2501
WMA12 Mark Scheme

Question Number	Scheme	Marks
1 (a)	$254 = 2+3(n-1) \Rightarrow n = ...$	M1
	85	A1
		(2)
(b)	$\frac{"85"}{2}(2\times 2+("85"-1)\times 3)$	M1
	10 880	A1
		(2)
		4 marks

(a)

M1: Attempts to find the number of terms.

Look for $254 = 2+3(n-1) \Rightarrow n = ...$, $3n-1 = 254 \Rightarrow n = ...$ or $\left(\frac{254-2}{3}\right)+1$ o.e. condoning slips such as writing 254 as 245. The values of a and d must be correct.

Sight of either $\frac{254-2}{3}$, $\frac{252}{3}$ or 84 without any incorrect working can score this mark.

Note that $\frac{254}{3} = 84$ or similar calculations that give non-integer solutions (with no further working e.g. trial and error attempts) score M0 even if rounded

A1: 85.

An answer of 85 **with no incorrect working** or without any working scores both marks

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Alt (a) Trial and error attempts

You may see candidates using $2+(n-1)\times 3$ with various values for n

M1: Attempts $2+(n-1)\times 3$ with $n = 85$

A1: Achieves $2+84\times 3 = 254$ **and** concludes 85 (terms)

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(b)

M1: Attempts the sum formula $\frac{n}{2}(2a+(n-1)d)$ or $\frac{n}{2}(a+l)$ following through on their value for n.

Look for $\frac{"85"}{2}(2\times 2+("85"-1)\times 3)$, $\frac{"85"}{2}(2\times 254+("85"-1)\times -3)$ or $\frac{"85"}{2}(2+254)$

The values of a, d and l if used must be correct (condoning a slip such as 245 for 254) but follow through on their value of n from part (a)

A1: 10 880.

This may be awarded even following an 85 in (a) that was found using an incorrect method

10 880 without working or following listing scores both marks.

Listing leading to an incorrect answer scores M0 A0

Question Number	Scheme	Marks
2 (a)	$(2-5x)^8 = 2^8 + {}^8C_1(2)^7(-5x)^1 + {}^8C_2(2)^6(-5x)^2 + {}^8C_3(2)^5(-5x)^3 + \ldots$	
	Constant term 2^8 OR 256	B1
	Correct form for terms 3 or 4: ${}^8C_2(2)^6(\pm 5x)^2$ or ${}^8C_3(2)^5(\pm 5x)^3$	M1
	Correct un-simplified expansion for 2 out of 3 terms from terms 2, 3 and 4: $\ldots + 8(2)^7(-5x)^1 + 28(2)^6(-5x)^2 + 56(2)^5(-5x)^3 + \ldots$	A1
	Correct simplified expansion $256 - 5120x + 44800x^2 - 224000x^3 + \ldots$	A1
		(4)
(b)	-0.01	B1
		(1)
		(5 marks)

(a)

B1: Correct constant term, 2^8 OR 256. It must be on its own and not as ${}^8C_8 2^8$ for example

M1: Correct form for term 3 or term 4. Allow with C notation (o.e).
Examples of correct forms include:
For term three ${}^8C_2(2)^6 \times (-5x)^2$ and $28 \times 64 \times \pm 25x^2$
For term four $\binom{8}{3} \times (2)^5 \times -125x^3$ and $\pm 224000x^3$
Condone a missing sign or bracket. So, allow terms such as ${}^8C_2(2)^6 \times 5x^2$ and $56 \times (2)^5 \times 125x^3$
Look for the correct binomial coefficient, the correct power of 2 and the correct power of x on $5x$

A1: Two out of three correct un-simplified (or simplified) terms from terms 2, 3 or 4
$\ldots + 8(2)^7(-5x)^1 + 28(2)^6(\pm 5x)^2 + 56(2)^5(-5x)^3 + \ldots$
The binomial coefficient must now be numerical.

A1: $256 - 5120x + 44800x^2 - 224000x^3 + \ldots$ which must be in simplest form.
Accept $256 + -5120x + 44800x^2 + -224000x^3 + \ldots$ or a list $256, -5120x, 44800x^2, -224000x^3$.
Withhold this mark if they decide to divide by 4 for example, but condone the sight of extra terms, for example in x^4 or higher.

(b)

B1: -0.01 o.e. such as $-\frac{1}{100}$. This must be clearly stated and not embedded in the $(2-5x)^8$
Ignore any attempts to calculate the result

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Alt to (a) via a common factor

B1: Takes out a common factor of 2, scored for $2^8(1 \pm kx)^8$

M1: Correct "form" for terms 3 or 4 in expansion of $\left(1-\frac{5}{2}x\right)^8$. Look for $\frac{8\times 7}{2}\left(\pm\frac{5}{2}x\right)^2$ or $\frac{8\times 7\times 6}{3!}\left(\pm\frac{5}{2}x\right)^3$
As with the main method, condone missing brackets etc. Condone with C notation as in main scheme.

A1: Correct simplified or unsimplified expansion of $\left(1-\frac{5}{2}x\right)^8$ FYI it is $1 - 20x + 175x^2 - 875x^3 + \ldots$

A1: $256 - 5120x + 44800x^2 - 224000x^3 + \ldots$. Accept as a list.

Question Number	Scheme (OPEN TOPPED)	Scheme (CLOSED CONTAINER)	Marks
3.(a)	$120=3x^2y \Rightarrow y=\dfrac{40}{x^2}$	$120=3x^2y \Rightarrow y=\dfrac{40}{x^2}$	M1, A1
	$A=3x^2+8xy$	$A=6x^2+8xy$	
	$=3x^2+8x\times\dfrac{40}{x^2}=3x^2+\dfrac{320}{x}$	$=6x^2+8x\times\dfrac{40}{x^2}=6x^2+\dfrac{320}{x}$	dM1, A1
			(4)
(b)	$\dfrac{dA}{dx}=6x-\dfrac{320}{x^2}$	$\dfrac{dA}{dx}=12x-\dfrac{320}{x^2}$	M1, A1ft
	$\dfrac{dA}{dx}=6x-\dfrac{320}{x^2}=0$	$\dfrac{dA}{dx}=12x-\dfrac{320}{x^2}=0$	
	$\Rightarrow x^3=\dfrac{160}{3} \Rightarrow x=\text{awrt } 3.76$	$\Rightarrow x^3=\dfrac{80}{3} \Rightarrow x=\text{awrt } 2.99$	dM1, A1
			(4)
(c)	Attempts $\left.\dfrac{d^2A}{dx^2}\right\vert_{x=3.76}=6+\dfrac{640}{x^3}$	Attempts $\left.\dfrac{d^2A}{dx^2}\right\vert_{x=2.99}=12+\dfrac{640}{x^3}$	M1
	$=6+\dfrac{640}{3.76^3}=..$	$=12+\dfrac{640}{2.99^3}=..$	
	$\left.\dfrac{d^2A}{dx^2}\right\vert_{x=3.76}=\text{awrt } 18>0$ MINIMUM*	$\left.\dfrac{d^2A}{dx^2}\right\vert_{x=2.99}=\text{awrt } 36>0$ MINIMUM*	A1*
			(2)
			(10 marks)

Note that candidates may be confused regarding open topped or closed container so both cases can potentially score full marks if the case is consistently applied.

Open Container   Closed Container

(a)

M1: Sets $120=kx^2y$ o.e such as $120=k\,x\times x\times y$

A1: $y=\dfrac{40}{x^2}$ or $xy=\dfrac{40}{x}$ or equivalent such as $y=\dfrac{120}{3x^2}$

It can be scored from an embedded expression within an attempt at A.

It can be implied following a correct $120=3x^2y$ o.e. and a correct expression for A in terms of x

dM1: Achieves an expression of the form $(A=)\,px^2+qxy,\ p,q>0$ **AND** substitutes in their y or xy of the form $y=\dfrac{K}{x^2}$ or $xy=\dfrac{K}{x}$ which may be implied by a correct expression for A in terms of x

A1: Achieves the answer of Area / $A=3x^2+\dfrac{320}{x}$ **or** Area/ $A=6x^2+\dfrac{320}{x}$

The LHS A = or an equivalent such as 'Area =' must appear at some point.

Condone A appearing, for example, as SA or S or Area (of sheet metal) or other but a LHS must be seen.

Mark parts (b) and (c) together.

(b) Condone $\frac{dA}{dx}$ appearing as $\frac{dy}{dx}$, y' or A' or the absence of a LHS **throughout part (b)**

M1: $\frac{dA}{dx} = \alpha x \pm \frac{\beta}{x^2}$.

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1ft: $\frac{dA}{dx} = 6x - \frac{320}{x^2}$ o.e. or $\frac{dA}{dx} = 12x - \frac{320}{x^2}$ following through on their numerical P and Q

It can be scored for made up values of P and Q

dM1: Sets $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ and proceeds to a numerical $x^3 = k$ $k > 0$ but allow $\alpha x^3 = \beta$ $\alpha\beta > 0$

Condone $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ leading to a correct value for x for their $\frac{dA}{dx} = 0$

A1: $x =$ awrt 3.76 or $x =$ awrt 2.99

(c) Condone $\frac{d^2A}{dx^2}$ being written as A'' throughout part (c)

M1: Achieves $\frac{d^2A}{dx^2} = \delta \pm \frac{\varepsilon}{x^3}$ and

- Either attempts to find its value at their 3.76 or 2.99
- Or states it is positive (even following an incorrect value)

Condone $\frac{d^2A}{dx^2}$ as $\frac{d^2y}{dx^2}$ for this mark only

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1*: Scored for

- A correct $\frac{d^2A}{dx^2}$ including the LHS. So look for $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3}$ or $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3}$
- A correct statement and conclusion

Either $\left.\frac{d^2A}{dx^2}\right|_{x=3.76} =$ awrt $18 > 0$ and MINIMUM $\left.\frac{d^2A}{dx^2}\right|_{x=2.99} =$ awrt $36 > 0$ and MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ when $x = 3.76$, MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ when $x = 2.99$ MINIMUM

The mark can only be awarded following $x =$ awrt 3.76 or $x =$ awrt 2.99

Condone statements like '$x = 3.76$ is the minimum value of A as $\frac{d^2A}{dx^2} > 0$'.

Look for the key aspects '$\frac{d^2A}{dx^2} > 0$' and 'minimum'. If in doubt use review

Question Number	Scheme	Marks
4 (a)	$\frac{a\left(1-r^3\right)}{1-r}=70.2$, $\frac{a}{1-r}=75$	B1, B1
	Sub (2) in (1) $\quad 75\left(1-r^3\right)=70.2 \Rightarrow r^3=(0.064)$	M1
	$r=0.4$	A1
		(4)
(b)	Substitutes $r=0.4$ into $\frac{a}{1-r}=75$ and finds a	M1
	$a=45$	A1
		(2)
		(6 marks)

(a)

B1: One correct equation. Most of the time it is one of

- $\frac{a}{1-r}=75$
- $\frac{a\left(1-r^3\right)}{1-r}=70.2$, $a+ar+ar^2=70.2$ o.e.

B1: Two different equations that allow the problem to be solved.

Another more complicated option that can be used (with either of the above bullets) is $\frac{a}{1-r}-\frac{a\left(1-r^3\right)}{1-r}=4.8$

M1: Combines both necessary equations (condoning numerical slips e.g. 72.0 for 70.2) and proceeds to an equation of the form $\delta r^3=\beta$. Also condone the equations of the same form with sign slips $\frac{a}{r-1}=75$

Working is necessary here so candidates cannot just write down the equations and the solution without sight of the line $\delta r^3=\beta$ or $\delta r^3-\beta=0$.

A more complicated method involves substituting $a=75-75r$ into $a-ar^3=70.2(1-r)$ to produce a simplified quartic equation in *r*. FYI the correct simplified equation is $75r^4-75r^3-4.8r+4.8(=0)$. If they achieve a simplified quartic equation in *r* and write down a value for *r* $\left(r\neq 1, r\in\mathbb{R}\right)$ that solves their equation they can score M1. So, if they write down the correct simplified equation followed by $r=0.4$ from their calculator they can score M1, A1.

If you spot a method that does not proceed via one of these two methods and you feel deserves credit, then please send to review

A1: $r=0.4$ only, following award of the method mark.

(b)

M1: Substitutes their value of r, $|r|<1$, found in part (a) into one of their correct equations and find a value for *a*.

Candidates cannot just make up a value. It must follow a value in *r* found from an equation in just *r*.

A1: $a=45$ Can only be awarded if two correct equations are written down and $r=0.4$

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Part (b) can be done via substituting $r=1-\frac{a}{75}$ into $a+ar+ar^2=70.2$ for example, to produce an equation in just *a*, FYI $\frac{a^3}{5625}-\frac{a^2}{25}+3a-70.2=0$ This can score M1 for setting up and solving a cubic with A1 for the correct cubic with $a=45$

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Question Number	Scheme	Marks
5 (a)	$\mathrm{f}(x)=3x^3+ax^2-10x+b$	
	$\mathrm{f}\left(\frac{4}{3}\right)=0\Rightarrow 3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0$	M1
	$\frac{64}{9}+\frac{16}{9}a-\frac{40}{3}+b=0\Rightarrow 64+16a-120+9b=0\quad\Rightarrow 16a+9b=56$ *	A1*
		(2)
(b)	$\mathrm{f}(2)=b\Rightarrow 3\times(2)^3+a(2)^2-10(2)+b=b$ leading to an equation in just a	M1
	$4a=-4\Rightarrow a=-1$	A1
	Substitutes $a=$ '-1' in $-16+9b=56\Rightarrow b=...$	M1
	$\Rightarrow b=8$	A1
		(4)
(c)	$\mathrm{f}(x)=3x^3-x^2-10x+8=(3x-4)(x^2+x-2)$	M1, A1
	$=(3x-4)(x+2)(x-1)$	A1
		(3)
		(9 marks)

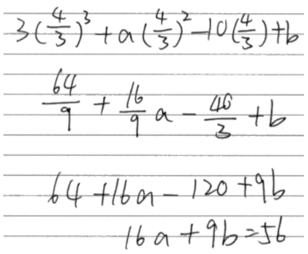
(a)

M1: Sets $\mathrm{f}\left(\frac{4}{3}\right)=0$ producing an equation in *a* and *b*. Condone slips but the intention must be clear.

For example, $3\times\frac{4}{3}^3+a\times\frac{4}{3}^2-10\times\frac{4}{3}+b=0$ is incorrect but the intention is clear and scores M1, A0

Finding a simplified $\mathrm{f}\left(\frac{4}{3}\right)=\frac{64}{9}+\frac{16}{9}a-\frac{40}{3}+b=\frac{16}{9}a-\frac{56}{9}+b$ before setting it equal to 0 is acceptable

Candidates who find $\mathrm{f}\left(\frac{4}{3}\right)$ and proceed to a correct equation without ever stating $\mathrm{f}\left(\frac{4}{3}\right)=0$ or setting their expression for $\mathrm{f}\left(\frac{4}{3}\right)$ equal to 0 can be awarded M1, A0 by implication.



Example of M1 A0*:

Candidate attempts $\mathrm{f}\left(\frac{4}{3}\right)$ which they then simplify (and then multiply by 9)

The final line implies $9\times\mathrm{f}\left(\frac{4}{3}\right)=0$ and so implies $\mathrm{f}\left(\frac{4}{3}\right)=0$

Setting $\mathrm{f}\left(-\frac{4}{3}\right)=0$ is M0

A1*: Shows that $16a+9b=56$. This is a given answer so there must be no incorrect lines (no recovery) or working. You do not explicitly need to see the '$\mathrm{f}\left(\frac{4}{3}\right)$' just its result, see below.

It is acceptable to have just one correct intermediate line with either collected terms or non-fractional coefficients between $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0$ and $16a+9b=56$

For example: $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0\Rightarrow\frac{16}{9}a+b-\frac{56}{9}=0\Rightarrow 16a+9b=56$ scores M1 A1

as does $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0\Rightarrow 64+16a-120+9b=0\Rightarrow 16a+9b=56$

(b)
M1: Attempts to set $\mathrm{f}(2)=b$ and proceeds to form an equation in just a.

Condone slips but setting $\mathrm{f}(-2)=b$ is M0

A1: $a=-1$

M1: Substitutes their value of a found from the equation $\mathrm{f}(\pm 2)=b$ into $16a+9b=56$ and finds a value for b

A1: $b=8$

(c)
M1: Attempts to divide $\mathrm{f}(x)$ by $(3x-4)$. Alternatively attempts to factorise out $(3x-4)$

Look for two 'correct' terms via either method.

E.g. via factorisation look at first and last terms $3x^3+ax^2-10x+b=(3x-4)\left(x^2+\ldots x\pm\frac{"b"}{4}\right)$

$$3x-4 \overline{\smash{)}\, 3x^3+ax^2-10x+b} \quad \text{quotient: } x^2+\left(\frac{a+4}{3}\right)x\ldots\ldots\ldots\ldots$$

via division look for first two terms, for their numerical values of a and b

The demand of the question is 'Hence', so the candidates must use the information given, that is $(3x-4)$ is a factor. Do not accept division or factorisation by other factors.

A1: Achieves the correct quadratic factor of (x^2+x-2)

A1: Fully factorised form required $(3x-4)(x+2)(x-1)$ all together on one line.

It can only be scored following M1. ISW if you see the addition of $=0$ and perhaps roots

You will see attempts that factorise attempt to $\mathrm{f}(x)$ via factors formed via calculator methods. These should be scored M0, A0, A0

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Alt (c).
M1: Candidates can attempt to divide by $(x-1)$ or $(x+2)$ only if they have firstly proven/shown that $\mathrm{f}(1)=0$ or $\mathrm{f}(-2)=0$. This can be marked in a similar way to the main scheme and can only really follow a correct $\mathrm{f}(x)$. So expect to see $\mathrm{f}(x)=3x^3-x^2-10x+8$ with, for example, $\mathrm{f}(1)=3-1-10+8=0 \Rightarrow (x-1)$ is a factor. Then division by $(x-1)$ with similar conditions as the main scheme.

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Inevitably we will see attempts at parts (a) and (b) via a table or by division. This is acceptable.
For example:
In part (a), award M1 for an attempt at

$$3x-4 \overline{\smash{)}\, 3x^3+ax^2-10x+b} \quad \text{quotient: } x^2\ldots\ldots\ldots\ldots$$

that achieves a quadratic quotient and a remainder which is set = 0

Then award A1 if it is correct. For example, $b+4\left(\frac{4}{9}a-\frac{14}{9}\right)=0 \Rightarrow 16a+9b=56$

In part (b), award M1 for an attempt at

$$x-2 \overline{\smash{)}\, 3x^3+ax^2-10x+b} \quad \text{quotient: } 3x^2+\ldots\ldots$$

that achieves a quadratic quotient and a remainder which is set = b

Then award A1 if it is correct. For example, $4a+4+b=b \Rightarrow a=-1$

If in doubt then please send to review

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Question Number	Scheme	Marks
6(a)	States or uses $M=(5,4)$ **or** grad $AB=\frac{1}{3}$ o.e.	B1
	States or uses $M=(5,4)$ **and** grad $AB=\frac{1}{3}$ o.e.	B1
	Equation of l is $y-4=-3(x-5)$	M1
	$y=-3x+19$	A1
		(4)
(b)	$k=-2$	B1 ft
		(1)
(c)	Attempts radius or radius ^{2} E.g. $(11-7)^2+(6--2)^2$	M1
	States $(x-7)^2+(y+'2')^2=(11-7)^2+(6--2)^2$	dM1
	$(x-7)^2+(y+2)^2=80$	A1
		(3)
		(8 marks)

(a)

B1: States or uses $M=(5,4)$ **or** grad $AB=\frac{1}{3}$ o.e. such as $\frac{4}{12}$

Note that gradient AB may be implied by use of a gradient for l of -3 o.e.

B1: States or uses $M=(5,4)$ **and** grad $AB=\frac{1}{3}$ o.e. such as $\frac{4}{12}$

Note that gradient AB may be implied by use of a gradient for l of -3 o.e.

M1: Attempts equation of l using their M and their correctly adapted gradient.

The gradient used must be the negative reciprocal of their gradient of AB. If it just appears without any working it must be correct.

If the form $y=mx+c$ is used this mark is only scored when they proceed as far as $c=\ldots$

A1: CAO $y=-3x+19$

(b)

B1ft: $k=-2$ or follow through on the value of their $'-3'\times 7+'19'$

Allow this to be scored if the coordinates of P are given.

For example as $(7,-2)$ or even $y=-2$ following use of 7

(c)

M1: Attempts either the radius^2 or the radius using their $(7,k)$ and one of the given points

E.g. $(11-7)^2+(6-'-2')^2$ or $\sqrt{(7-(-1))^2+('-2'-2)^2}$

It does not need to be correctly identified as r or r^2

dM1: Attempts equation of circle using their radius ^{2} and their $(7,k)$

Note that $(x-7)^2+(y-'k')^2=(11-7)^2+(6-'k')^2$ would score M1 dM1

A1: $(x-7)^2+(y+2)^2=80$ The rhs of the equation must be simplified and ISW after a correct answer.

Alt (c)

M1: Writes equation of circle as $x^2+y^2-14x-2'k'y=\lambda$ following through on their k

dM1: Substitutes $(-1,2)$ or $(11,6)$ into the equation of the circle and finds a value for λ

A1: $x^2+y^2-14x+4y=27$

Question Number	Scheme	Marks
7 (i)	$\frac{1}{2}\left\{\log 4+\log 9+2\left(\log 5+\log 6+\log 7+\log 8\right)\right\}$	B1
	$=\frac{1}{2}\log\left(4\times 9\right)+\log(5\times 6\times 7\times 8)\quad =\log 10080$	M1 A1
		(3)
(ii)	$2\log_5\left(5-a\right)-\log_5\left(a+25\right)=1$	
	$2\log_5\left(5-a\right)=\log_5\left(5-a\right)^2$ **OR** $1=\log_5 5$	B1
	$\log_5\frac{\left(5-a\right)^2}{\left(a+25\right)}=1$	M1
	$a^2-15a-100=0$	A1
	$\left(a-20\right)\left(a+5\right)=0$	dM1
	$a=-5$	A1
		(5)
		(8 marks)

(i) Do not be concerned if the 10's completely disappear or disappear and reappear from the $\log_{10}$

B1: For writing down a correct expression for the approximate area using the trapezium rule.
The bracketing must be correct (condoning a missing trailing bracket) or implied to be correct by subsequent work. It is not implied by answers rounding to 4.0
Allow the calculation with separate trapezia. E.g.

$$\frac{1}{2}\left(\log 4+\log 5\right)+\frac{1}{2}\left(\log 5+\log 6\right)+\frac{1}{2}\left(\log 6+\log 7\right)+\frac{1}{2}\left(\log 7+\log 8\right)+\frac{1}{2}\left(\log 8+\log 9\right)$$

M1: Attempts some correct log work seen at least once within an attempted trapezium rule.
This will usually be for a correct use of the addition law at some point but could be for $2\log 5=\log 25$.
It cannot be scored for $\log k=4.00\Rightarrow k=10^{4.00}$

A1: $\log 10080$ following the award of the M mark.
This cannot be scored by candidates who approximate the result to be 4.003…for instance and then use their calculators to get the $\log 10080$

(ii) Do not be concerned if a different variable is used or the base 5 is omitted

B1: For $2\log_5\left(5-a\right)=\log_5\left(5-a\right)^2$ **OR** replacing 1 by $\log_5 5$

M1: Correctly combines any two terms which will usually be

$$2\log_5\left(5-a\right)-\log_5\left(a+25\right)=1\Rightarrow \log_5\frac{\left(5-a\right)^2}{\left(a+25\right)}=...$$

Alternatives exist. E.g. $2\log_5\left(5-a\right)=\log_5\left(a+25\right)+1\Rightarrow ...=\log_5\left(5a+125\right)$

A1: Correct 3TQ $a^2-15a-100=0$ o.e.

dM1: Correct attempt to find at least one value of *a*. It is dependent upon previous M and having reached a quadratic equation. Apply usual rules for solving a quadratic.
Allow calculator solutions but you will need to check that they are/it is correct

A1: $a=-5$ ONLY
If $a=20$ is written down it must be rejected

Note that a solution where the log terms are incorrectly combined can score
SC 10001 as long as they reach $a=-5$ only

$$\frac{\log_5\left(5-a\right)^2}{\log_5\left(a+25\right)}=1\Rightarrow a^2-15a-100=0\Rightarrow a=-5$$

Question Number	Scheme	Marks
8(i)	Provides a counter example E.g. $\sqrt{2}\times\sqrt{8}=4$	B1
		(1)
(ii)	Knows that odd numbers are of the form $2k+1, \quad (k\in\mathbb{N})$	B1
	Attempts $(2k+1)^3+3(2k+1)+2=....$	M1
	$=8k^3+12k^2+12k+6$	A1
	$=4\times\left(2k^3+3k^2+3k+1\right)+2$ Which is even but not a multiple of 4. Hence proven	A1*
		(4) **(5 marks)**

(i)
B1: Provides a counter example and shows that the statement is not always true.
The correct result must be written out showing that it is rational (that is, giving it in a rational form)
So $\sqrt{2}\times\sqrt{8}=\sqrt{16}$ is insufficient but $\frac{\pi}{2}\times\frac{6}{\pi}=\frac{6}{2}$ is acceptable

(ii)
A proof attempted by substituting odd numbers n = 1, 3, 5 etc into n^3+3n+2 will not score any marks

B1: Knows that odd numbers are of the form $2k\pm1$ using any variable apart from n
Other variations may be used, for example $2k\pm3$, $2k\pm5$, etc but will be much less common.
Note that use of $4k+1$ is flawed as it doesn't pick up all the odd numbers so on its own scores B0

M1: Attempts to expand $(2k+1)^3+3(2k+1)+2$ or $(2k-1)^3+3(2k-1)+2$ using any variable including n.
Generally, look for a minimum of $(2k+1)^3+3(2k+1)+2\rightarrow$ a cubic expression in k
As with the B mark, other variations may be used, for example $2k\pm3$ but will be much less common.
The use of $n=4k+1$ in n^3+3n+2 and expanded to a cubic form can be awarded this M mark only

A1: A correct expansion for $(2k+1)^3+3(2k+1)+2$ Allow the use of any variable including n
Look for $8k^3+12k^2+12k+6$ …
but …allow any correct expression that allows the problem to be solved e.g. $4(2k+1)\left(k^2+k+1\right)+2$
FYI the correct expansion for $(2k-1)^3+3(2k-1)+2$ is $8k^3-12k^2+12k-2$

A1*: Requires the candidate to have fully correct algebra, scored all 3 previous marks and
1) prove/show that the result is even
2) prove/show that the result is not a multiple of 4
3) give a minimal conclusion

Examples of expressions, for $n = 2k + 1$, that show the result is even;

- $8k^3 + 12k^2 + 12k + 6$ with a statement saying that it is a sum of even numbers
- $\left(8k^3 + 12k^2 + 12k + 6\right) \div 2 = 4k^3 + 6k^2 + 6k + 3$
- $8k^3 + 12k^2 + 12k + 6 = 2\left(4k^3 + 6k^2 + 6k + 3\right)$

Examples of expressions, for $n = 2k + 1$, that show the result is not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 2\left(4k^3 + 6k^2 + 6k + 3\right)$ with some statement alluding to the fact that another factor of 2 cannot be taken out of the $\left(4k^3 + 6k^2 + 6k + 3\right)$
- $\left(8k^3 + 12k^2 + 12k + 6\right) \div 4 = 2k^3 + 3k^2 + 3k + \frac{3}{2}$
- $8k^3 + 12k^2 + 12k + 6 = 4\left(2k^3 + 3k^2 + 3k + 1.5\right)$

Examples of expressions, for $n = 2k + 1$, that show the result is both even but not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 4\left(2k^3 + 3k^2 + 3k + 1\right) + 2$
- $8k^3 + 12k^2 + 12k + 6 = 4 \times \left(2k^3 + 3k^2 + 3k\right) + 6$

when $n=2k+1$
n^3+3n+2
$=(2k+1)^3+3(2k+1)+2$
$=(4k^2+4k+1)\cdot(2k+1)+6k+3+2$
$=8k^3+8k^2+2k+4k^2+4k+1+6k+5$
$=8k^3+12k^2$ ~~$+6k$~~ $+12k+6$
~~$=4(2k^3)$~~
$=4(2k^3+3k^2+3k+1)+2$
$\therefore$ It's always even but never a multiple of 4.

This is acceptable for all 4 marks.
It uses an expression that proves/shows both results and is followed by an acceptable conclusion

ii) $n = 2x+1$
n^3+3n+2
$=(2x+1)^3+3x(2x+1)+2$
$=(2x+1)^2(2x+1)+6x+3+2$
$=(4x^2+4x+1)(2x+1)+6x+5$
$=8x^3+4x^2+8x^2+4x+2x+1+6x+5$
$=8x^3+12x^2+6x+6x+6$
$=8x^3+12x^2+12x+6$
It is an even number!
$2(4x^3+6x^2+6x+3)$
~~It is not a multiple of 4~~
It is not a multiple of 4!
$=4(2x^3+3x^2+3x)+6$
~~6 cannot be~~
4 is not a factor of 6.

This is acceptable for all 4 marks.
It uses two different expressions to show both the results. The fact that the conclusion is split over a couple of lines is not an issue.

Question Number	Scheme	Marks
	$y=\dfrac{9x^2\left(5-\sqrt{x}\right)}{5}$	
9(a)	$y=9x^2-\frac{9}{5}x^{\frac{5}{2}}\Rightarrow\frac{dy}{dx}=18x-\frac{9}{2}x^{\frac{3}{2}}$	M1, A1
	$18x-\frac{9}{2}x^{\frac{3}{2}}=0\Rightarrow x^{\frac{1}{2}}=(4)$	dM1
	$(16, 460.8)$	A1, A1
		(5)
(b)	$5-\sqrt{x}=0\Rightarrow x=25$	M1, A1
		(2)
(c)	$\int 9x^2-\frac{9}{5}x^{\frac{5}{2}}dx=3x^3-\frac{18}{35}x^{\frac{7}{2}}$	M1A1
	Shaded area = $\underline{(25-16)\times 460.8}-\left[3x^3-\frac{18}{35}x^{\frac{7}{2}}\right]_{16}^{25}$	<u>M1</u>, dM1
	=1312.7	A1
		(5)
		(12 marks)

(a)

M1: Attempts to differentiate. In most cases this will involve multiplying out and differentiating each term. Look for a sum of two terms with one correct index.

So, score for either $y=ax^2\pm\ldots\Rightarrow\frac{dy}{dx}=px\pm\ldots$ or $y=\ldots\pm bx^{\frac{5}{2}}\Rightarrow\frac{dy}{dx}=\ldots\pm qx^{\frac{3}{2}}$

It is possible to differentiate using the product rule.

In this case look for the correct form. E.g $y=\frac{9x^2\left(5-\sqrt{x}\right)}{5}\Rightarrow\frac{dy}{dx}=\left(5-\sqrt{x}\right)\times\ldots x+\ldots x^2\times\ldots x^{-\frac{1}{2}}$

You may see attempts using the product or quotient rules so look carefully at what is written

A1: $\frac{dy}{dx}=18x-\frac{9}{2}x^{\frac{3}{2}}$ o.e but allow this un-simplified.

FYI, using the product rule, the correct unsimplified answer is $\left(5-\sqrt{x}\right)\times\frac{18x}{5}+\frac{9x^2}{5}\times-\frac{1}{2}x^{-\frac{1}{2}}$

dM1: Sets their $px\pm qx^{\frac{3}{2}}=0$ or $px\pm qx^{\frac{1}{2}}=0$ (from a slip on the fractional index) and proceeds using correct index work to $\alpha x^{\pm\frac{1}{2}}=\beta$

Other methods exist, such as squaring, but look for correct index work with the coefficients squared as well. The demand of candidates is to show their working and it will be applied in this question on this mark.

If candidates go from $18x-\frac{9}{2}x^{\frac{3}{2}}=0$ to $x=16$ without intermediate working they will lose this mark.

They can be awarded the next two marks however.

A1: x coordinate of 16 **following correct differentiation and** $\frac{dy}{dx}=0$.

Note that errors stemming from a misunderstanding of the $\div 5$ can lead to $x=16$ but should be scored A0

A1: $(16, 460.8)$ following award of previous A1. Allow this to be scored from correct coordinate in part (c)

Also allow this to be given separately as $x = 16$, $y = \dfrac{2304}{5}$ o.e.

(b)

M1: Deduces that $5 - \sqrt{x} = 0$ or $\sqrt{x} = 5$. Squaring methods are also possible (with coefficients squared) usually leading to $25 - x = 0$ o.e.

A1: $x = 25$ following award of the M1

25 on its own without any working scores M0 A0

(c) In epen this is marked M, A, M, A, A but is now being scored M, A, M, dM, A

Method integrating just curve

M1: Attempts to integrate. In most cases this will involve multiplying out and integrating each term. Look for a sum of two terms with one correct index. Ignore any spurious integral signs if it is obvious they have integrated.

So score for either $\int\left(ax^2 \pm ...\right)dx \Rightarrow rx^3 \pm ....$ or $\int\left(... \pm bx^{\frac{5}{2}}\right)dx \Rightarrow ... \pm sx^{\frac{7}{2}}$

This can be scored in a similar way if they have attempted (line – curve)

A1: $3x^3 - \dfrac{18}{35}x^{\frac{7}{2}}$ but allow this unsimplified

If (line – curve) has been attempted look for $... - \underline{\underline{\left(3x^3 - \dfrac{18}{35}x^{\frac{7}{2}}\right)}}$ OR $... \underline{\underline{-3x^3 + \dfrac{18}{35}x^{\frac{7}{2}}}}$

M1: Correct method of finding the **area of rectangle** or **area of right-angled triangle** (with *MP* as the hypotenuse).

E.g. For the rectangle look for $('25' - '16') \times '460.8'$ or $\displaystyle\int_{16}^{25} ('460.8' - ....)\,dx = ['460.8'x - ...]_{16}^{25} = ...$

dM1: Complete attempt to find area of R involving the correct combination of areas, usually (area of rectangle – area under curve) or (2 × area of triangle – area under curve).

It is dependent upon both previous M's. For the area under the curve look for a minimum of one term being correctly integrated and limits of their 16 from (a) and their 25 from (b)

A1: AWRT 1312.7

……..

Other more complicated methods are possible; If they integrate (line – curve) we score as follows

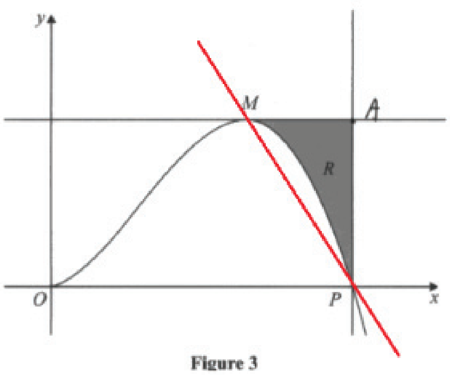

Figure 3

The first M1, A1 can be awarded as in the main method.
The following M1 is a correct method for triangle AMP
The dM1is for the full method to find area of R as before.

……..

Question Number	Scheme	Marks
10 (a)	Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ o.e. E.g. $\cos\theta\left(3\tan\theta + \dfrac{2}{\tan\theta}\right) \equiv \cos\theta\left(3\dfrac{\sin\theta}{\cos\theta} + \dfrac{2\cos\theta}{\sin\theta}\right)$	M1
	Uses $\sin^2\theta + \cos^2\theta = 1$ E.g. $\equiv 3\sin\theta + \dfrac{2\cos^2\theta}{\sin\theta} \equiv 3\sin\theta + \dfrac{2\left(1-\sin^2\theta\right)}{\sin\theta}$	dM1, A1
	$\equiv 3\sin\theta + \dfrac{2}{\sin\theta} - 2\sin\theta \equiv \sin\theta + \dfrac{2}{\sin\theta}$	A1*
		(4)
(b)	$\sin x + \dfrac{2}{\sin x} = 4\sin x - 5 \Rightarrow 3\sin^2 x - 5\sin x - 2 = 0$	M1, A1
	$\Rightarrow \sin x = 2, -\dfrac{1}{3} \Rightarrow x = 3.5$ for example.	dM1
	$\Rightarrow x = 3.48, 5.94$	A1
		(4)
		(8 marks)

(a) Allow use of $=$ for $\equiv$ within this question.

Allow workings on consecutive lines to follow without equals signs or with arrows.

M1: Uses $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ o.e. correctly at least once

dM1: Uses $\cos^2\theta \equiv \pm 1 \pm \sin^2\theta$ following the award of the previous M1

You may see $\cos\theta\left(\dfrac{3\sin^2\theta + 2\cos^2\theta}{\sin\theta\cos\theta}\right) \equiv \cos\theta\left(\dfrac{3\sin^2\theta + 2\left(\pm 1 \pm \sin^2\theta\right)}{\sin\theta\cos\theta}\right)$ condoning slips in coefficients

A1: A correct intermediate line in just $\sin\theta$ E.g. $3\sin\theta + \dfrac{2\left(1-\sin^2\theta\right)}{\sin\theta}$

A1*: Proceeds correctly to the given answer with no errors or omissions.

Withold this mark for notational errors. E.g writing $\sin^2\theta$ as $\sin\theta^2$

Withold this mark if you don't see both sides of the identity (or their equivalents e.g. LHS and RHS) within their solution.

Condone one or two missing θ's or changes of variable in say, $\sin\theta$, but penalise persistent offences, i.e. > 2 occurrences.

(b)

M1: Uses part (a) and proceeds to a 3TQ in sin$x$. Condone slips in coefficients.

The multiplication by sin$x$ must be seen/have occurred on at least three of the four terms

Allow restarts that essentially proceeds to the same equation

A1: Correct quadratic equation. The = 0 may be implied by subsequent work

dM1: Solves their quadratic equation in sin$x$ and proceeds to one solution in the range to 1dp

If their quadratic equation in sin$x$ is incorrect you may need to use a calculator to check their roots and their value for x. Allow this method to be scored for angles in degrees.

So, for example, $\sin x = -\dfrac{1}{3} \Rightarrow x =$ awrt 199.5 or 340.5 but not -0.339 radians as it is not in the range

A1: AWRT $3.48, 5.94$ and no others in the range.

This can only be awarded after the award of all previous marks in (b)

Answers without working scores 0 marks

Other methods exist for part (a) but many are more complicated and may involve WMA13 methods. The same principles of scoring can be applied. If unsure use review

Example 1 using WMA13 trigonometry

Assume $\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\Rightarrow 3\tan\theta+\frac{2}{\tan\theta}\equiv\frac{\sin\theta}{\cos\theta}+\frac{2}{\sin\theta\cos\theta}$

$$\equiv\tan\theta+\frac{2}{\sin\theta/\cos\theta\times\cos^2\theta}$$ M1

$$\equiv\tan\theta+\frac{2\sec^2\theta}{\tan\theta}$$

$$\equiv\tan\theta+\frac{2\left(1+\tan^2\theta\right)}{\tan\theta}$$ dM1, A1

$$\equiv 3\tan\theta+\frac{2}{\tan\theta}$$

Hence $\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}$ A1*

Example 2 using both sides of the identity.

$$\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\Rightarrow\sin\theta\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin^2\theta+2$$

$$\Rightarrow\sin\theta\cos\theta\left(3\frac{\sin\theta}{\cos\theta}+\frac{2\cos\theta}{\sin\theta}\right)\equiv\sin^2\theta+2$$ M1

$$\Rightarrow 3\sin^2\theta+2\cos^2\theta\equiv\sin^2\theta+2$$

$$\Rightarrow 3\sin^2\theta+2\left(1\text{-}\sin^2\theta\right)\equiv\sin^2\theta+2$$ dM1, A1

$$\Rightarrow\sin^2\theta+2\equiv\sin^2\theta+2$$

Scores the final A1* when they state Hence $\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}$ A1*