Question Number	Scheme		Marks
1.			
	a | *b* | *c* | *abc*	One correct set *a*, *b* & *c*	B1
	2 | **15** | **3** | **90**		
	3 | **9** | **8** | **216**	Two correct rows	M1
	4 | **1** | **15** | **60**		
	All values of *abc* are multiples of 6	Fully correct + statement	A1
			(3 marks)

Main method: Numerical answer finding all known solutions

B1: Any correct set of values for *a*, *b* and *c*. You can ignore any attempt at the product *abc*

M1: Any two fully correct rows including the correct calculations for the products.
Condone any extra or incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for *abc*
- any extra (or incorrect) rows that may be present must be deleted or somehow excluded
- a minimal statement showing that they are aware that 90, 216 and 60 are multiples of 6.

  E.g they are all divisible by 6, use of a tick ✓, proven, QED, calculations such as $\frac{90}{6}=15$. They can be divided by 6 (on its own) is insufficient without further clarification such as "to give whole numbers"
- no incorrect calculations. E.g if their attempt to prove that 90 being a multiple of 6 includes $\frac{90}{6}=10$ then it is A0

Algebraic solution or solutions that rely on explanations: These will be rare and unlikely to lead to completely correct solutions.

Example of an attempt that has some merit based upon odds and evens

If *a* is even
Then $a=2n$, leading to $c=4n^2-1=(2n+1)(2n-1)$ and $b=21-4n^2-2n$
giving $abc=2n(21-4n^2-2n)(2n+1)(2n-1)$

If *a* is odd.
Then $a=2n+1$, leading to $c=4n^2+4n=4n(n+1)$ and $b=19-4n^2-6n$
giving $abc=(2n+1)(19-4n^2-6n)4n(n+1)$

B1: Attempts both odd and even and achieves correct algebraic expressions for *a*, *b* and *c* in both cases.
M1: Correct products with attempts when $n=1, 2$ and achieves two fully correct rows
A1: Fully correct with no additional rows

Question Number	Scheme	Marks
2.(a)	E.g. $64\times\left(-\frac{1}{2}\right)^3=-8$ ✓ *	M1, A1*
		(2)
(b)	Finds the value of *a*. E.g. $64\div\left(-\frac{1}{2}\right)^2=256$	M1, A1
	Uses $S_\infty=\frac{a}{1-r}=\frac{256}{1-\left(-\frac{1}{2}\right)}=\frac{512}{3}$	M1, A1
		(4)
		(6 marks)

(a)

M1: Any acceptable attempt to prove the statement.

This usually involves getting to an equation in *r*…

E.g. I $ar^2=64,\quad ar^5=-8\ \Rightarrow 64r^3=-8$ o.e. such as $ar^2=64,\quad ar^5=-8\ \Rightarrow r^{-3}=-8$

E.g. II $ar^2=64,\quad ar^5=-8\ \Rightarrow \frac{64}{r^2}=\frac{-8}{r^5}$

(the numbers used in the attempt, that is 64 and -8 must be correct)

but could involve using the given answer

E.g. III $64\times\left(-\frac{1}{2}\right)^3$, $64\times-\frac{1}{2}\times-\frac{1}{2}\times-\frac{1}{2}$ or $-8\div\left(-\frac{1}{2}\right)^3$

Condone the following types of slips/errors

E.g. I or II with incorrect indices on terms as long as they differ by 3;

$ar^3=64,\quad ar^6=-8\ \Rightarrow r^3=-\frac{8}{64}$

E.g. III with missing brackets $64\times-\frac{1}{2}^3$

A1*: Fully correct proof. A minimal proof could be $r^3=-\frac{8}{64}\Rightarrow r=-\frac{1}{2}$ o.e

As the result is given a direct proof must have a penultimate line equivalent of $r^3=-\frac{8}{64}$ or $r^{-3}=-8$

If they state $64r^3=-8$ followed by $r=-\frac{1}{2}$ without any intermediate work then it is M1 A0

If they assume the answer then expect to see equivalent to $64\times\left(-\frac{1}{2}\right)^3=-8$ which must be followed by a concluding statement (tick sufficient) is required.

Equivalent work would be $64\xrightarrow{\times-\frac{1}{2}}-32\xrightarrow{\times-\frac{1}{2}}16\xrightarrow{\times-\frac{1}{2}}-8$ ✓

This mark cannot be awarded from incorrect indices on *r* or missing brackets (that are necessary)

(b) **The marks for the following can only be awarded from work in part (b)**

M1: Uses a suitable method to find the value of a allowing $r = \pm\frac{1}{2}$

E,g. Attempts $a \times \left(\pm\frac{1}{2}\right)^2 = 64 \Rightarrow a = ...$

Condone a missing bracket e.g. $a \times -\frac{1}{2}^2 = 64 \Rightarrow a = ...$

A1: States or implies that $a = 256$ via the correct $r = -\frac{1}{2}$.

Stating the value scores the two marks. Note that $a \times \frac{1}{4} = 64 \Rightarrow a = 256$ is M1, A1 (nothing incorrect)

This can be awarded following a missing bracket. (It isn't a proof)

M1: Uses $S_\infty = \frac{a}{1-r}$ with a value of $a \neq 64, -8$ and $r = \pm\frac{1}{2}$

So award $S_\infty = \frac{'a'}{1-\frac{1}{2}} = ...$ or $S_\infty = \frac{'a'}{1-\left(-\frac{1}{2}\right)} = ...$ but if an incorrect formula is stated score it M0

A1: $\frac{512}{3}$ o.e . ISW after a correct answer.

Condone $170.\dot{6}$ but do not allow awrt 170.7 unless the exact correct answer is seen first.

Question Number	Scheme	Marks
	$f(x)=(3x^2-4x-5)(x-k)-5$	
3 (a)	States -5	B1
		(1)
(b)	Sets $f(-2)=25 \rightarrow (3\times4-4\times-2-5)(-2-k)-5=25$	M1
	$15(-2-k)=30 \Rightarrow -2-k=2 \Rightarrow k=-4$ *	A1*
		(2)
(c)	$(3x^2-4x-5)(x+4)-5=3x^3+8x^2-21x-25$	B1
	Attempts $3x-1\overline{)3x^3+8x^2-21x-25}$ to achieve quotient of $...x^2+...x+...$ and a remainder that is a constant	M1
	Quotient = x^2+3x-6 OR Remainder = -31	A1ft
	Quotient = x^2+3x-6 AND Remainder = -31	A1
		(4)
		(7 marks)

(a)
B1: States -5
You may see $f(k)$ or a division sum attempted, which is fine. You are just awarding the sight of -5
Don't be too concerned about what -5 is called. You may see $f(x)$, $f(k)$ or even x
(b)
M1: Sets $f(-2)=25$ to form an equation in k Condone sign slips within the calculations but setting $f(-2)=-25$ or $f(2)=25$ is M0. Some may attempt to substitute -2 into the expanded form of $f(x)=3x^3-(4+3k)x^2+(4k-5)x+5k-5$ which is fine. Condone slips but expect to see linear coefficients for k in each of x^2, x and the constant terms.
Don't be concerned about poor attempts to solve the equation in k for this mark
A1*: Shows sufficient working with no errors to prove that $k=-4$.
The answer must follow a simplified and (easily) solvable equation.
(See scheme for possible route but allow simplified linear equations like $-30-15k=30$)

....

M1: Alternatively substitutes $k=-4$ into $f(x)$ to get $(3x^2-4x-5)(x+4)-5$ and attempt the value of $f(-2)$
A1: Shows that $f(-2)=(12+8-5)(-2+4)-5=15\times2-5=25$ and makes a minimal conclusion e.g. ✓

....

(c)

B1: Multiplies out $(3x^2-4x-5)(x+4)-5$ to achieve a simplified $3x^3+8x^2-21x-25$

M1: Scored for a full method of finding **both** the quotient and the remainder.

The two main ways of scoring this mark are;

Either attempting to divide their expanded $\mathrm{f}(x)$ by $(3x-1)$ to form a quotient of the form $Ax^2+B.x+C$ and a remainder that is a constant.

Alternatively, setting up an identity of the form $(Ax^2+B.x+C)(3x-1)+R$ followed by an attempt to find the values of *A*, *B*, *C* and *R*.

Watch for candidates who find $\mathrm{f}\left(\frac{1}{3}\right)=-31$ and then adapt $\mathrm{f}(x)$ to $3x^3+8x^2-21x+6$. The M mark would be scored for dividing their $3x^3+8x^2-21x+6$ by $(3x-1)$ to form a quotient of the form $Ax^2+B.x+C$ and a remainder that is a constant (which should be 0 in a correct division).

A1ft: For either the quotient = x^2+3x-6. Allow this to be scored within the division sum

Or the remainder = -31 (may be scored within the division sum)

It must follow M1 so attempting $\mathrm{f}\left(\frac{1}{3}\right)$ and reaching -31 will not score this mark without further work

Follow through on finding a correct quotient or remainder for their $3x^3+8x^2-21x-25$ (4 term cubic expression). Note that any $3x^3+8x^2-21x+k$ will have a quotient of x^2+3x-6

A1: States that the Quotient = x^2+3x-6 AND the Remainder = -31. These cannot be scored within a division sum and must be stated separately to this. All previous marks must have been scored in (c) so cannot be awarded following an incorrect $\mathrm{f}(x)$

Question Number	Scheme	Marks
4. (a)	$\log_3(a+1)-\log_3 a=4 \Rightarrow \log_3\left(\frac{a+1}{a}\right)=4$	M1
	$\Rightarrow \left(\frac{a+1}{a}\right)=3^4$	A1
	$\Rightarrow a+1=81a \Rightarrow a=\frac{1}{80}$	dM1, A1
		(4)
(b)	Area $\approx \frac{1}{2}\left\{\log_3\left(\frac{2}{1}\right)+\log_3\left(\frac{6}{5}\right)+2\left(\log_3\left(\frac{3}{2}\right)+\log_3\left(\frac{4}{3}\right)+\log_3\left(\frac{5}{4}\right)\right)\right\}$	M1, A1
	$=\frac{1}{2}\log_3\left(\frac{2}{1}\times\frac{6}{5}\times\frac{3^2}{2^2}\times\frac{4^2}{3^2}\times\frac{5^2}{4^2}\right)=\frac{1}{2}\log_3 15=\log_3\sqrt{15}$	dM1, A1
		(4)
(c)	States 'increase the number of strips'	B1
		(1)
		(9 marks)

(a) Allow with $a \leftrightarrow x$ as seen in the notes

M1: Sets $\log_3(x+1)-\log_3 x=4$ and then applies the subtraction law. Condone the omission of the 3 in $\log_3$

Alternatively writes the equation $\log_3(x+1)=4+\log_3 x$ as $\log_3(x+1)=\log_3 k+\log_3 x \quad (k\neq 4)$ and then uses the addition law $\log_3(x+1)=\log_3(kx)$

A1: Correct equation not involving logs. For example, $\left(\frac{x+1}{x}\right)=3^4$

dM1: Correct attempt to solve an equation of the form $\frac{x+1}{x}=k$ o.e. where k is an integer $k\neq 3$ or 4.

Look for $\frac{x+1}{x}=k \Rightarrow x=\frac{1}{"k"-1}$ where $k\neq 3$.

A1: a or $x=\frac{1}{80}$ or 0.0125 following the award of M1,A1, dM1.

Working must be seen as required by the question.

Special Case:
You may see attempts that proceed towards the correct answer using incorrect laws.

E.g. $\log_3(x+1)-\log_3 x=4 \Rightarrow \frac{\log_3(x+1)}{\log_3 x}=4 \Rightarrow \left(\frac{x+1}{x}\right)=3^4 \Rightarrow x=\frac{1}{80}$

Candidates who achieve the correct answer using the above incorrect line can score SC 0, 0, 1, 1

(b)

Condone the omission of the 3 in $\log_3$ except for the final A mark where they must arrive at $\log_3\sqrt{15}$.

Any use of calculators to produce decimal equivalents to these values will result in a maximum score of 1,0,0,0 marks as an exact value could never be achieved.

M1: Score for a reasonable attempt at applying the trapezium rule.

Look for **an attempt** at the correct form $\frac{h}{2}\{\text{first term}+\text{last term}+2(\text{sum of the other terms})\}$ with at least the following

(1) Correct first **or** last term

(2) At least two correct 'other terms'

(3) $h=1$

The bracketing must be correct but condone a missing trailing bracket.

So, look for a minimum of $\frac{1}{2}\{a+e+2\times(b+c+d)$ with a or e correct, two out of three of b, c and d correct with the missing trailing bracket

FYI the decimal solution is $\frac{1}{2}\{0.631+0.166+2(0.369+0.262+0.203)\}$ allow awrt 2 dp

A1: A fully correct application of the trapezium rule in un-simplified form.

Amongst various correct alternatives are

$$\text{Area}\approx\frac{1}{2}\left\{\log_3\left(\frac{2}{1}\right)+\log_3\left(\frac{6}{5}\right)+2\left(\log_3\left(\frac{3}{2}\right)+\log_3\left(\frac{4}{3}\right)+\log_3\left(\frac{5}{4}\right)\right)\right\}$$

$$\text{Area}\approx\frac{1}{2}\{\log_3 2+\log_3 6-\log_3 5+2(\log_3 3-\log_3 2+\log_3 4-\log_3 3+\log_3 5-\log_3 4)\}$$

$$\text{Area}\approx\frac{1}{2}\{\log_3 2+\log_3 6-\log_3 5+2(-\log_3 2+\log_3 5)\}$$

$$\text{Area}\approx\frac{1}{2}\left\{\log_3 2+\log_3\frac{3}{2}\right\}+\frac{1}{2}\left\{\log_3\frac{3}{2}+\log_3\frac{4}{3}\right\}+\frac{1}{2}\left\{\log_3\frac{4}{3}+\log_3\frac{5}{4}\right\}+\frac{1}{2}\left\{\log_3\frac{5}{4}+\log_3\frac{6}{5}\right\}$$

FYI: A fully simplified expression for the approximate area is $\frac{1}{2}\{\log_3 6+\log_3 5-\log_3 2\}$

Condone the omission of the trailing bracket as long as subsequent work implies its presence.

dM1: Correct use of logs in an attempt to fully combine their terms following the award of the previous M.

This must include sight of at least one correct use of two of the three log laws.

So, use of just addition and subtraction laws is fine as would addition and power laws

This mark can be implied if they go directly from a correct exact expression to the correct answer.

Candidates must proceed as far as $\frac{1}{2}\log p$ or $\log q$

A1: $\log_3\sqrt{15}$ following M1, A1 dM1

(c)

B1: States

- 'increase the number of strips'
- 'decrease the width of the strips'
- 'make more strips'

without any incorrect statements such as increase the strips, so try $\int_1^{20}\log_3(x+1)-\log_3 x\,dx$

Do not allow vague statements such as 'add more numbers of x'

Question Number	Scheme	Marks
5. (a)	$u_2 = 1 - \frac{1}{4} = \frac{3}{4}, \quad u_3 = 1 - \frac{4}{3} = -\frac{1}{3},$	M1 A1
	$u_4 = 1 - \left(-\frac{3}{1}\right) = 4$ with statement saying the sequence repeats every 3 terms o.e	A1*
		(3)
(b)	$\sum_{n=1}^{180}(5n+3) = \frac{180}{2}(8+903) = (81990)$	M1
	$\sum_{n=1}^{180}(u_n) = 60 \times \left(4 + \frac{3}{4} + -\frac{1}{3}\right) = (265)$	M1
	$\sum_{n=1}^{180}(5n+3+u_n) = 81990 + 265 = 82255$	A1, A1
		(4)
		(7 marks)

(a)

M1: Attempts to use the iterative sequence correctly at least once.

Most often this is for $u_2 = 1 - \frac{1}{4}$ or $u_2 = \frac{3}{4}$ o.e.

Condone omission of the lhs as long as it is obvious. E.g. 4, 0.75

A1: Achieves a correct value for both u_2 and u_3. Condone $u_3 = -0.\dot{3}$

Do not allow $u_3 = -0.33$ unless you see the exact value beforehand.

Allow $(4), \frac{3}{4}, -\frac{1}{3}$ if it is obviously values for both u_2 and u_3.

A1*: This is a show that question and requires

- correct values for u_2 and u_3
- a correct calculation and value seen for u_4 E.g. $u_4 = 1 - \left(-\frac{3}{1}\right) = 4$
- a minimal reason/statement 'the sequence repeats every 3 terms' o.e such as 'as $u_4 = u_1$,
- a minimal conclusion such as sequence is periodic of order 3' or 'hence true'

(b) **All stages of working must be shown so don't accept just answers as evidence of method**

M1: Establishes a viable method to find $\sum_{n=1}^{180}(5n+3)$ using the fact that it is an AP.

Look for use of $\frac{n}{2}\{a+l\}$ or $\frac{n}{2}\{2a+(n-1)d\}$ o.e. with $n=180, a=3,5\ or\ 8$ and $d=5$

Alternatively attempts $\sum_{n=1}^{180}(5n+3)=\sum_{n=1}^{180}5n+\sum_{n=1}^{180}3=5\times\frac{180}{2}\{180+1\}+180\times3$ o.e.

Do not just accept $\sum_{n=1}^{180}(5n+3)=81990$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180}(5n+3+u_n)$

M1: Establishes a viable method to find $\sum_{n=1}^{180}(u_n)$.

For example, as it repeats every 3 terms look for $60\times(4+u_2+u_3)$ o.e such as $60\times4+60\times u_2+60\times u_3$

Do not accept $\sum_{n=1}^{180}(u_n)=265$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180}(5n+3+u_n)$

A1: Scored for either $\sum_{n=1}^{180}(5n+3)=81990$ or $\sum_{n=1}^{180}(u_n)=265$ following the award of the equivalent M mark

It can be implied, so for instance 82255 following the award of both M's but it is the value, not the calculation that must be seen or implied.

A less obvious award would be for $\sum_{n=1}^{180}(u_n+3)=60\times\left(4+\frac{3}{4}+-\frac{1}{3}+9\right)=805$ which implies the 265

A1: 82255 which must follow the award of M1, M1 and A1

Question Number	Scheme	Marks
6 (i) (a)	Uses $\sin^2\theta+\cos^2\theta=1$ with $\cos\theta=\frac{1}{\sqrt{5}}\Rightarrow\sin^2\theta=\frac{4}{5}$	M1
	$\Rightarrow\sin\theta=-\frac{2}{\sqrt{5}}$	A1
(b)	Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ with $\cos\theta=\frac{1}{\sqrt{5}}$ and their $\sin\theta=-\frac{2}{\sqrt{5}}$	M1
	$\Rightarrow\tan\theta=-2$	A1
		(4)
(ii) (a)	1 (m)	B1
		(1)
(b)	$30t-40=180\Rightarrow t=7.33$ Hence 7:20 **am**	M1, A1, A1
		(3)
(c)	$4+3\cos(30T-40)°=3.5\Rightarrow\cos(30T-40)°=-\frac{1}{6}$	M1
	$\Rightarrow 30T-40=99.6,\ 260.4,\ \underline{\underline{459.6}}$	A1
	$\Rightarrow T=\frac{"459.6"+40}{30}=16.65$	dM1, A1
		(4)
		(12 marks)

(i)

Clear use of trigonometric identities must be shown. Answers without method don't score marks

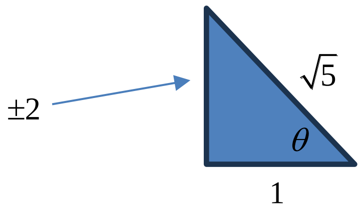
Look for work within the body of the script which may be common

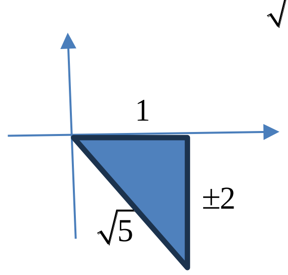
(a)

M1: Uses $\sin^2\theta+\cos^2\theta=1$ with $\cos\theta=\frac{1}{\sqrt{5}}\Rightarrow\sin^2\theta+\frac{1}{5}=1$ o.e.

Alternatively shows a 'triangle' method and uses Pythagoras' theorem to reach $\sin\theta=\pm\frac{2}{\sqrt{5}}$

E.g.





A1: $\sin\theta=-\frac{2}{\sqrt{5}}$ or simplified equivalent such as $-\frac{2\sqrt{5}}{5}$ following the award of the M1. ISW after a correct answer. Note that $\sin\theta=-\sqrt{\frac{4}{5}}$ is A0 as it is not simplified

(b)

M1: Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ with $\cos\theta = \frac{1}{\sqrt{5}}$ and either $\sin\theta = \frac{2}{\sqrt{5}}$ or $\sin\theta = -\frac{2}{\sqrt{5}}$ from (a) which must have been awarded from a correct method.

Alternatively, from a triangle method, shows the opposite side = 2 in a right-angled triangle and takes the correct ratio

A1: $\tan\theta = -2$ following the award of the M1 in both (i) and (ii)

(ii)(a)

B1: 1 (m) Units are not necessary here but incorrect units e.g. 1 cm would be B0

(ii)(b)

M1: Sets $30t - 40$ equal to any value in degrees where the cosine is -1

So $30t - 40 = 180(2n+1)$, for any $n \in \mathbb{Z}$ is acceptable. Condone $30t - 40 = (2n+1)\pi$,

Condone the result obtained from differentiation with $\sin(30t-40) = 0 \Rightarrow 30t - 40 = 180(2n+1)$

A1: $30t - 40 = 180 \Rightarrow t = 7\frac{1}{3}$ or 7.33 but condone 7.3. It must follow the award of the M mark

A1: Achieves 7:20 am or 07:20 or any other acceptable way of establishing the exact **time of day** time in hours and minutes. So 7 hours 20 minutes after midnight is fine. **Condone 7:20 but 7:20pm is A0 and 7 hours 20 minutes without reference to after midnight is A0. It must follow correct working and cannot be rounded from a different time value such as 7:19.5**

(ii)(c) Radian attempts can only usually score the first M1.

M1: Sets $4 + 3\cos(30T - 40)°$ equal to 3.5 and proceeds to $\cos(30T - 40)° = k, \quad |k| < 1$

A1: Achieves $30T - 40$ equal to any of $99.6, 260.4, 459.6$

dM1: Proceeds from $30T - 40$ equal to any of awrt $100, 260, 460$ AND uses correct order of operations and working leading to a value for T.

For example, allow for $30T - 40 = \text{awrt } 100 \Rightarrow T = \frac{140}{30}$ or awrt 4.7

$$30T - 40 = \text{awrt } 260 \Rightarrow T = \text{awrt } 10$$

$$30T - 40 = \text{awrt } 460 \Rightarrow T = \frac{500}{30} \text{ or awrt } 16.7$$

A1: awrt 16.65 following the award of M1, A1 dM1

ISW after a correct answer for T. Ignore any attempts to convert the T value to a time.

A clear method must be shown here, so solutions produced entirely from a calculator will not score.

E.g. $4 + 3\cos(30T - 40)° = 3.5 \Rightarrow T = 16.65$ is 0 marks

A minimal acceptable response for 4 marks would be

$$4 + 3\cos(30T - 40)° = 3.5 \Rightarrow \cos(30T - 40)° = -\frac{1}{6}$$

$$\Rightarrow 30T - 40 = 459.6$$

$$\Rightarrow T = 16.65$$

Question Number	Scheme	Marks
7 (a)	States either $kn=-24$ (1) or $\frac{n(n-1)}{2}k^2=270$ (2)	M1
	States both $kn=-24$ (1) and $\frac{n(n-1)}{2}k^2=270$ (2)	A1
	Substitutes $k=-\frac{24}{n}$ in equation (2) $\Rightarrow \frac{n(n-1)}{2}\left(-\frac{24}{n}\right)^2=270\Rightarrow n=...$	M1
	$n=16$	A1
	Uses their $n=16$ in $kn=-24\Rightarrow k=-\frac{24}{16}=-\frac{3}{2}$	dM1, A1
		(6)
(b)	$p=\frac{n(n-1)(n-2)}{3!}k^3=\frac{16\times15\times14}{6}\times\left(-\frac{3}{2}\right)^3=...$	M1
	$=-1890$	A1
		(2)
		(8 marks)

(a)

M1: States either $kn=-24$ (1) or $\frac{n(n-1)}{2}k^2=270$ (2) accepting equivalents but see note below

This mark cannot be awarded from $\binom{n}{1}k=-24$ or ${}^nC_2\left(kx\right)^2=270x^2$, with factorials

$\frac{n!}{(n-2)!}k^2=540$

or indeed versions including x terms such as $knx=-24x$ unless followed by a correct $kn=-24$

A1: States both $kn=-24$ (1) and $\frac{n(n-1)}{2}k^2=270$ (2) accepting equivalents

M1: Forms an equation in one variable leading to a value for k or n

It is dependent upon having achieved $kn=\pm24$ and $\frac{n(n-1)}{...}k^2=270$

The process of forming the equation must be credible so look for $k=\frac{\pm24}{n}$ being substituted and

NOT $k=\pm24-n$ FYI the equation in k is $\Rightarrow -\frac{24}{k}\left(-\frac{24}{k}-1\right)\times\frac{k^2}{2}=270\Rightarrow k=...$

A1: For either $n=16$ following a correctly solved equation in n.

or $k=-\frac{3}{2}$ if the equation in k was formed.

Look for correct working leading to the value for n. For example, forming a linear equation is sufficient. $\frac{n(n-1)}{2}\left(-\frac{24}{n}\right)^2=270\Rightarrow\frac{288(n-1)}{n}=270\Rightarrow 288n-288=270n\Rightarrow n=16$

dM1: Uses $k=\pm\frac{24}{n}$ o.e. with their $n=$'16' to find k or their $k=$'$-\frac{3}{2}$' to find n

This mark may be implied. It is dependent upon the **previous** M

So, if they have scored the previous M, achieved $n=8$ and $k=\frac{24}{n}$ award for $k=3$

A1: $n=16$ and $k=-\frac{3}{2}$ following the award of all previous marks

………

The demand of the question is that they show all stages of their working.

Example I

If a candidate states both equations $kn=-24$, $\frac{n(n-1)}{2}k^2=270$, shows **a correct intermediate line in a single variable** then follows this with correct $n=16$ and $k=-\frac{3}{2}$ **without the necessary working** they can be awarded SC: 1, 1, 1, 0, 1, 0

Example II

If a candidate states both equations $kn=-24$, $\frac{n(n-1)}{2}k^2=270$ and follows this with correct $n=16$ and $k=-\frac{3}{2}$ without any necessary intermediate equations and working they can be awarded SC: 1, 1, 0, 1, 0, 0

If the SC is seen in part (a), they can go on to score both marks in (b) for correct answers

……….

(b)

M1: Uses $p=\frac{n(n-1)(n-2)}{3!}k^3$ with their values for *n* and *k* found from part (a) to find a value for *p*.

This can be implied by a 'correct' answer for their values of *n* and *k*.

Allow fractional values for both *n* and *k*

Allow this to be scored by candidates who perhaps have only one equation, usually $kn=-24$ and go on to state for example that $n=4$ and $k=-6$ and then find $p=-864$

Values cannot be made up without an attempt in part (a) scoring at least one mark

A1: –1890 following correct values for *n* and *k*.

It must follow correct equations in part (a) but allow the values to be found from equations containing terms including factorial and/or combination notation.

Question Number	Scheme	Marks
8. (a)	$-5 < x < \frac{2}{3}$	M1, A1
		(2)
(b)	$(2x-7)$	B1
		(1)
(c)	$f'(x) = 2(3x-2)(x+5) = 6x^2 + 26x - 20$	
	$f(x) = 2x^3 + 13x^2 - 20x + c$	M1, A1
	$x = \frac{7}{2}, \quad y / f(x) = 0 \Rightarrow c = (-175)$	dM1, A1
	$f(x) = 2x^3 + 13x^2 - 20x - 175 = (2x-7)(x^2 + 10x + 25)$	ddM1
	$= (2x-7)(x+5)^2$	A1
		(6)
		(9 marks)

(a)
M1: Achieves the correct critical values. Condone for this mark -5 and awrt 0.67
They may be seen as part of an incorrect inequality
A1: $-5 < x < \frac{2}{3}$ or $-5 \leqslant x \leqslant \frac{2}{3}$. Either of these on their own (with consistent inequality use) scores both marks
Accept other valid alternatives such as $x \in \left(-5, \frac{2}{3}\right)$ and $\{x : x > -5\} \cap \left\{x : x < \frac{2}{3}\right\}$ but not $x < \frac{2}{3}$ or $x > -5$

(b)
B1: States $(2x-7)$ or $A = 2, B = -7$ which must be seen in (b).
Condone candidates who write $\left(x - \frac{7}{2}\right) = (2x-7)$ Note that stating $\left(x - \frac{7}{2}\right)$ on its own is B0

(c) Some candidates integrate in part (b). Allow these candidates to score the marks in (c) as long as they re-write $f(x)$ in (c) or do some extra work involving $f(x)$ in (c)

M1: Multiplies out $f'(x)$ to form a quadratic expression and attempts to integrate.
Look for $f'(x) = 2(3x-2)(x+5) = ax^2 + bx + c \rightarrow f(x) = px^3 + qx^2 + rx....(+c)$ with or without the + c
A1: $f(x) = 2x^3 + 13x^2 - 20x + c$ with or without the + c
dM1: Substitutes $x = \frac{7}{2}, \quad y / f(x) = 0 \Rightarrow c = ...$
Alternatives exist via division using the fact that $(2x-7)$ is a factor of $2x^3 + 13x^2 - 20x + c$
A1: $f(x) = 2x^3 + 13x^2 - 20x - 175$ but allow $f(x) = 2x^3 + 13x^2 - 20x + c$ followed by $c = -175$

ddM1: Attempts to divide or factorise out a factor of $(2x-7)$ or $\left(x-\frac{7}{2}\right)$ from their $f(x)=2x^3+13x^2-20x-175$

Awarded for 'correct' first and last terms or correct first two terms for their $f(x)$ condoning sign slips

E.g. $2x^3+13x^2-20x-175=(2x-7)(x^2+...x\pm 25)$ or $2x^3+13x^2-20x-175=\left(x-\frac{7}{2}\right)(2x^2+...x\pm 50)$

It is dependent upon both previous M's so cannot be awarded if they don't have a value for c
Mark their final response here as there may be working before the final answer

A1: CSO Requires working to be seen (see bold sentences in question) so look for

- $f(x)=(2x-7)(x+5)^2$ or $f(x)=(2x-7)(x+5)(x+5)$ following correct $f(x)$
- AND correct intermediate factorisation $(2x-7)(x^2+10x+25)$

Going directly from $2x^3+13x^2-20x-175$ to $(2x-7)(x+5)^2$ without sight of working would result in the loss of the last two marks

Note that the last 4 marks can be scored without finding the value of *c* BUT the constant *c* must be present.

We know that $f(x)=2x^3+13x^2-20x+c$ and that $(2x-7)$ is a factor

So $2x^3+13x^2-20x+c=(2x-7)(Ax^2+Bx+C)$ which may be attempted via division

$2x^3+13x^2-20x+c=(2x-7)(1x^2+10x+25)=(2x-7)(x+5)^2$ o.e

The 6 marks in part (c) are awarded as follows

M1, A1: For $f(x)=2x^3+13x^2-20x+c$ with or without the + c

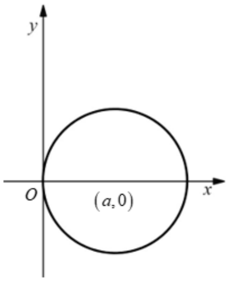
dM1: For setting their $f(x)='2'x^3+'13'x^2-'20'x+c=(2x-7)(Ax^2+Bx+C)$ and correctly deduces the value for A for their $f(x)$. You may see a division attempted here which is equivalent work

A1: Correctly deduces $2x^3+13x^2-20x+c=(2x-7)(x^2+10x+C)$

Via division, the $(x^2+10x+...)$ can be awarded implied from within the calculation

ddM1: Correctly deduces $2x^3+13x^2-20x+c=(2x-7)(x^2+10x+25)$

A1: $f(x)=(2x-7)(x+5)^2$ o.e.

Question Number	Scheme	Marks
9(a)		B1
		(1)
(b)	$(x \pm a)^2 + y^2 = ...$	M1
	$(x \pm a)^2 + y^2 = a^2$	A1
	Uses $(5, 6)$ in $(x \pm a)^2 + y^2 = a^2$ to form and solve an equation in a	dM1
	E.g. $(5-a)^2 + 36 = a^2 \Rightarrow 10a = 61 \Rightarrow a = 6.1$	
	$(x-6.1)^2 + y^2 = 6.1^2$	A1
		(4)
		(5 marks)

(a)

B1: A circle lying in quadrants 1 and 4 only which

- passes through (0, 0)
- is symmetrical about the x-axis

Condone slips of the pen. Mark this positively.
Ignore any reference to the point $(5, 6)$
Ignore any reference to the coordinates of the centre

(b) Way One: Candidates start from an appropriate equation involving x^2 and y^2 then use (5, 6)

The following equations are mostly written with a constant a. Any suitable constant may be used. The sign of the constant is also not important so accept $(x+a)^2 + y^2 = ...$ or $(x-a)^2 + y^2 = ...$ as well as $(a-x)^2 + y^2 = ...$

M1: Allowable left-hand side for the equation of C For example $(x \pm a)^2 + y^2 = ...$ or $(a-x)^2 + y^2 = ...$

This must involve a constant and not be $(x-5)^2 + y^2 = ...$ for example

A1: Correct form for the equation involving a **single** constant.

Examples of correct equations are $(x-a)^2 + y^2 = a^2$, $(x+k)^2 + y^2 = k^2$ and $x^2 + y^2 = \pm 2px$

If they write $(x-a)^2 + y^2 = r^2$ and $r^2 = a^2$ this can be implied

dM1: Uses $(5, 6)$ in $(x \pm a)^2 + y^2 = a^2$ o.e. to form and solve an equation in a.

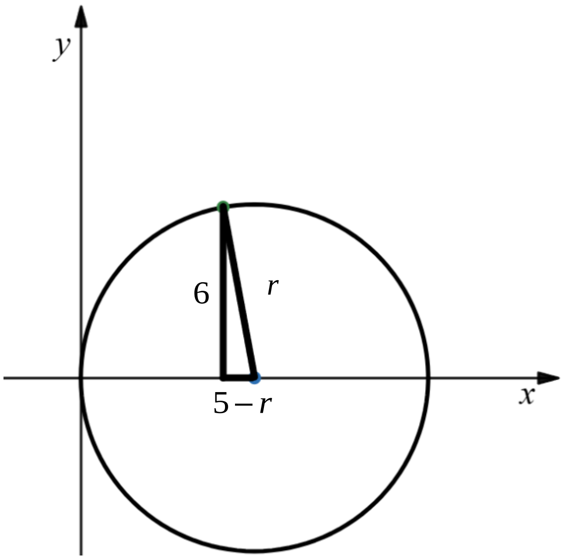
E.g. $(5-a)^2 + 36 = a^2 \Rightarrow \pm 10a = 61 \Rightarrow a = ...$

$5^2 + 6^2 = \pm 10p \Rightarrow p = ...$

A1: $(x-6.1)^2 + y^2 = 6.1^2$ o.e. such as $x^2 + y^2 = 12.2x$

....

Way Two: Candidates use Pythagoras' theorem to set up an equation in single variable



M1: Applies Pythagoras's theorem leading to the equation $(r-5)^2+36=r^2$ or $(5-r)^2+36=r^2$

This may be awarded for any variable

A1: Solves the above equation to find correct value $r=6.1$

dM1: Uses a correct equation of circle $(x-'6.1')^2+y^2='6.1'^2$ following through on their 6.1.

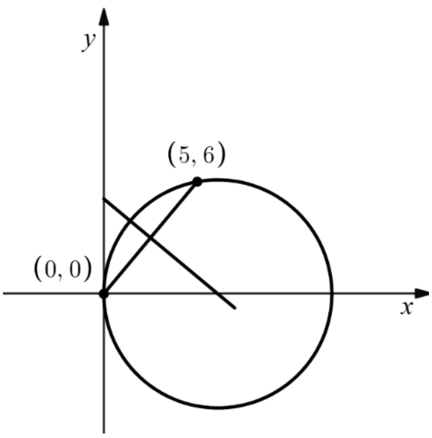
This is dependent on the 6.1 being a solution of a correct equation (previous M1)

A1: $(x-6.1)^2+y^2=6.1^2$ o.e.

Way Three: Bisector of chord method

M1: Attempts to find where the perpendicular bisector cuts the *x*-axis

Look for a good attempt using mid points and perpendicular gradient work.



A1: Achieves equation of perpendicular bisector as $y=-\frac{5}{6}x+\frac{61}{12}$

and point of intersection with the *x*-axis to be $\frac{61}{10}$ o.e

dM1: Correct form of equation with $(x-'6.1')^2+y^2='6.1'^2$

A1: $(x-6.1)^2+y^2=6.1^2$ o.e.

Question Number	Scheme	Marks
10 (a)	$y=2x+\frac{64}{x^2}-3$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{128}{x^3}$	M1, A1
	Attempts to solve $\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow x^3=64\Rightarrow x=4$ *	dM1, A1*
		(4)
(b)	$\int 2x+\frac{64}{x^2}-3\,\mathrm{d}x=x^2-\frac{64}{x}-3x \quad (+c)$	M1, A1
	Finds the y values at both x = 2 and x = 4. M = (0, 17) and N = (0, 9)	B1
	Full attempt at area = $\left[x^2-\frac{64}{x}-3x\right]_2^4-9\times2+2\times(17-9)=22-18+16=20$	dM1, A1
		(5) **(9 marks)**

(a)

M1: Attempts $\frac{\mathrm{d}y}{\mathrm{d}x}$ with one correct aspect. Allow for $2x\to2$ or $x^{-2}\to x^{-3}$ but not $3\to0$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{128}{x^3}$ o.e which may be left un simplified. The lhs is not required

dM1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=p\pm qx^{-3}=0$ and proceeds to $\alpha x^{\pm3}=\beta$

Also allow candidates to substitute 4 into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=p\pm qx^{-3}$ and find its value.

A1*: Shows that the x coordinate of P is 4. This is a given answer so there must be sufficient evidence

For a direct proof you should see all of $2-\frac{128}{x^3}=0$, $x^3=64$ (condone $2x^3=128$) and $x=4$

or $2-\frac{128}{x^3}=0$, $x^{-3}=\frac{1}{64}$ (condone $x^{-3}=\frac{2}{128}$) and $x=4$

For a substitution method look for $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{128}{x^3}$ and at P, $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{128}{4^3}=0$, hence proven

(o.e.)

(b)

M1: Attempts to integrate $2x+\frac{64}{x^2}-3$ scored for increasing a correct index by 1 including $3 \to 3x$

A1: $\int 2x+\frac{64}{x^2}-3\,\mathrm{d}x = x^2-\frac{64}{x}-3x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both x = 2 and x = 4. M = (0, 17) and N = (0, 9)

This mark may be implied by an area calculation $\left[x^2-\frac{64}{x}-3x\right]_2^4 - \underline{9}\times 2 + 2\times \underline{\underline{8}}$

Evidence may be awarded from the diagram or from coordinates for P and Q

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips

Total area $='22'-'18'+'16'=\ldots$

A1: 20

Alt method to b using (curve – line) can be marked in a similar way

M1: Attempts to integrate $\left(2x+\frac{64}{x^2}-3\right)-(k)$ scored for increasing a correct index by 1

A1: $\int 2x+\frac{64}{x^2}-12\,\mathrm{d}x = x^2-\frac{64}{x}-12x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both x = 2 and x = 4. M = (0, 17) and N = (0, 9)

This mark may be implied by an area calculation $\left[x^2-\frac{64}{x}-12x\right]_2^4 + (\underline{17-9})\times 2$

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

Shaded area under curve $R_1 = \int_2^4 \left(2x+\frac{64}{x^2}-3\right)-(9)\,\mathrm{d}x = \left[x^2-\frac{64}{x}-12x\right]_2^4 = 4$

Total area = 4 + 16 = 20

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips (such as 4+16 =22)

A1: 20