x+4)$ and achieves an acceptable 3 term quadratic factor

If division is used look for first two terms $4x^2 - 3x + ...$

If they are using inspection, look for first and last terms $(x+4)\left(4x^2 + ...x + 2\right)$

M1: Makes a valid attempt at finding the roots or number of roots of their 3-term quadratic factor resulting from factoring out or dividing by (x+4) . Look for an attempt at the value of "$b^2 - 4ac$" or an attempt at the correct quadratic formula o.e. such as completing the square. It is dependent upon them having a quadratic factor of the form $4x^2 + bx + c,\ b \neq 0,\ c \neq 0$

A1*: This requires all of the following;

- a correct division (o.e.) leading to the correct quadratic factor
- a correct 'proof' showing that $4x^2 - 3x + 2$ hasn't any real roots e.g. "$b^2 - 4ac$" $= 9 - 32 < 0$ or attempting to solve $4x^2 - 3x + 2 = 0$ via the quadratic formula followed by 'not possible' o.e.
- a minimal reason/statement showing that $f(x) = 0$ has only one real solution. This could be a statement that the one (real) solution is $x = -4$ or reasons that the one real solution is as a result of $(x+4) = 0$

There can be no incorrect statements in the body of their proof. Eg. The real solution is $(x+4)$

(c) **This is a non-calculator question/part question**

M1: Attempts to differentiate and find the roots. This requires all of

- Differentiation to a 3TQ expression with at least one correct term. FYI $\left(\mathrm{f}'(x)\right) = 12x^2 + 26x - 10$
- A valid non calculator attempt at finding the two critical values of $\mathrm{f}'(x) = 0$. Look for a valid attempt to factorise or use the quadratic formula in an attempt to solve $\mathrm{f}'(x) = 0$. Condone attempts in which the factor of 2 disappears. E.g. $12x^2 + 26x - 10 = 0 \Rightarrow (2x+5)(3x-1) = 0$

dM1: Finds the inside region for their quadratic. It is dependent upon the previous M mark

A1: $-\frac{5}{2} < x < \frac{1}{3}$ or $-\frac{5}{2} \leqslant x \leqslant \frac{1}{3}$ o.e. such as $\left(-\frac{5}{2}, \frac{1}{3}\right)$

SC: Candidates who use a calculator can be awarded SC 1,1,0 for an otherwise completely correct solution.

E.g. $\mathrm{f}'(x) = 12x^2 + 26x - 10 < 0 \Rightarrow -2.5 < x < \frac{1}{3}$

Question Number	Scheme	Marks
5(i)	$4\tan\theta + 5\sin\theta = 0$	
	States or uses $\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow 4\frac{\sin\theta}{\cos\theta} + 5\sin\theta = 0$	M1
	$\sin\theta(4 + 5\cos\theta) = 0$	
	$\cos\theta = -\frac{4}{5}$	dM1
	$\cos\theta = -\frac{4}{5} \Rightarrow \theta = \text{awrt } 143° \text{ or awrt } 217°$	A1
	$\cos\theta = -\frac{4}{5} \Rightarrow \theta = \text{awrt } 143.1° \text{ and awrt } 216.9°$	A1
	$\sin\theta = 0 \Rightarrow \theta = 180°, 360°$	B1
		(5)
(ii)	$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\cos x} \overset{\times \sin x \cos x}{\Rightarrow} \sin^2 x + \cos^2 x = 5\sin x$	
	$5\sin x = 1$	M1, A1
	$x = \text{awrt } 0.201, 2.94$	dM1, A1
		(4)
		(9 marks)

(i) Condone a lack of a degrees symbol throughout. Condone solutions that drop θ's in their working

M1: States $\tan\theta = \frac{\sin\theta}{\cos\theta}$ or attempts to use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ in the given equation.

Condone slips in coefficients but writing $4\tan\theta$ as $\frac{4\sin\theta}{4\cos\theta}$ is M0.

dM1: Attempts all of the following

- uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ condoning slips in the coefficients
- multiplies by $\cos\theta$ and proceeds to a linear equation in $\cos\theta$ via factorising or cancelling out the term in $\sin\theta$
- achieves $\cos\theta = k$ where $|k| < 1$

A1: Achieves $\cos\theta = -\frac{4}{5}$ and finds one correct value for θ to the nearest degree or in radians to one dp. So, allow for example, awrt 2.5(0) following $\cos\theta = -\frac{4}{5}$

A1: Achieves $\cos\theta = -\frac{4}{5}$ and finds both θ values in degrees correct to one dp.

awrt 143.1° and awrt 216.9°

Any extra values in the range (for $\cos\theta = -\frac{4}{5}$) is A0

B1: Correctly achieves $\sin\theta = 0$ for their equation followed by $\theta = 180°, 360°$

Condone 0 appearing as a solution

……………………………………………………………………………………………………

Alternative approaches exist and can be marked in a similar way:
If you see an approach that deserves credit and you don't know how to score it, please send to review.
Alt I, II and III are examples of alternative approaches.

Alt I Using $\sin\theta = \tan\theta\cos\theta$

$4\tan\theta + 5\sin\theta = 0 \Rightarrow 4\tan\theta + 5\tan\theta\cos\theta = 0 \Rightarrow \tan\theta(4 + 5\cos\theta) = 0$

then solve $\cos\theta = -\frac{4}{5}$ and $\tan\theta = 0$

M1: Writing $\sin\theta$ as $\cos\theta\tan\theta$

dM1: Factorises out $\tan\theta$ and proceeds to $\cos\theta = k$ where $|k| < 1$

Alt II Squaring approach

$4\sin\theta + 5\sin\theta\cos\theta = 0 \Rightarrow 4\sin\theta = -5\sin\theta\cos\theta \Rightarrow 16\sin^2\theta = 25\sin^2\theta\cos^2\theta$

This will produce $\sin\theta = \pm\frac{3}{5}$ and $\sin\theta = 0$

M1: As main method

dM1: Squaring each term including the coefficients, factorising and cancelling leading to $\sin\theta = k$ where $|k| < 1$ Note that an approach $a + b = 0 \Rightarrow a^2 + b^2 = 0$ would score

dM0. It must be $a + b = 0 \Rightarrow a^2 - b^2 = 0$

B1: Allow this to be scored even from an approach where dM1has not been awarded

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Alt III using Pythagoras' theorem

$$4\frac{\sin\theta}{\cos\theta} + 5\sin\theta = 0 \Rightarrow 4\frac{\sin^2\theta}{\cos\theta} + 5\sin^2\theta = 0 \Rightarrow 4\frac{(1-\cos^2\theta)}{\cos\theta} + 5(1-\cos^2\theta) = 0 \Rightarrow 5\cos^3\theta + 4\cos^2\theta - 5\cos\theta - 4 = 0$$

Then solve by any means to find values for $\cos\theta$

$\Rightarrow \cos\theta = \pm1, -\frac{4}{5} \Rightarrow \theta = 180°, 360°, \text{awrt } 143.1°$ and awrt $216.9°$

M1: As main method. dM1: Correct method of producing a 4 term cubic in $\cos\theta$ followed by a valid attempt to solve to produce value(s) for $\cos\theta$

(ii) Condone solutions that drop x's in parts of the working and notational errors e.g. $\sin x^2 \leftrightarrow \sin^2 x$ as long as the candidate proceeds in a manner equivalent to the scheme

M1: Attempts to multiply by $\sin x \cos x$, seen on the two terms on the lhs of the equation and then uses $\sin^2 x + \cos^2 x = 1$. Condone slips when rearranging this identity as long as the equation leads to $\sin x = k, |k| < 1$ o.e

Alternatively uses a common denominator approach and simplifies $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ to $\frac{1}{\sin x \cos x}$

A1: Proceeds to $5\sin x = 1$ or exact equivalent

dM1: Correct attempt to solve $5\sin x = 1$, implied by one correct solution to 2 sf in radians or 1 dp in degrees within the given range. 0 to So $5\sin x = 1 \Rightarrow \sin x = \frac{1}{5} \Rightarrow x = 0.2$ only is dM0 but x = 0.20 or 2.9 is dM1.

Note that the degree answer is 11.5.
Also, solutions such as 0.064π (radians) are acceptable

Allow a miscopy on the 5. So, for example, if they had written $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{3}{\cos x}$ it would be for a solution of $3\sin x = 1$.

A1: $x = \text{awrt } 0.201, 2.94$ and no extras within the given domain (condoning the appearance of $\frac{\pi}{2}$ which occurs from a solution of $\cos x = 0$). Award as soon as you see the two correct solutions, so long as there are no additional solutions inside the range.

Also allow multiples of π so accept $x = 0.064\pi, 0.936\pi$ with the decimal aspect being accurate to at least 3dp

Question Number	Scheme	Marks
6 (a)	Centre of circle is midpoint of $(-2,18)$ and $(14,6) = (6,12)$	B1
	Attempts radius^2 or diameter^2. E.g. $D^2 = (14--2)^2 + (6-18)^2 = 400$	M1
	Radius^2 = 100	A1
	$(x-6)^2 + (y-12)^2 = 100$	M1, A1
		(5)
(b)	Recognises equation of C_2 is $x^2 + y^2 = k^2$	B1
	Attempts to find one value of k or k^2 Look for $\sqrt{"6"^2 + "12"^2} \pm \sqrt{"100"}$	M1
	$x^2 + y^2 = \left(6\sqrt{5}+10\right)^2$ or $x^2 + y^2 = \left(6\sqrt{5}-10\right)^2$ o.e.	A1
	$x^2 + y^2 = \left(6\sqrt{5}+10\right)^2$ and $x^2 + y^2 = \left(6\sqrt{5}-10\right)^2$ o.e.	A1
		(4) **(9 marks)**

(a) **The demand here is that the method must be made clear**

B1: Correctly finds the midpoint of the two given points to find the centre of the circle.
It can be implied within the circle formula $(x-6)^2 + (y-12)^2 = r^2$

M1: Attempts to find the radius, radius^2 , diameter or diameter^2.
It does not need to be correctly assigned to either the radius or the diameter
So $(14--2)^2 + (6-18)^2$, $16^2 + 12^2$, $\sqrt{8^2 + 6^2}$ or radius = 10 are acceptable attempts that score M1

A1: Correctly calculates the radius^2 = 100 o.e. or the radius = 10
Alternatively states that the diameter = 20 or diameter^2 = 400
It must be simplified and correctly assigned (radius or diameter) but may be implied from within a circle formula $(x \pm '6')^2 + (y \pm '12')^2 = 100$

M1: Attempts correct form for circle using $(x-a)^2 + (y-b)^2 = r^2$
where (a, b) is their m.p. of *AB* and r is the value of what they think is the radius of the circle.
If there is no statement i.e, radius(*r*) = ... or diameter (*d*) = ... , the value used must be the equivalent of the distance between the mid-point of *AB* and either point *A* or point *B*.
Look for $(x-"6")^2 + (y-"12")^2 = "100"$ or $(x-"6")^2 + (y-"12")^2 = \frac{"400"}{4}$
So if they clearly state something that is incorrect, e.g. $r^2 = 10$, thus losing the previous A mark, they can score this method mark for $(x-6)^2 + (y-12)^2 = 10$

A1: $(x-6)^2+(y-12)^2=100$, $(x-6)^2+(y-12)^2=10^2$ but not $(x-6)^2+(y-12)^2=\left(\sqrt{100}\right)^2$

ISW after a correct answer.

The method used must have been made clear for A1 to be scored. So, look for all of the following

- a correct calculation of the mid-point of *AB* or alternatively words such as 'centre of circle is $(6, 12)$'
- a correct calculation for radius (radius^2) or diameter (diameter^2) or alternatively words such as 'radius of circle is 10'
- a correct equation

(b)

B1: Recognises equation of C_2 is $x^2+y^2=k^2$. Allow for any $x^2+y^2=p, \quad p>0$

Accept $(x\pm 0)^2+(y\pm 0)^2=p, \quad p>0$ Allow 'letters' here so long as the rhs is positive, so k^2 is fine but *p* isn't unless *p* is defined to be positive.

M1: Attempts to find at least one value for the radius of C_2.

For example, score for a radius of $\sqrt{a^2+b^2}\pm r$ for their $(x-a)^2+(y-b)^2=r^2$

A1: $x^2+y^2=\left(6\sqrt{5}+10\right)^2$ or $x^2+y^2=\left(6\sqrt{5}-10\right)^2$ or exact equivalents including versions such as $(x\pm 0)^2+(y\pm 0)^2=\left(6\sqrt{5}+10\right)^2$. ISW after sight of a correct answer.

Expanded equivalents are $x^2+y^2=280+120\sqrt{5}$ or $x^2+y^2=280-120\sqrt{5}$ but isw after a correct answer

A1: $x^2+y^2=\left(6\sqrt{5}+10\right)^2$ and $x^2+y^2=\left(6\sqrt{5}-10\right)^2$ o.e. including versions such as $(x\pm 0)^2+(y\pm 0)^2=\left(6\sqrt{5}+10\right)^2$. ISW after sight of correct answers.

Expanded equivalents are $x^2+y^2=280+120\sqrt{5}$ and $x^2+y^2=280-120\sqrt{5}$ but isw after a correct answer

(b) Longer alternative

Substitutes $y=2x$ into $(x-6)^2+(y-12)^2=100$ and solves

E.g. $(x-6)^2+(2x-12)^2=100\Rightarrow 5(x-6)^2=100\Rightarrow (x-6)^2=20\Rightarrow x=6\pm 2\sqrt{5}$

Hence points of intersection are $\left(6-2\sqrt{5}, 12-4\sqrt{5}\right)$ and $\left(6+2\sqrt{5}, 12+4\sqrt{5}\right)$

Radius for smaller circle is found via $r^2 = \left(6-2\sqrt{5}\right)^2 + \left(12-4\sqrt{5}\right)^2 = 280-120\sqrt{5}$

Radius for larger circle is found via $r^2 = \left(6+2\sqrt{5}\right)^2 + \left(12+4\sqrt{5}\right)^2 = 280+120\sqrt{5}$

The same scoring traits can be applied

B1: Recognises equation of C_2 is $x^2 + y^2 = k^2$. Allow for any $(x \pm 0)^2 + (y \pm 0)^2 = p, \quad p > 0$

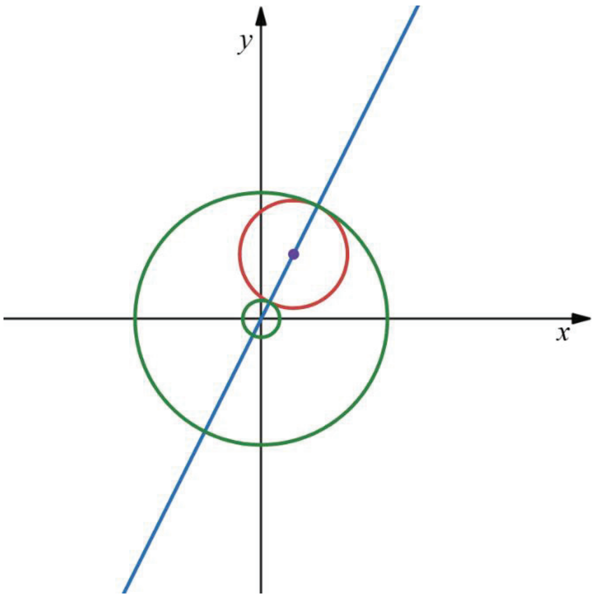
M1: Attempts to find at least a value of r or a value of r^2 for C_2.

For example,

- find at least one intersection point of the line through (0, 0) and their (6, 12) and the circle
- find the distance from the origin to one of the points of intersection

A1: $x^2 + y^2 = 280-120\sqrt{5}$ or $x^2 + y^2 = 280+120\sqrt{5}$ or exact equivalents

A1: $x^2 + y^2 = 280-120\sqrt{5}$ and $x^2 + y^2 = 280+120\sqrt{5}$ or exact equivalents



Question Number	Scheme	Marks
	$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44$	
7.(a)	$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \Rightarrow \left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$	M1, A1
		(2)
(b)	Subs $x=9$ into $\frac{dy}{dx}$ and sets = 0 $\Rightarrow 3\times81 - 10\times27 - \frac{k}{6} + 28 = 0$	M1
	$243 - 270 - \frac{k}{6} + 28 = 0 \Rightarrow \frac{k}{6} = 1 \Rightarrow k = 6$ *	A1*
		(2)
(c)	$\int x^3 - 4x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + 28x - 44\,dx = \frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x$	M1, A1, A1
	Correct value of $y = -53$ at *T*	B1
	Shaded area = $\text{"53"}\times 9 + \left[\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x\right]_0^9$	dM1
	= 248	A1
		(6)
		(10 marks)

(a)

M1: Attempts to differentiate. Look for **two** correct terms, one of which could be $-44 \to 0$.
Allow the coefficients to be un-simplified but the indices must be processed

A1: $\left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$ or other simplified equivalent. ISW after a correct answer

(b)

M1: Substitutes $x=9$ into their $\frac{dy}{dx}$ and sets = 0. Look for embedded values, or 4 simplified terms set = 0

For the M1 only, allow a substitution of $x=9$ into their $\frac{dy}{dx}$ which develops into an equation that implies $\frac{dy}{dx}$ = 0. E.g. $\frac{dy}{dx} = 3\times9^2 - 10\times9^{\frac{3}{2}} - \frac{k}{2}\times9^{-\frac{1}{2}} + 28 = \frac{k}{6} - 1$ So $\frac{k}{6} = 1$

Alternatively substitutes $x=9$ and $k=6$ into their $\frac{dy}{dx}$ and finds the value of $\frac{dy}{dx}$ with conditions as above.

A1*: Shows that $k=6$

Via the main method you must see

- an un-simplified equation such as $3\times 81-270-\frac{1}{6}k+28=0$
- a simplified equation such as $\frac{k}{6}=1$ o.e.
- and the given answer $k=6$

Via the alternative method you must see

- an un-simplified expression for $\frac{\mathrm{d}y}{\mathrm{d}x}=3\times 81-270-\frac{1}{6}\times 6+28$
- before proceeding to $\frac{\mathrm{d}y}{\mathrm{d}x}=0$
- followed by a conclusion stating that there is a stationary point when $k=6$

(c) Main method: Finds both areas and subtracts

M1: Attempts to integrate. Look for two correct indices which must be processed

A1: Two correct terms but allow the coefficients to be left un-simplified

A1: $\frac{1}{4}x^4-\frac{8}{7}x^{\frac{7}{2}}-4x^{\frac{3}{2}}+14x^2-44x$ allowing coefficients to be left un-simplified.

Condone an additional +c

B1: $y=-53$ at *T* May be awarded at any point in the question, even from the diagram

dM1: Look for a FULL method that involves (area of rectangle –area under curve) either way around to find a value for the area of *R*. Condone slips but it is dependent upon the previous M.

The area of the rectangle must be found using $9\times$ (an attempt at the *y* value of the curve at *x* = 9), and not for instance using 9×44 as 44 is the intersection of the curve with the *y* -axis.

You don't need to see working for the evaluation of $\left[\frac{1}{4}x^4-\frac{8}{7}x^{\frac{7}{2}}-4x^{\frac{3}{2}}+14x^2-44x\right]_0^9$

A1: AWRT 248. Allow the exact answer $\frac{6939}{28}$ Note that -248 is A0

(c) Alt method via integration of line – curve (or curve –line)

This could also be attempted by translating the curve upwards by 53 units

The following example shows a correct line – curve approach

$$\int '-53'-\left(x^3-4x^{\frac{5}{2}}-6x^{\frac{1}{2}}+28x-44\right)\mathrm{d}x=\int -9-x^3+4x^{\frac{5}{2}}+6x^{\frac{1}{2}}-28x\,\mathrm{d}x$$

$$-9x-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2$$

Shaded area = $-9\times 9-\frac{1}{4}\times 9^4+\frac{8}{7}\times 9^{\frac{7}{2}}+4\times 9^{\frac{3}{2}}-14\times 9^2=\frac{6939}{28}$

Area to 3sf = 248 which must be +ve

M1: Attempts to integrate an expression of the form $\pm\left(-x^3+4x^{\frac{5}{2}}+6x^{\frac{1}{2}}-28x\pm p\right)$.

Look for two correct indices which must be processed. Condone bracketing errors
Allow even with *p* = 97 or 0

A1: Two correct terms of $\pm\left(-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2\pm px\right)$ but allow the coefficients to be left un-simplified. Follow through on their value of *p* which may be 97 or 0

A1: Correct integration. So look for $\pm\left(-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2\pm px\right)$, allowing the coefficients to be left un-simplified. Follow through on their value of *p* which may be 97 or 0. Condone an additional + c

B1: For $y=-53$ at *T*. May be awarded at any point in the question, even from the diagram

dM1: Attempts the value of $\pm\left[-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2-9x\right]_0^9$. No intermediate working is required.

This is dependent upon the first M and also obtaining '9*x*'

A1: AWRT 248. Allow the exact answer $\frac{6939}{28}$

Note that final answers of -248 and $|-248|$ are A0

Question Number	Scheme	Marks
8 (i)	$S_n = a + ar + ar^2 + ...ar^{n-1}$	B1
	$S_n = a + ar + ar^2 + ...ar^{n-1}$ **and** $rS = ar + ar^2 + ar^3 + ...ar^n$	
	Finds S_n and rS_n and subtracts. E.g. $S - rS = ...$	M1
	Completes proof $\Rightarrow S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*
		(3)
(ii) (a)	Attempts $U = 150 \times 0.92^n$ with $n = 5 \text{ or } 6$	M1
	$\Rightarrow (U_6) = 150 \times (0.92)^6 = 90.95\ (\approx 91$ litres) *	A1*
		(2)
(b)	Attempts $S_n = \dfrac{a(1-r^n)}{(1-r)}$ with $n = 39/40/41$ $a = 150/138$ and $r = 0.92$	M1
	$S = \dfrac{138(1-0.92^{40})}{(1-0.92)}$, $S = 150 \times \dfrac{0.92(1-0.92^{40})}{(1-0.92)}$ OR $S = \dfrac{150(1-0.92^{41})}{(1-0.92)} - 150$	A1
	1664 litres	A1
		(3)
		(8 marks)

Do not be too concerned with labelling here. It should be obvious which part of the question candidates are attempting to answer

(i) Condone $S \leftrightarrow S_n$ throughout.

A proof via the infinite sum formula gains no credit unless you see a correct expression for S_n.

B1: Scored for a correct expression for S_n. For example, $S_n = a + ar + ar^2 + ...ar^{n-1}$ including the "S_n" with a minimum of 3 terms and must include the first and the nth term.

$S_n = a + ar + ...ar^{n-1}$ would be acceptable but $S_n = a + ...ar^{n-1}$ is not.

It cannot be scored if some terms are incorrect $S_n = a + ... + \underline{\underline{ar^n}} + ar^{n-1}$

Or they are not in the correct order $S_n = a + ...ar^{n-1} + ar^{n-2}$

M1: States $S_n = a + ar + ar^2 + ...ar^{n-1}$ (minimum of 3 **correct** terms including **first** and **last** terms)

and $rS_n = ar + ar^2 + ...ar^n$ (minimum of 3 correct terms including first and last terms)

followed by an attempt to subtract either way around

A1*: Fully correct proof. Look for the following 3 lines (or their equivalents) following B1, M1

- $S_n - rS_n = a - ar^n$
- $S_n(1-r) = a(1-r^n)$
- $S_n = \dfrac{a(1-r^n)}{(1-r)}$ (Note that this is a given answer)

Special case: Candidates who do not have the '+' signs in their work

Example

$S_n : a \quad ar \quad ar^2 \quad .... \quad ar^{n-1}$

$rS_n : ar \quad ar^2 \quad ar^3 \quad .... \quad ar^n$

$rS_n - S_n = ar^n - a$

$$(r-1)S_n = a(r^n - 1) \Rightarrow S_n = \frac{a(r^n-1)}{(r-1)} \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}$$

Scores SC B0, M1, A1

(Note that this candidate subtracts the other way around, but the given answer must be produced and follow correct working)

(ii) (a)

M1: Attempts $U_n = 150 \times 0.92^n$ with $n = 5 \text{ or } 6$.

You may see variations on this such as $150 \times (92\%)^5$

Allow step by step solutions such as

$$150 \xrightarrow{\times 0.92} 138 \xrightarrow{\times 0.92} 126.96 \xrightarrow{\times 0.92} 116.8032 \xrightarrow{\times 0.92} 107.458944 \xrightarrow{\times 0.92} 98.86222848 \xrightarrow{\times 0.92} 90.9532502$$

(with accuracy at least 1dp for the M1)

A1*: Awrt 90.95 but accept the truncated value 90.9. You should expect to see an answer that isn't exactly 91

ISW after sight of a correct answer

Note that candidates who just write $150 \times 0.92^6 = 91$ then award M1, A0 as this is incorrect

If attempted via step by step solutions (listing) then intermediate answers must be correct to 2dp rounded or truncated.

(b)

M1: Attempts $S_n = \frac{a(1-r^n)}{(1-r)}$ with $n = 39/40/41$, $a = 150/138$ and $r = 0.92$ o.e. such as 92%

A1: A correct expression. You may see variations of the following

$S = \frac{138(1-0.92^{40})}{(1-0.92)}$, $S = 150 \times \frac{92\%(1-92\%^{40})}{(1-92\%)}$ OR $S = \frac{150(1-0.92^{41})}{(1-0.92)} - 150$

A1: cao 1664 litres

Question Number	Scheme	Marks
9 (i)	$2\log_3(4x+5)-\log_3(x+3)=2 \Rightarrow \log_3\dfrac{(4x+5)^2}{(x+3)}=2$	M1, M1
	$\Rightarrow \dfrac{(4x+5)^2}{(x+3)}=9$	A1
	$\Rightarrow 16x^2+31x-2=0$	
	$(16x-1)(x+2)=0 \Rightarrow x=\dfrac{1}{16}$ only	dM1, A1
		(5)
(ii) (a)	States that $\log a+\log b=\log(ab)$ or else uses rule and proceeds from given equation $\log a+\log b=\log(a+b)$ to $\log ab=\log(a+b)$	B1
	Deduces $ab=a+b \Rightarrow ab-a=b \Rightarrow a(b-1)=b \Rightarrow a=\dfrac{b}{b-1}$ *	M1, A1*
(b)	States either $b>1$ or $b\neq 1$ as a would not be defined	B1
	$b>1$ as logs only exist for positive numbers	B1
		(5)
		(10 marks)

(i) **We are now scoring this question M1, M1, A1, dM1, A1**

M1: Usually scored for the power law of logs $2\log_3(4x+5)\to\log_3(4x+5)^2$ (allow without the base 3)

But may be awarded for writing 2 as $\log_3 9$

M1: For combining two terms of the original equation.

E.g $\log_3\dfrac{(4x+5)^2}{(x+3)}=2$ (allow without the base 3) or $2\log_3(4x+5)=\log_3 9(x+3)$

A1: A correct equation not involving logs. E.g. $\dfrac{(4x+5)^2}{(x+3)}=9$ or $(4x+5)^2=9(x+3)$

dM1: Requires all of the following

- A starting equation of the form (or equivalent to the form) $\dfrac{(4x+5)^2}{(x+3)}=k,\quad k>0$
- An intermediate equation that is a 3TQ
- A correct attempt to solve the 3TQ by any means condoning the use of a calculator here.

It is dependent upon scoring at least one of the previous M marks

A1: CSO $\dfrac{1}{16}$ oe only. The decimal equivalent is 0.0625

'Correct' solutions with missing or incorrect lines

Example I: Incorrect statement

$2\log_3(4x+5)-\log_3(x+3)=2 \Rightarrow \dfrac{\log_3(4x+5)^2}{\log_3(x+3)}=2 \Rightarrow \dfrac{(4x+5)^2}{(x+3)}=9$ leading to full and correct solution

Score Special Case M1, M0, A1, dM1, A0 for 3 out of 5

Example II: Missing lines

$2\log_3(4x+5)-\log_3(x+3)=2 \Rightarrow \log_3\dfrac{(4x+5)^2}{(x+3)}=2 \Rightarrow \dfrac{(4x+5)^2}{(x+3)}=9$

Or even $2\log_3(4x+5)-\log_3(x+3)=2 \Rightarrow \dfrac{(4x+5)^2}{(x+3)}=9$ leading to full and correct solution

can be awarded all the marks.

(ii) You can condone the omission of the base

(a) Main method

B1: States or uses $\log a+\log b=\log(ab)$ o.e. It is implied when $\log(ab)=\log(a+b)$ is written down or $ab=a+b$ but it cannot be awarded following incorrect work. Condone $\log a+b$ for $\log(a+b)$ if the intention is clear (but the use of this would mean that the A1* mark is not scored)

Alternatives exist such as $\log(a+b)-\log b=\log(\frac{a+b}{b})$

M1: Following correct log work, candidate removes the logs, correctly deduces $ab=a+b$, and then attempts to collect terms in *a*. To award this mark the two terms in *a* must be moved to the same side of the equation and the *a* factorised out. It can be awarded when the term in '*a*' is isolated. E.g $ab\pm a=b \Rightarrow a(b\pm1)=b$

Alternatively removes the logs, deduces $a=\dfrac{a+b}{b}$ and then attempts to collect terms in *a*

Acceptable alternatives exist. E.g. $ab=a+b \overset{\div a}{\Rightarrow} \underline{\underline{b=1+\dfrac{b}{a}}} \Rightarrow \dfrac{b}{a}=b-1 \Rightarrow a=\dfrac{b}{b-1}$

The method mark can be awarded here when there is a single term in *a*. See double underlined expression

A1*: Correctly proves that $a=\dfrac{b}{b-1}$ via $a(b-1)=b$ or $ab-a=b$.

This is a given answer and **ALL** appropriate lines must be seen and be correct.

Condone a missing trailing bracket.

The appropriate lines that must be seen (as a minimum) via the standard approach are;

$$\log ab=\log(a+b) \Rightarrow ab=a+b \Rightarrow a(b-1)=b \Rightarrow a=\frac{b}{b-1}$$

(a) Alt method

B1: Substitutes $a=\frac{b}{b-1}$ in $\log a+\log b$ and proceeds to $\log(\frac{b^2}{b-1})$

M1: Substitutes $a=\frac{b}{b-1}$ in $\log(a+b)$ and proceeds to $\log\left(\frac{b}{b-1}+b\right)$ and attempts to write as a single fraction

A1*: Shows that $\log a+\log b=\log(ab)$ when $a=\frac{b}{b-1}$ and makes a minimal statement.

This is a given answer and all appropriate lines must be seen and be correct (including any bracketing).

(ii)(b)

B1: For either (i) giving the full restriction on *b*. E.g.

$b>1$, '*b* is more than one' or 'it must be bigger than one'. No (correct) explanation required.

or (ii) stating that $b\neq 1$ with a reason such as $\frac{b}{b-1}\to\infty$ or $b-1=0$.

Allow reasons such as 'the denominator cannot be 0' 'when *b* = 1 you get a maths error'

Do not accept ambiguous statements such as $b\neq 1$ because it cannot be 0

or (iii) stating that $b \geqslant 1$ with a reason such as $b-1$ cannot be negative

B1: $b>1$ or '*b* must be more than one' as logs only exist for positive numbers.

Allow '$b>1$ as *a* must be greater than 0'

Allow variations that imply this so accept $b>1$ as $\frac{b}{b-1}$ needs to be positive or $b-1$ needs to be positive

There really needs to be some words here and the statement and explanation must be given together and in a logical order. A minimum acceptable response could be 'as *a* > 0, *b* >1'

Do not accept incorrect reasons such as '$b>1$ as *a* must be greater than or equal to 0'

Do not award this mark if there are incorrect statements along with correct ones. If unsure use review.