

Question Number	Scheme	Marks
1(a)	$(1-4x)^7$ Correct form for term 3 or term 4: Either $\frac{7 \times 6}{2}(\dots x)^2$ <b>or</b> $\frac{7 \times 6 \times 5}{6}(\dots x)^3$ o.e. $\frac{7 \times 6}{2}(-4x)^2$ <b>or</b> $\frac{7 \times 6 \times 5}{6}(-4x)^3$ o.e. $= \underline{1-28x} + 336x^2 - 2240x^3$	M1 A1 <u>B1</u> , A1 <b>(4)</b>
(b)	Sets "336" $\times 5 - "28" k = 1316$ $(k =) 13$	M1 A1 <b>(2)</b> <b>(6 marks)</b>

**(a) Note that a correct expression scores 4/4**

M1: Correct form for  $x^2$  or  $x^3$ . Look for the correct binomial coefficient with the correct power of  $x$ . You do not need to be concerned with the  $-4$  which may have been omitted.

Allow alternative equivalent notation for the binomial coefficient for this mark e.g.  ${}^7C_2$  or  ${}^7C_5$  or  $\binom{7}{2}$

or  $\frac{7!}{2!5!}$  etc

Condone invisible brackets for this mark e.g.  ${}^7C_2 1^5 - 4x^2$

A1: Correct  $x^2$  **or**  $x^3$  term (which may be unsimplified). They must have a numerical expression or value for their binomial coefficient for this mark. i.e. do not allow e.g.  ${}^7C_3$

Do not condone poor bracketing for this mark but may be implied by further work.

B1: For the series starting  $1-28x$  (terms may be in any order or as a list)

Condone  $1-28x^1$  but not e.g.  $1+-28x$

isw if they subsequently incorrectly attempt to simplify their answer.

A1:  $336x^2 - 2240x^3$  Do not accept e.g.  $336x^2 + -2240x^3$  unless already penalised for B1.

Allow the terms to be written as a list.

isw once the correct simplified terms have been found

**(b)**

M1: Adds the correct two terms using their coefficients from (a) and sets equal to 1316. (Allow misread)

Look for "336"  $\times 5 \pm "28" k = 1316$  i.e. they do not need to proceed to a value for  $k$  for this mark

If they still have  $x$  and  $x^2$  present in the equation then these must be removed or implied by further work to score the mark. Condone transcription errors from their part (a).

If just a value is stated following an incorrect part (a), then you will need to check this.

A1:  $(k =) 13$

Question Number	Scheme	Marks
<b>2(a)</b>	Midpoint of $(-2, 5)$ and $(4, 15)$ is $\left(\frac{-2+4}{2}, \frac{5+15}{2}\right) = (1, 10)$ Attempts radius <sup>2</sup> or diameter <sup>2</sup> : e.g. $D^2 = (4 - -2)^2 + (15 - 5)^2 = 136$ Radius <sup>2</sup> = 34 $(x-1)^2 + (y-10)^2 = 34$	B1 M1 A1 M1, A1 (5)
<b>(b)</b>	$(1, 10 - \sqrt{34})$	B1, dB1 (2) (7 marks)

**(a)**

- B1: Finds the midpoint of the two given points. Must be simplified to score this mark.  
Can be implied by circle formula. Allow e.g.  $x = 1$ ,  $y = 10$  or may be without brackets. May be seen on a diagram.
- M1: Attempts to find the radius<sup>2</sup> or diameter<sup>2</sup>. Can be implied by the circle formula e.g.  
 $(x \pm \dots)^2 + (y \pm \dots)^2 = 34$   
 It does not need to be correctly labelled but they must be attempting to find the difference between the  $x$  coordinates and the difference between the  $y$  coordinates before squaring.  
 Condone square rooting to find the radius or diameter (which may be all that is seen)  
 e.g.  $2\sqrt{34}$ ,  $\sqrt{136}$  (=11.66...),  $\sqrt{34}$  (=5.83...). Allow decimals for this mark.  
 May be seen embedded within  $x^2 + y^2 + 2fx + 2gy + c = 0$

- A1: Calculates the radius<sup>2</sup> = 34 o.e. (or allow to be implied by radius =  $\sqrt{34}$ )  
 Can be implied by circle formula or a value of  $c = 67$  in  $x^2 + y^2 + 2fx + 2gy + c = 0$

- M1: Attempts correct form for the circle  $(x \pm "1")^2 + (y \pm "10")^2 = "34"$  or  $(x \pm "1")^2 + (y \pm "10")^2 = \frac{"136"}{4}$   
 Do not allow this mark if they have clearly attempted to use their radius instead of their radius<sup>2</sup>  
 If they have found the length of the diameter or diameter<sup>2</sup> however they have labelled it, then they must have attempted to divide by 4.  
 They may alternatively attempt  $x^2 + y^2 + 2fx + 2gy + c = 0$  where  $f = "\pm 1"$ ,  $g = "\pm 10"$  and  $c = "\pm 67"$   
 If a centre has not been found or attempted with just values appearing in the equation then they must be correct.

- A1:  $(x-1)^2 + (y-10)^2 = 34$  but not  $(x-1)^2 + (y-10)^2 = (\sqrt{34})^2$  isw after a correct answer  
 Also allow equivalent expressions e.g.  $x^2 + y^2 - 2x - 20y + 67 = 0$

**(b) Note coordinates may be seen on a diagram. Do not allow  $x = 1$  if just the midpoint from (a) is stated.**

- B1: Either  $x = 1$  or  $y = 10 - \sqrt{34}$  Condone lack of labelling for this mark.

Isw after an exact coordinate is seen.

- dB1: Both  $x = 1$  and  $y = 10 - \sqrt{34}$  Isw after both exact coordinates are seen. **Note B0B1 is not possible.**

Question Number	Scheme	Marks
3	$\int_1^4 3x + \frac{16}{x^2} - 8 \, dx = \frac{3}{2}x^2 - \frac{16}{x} - 8x \quad (+c)$ <p>e.g. Area = <math>\frac{3}{2}(5+11) - \left[ \frac{3}{2}x^2 - \frac{16}{x} - 8x \right]_1^4 = 24 - \left( -12 + \frac{45}{2} \right) = \frac{27}{2}</math></p>	M1, A1, A1  dM1, A1  <b>(5)</b> <b>(5 marks)</b>

M1: Attempts to integrate the curve and achieves at least one term with the correct index i.e.  $x \rightarrow x^2$ ,  $x^{-2} \rightarrow x^{-1}$  or  $-8 \rightarrow \pm 8x$  (indices do not need to be processed for this mark e.g.  $x \rightarrow x^{1+1}$ )

A1: Two correct terms from  $\frac{3}{2}x^2 - \frac{16}{x} - 8x$ , may be unsimplified but indices must be processed.

Allow equivalent forms.

A1:  $\frac{3}{2}x^2 - \frac{16}{x} - 8x$ , may be unsimplified. Allow the presence of the constant of integration.

Do not be concerned with spurious notation or any limits.

dM1: Full method including correct area (value or expression) of trapezium. The expression with correct limits substituted in or partially evaluated as evidence is sufficient for this mark and condone subtracting either way round. They cannot just proceed to the final answer without additional evidence of using the limits. Note area of trapezium = 24, area under curve = 10.5.

Condone slips in evaluating some of the terms provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value for the integral then we must see that calculated correctly before they state the area is 13.5

#### Alternative: line – curve

$$\int_1^4 (13 - 2x) - \left( 3x + \frac{16}{x^2} - 8 \right) dx = \int_1^4 \left( -5x - \frac{16}{x^2} + 21 \right) dx = \left[ -\frac{5x^2}{2} + \frac{16}{x} + 21x \right]_1^4 = 48 - \frac{69}{2} = \frac{27}{2}$$

**Note: They may not fully simplify/collect terms in this method so score according to the main scheme for the first 3 marks.**

M1: Attempts to integrate and achieves at least one term with the correct index i.e.  $x \rightarrow x^2$ ,  $x^{-2} \rightarrow x^{-1}$  or " $21$ "  $\rightarrow$  " $\pm 21$ "  $x$  (allow slips in their collecting terms). Indices do not need to be processed for this mark.

A1: Two correct terms from  $-\frac{5x^2}{2} + \frac{16}{x} + 21x$ , which may be unsimplified but indices must be processed.

Allow equivalent forms including terms which have not been collected.

A1: Correct integration, may be unsimplified. Allow the presence of the constant of integration. Allow equivalent forms including terms which have not been collected.

Do not be concerned with spurious notation or any limits.

dM1: Full method (condoning an incorrect equation for the line). Scored for

$$\text{e.g. } \int_1^4 (13-2x) - \left(3x + \frac{16}{x^2} - 8\right) dx = \left[ \left(13x - x^2\right) - \left(\frac{3}{2}x^2 - \frac{16}{x} - 8x\right) \right]_1^4 = \dots$$

The expression with correct limits substituted in or partially evaluated is sufficient for this mark and condone subtracting either way round.

Condone slips in finding the straight line, evaluating some terms or collecting terms incorrectly provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value then we must see that calculated correctly before they state the area is 13.5

**There will be variations of these two methods but should mark similarly. Send to review if you are unsure.**

**Note for those who attempt curve-line**

$$\int_1^4 \left(3x + \frac{16}{x^2} - 8\right) - (13 - 2x) - dx = \int_1^4 \left(5x + \frac{16}{x^2} - 21\right) dx = \left[ \frac{5x^2}{2} - \frac{16}{x} - 21x \right]_1^4 = -48 + \frac{69}{2} = -\frac{27}{2}$$

Allow to score M1A1A1dM1A0 but accept the final A1 to be scored if they subsequently state the area is  $\frac{27}{2}$

(i.e. they cannot just write  $\frac{27}{2}$  as the answer to this integral)

Question Number	Scheme	Marks
4(a)	$f(x) = ax^3 + bx^2 + 5x - 3$ Sets $f(-3) = 0 \rightarrow a \times (-3)^3 + b \times (-3)^2 + 5 \times (-3) - 3 = 0$ $\Rightarrow -27a + 9b = 18 \Rightarrow b = 3a + 2 *$	M1 A1* (2)
(b)	Sets $f\left(\frac{1}{2}\right) = \frac{7}{4} \rightarrow a \times \left(\frac{1}{2}\right)^3 + b \times \left(\frac{1}{2}\right)^2 + 5 \times \left(\frac{1}{2}\right) - 3 = \frac{7}{4}$ $a + 2b = 18$ Solves $b = 3a + 2$ with their $a + 2b = 18 \Rightarrow a = \dots, b = \dots$ $a = 2, b = 8$	M1 A1 M1 A1 (4)
(c)	e.g. $\frac{2x^3 + 8x^2 + 5x - 3}{x - 2} = \left( \dots x^2 + \dots x + \dots \right) + \text{Remainder}$ (Quotient =) $2x^2 + 12x + 29$ Remainder = 55	M1 A1 B1 (3) (9 marks)

(a)

M1: Sets  $f(-3) = 0$  and proceeds to an equation in  $a$  and  $b$  Condone sign slips.

Score when you see embedded values within the equation or two correct simplified terms on the lhs of the equation. The  $= 0$  may be implied by further work which is not the given answer.

A1\*: Completes proof with no errors including brackets and at least one intermediate stage of working such as  $-27a + 9b = 18$  **and**  $= 0$  **must be seen** somewhere in their solution.

**Alt I**

M1: Attempts to divide by  $x + 3$  and sets their remainder in  $a$  and  $b = 0$ . May be seen in a variety of forms such as the grid or box method. Condone slips provided they are attempting to divide by  $x + 3$  and set their remainder in  $a$  and  $b$  equal to 0. The  $= 0$  may be implied by further work which is not the given answer.

$$\begin{array}{r}
 \phantom{x+3} \overline{ax^2 + (b-3a)x + 5 - 3(b-3a)} \\
 x+3 \overline{) ax^3 \phantom{+ bx^2} + 5x \phantom{- 3}} \\
 \underline{ax^3 \phantom{+ 3ax^2}} \\
 (b-3a)x^2 \phantom{+ 5x} \\
 \underline{(b-3a)x^2 \phantom{+ 3(b-3a)x}} \\
 (5-3(b-3a))x \phantom{- 3} \\
 \underline{(5-3(b-3a))x + 15 - 9(b-3a)} \\
 \phantom{(5-3(b-3a))x + 15 - 9(b-3a)} - 3
 \end{array}
 \Rightarrow -3 - (15 - 9(b-3a)) = 0$$

A1\*: Completes proof with no errors seen and an intermediate stage of working such as  $-18 + 9(b-3a) = 0 \Rightarrow b = 3a + 2 *$  ( $= 0$  **must be seen** somewhere in their solution)

**Alt II**

M1: Substitutes in  $b=3a+2$ , sets  $f(-3)=0 \rightarrow$  equation in  $a$  or  $b$  only Condone sign slips.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation.

A1\*: Completes proof and achieves  $0=0$  concluding that  $b=3a+2$

**(b) Note they may have substituted in  $b=3a+2$  early so the equation will be in  $a$  only or  $b$  only**

M1: Sets  $f\left(\frac{1}{2}\right)=\frac{7}{4}$  o.e. and proceeds to an equation in  $a$  and  $b$ .

Condone slips provided they set their remainder in  $a$  and  $b$  equal to  $\frac{7}{4}$  o.e. (e.g.  $f\left(\frac{1}{2}\right)-\frac{7}{4}=0$ )

May use algebraic division or e.g. grid method.

$$\begin{array}{r}
 \frac{a}{2}x^2 + \frac{1}{2}\left(b + \frac{a}{2}\right)x + \frac{5}{2} + \frac{1}{4}\left(b + \frac{a}{2}\right) \\
 2x-1 \overline{) ax^3 \quad + bx^2 \quad + 5x \quad - 3} \\
 \underline{ax^3 \quad - \frac{a}{2}x^2} \phantom{+ 5x} \\
 \left(b + \frac{a}{2}\right)x^2 \phantom{+ 5x} \\
 \underline{\left(b + \frac{a}{2}\right)x^2 \quad - \frac{1}{2}\left(b + \frac{a}{2}\right)x} \phantom{- 3} \\
 \left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x \quad - 3 \\
 \underline{\left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x - \frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)} \\
 \Rightarrow -3 - \left(-\frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)\right) = \frac{7}{4}
 \end{array}$$

A1:  $a+2b=18$  or other simplified equivalent including  $\frac{1}{8}a + \frac{1}{4}b = \frac{9}{4}$  which may only be in  $a$  or only in  $b$ .

e.g.  $7a=14$

M1: Solves  $b=3a+2$  with their  $a+2b=18 \Rightarrow a=..., b=...$

You do not need to be concerned with the mechanics of the rearrangement. Award as long as values appear for both  $a$  and  $b$  (both non zero) (which may be found using a calculator)

A1:  $a=2, b=8$

**(c) In EPEN this is M1A1A1 but we are marking this M1A1B1**

M1: Attempts to divide the cubic with their values for  $a$  and  $b$  by  $(x-2)$  and proceeds to a quadratic

quotient  $ax^2 + \dots x + \dots$  where  $a$  is the value found in (b). It is not a requirement to proceed to a value for  $R$  for this mark, but their method must allow for it to be found. If  $R$  is stated as 0 then M0.

Condone slips.

If they attempt  $2x^3 + 8x^2 + 5x - 3 = (x-2)(ax^2 + bx + c) + R \Rightarrow a=..., b=..., c=..., (R=...)$

A1: Quotient  $= 2x^2 + 12x + 29$  which can be extracted from their working e.g. on the top line of their algebraic division or  $(x-2)(2x^2 + 12x + 29) + R$  isw

B1: Remainder  $= 55$  which can be extracted from their working. isw

May also be written in the expression  $(x-2)(2x^2 + 12x + 29) + 55$

Question Number	Scheme	Marks
<b>5(a)</b>	Identifies $h = 1.5$ $\text{Area} = \frac{1.5}{2} \{12 + 0.023 + 2(4.243 + 1.5 + 0.530 + 0.188 + 0.066)\}$ $= 18.8$	B1 M1 A1 (3)
<b>(b) (i)</b>	$\int_{-2}^7 3\left(\frac{1}{2}\right)^{x+2} dx = \frac{1}{4} \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx = \frac{1}{4} \times "18.8" = 4.7$	B1ft
<b>(ii)</b>	$\int_{-2}^7 (2^{-x} + 2x) dx = \frac{1}{3} \times "18.8" + [x^2]_{-2}^7 = 51.3$	M1, A1ft (3) (6 marks)

**(a) Note that awrt 18.8 with no working is 0 marks**

B1: Identifies  $h = 1.5$  or may be implied by e.g. 0.75 oe outside of the bracket in the expression for the area

M1: Correct method of finding the area. Requires  $\frac{1}{2} \times \text{their } h \times \text{correct bracket structure}$  but condone the omission of a trailing bracket. Allow  $h = 1$  if stated but if only an expression is seen then it must be correct. Allow equivalent expressions e.g. they may add together individual trapezia. Condone invisible brackets to be implied by further work or awrt 18.8 but do not allow just awrt 18.8 with no working seen to score this mark. Condone slips on the digits of the heights, e.g. 4.234 for 4.243

A1: 18.8 or better (18.80775) following M1. i.e. if they have carried out the calculation and their 18.8 does not come from 18.0775 (allow truncating or rounding) then A0. Isw  
 Note that the calculator answer is 17.2785...

**(b) Note in part (b) they may have rounded their value in part (a) as their final answer (e.g. 18.8) but then used an earlier unrounded value in part (b). They must give an answer in part (b) that is at least 1dp for the B1ft in (i) and/or A1ft (ii) to be awarded. You will need to check their follow through answers if (a) is incorrect.**

**Do not accept solutions in (b) which involve re-doing the table of values and attempting the trapezium rule again.**

**(b)(i) Note if they have awrt 17.3 in (a) then this mark cannot be scored unless we see  $\frac{1}{4} \times \text{awrt } 17.3$**

B1ft: Calculates  $\frac{1}{4} \times "18.8" = "4.7"$

**Note that the calculator answer is 4.3196...**

**(b)(ii) Note if they have awrt 17.3 in (a) then A1ft can be scored following M1.**

M1: Recognises that  $\int_{-2}^7 (2^{-x}) dx = \frac{1}{3} \times \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx$  AND integrates  $2x$  to  $x^2$ . We must see the  $x^2$  or allow to be implied by limits substituted in and subtracting e.g.  $7^2 - (-2)^2$  (or  $7^2 - 2^2$  or  $49 - 4$ ) but not just 45

A1ft: 51.3 or ft on their answer to part (a) by adding 45 to  $\frac{1}{3} \times "18.8"$ . Allow fractions e.g.  $\frac{769}{15}$

**Note that the calculator answer is 50.7595...**

Question Number	Scheme	Marks
6. (i)	$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$ <p>One correct law applied – examples (not exhaustive):</p> $2\log_2(4-x) = \log_2(4-x)^2, \log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - \log_2 2$ $3 = \log_2 8$ <p>Correctly combines two original terms e.g. <math>2\log_2(4-x) = \log_2(4x+44)</math></p> <p>Correct equation not involving logs <math>(4-x)^2 = 4x+44</math></p> $x^2 - 12x - 28 = 0 \Rightarrow (x-14)(x+2) = 0 \Rightarrow x = \dots$ $\Rightarrow x = -2 \text{ only}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>(5)</p>
(ii)	<p>Combines <math>y = 6 \times 3^x</math> and <math>y = 3^{2x+1} \Rightarrow 3^{2x+1} = 6 \times 3^x</math></p> $\Rightarrow 3^x \times 3 = 6$ $\Rightarrow 3^x = 2 \Rightarrow x = \log_3 2$ $y = 6 \times 3^{\log_3 2} = 6 \times "2"$ <p>coordinates of P are: <math>(\log_3 2, 12)</math></p>	<p>M1</p> <p>dM1, A1</p> <p>ddM1</p> <p>A1</p> <p>(5)</p> <p>(10 marks)</p>

(i)

B1: Sight of one correct log law applied. See scheme. There will be other equivalent examples e.g.  $\log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - 1$ . This mark can still be awarded if they have made errors before applying the law and do not withhold this mark for any subsequent errors.

M1: Correctly combines two of the three original terms of the equation. e.g.  $\log_2\left(\frac{2(4-x)^2}{x+11}\right) = 3$

May be implied by a correct equation in any form not involving logs provided it has not come from incorrect log work. e.g.  $\frac{\log_2(4-x)^2}{\log_2\left(\frac{x+11}{2}\right)} = 3 \Rightarrow \frac{(4-x)^2}{\frac{x+11}{2}} = 8$  is M0

A1: Correct equation in any form not involving logs which has not come from incorrect log work.

dM1: Solves a quadratic equation via factorisation, the formula or completing the square (usual rules apply). They are not allowed to just state the roots. Dependent on the previous method mark.

A1:  $x = -2$  only (the 14 if found must be rejected or not included in their final answer)



- (ii) **Note that if logarithms are written as decimals instead of keeping their values exact then**  
**- if they find  $x$  first the maximum score is M1dM0A0ddM0A0**  
**- if they find  $y$  first the maximum score is M1dM1A1ddM0A0**

M1: Sets up a correct equation in  $x$  or  $y$ . Most likely in  $x$  but it is possible to form an equation in  $y$

$$\text{e.g. } y = 3^{2x+1} = 3 \times (3^x)^2 = 3 \times \left(\frac{y}{6}\right)^2$$

dm1: A correct method of solving the equation in  $x$  (or  $y$ ) to find either

- an unsimplified expression or value for  $x$  in terms of logs, or
- an expression or value for  $y$  not involving logs

It is dependent on the previous method mark. Condone arithmetical slips but any log/index work must be correct.

There may be some alternative approaches here to solving:

e.g. taking logarithms of base 3:

$$\log_3(3^{2x+1}) = \log_3(6 \times 3^x) \Rightarrow 2x+1 = \log_3 6 + \log_3 3^x \Rightarrow 2x+1 = 1 + \log_3 2 + x \Rightarrow x = \dots$$

e.g. forming a quadratic in  $3^x$  (or another variable)

$$3 \times 3^{2x} - 6 \times 3^x = 0 \Rightarrow 3^x(3^x - 2) = 0 \Rightarrow 3^x = 2 \Rightarrow x = \dots$$

$$\text{If solving an equation in } y \text{ then e.g. } y = 3 \times \left(\frac{y}{6}\right)^2 \Rightarrow \cancel{y} = 3 \times \frac{\cancel{y}}{36} \Rightarrow y =$$

A1: Correct  $x$  coordinate ( $\log_3 2$  or equivalent expressions such as  $\log_3 6 - 1$  or  $\frac{\log 2}{\log 3}$  o.e.) **or**  $y$  coordinate which must be 12

ddM1: Uses a correct method to find values for both  $x$  **and**  $y$ . Condone arithmetical slips but any log/index work must be correct.

It is dependent on the previous two method marks.

May be implied by a correct coordinate for their  $x$  or  $y$ . Do not allow logarithms to be written as decimals.

A1:  $(\log_3 2, 12)$  Accept if written separately e.g.  $x = \log_3 2$ ,  $y = 12$ . isw if they incorrectly state the values of  $a$  and  $b$  after correct coordinates are seen.  
 Condone missing brackets but do not allow the coordinates to be the wrong way round.

Question Number	Scheme	Marks
7. (i)	10+8+6.4+...	
(a)	Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r=\frac{8}{10}$ $S_{\infty} = 50$	M1 A1 (2)
(b)	Sets $10 \times 0.8^{k-1} < 0.0005 \Rightarrow 0.8^{k-1} < 0.00005$ e.g. $\Rightarrow k-1 > \frac{\log(0.00005)}{\log(0.8)}$ or e.g. $k-1 > \log_{0.8} 0.00005$ (see notes) $\Rightarrow k-1 > 44.38 \Rightarrow (k=) 46$	M1 M1 A1 (3)
(ii)	Sets $850 + (n-1) \times -7 = 0 \Rightarrow n = 122.4$ States or implies that $n=122$ $S = \frac{122}{2} \{2 \times 850 + 121 \times -7\} = 52033$	M1 A1 dM1, A1 (4)
		(9 marks)

(i) (a)

M1: Attempts  $S_{\infty} = \frac{a}{1-r}$  with  $a=10, r=\frac{8}{10}$  but condone slips on an incorrectly evaluated "r" provided the method to finding it is correct and  $-1 < r < 1$ . The expression with the values substituted in is sufficient.

A1:  $(S_{\infty} =) 50$

(b) Note that candidates may misread the number of 0s in 0.0005 but can still score M1M1A0

M1: Sets  $10 \times "0.8"^{k-1} < 0.0005$  and proceeds to  $"0.8"^{k-1} < 0.00005$  (or they may manipulate correctly to  $"0.8"^{k-1} < 0.00005 \times 0.8$ ). If they take logs before this stage then they will need to proceed to e.g.  $\log "0.8"^{k-1} < \log 0.0005 - \log 10$

Condone other signs being used e.g. = or >

M1: Uses a correct method to solve an equation or inequality of the type  $p^k \dots q$  or  $p^{k-1} \dots q$

It cannot be for just stating the value for  $k$ . They only need to proceed as far as  $k-1$  on one side of an equation or inequality. Evidence of using logs is required which cannot be e.g. 44.38... otherwise M0A0

Do not be concerned with the direction of inequalities, if used, in their solution for this mark.

A1:  $(k=) 46$  from correct working.

If inequalities are used in their method then the direction must be correct.

(ii) When using either formula condone missing brackets around  $(-7)$

e.g.  $850 + (n-1) \times -7$  or e.g.  $S_n = \frac{122}{2} [2 \times 850 + (122-1) \times -7]$

M1: Attempts to find the value of  $n$  that maximises  $S_n$

May set  $850 + (n-1) \times -7 = 0 \Rightarrow -7n + 857 = 0 \Rightarrow n = \dots$  or show trial calculations of  $u_{122} = 3$  and  $u_{123} = -4$ . May be implied by 122.

Alternatively sets  $S_n = \frac{n}{2} [2 \times 850 + (n-1) \times -7]$  and either

- differentiates and sets  $= 0$ :  $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 \Rightarrow \frac{1707}{2} - 7n = 0 \Rightarrow n = \dots$
- uses roots:  $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow n \left( \frac{1707}{2} - \frac{7}{2}n \right) = 0 \Rightarrow n = 0, n = \dots$  and then finds the midpoint of the two roots
- completes the square:  $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow -\frac{7}{2} \left( n^2 - \frac{1707}{7}n \right) \Rightarrow -\frac{7}{2} \left( n - \frac{1707}{14} \right)^2 \pm \dots \Rightarrow n = \dots$

to find the position of the turning point (approximately 121.9)

In all cases condone the use of  $n$  instead of  $n-1$

A1: States or implies that  $n = 122$  (if there are multiple attempts using different values of  $n$  then the others must be rejected for this mark to be scored)

dM1: Attempts  $S_n$  using a correct formula (they may attempt to find the last term and use  $S_n = \frac{n}{2}[a + L]$ )

It is dependent upon the previous M.

Condone the incorrect value of  $n$  being selected but it must be a positive integer.

May see multiple attempts using different values of  $n$  which is acceptable.

A1: 52033 only (i.e. if they have multiple attempts using different values then this answer must be clearly identified or the others all rejected)

Question Number	Scheme	Marks
8 (i)	$3 \tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3} \Rightarrow 2x + \frac{\pi}{5} = \arctan\left(\frac{\sqrt{3}}{3}\right)$ $\Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \dots$ $\Rightarrow x = \frac{29\pi}{60}, \frac{59\pi}{60}$	M1 dM1 A1 (3)
(ii)	$5 \sin \theta \tan \theta = \cos \theta + 4$ $\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow 5 \sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$ $\Rightarrow 5(1 - \cos^2 \theta) = \cos^2 \theta + 4 \cos \theta$ $\Rightarrow 6 \cos^2 \theta + 4 \cos \theta - 5 = 0$ $\Rightarrow \cos \theta = \frac{-4 \pm \sqrt{16 + 120}}{12} (= 0.63849\dots) \Rightarrow \theta =$ $\Rightarrow \theta = 50.3^\circ, 309.7^\circ$	M1 dM1 A1 ddM1 A1 (5) (8 marks)

**In both (i) and (ii) condone poor notation, omission of  $\theta$  or  $x$  in their working provided the intention is clear or recovered/implied by further work.**

(i)

M1: Correct order of operations  $2x + \frac{\pi}{5} = \arctan\left(\frac{\sqrt{3}}{3}\right)$  implied by  $2x + \frac{\pi}{5} = \frac{\pi}{6}$  or any correct angle.

Condone decimal values here e.g.  $2x + 0.628\dots = 0.52\dots$

Allow to work consistently in degrees. e.g.  $2x + 36 = 30$

Alternatively, attempts to use  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ , squares both sides, attempts to use

$\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$  and proceeds to a quadratic equation in sine or cosine

e.g.

$$\frac{3 \sin\left(2x + \frac{\pi}{5}\right)}{\cos\left(2x + \frac{\pi}{5}\right)} = \sqrt{3} \Rightarrow 3 \sin\left(2x + \frac{\pi}{5}\right) = \sqrt{3} \cos\left(2x + \frac{\pi}{5}\right) \Rightarrow 9 \sin^2\left(2x + \frac{\pi}{5}\right) = 3 \left(1 - \sin^2\left(2x + \frac{\pi}{5}\right)\right)$$

$$\Rightarrow 12 \sin^2\left(2x + \frac{\pi}{5}\right) = 3 \Rightarrow \sin\left(2x + \frac{\pi}{5}\right) = (\pm) \frac{1}{2} \Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \dots$$

dM1: Correct order of operations  $2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$  to find a value for  $x$  which does not need to be in the given range (e.g. condone  $-\frac{\pi}{60}$ )

Condone decimal answers here (awrt  $-0.05$ , awrt  $1.5$ , awrt  $3.1$ ). It is dependent on the previous method mark. Allow to work consistently in degrees. e.g.  $2x + 36 = 30 \Rightarrow x = \frac{30 \pm 36}{2} = \dots$

A1:  $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$  and no other angles in the range. Must be exact and in radians.

**Alternative: Using compound angle formula**

M1: Forms the equation  $\tan 2x = \frac{\frac{\sqrt{3}}{3} - \tan\left(\frac{\pi}{5}\right)}{1 + \frac{\sqrt{3}}{3} \tan\left(\frac{\pi}{5}\right)}$  Allow to work consistently in degrees

dM1: Correct order of operations (arctan followed by dividing by 2)  
Allow to work consistently in degrees

A1:  $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$  and no other angles in the range. Must be exact and in radians.

(ii) **Note that there may be other credit worthy methods relying on identities covered in later units. If seen then send to review.**

M1: Attempts to use  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  leading to an equation in  $\sin \theta$  and  $\cos \theta$

e.g.  $5\sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$  Condone slips.

dM1: Multiplies through by  $\cos \theta$  (on at least 2 of the 3 terms in the equation) and attempts to use  $\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$  to set up a quadratic equation in  $\cos \theta$  (may be in  $\sin \theta$  if they make an earlier slip so allow this to score as well)

Condone slips e.g. not multiplying both terms by 5 if  $5(1 - \cos^2 \theta)$  is not seen on the left hand side of the equation.

Terms do not need to be collected for this mark.

Condone arithmetical slips in their rearrangement.

A1:  $6\cos^2 \theta + 4\cos \theta - 5 = 0$  o.e The  $= 0$  may be implied by further work.  
Requires 3 terms but allow the constant term to be on the other side of the equation.

ddM1: Solves their **three-term** quadratic equation in  $\cos \theta$  (or condone  $\sin \theta$ ) and proceeds to a value for  $\theta$

**Allow use of a calculator to solve the quadratic** otherwise usual rules apply for factorising, using the formula or completing the square.

We must see

- a correct method used to solve the quadratic, or can be implied by a correct root to 1dp for their quadratic equation
- a correct angle for their root by taking arccos of the root (allow to the nearest integer in degrees) You may need to check this.

A correct angle on its own does not imply this mark.

A1:  $(\theta =)$  awrt  $50.3^\circ$ , awrt  $309.7^\circ$  and no others in the range. Must be in degrees.

Question Number	Scheme	Marks
9 (a)	$\text{Attempts } 12 = 3x^2 \times L \Rightarrow L = \frac{4}{x^2} \text{ or } Lx = \frac{4}{x}$ $S = 6x^2 + 5xL = 6x^2 + \frac{20}{x}$	M1, A1 dM1, A1 <b>(4)</b>
(b)	$\frac{dS}{dx} = 12x - \frac{20}{x^2}$ $\frac{dS}{dx} = 0 \Rightarrow x^3 = \frac{5}{3} \Rightarrow x = 1.18563\dots$ $S _{x=1.18563} = 6 \times "1.18563"^2 + \frac{20}{"1.18563"} = 25.3(\text{m}^2)$	M1, A1ft dM1 ddM1, A1 <b>(5)</b>
(c)	$\left( \frac{d^2S}{dx^2} = \right) 12 + \frac{40}{x^3}$ <p>Justifies <math>\frac{d^2S}{dx^2} &gt; 0</math> (at <math>x = 1.18563\dots</math>), hence minimum *</p>	M1 A1* <b>(2)</b> <b>(11 marks)</b>

(a)

M1: Attempts to use a volume formula to make  $L$  or  $Lx$  the subject. May be seen substituted into a surface area expression which is not in the required form of the final answer.

Allow  $12 = \alpha x^2 L \Rightarrow L = \frac{\beta}{x^2} \text{ or } Lx = \frac{\beta}{x}$

A1:  $L = \frac{4}{x^2}$  OR  $Lx = \frac{4}{x}$  o.e. Maybe seen substituted into the equation for the surface area.

dM1: Substitutes  $L = \frac{"4"}{x^2}$  OR  $Lx = \frac{"4"}{x}$  into  $S = Rx^2 + TxL$  which may be unsimplified to produce a formula of the correct form. It is dependent on the previous method mark.

A1:  $S = 6x^2 + \frac{20}{x}$

Must see " $S =$ " or e.g. "(surface) area =" connected at some point with  $6x^2 + \frac{20}{x}$

**They may work algebraically using  $P$  and  $Q$ . They can still score M1A1ft in (b) but they would have to substitute in their values to score any further marks in (b) or any marks in (c)**

- (b) Condone other letters used instead of  $S$  and  $x$  for  $\frac{dS}{dx}$  including  $y$  for  $S$ .**

**Accept use of e.g.  $S'$**

M1: Correct form for the derivative  $\left(\frac{dS}{dx} = \right) ax + \frac{b}{x^2}$

A1ft: Follow through on the derivative of their  $S = Px^2 + \frac{Q}{x} \Rightarrow 2Px - \frac{Q}{x^2}$

dM1: Solves  $\frac{dS}{dx} = 0 \Rightarrow x = \dots$  Do not be concerned by the mechanics of the rearrangement as long as a value for  $x$  is achieved.  $=0$  may be implied by further work or if just a correct value for  $x$  then you may need to check this. It is dependent on the previous method mark.

ddM1: Substitutes their value of  $x$  in  $S$  to find  $S$ . Condone slips. It is dependent on the previous two method marks.

If only a value is stated you may need to check this.

A1: awrt 25.3 ( $m^2$ ) following correct differentiation. Units not required.

- (c) Note the final A1\* can only be scored using a correct expression for  $\frac{d^2S}{dx^2}$  or  $\frac{dS}{dx}$  (depending on the approach taken). Condone other letters used instead of  $S$  and  $x$  for  $\frac{d^2S}{dx^2}$  including  $y$  for  $S$ . Accept use of e.g.  $S''$**

If second differentiation is seen in (b) then it must be seen or used in (c) to score.

M1: Differentiates and finds  $\frac{d^2S}{dx^2}$  of the form  $M + \frac{N}{x^3}$  (do not be concerned about signs for this mark).  
Alternatively, substitutes in values of  $x$  above and below their 1.1856.. and proceeds to find values for  $\frac{dS}{dx}$  at these points.

A1\*: **Either** justifies  $\frac{d^2S}{dx^2} > 0$ , which must be a correct expression AND gives a minimal conclusion

Requires one of:

- a correct value for  $\frac{d^2S}{dx^2}$  (awrt 36 if using 1.18(56..) but allow awrt 35.1 if using 1.2) from a correct expression, a correct comparison with 0 and a conclusion e.g. min
- deduces that as 12 and 40 are both positive and the correct value of  $x$  (awrt 1.2)  $> 0$  then  $\frac{d^2S}{dx^2} > 0$  with a conclusion e.g. proven

**Or** justifies there is a change of sign for  $\frac{dS}{dx}$  AND gives a minimal conclusion

Requires both calculations to be correct using a correct expression for  $\frac{dS}{dx}$  (you will need to check this)

Question Number	Scheme	Marks
<b>10</b>	Sets $m = 3p + 1$ or $m = 3p + 2$ ( $p > 0$ ) o.e. and attempts $m^2 + 3m + 2$ E.g. $m^2 + 3m + 2 = (3p + 1)^2 + 3(3p + 1) + 2 = 9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$ Sets $m = 3p + 1$ and $m = 3p + 2$ ( $p > 0$ ) and attempts $m^2 + 3m + 2$ E.g. $m^2 + 3m + 2 = (3p + 2)^2 + 3(3p + 2) + 2 = 9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$ And gives a minimal conclusion *	M1 A1 dM1 A1* <b>(4 marks)</b>

**Main scheme algebraic method using e.g.  $m = 3p + 1$  and  $m = 3p + 2$**

**You will need to look at both cases and mark the one which is fully correct first.**

**Allow a different variable to  $p$  and may be different letters for the two cases.**

**Condone use of  $m$  as a variable for the first three marks.**

**There should be no errors in the algebra for the A marks including invisible brackets.**

**Note that there are other allowable pairs of combinations covering the two distinct cases**

**e.g.  $m = 3p - 1$  and  $m = 3p + 1$  or e.g.  $m = 3p - 2$  and  $m = 3p + 2$**

**but not e.g.  $m = 3p - 1$  and  $m = 3p + 2$**

**Note: An attempt substituting in values rather than using algebra will score 0 marks.**

**M1:** Sets  $m = 3p + 1$  o.e **or**  $m = 3p + 2$  o.e., expands  $m^2 + 3m + 2$  and achieves a quadratic expression which may be unsimplified

**A1:** Correct quadratic expression with constant terms collected for  $m^2 + 3m + 2$  for either of the two cases and shows or gives a reason why the expression is a multiple of 3. They must have fully multiplied out or the quadratic expression must have a factor of 3 taken out.

Do not isw if they simplify their quadratic incorrectly.

e.g.  $9p^2 + 15p + 6$  and states each term is a multiple of 3 or alternatively, correctly divides by 3 and concludes that it is divisible by 3

e.g.  $3(3p^2 + 5p + 2)$  or e.g.  $3(3p^2 + 2p) + 3(3p + 1) + 3$

e.g.  $9p^2 + 21p + 12$  and states each term is a multiple of 3 or alternatively correctly divides by 3 and concludes that it is divisible by 3

e.g.  $3(3p^2 + 7p + 4)$

**dM1:** Sets  $m = 3p + 1$  o.e **and**  $m = 3p + 2$  o.e. and expands  $m^2 + 3m + 2$  and achieves two quadratic expressions which may be unsimplified. See first M1 for guidance. It is dependent on the previous method mark.



A1\*: Requires

- correct quadratic expression for  $m^2 + 3m + 2$  for both cases which is not in terms of  $m$
- shows or gives a reason for why the expressions are multiples of 3 (see first A1 for guidance)
- makes a concluding overall statement. “Hence  $m^2 + 3m + 2$  is always a multiple of 3 for all  $m$  that are not multiples of 3” (or equivalent) Allow generic conclusions e.g. “proven”, “hence true”

Withhold the final mark if they have a spurious = 0 on the end of their quadratic expressions.

Below is a table for the different possible cases and the quadratic expressions:

	$m^2 + 3m + 2$
$3p - 2$	$9p^2 - 3p = 3(3p^2 - p)$
$3p - 1$	$9p^2 + 3p = 3(3p^2 + p)$
$3p + 1$	$9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$
$3p + 2$	$9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$

**Factorising first then using logic**

**Note the same marking principle can be applied as in the main scheme and notes**

$$m^2 + 3m + 2 = (m+1)(m+2)$$

Sets e.g.  $m = 3p + 1$  leading to  $m^2 + 3m + 2 = (3p+1+1)(3p+1+2) = 3(3p+2)(p+1)$

Sets e.g.  $m = 3p - 1$  leading to  $m^2 + 3m + 2 = (3p-1+1)(3p-1+2) = 3p(3p+1)$

**Alternative using algebra with logic**

$$m^2 + 3m + 2 = (m+1)(m+2)$$

e.g.  $(m+1)(m+2)$  represents two consecutive numbers and if  $m$  is not a multiple of 3 then one of  $(m+1)$  or  $(m+2)$  must be a multiple of 3 hence  $m^2 + 3m + 2$  is a multiple of 3.

If you see this approach or other attempts that you think may be credit-worthy then send to review.