Question Number	Scheme	Marks
1(a)	$h = 1.25$	B1
	$A \approx \frac{1}{2}\times 1.25\{3.479+5.182+2(6.101+7.448+6.823)\}$	M1
	$= 30.9$	A1
		(3)
(b)	$\int_{0.5}^{5.5}(\mathrm{f}(x)+4x)\,\mathrm{d}x = 30.9+\left[2x^2\right]_{0.5}^{5.5} = 30.9+2\times 5.5^2-2\times 0.5^2$ **or** $\int_{0.5}^{5.5}(\mathrm{f}(x)+4x)\,\mathrm{d}x = \int_{0.5}^{5.5}\mathrm{f}(x)\,\mathrm{d}x+\frac{1}{2}(4\times 0.5+4\times 5.5)\times 5 = ...$	M1
	$= 30.9 + 60 = 90.9$	A1ft
		(2)
		Total 5

(a)

B1: Correct strip width of 1.25. Allow equivalent numerical expressions e.g. $\frac{5.5-0.5}{4}$.
May be implied by sight of e.g. $\frac{1.25}{2}$ or $\frac{5}{8}$ or 0.625 in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h=-1.25$ is B0 unless they recover and subsequently use $h=+1.25$

M1: Correct application of the trapezium rule with their h (which may be 1).
This requires a correct inner bracket structure $3.479+5.182+2(6.101+7.448+6.823)$ multiplied by $\frac{1}{2}h$. Condone slips copying values from the table or the omission of the final bracket on the rhs e.g. $\frac{1}{2}\times"1.25"(3.479+5.182+2(6.101+7.448+6.823)$ is M1 but $\frac{1}{2}\times"1.25"\times 3.479+5.182+2(6.101+7.448+6.823)$ is M0 unless the missing brackets are recovered or implied by the correct answer (you may need to check).
Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)
e.g. $\frac{1}{2}\times"1.25"(3.479+6.101)+\frac{1}{2}\times"1.25"(6.101+7.448)+\frac{1}{2}\times"1.25"(7.448+6.823)+\frac{1}{2}\times"1.25"(6.823+5.182)$

A1: For awrt 30.9. isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h=-1.25$ then maximum awarded is B0M1A0. The A mark cannot be awarded without both B1M1 being awarded in this question. The exact answer is $\frac{9881}{320}$ and scores A0 unless followed by awrt 30.9
For reference, note that use of $h = 1$ (from using 5 strips) gives 24.7… or $\frac{9881}{400}$

(a), $\int_{0.5}^{5.5} 1.25\,(3.479+6.101+7.448+6.823+5.182) = 36.3.$ Scores B1M0A0 (BOD)

(b)

M1: Attempts their answer to (a) + $\left[...x^2\right]_{0.5}^{5.5}$ = their answer to $(\text{a})+...5.5^2-...0.5^2$

May be implied by e.g. their answer to $(\text{a})+2\times5.5^2-2\times0.5^2$ if the integration is not seen explicitly.

Or

their answer to (a) $+\frac{1}{2}(4\times0.5+4\times5.5)\times5$ (trapezium)

oe e.g. their answer to (a) $+(4\times0.5\times5)+\frac{1}{2}(5\times(5\times5.5-5\times0.5))$ (rectangle + triangle)

May be implied by their answer to (a) + 60

Condone clear misreads of the 4 in the 4x but do not condone a misread of e.g. 4x for 4.

Condone clear mis-copy/mis-read of limits as long as they are non-zero E.g. 5 for 5.5.

Condone poor notation e.g. $\int_{0.5}^{5.5}(\text{f}(x)+4x)\,\text{d}x=30.9+\int_{0.5}^{5.5}2x^2\,\text{d}x=30.9+2\times5.5^2-2\times0.5^2$

A1ft: Correct answer of awrt 90.9 or correct ft e.g. 60 + their answer to part (a)

Allow exact or exact ft answers e.g. $\frac{29081}{320}$ for 90.9

Correct answer **only** scores no marks as the questions says "making your method clear"

Attempts to use the trapezium rule again by adding 4x to the y values score M0 in (b)

Note that $30.9+4(0.5+1.75+3+4.25+5.5)=90.9$ is fortuitous and scores M0

But $30.9+\frac{1}{2}\times\frac{5}{4}(2+22+2(7+12+17))=90.9$ scores M1A1

Condone an error with the strip width as in part (a) for this method but the ft is not available in this case.

<u>Examples of minimal acceptable working in (b) for both marks:</u>

$$30.9 + 60 = 90.9$$

$$30.9+\left[2x^2\right]_{0.5}^{5.5}=90.9$$

$$\int_{0.5}^{5.5}\text{f}(x)\,\text{d}x+\int_{0.5}^{5.5}4x\,\text{d}x=30.9+60=90.9$$

$$30.9+2\times5.5^2-2\times0.5^2=90.9$$

All score M1A1

Question Number	Scheme	Marks
2(a)	$u_1 = 7 \Rightarrow u_2 = k - 7$	B1
	$u_2 = k - 7 \Rightarrow u_3 = k + k - 7 \Rightarrow u_4 = k - (2k - 7)$ $u_5 = k + (-k + 7) = 7*$	M1A1*
		(3)
(b)	$\sum_{r=1}^{4} u_r = 30 \Rightarrow 7 + k - 7 + 2k - 7 - k + 7 = 30 \Rightarrow k = ...$	M1
	$k = 15$	A1cso
		(2)
(c)	$\sum_{r=1}^{150} u_r = 37 \times 30 + 7 + "15" - 7$	M1
	$= 1125$	A1cso
		(2)
		Total 7

General Guidance

Note that it is possible to obtain correct answers in all parts of this question fortuitously. The A marks should only be awarded following correct work.

(a)

B1: Correct expression for u_2 e.g. $k - 7$ with no incorrect work seen e.g. $u_2 = (-1)^7 \times 7 + k = -7 + k$ scores B0. Must be seen in part (a).

Allow e.g. $u_2 = (-1)^1 \times 7 + k$ or e.g. $u_2 = (-1) \times 7 + k$ or e.g. $u_2 = -1 \times 7 + k$

M1: Attempts to use the recurrence formula **to reach the 5th term** but condone **consistent** use of $(n + 1)$ for n on the $(-1)^n$ term so e.g. $u_2 = (-1)^2 \times 7 + k$ is condoned but e.g. $u_2 = (-1)^7 \times 7 + k$ is M0

Allow even if any slips mean their sequence is not of order 4.

A1*: Obtains $u_5 = 7$ with no errors or omissions. The "$u_5 =$" must appear at some point but allow e.g. "$u_{4+1} =$"

For reference the terms are: $u_1 = 7,\ u_2 = k - 7,\ u_3 = 2k - 7,\ u_4 = 7 - k,\ u_5 = 7$

(a) Alternatives:

[1] $u_2 = -u_1 + k,\ u_3 = -u_1 + 2k,\ u_4 = u_1 - k,\ u_5 = u_1 = 7$

B1: Implied by $u_2 = -u_1 + k$

M1: Attempts to use the recurrence formula **to reach the 5th term** but condone use of $(n + 1)$ for n on the $(-1)^n$ term as above

A1*: Fully correct work to reach $u_5 = 7$

(Allow the above approach to be used with a made up k or a wrong k as the result is independent of k)

[2] $u_5 = 7 \Rightarrow u_4 = 7 - k \Rightarrow u_3 = 2k - 7 \Rightarrow u_2 = k - 7 \Rightarrow u_1 = 7$ Hence true.

B1: $u_4 = 7 - k$

M1: Attempts to use the recurrence formula correctly backwards **to reach the 1st term** but condone use of $(n + 1)$ for n on the $(-1)^n$ term as above.

A1*: Fully correct work to reach $u_1 = 7$ with a (minimal) conclusion.

(b)

M1: Adds their first 4 terms, sets = 30 and solves a linear equation for k
Condone slips in copying their 4 terms if the intention is clear.
Allow even if their sequence is of order 2 e.g. $7,\ k-7,\ 7,\ k-7,\ ...$

A1: Correct value **from correct work and a correct sequence**.

(c)

Marks in (c) are only available if their sequence is of order 4

M1: **Correct** numerical attempt for the required sum using their k. May be implied by the correct answer or the correct ft answer unless an incorrect method is seen e.g. $37\times30+u_3+u_4$ or $38\times30-u_1-u_2$ score M0

Condone $\frac{150}{4}\times30$ or 75×15 or 37.5×30 following a **correct sequence** and the **correct value of k.**

Allow any equivalent **correct** method e.g. $38\times30-(2\times"15"-7+7-"15")$

A1: For 1125 **from correct work and a correct sequence**.
Correct answer **only** scores both marks.

For reference the **correct** sequence is:

n	1	2	3	4
u_n	7	$k-7$	$2k-7$	$7-k$
Value (with $k = 15$)	7	8	23	−8

<u>An unusual but correct method in (c) is:</u>

$$\sum_{r=1}^{150} u_r = 38\times7+38(15-7)+37(30-7)+37(7-15)=1125$$

<u>Example – Candidates who consistently use</u>

$$u_{n+1}=(-1)^{n+1}u_n+k$$

This gives:

(a) $u_1=7,\ u_2=k+7,\ u_3=-7,\ u_4=k-7,\ u_5=7$ which scores B0M1A0

(b) $\sum_{r=1}^{4} u_r=30\Rightarrow 7+k+7-7+k-7=30\Rightarrow k=15$ which scores M1A0

(c) $\sum_{r=1}^{150} u_r=37\times30+7+"15"+7=1139$ which scores M1A0

For reference this **incorrect** sequence is:

n	1	2	3	4
u_n	7	$k+7$	−7	$k-7$
Value (with $k = 15$)	7	22	−7	8

Question Number	Scheme	Marks
3(a)	$2(-3)^3-(-3)^2+A(-3)+B=55$ or e.g. $-54-9-3A+B=55$	M1
	$-54-9-3A+B=55$ $\Rightarrow 3A-B=-118$*	A1*
		(2)

(a)

M1: Attempts $f(-3)=55$. The -3 embedded in the expression set = 55 is sufficient.

Condone missing brackets e.g. $2-3^3--3^2+A(-3)+B=55$

Note that $2(-3)^3-(3)^2+A(-3)+B=55$ is acceptable.

May be implied by further work. If -3 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in -3 and set = 55 but $f(3)=55$ is M0

A1*: Completes proof with at least one intermediate simplified and correct line that is not the final line such as $-54-9-3A+B=55$ or $118=-3A+B$. Any incorrect lines written after the start of their 'proof' should be scored A0*
Must be the given equation and not e.g. $B=3A+118$ unless the correct equation is seen previously but condone $-B+3A=-118$

$2(-3)^3-(-3)^2+A(-3)+B=55 \Rightarrow 3A-B=-118$ scores M1A0

Tabular or division methods may be seen. The question does not demand use of the factor or remainder theorem so these methods should be rewarded.
The M1 would be scored for a full attempt and A1* for a correct proof.
If you cannot see what they are doing and find it hard to award, then use review.
e.g. long division:

$$\begin{array}{r} 2x^2-7x+A+21 \\ x+3\overline{)2x^3-x^2+Ax+B} \\ \underline{2x^3+6x^2} \\ -7x^2+Ax+B \\ \underline{-7x^2-21x} \\ (A+21)x+B \\ \underline{(A+21)x+3(A+21)} \\ B-3A-63=55 \end{array}$$

Score M1 for obtaining a quotient $...x^2 \pm ...x \pm ...A+...$ and a remainder $...A \pm ...B \pm ...$ set = 55 where "…" are non-zero constants. Then A1* for a fully correct proof.
e.g. tabular method:

	$2x^2$	$-7x$	$21+A$
x	$2x^3$	$-7x^2$	$(21+A)x$
3	$6x^2$	$-21x$	$63+3A$

So $B-(63+3A)=55$ etc.

Score M1 for obtaining a constant $...A+...$ where … are non-zero constants and then sets B – their constant = 55. Then A1* for a fully correct proof.

(b)	$2\left(\frac{5}{2}\right)^3-\left(\frac{5}{2}\right)^2+A\left(\frac{5}{2}\right)+B=0$	M1	
	$3A-B=-118,\ 5A+2B=-50$ $\Rightarrow A=...,$ **or** $B=...$	M1	
	$A=-26,\ B=40$	A1	
		(3)	

(b) Do not allow mis-reads of (2*x* – 5) in part (b).

M1: Attempts $\text{f}\left(\frac{5}{2}\right)=0$. The $\frac{5}{2}$ embedded in the expression set = 0 is sufficient.

$\text{f}\left(-\frac{5}{2}\right)=0$ scores M0.

May be implied by further work. Again, tabular or division methods may be seen.
e.g. long division:

$$\begin{array}{r} x^2+2x+\tfrac{1}{2}A+5 \\ 2x-5\overline{)2x^3-x^2+Ax+B} \\ \underline{2x^3-5x^2} \\ 4x^2+Ax+B \\ \underline{4x^2-10x} \\ (A+10)x+B \\ \underline{(A+10)x-5\left(\tfrac{1}{2}A+5\right)} \\ B+\tfrac{5}{2}A+25=0 \end{array}$$

Score M1 for obtaining a quotient $...x^2\pm...x\pm...A+...$ and a remainder $...A\pm...B\pm...$ set = 0 where "…" are non-zero constants.
e.g. tabular method:

	x^2	$2x$	$5+\frac{A}{2}$
$2x$	$2x^3$	$4x^2$	$(10+A)x$
-5	$-5x^2$	$-10x$	$-25-\frac{5A}{2}$

So $B-\left(-25-\frac{5A}{2}\right)=0$. Score M1 for obtaining a constant $...A+...$ where "…" are non-zero constants, and then sets *B* – their constant = 0

M1: Solves $3A-B=-118$ simultaneously with their equation in *A* and *B* from having attempted $\text{f}\left(\pm\frac{5}{2}\right)=0$ or long division/tabular method using the correct factor of (2*x* – 5) and obtains a value for *A* or *B*.
May be via a calculator and may be implied by their values. You do not need to check but the above conditions must be satisfied.
For reference, use of $\text{f}\left(-\frac{5}{2}\right)=0$ gives $A=-161,\ B=-365$

A1: Both correct values

(c)	$\mathrm{f}(x)=(x-7)\left(2x^2+\ldots x+\ldots\right)+\ldots$	M1
	$2x^2+13x+65$	A1
		(2)
		Total 7

(c) **Do not allow mis-reads of (x – 7) in part (c).**

M1: Uses any appropriate method e.g. long division/inspection/table with numeric non-zero A and B to obtain $2x^2 + px + q$ where p and q are non-zero.
There is no requirement to find the remainder but assuming (x – 7) is a **factor** is M0 (see example below)

A1: **Must follow correct values of A and B.**
Correct expression $2x^2+13x+65$ **which is clearly identified** e.g. by circling/underlining or e.g. Quotient = $2x^2+13x+65$, $Q=2x^2+13x+65$
Any remainder, correct or incorrect, can be ignored. (NB correct remainder is 495)

Long division for reference:

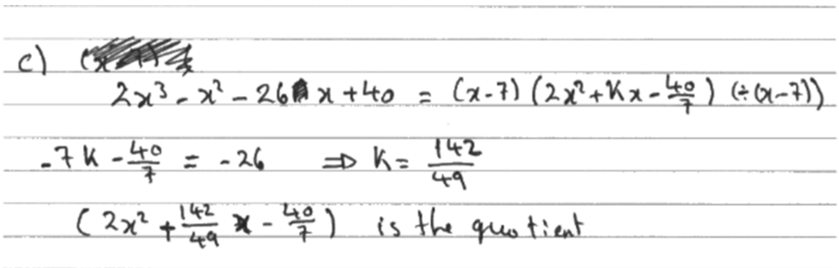
$$\begin{array}{r} 2x^2+13x+65 \\ x-7\,\overline{)\,2x^3-x^2-26x+40} \\ \underline{2x^3-14x^2} \qquad\qquad \\ 13x^2-26x+40 \\ \underline{13x^2-91x} \qquad \\ 65x+40 \\ \underline{65x-455} \\ 495 \end{array}$$

Tabular method for reference:

	$2x^2$	$13x$	65
x	$2x^3$	$13x^2$	$65x$
-7	$-14x^2$	$-91x$	-455

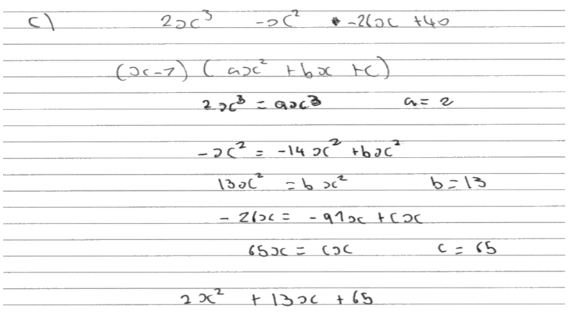
But note that **just** the above score M1A0 with no indication that the quotient is $2x^2+13x+65$

Some candidates assume that (x – 7) is a **factor** of f(x) and try to find the quadratic factor:


c) $2x^3 - x^2 - 26x + 40 = (x-7)(2x^2 + Kx - \frac{40}{7})$ $(\div(x-7))$
$-7K - \frac{40}{7} = -26 \Rightarrow K = \frac{142}{49}$
$(2x^2 + \frac{142}{49}x - \frac{40}{7})$ is the quotient

This scores M0 in part (c)

But work that doesn't assume (x – 7) is a factor is fine e.g.:


c) $2x^3 \quad -x^2 \quad -26x \quad +40$
$(x-7)(ax^2+bx+c)$
$2x^3 = ax^3 \qquad a=2$
$-x^2 = -14x^2 + bx^2$
$13x^2 = bx^2 \qquad b=13$
$-26x = -91x + cx$
$65x = cx \qquad c=65$
$2x^2 + 13x + 65$

Scores M1A1

Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3$ $\Rightarrow \left(\frac{dy}{dx} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = ...$	M1
	$x = \frac{9}{4}$ oe e.g. 2.25	A1
		(2)

(a)

M1: For $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$. Allow unprocessed e.g. $x^{\frac{1}{2}} \to x^{\frac{1}{2}-1}$ or $x^{-\frac{1}{2}} \to x^{-\frac{1}{2}-1}$.

A1: Correct simplified derivative. $\frac{dy}{dx} =$ is not required. Isw once a correct answer is seen.

Allow equivalent simplified expressions e.g. $2x^{-\frac{1}{2}} - 4.5x^{-\frac{3}{2}}$

$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} + c$ scores A0 as does $2x^{-\frac{1}{2}} + -\frac{9}{2}x^{-\frac{3}{2}}$

(b)

M1: Starts from a derivative of the form $Ax^{-\frac{1}{2}} - Bx^{-\frac{3}{2}}$ where $A \times B > 0$, sets = 0 (seen or implied) and solves via a correct method to obtain an expression or value of $x = \frac{B}{A}$.

A1: $x = \frac{9}{4}$ oe with no incorrect working seen in their method.

Condone if e.g. $\frac{dy}{dx} > 0$ is seen as long as they reach the correct value for x.

There must be no other values. **Must come from a correct derivative but see * note below.**

The minimum acceptable for both marks is $2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$

The following is condoned:

$$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{2}{x^{\frac{1}{2}}} - \frac{9}{2x^{\frac{3}{2}}} = 0 \Rightarrow \frac{4}{x} - \frac{81}{4x^3} = 0 \Rightarrow x = \frac{9}{4}$$

Ignore any attempts to find the y coordinate.

***Allow full recovery in (b) if the working is correct in (a) but isw has been applied e.g.**

(a) $\left(\frac{dy}{dx} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$ M1A1 (isw)

(b) $4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$ M1A1

4(c)(i)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right)-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}}+6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{x=\frac{9}{4}}=\right)-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}\left(=\frac{16}{27}(0.5925...)\right)$ $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)>0$ so (local) minimum	B1
		(2)
(d)	$0<x<\frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)

B1ft: Correct simplified second derivative.
Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers.

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=$ is not required.

Condone $-1x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of " + c" scores B0ft as with the first derivative.

(ii)

B1: Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x=\frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9}-\frac{8}{27}>0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ e.g.

$-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}=\frac{16}{27}>0$ so minimum is fine for this mark.

(d)

B1ft: For $0<x<\frac{9}{4}$. Allow $0<x\leqslant\frac{9}{4}$ and allow equivalent statements e.g. $\left(0,\frac{9}{4}\right)$, $\left(0,\frac{9}{4}\right]$.

Follow through their positive x value from part (b). Condone an answer of $x<\frac{9}{4}$ or $x\leqslant\frac{9}{4}$ Condone $0\leqslant x\leqslant\frac{9}{4}$

Do not allow $-\infty<x<\frac{9}{4}$

Question Number	Scheme	Marks
5(a)	$(2+ax)^6 = 2^6 + \binom{6}{1}2^5(ax) + \binom{6}{2}2^4(ax)^2 + \ldots$	M1
	$= 64 + 192ax + 240a^2x^2 + \ldots$	A1A1
		(3)
(b)	$\left(3+\frac{1}{x}\right)^2 = 9 + \frac{6}{x} + \frac{1}{x^2}$ or $9 + \frac{3}{x} + \frac{3}{x} + \frac{1}{x^2}$	B1
	$\mathrm{f}(x) = \left(9 + \frac{6}{x} + \frac{1}{x^2}\right)(64 + 192ax + 240a^2x^2 + \ldots) = \ldots$ Constant term is $9\times64 + 6\times192a + 240a^2$	M1
	$576 + 1152a + 240a^2 = 576 \Rightarrow 1152a + 240a^2 = 0$ $\Rightarrow 1152 + 240a = 0 \Rightarrow a = \ldots$	**d**M1
	$a = -\frac{24}{5}$	A1
		(4)
		Total 7

(a)

M1: Attempts the binomial expansion up to at least the second term to obtain the correct structure for the **2nd or 3rd term** i.e. the correct binomial coefficient (allow alternative notation) with the correct power of 2 and the correct power of ax.
The binomial coefficients do not have to be evaluated but must be correct if they are.
If awarding this mark for the x^2 term you can condone missing brackets e.g. ${}^6C_2 \times 2^4 \times ax^2$

Alternatively writes e.g. $(2+ax)^6 = 2^6\left(1+\frac{ax}{2}\right)^6 = 2^6\left(1 + \frac{6}{2}ax + \frac{6\times5}{2}\left(\frac{ax}{2}\right)^2 + \ldots\right)$

which can also score M1 for the expansion up to at least the second term with an acceptable structure for either the **2nd or 3rd term**. They do not have to multiply out the brackets for this mark but the 2^6 cannot be omitted unless it is later recovered.

Condone missing brackets e.g. $\frac{6\times5}{2}\times\frac{ax}{2}^2$

A1: For any 2 correct simplified terms in any order (Allow a^2x^2 or $(ax)^2$ for the third term **for this mark**)
Allow terms to be listed.
Do **not** allow $64x^0$ for 64 or $192ax^1$ for $192ax$

A1: All correct and in any order. Allow terms to be listed. Must be a^2x^2 not $(ax)^2$ for the 3rd term but allow (ax) for ax.
Apply isw if they divide through by e.g. 2 or 4 or 16 or set their expression = 0

Ignore any extra terms if attempted.

(b)

B1: Correct expansion of $\left(3+\frac{1}{x}\right)^2$ unsimplified or simplified.

Note that this may be implied by the omission of the "9" as $9\times64=576$ and so the "9" is not required.

Note that $3^2\left(1+2\left(\frac{1}{3x}\right)+\frac{1}{(3x)^2}\right)$ is correct.

Condone missing brackets if recovered e.g. $\left(3+\frac{1}{x}\right)^2=9+\frac{6}{x}+\frac{1}{x}^2=9+\frac{6}{x}+\frac{1}{x^2}$

M1: Uses their expansion in part (a) and their expansion of $\left(3+\frac{1}{x}\right)^2$ to extract the constant term. This depends on having obtained $\left(3+\frac{1}{x}\right)^2=\alpha+\frac{\beta}{x}+\frac{\gamma}{x^2}$ oe with $\alpha,\ \beta,\gamma$ non-zero and an attempt at $\alpha\times"64"+\frac{\beta}{x}\times"192ax"+\frac{\gamma}{x^2}"240a^2x^2"$ oe

An expression of this form is sufficient i.e. with the x's still included.

Note that the 9×64 may not be seen as this cancels with the 576 so this mark may be implied.

dM1: Sets their constant term = 576 and proceeds to obtain a non-zero value for a.

You do not need to be concerned about the processing for this mark.

This may be implied by e.g. $6\times192a+240a^2=0\Rightarrow a=...$ as $9\times64=576$

May be implied by their value(s).

Condone x = … here.

Depends on the previous method mark.

A1: Correct value. Allow equivalents e.g. –4.8, $-\frac{48}{10}$

The questions asks for the value of a so just look for the correct value e.g. "a = …" is not required but $x=-\frac{24}{5}$ scores A0.

If a = 0 is also given and not rejected, score A0

Question Number	Scheme	Marks
6	$2\log_4(x+1)=\log_4(x+1)^2$	B1
	e.g. $\log_4(12-2x)-\log_4(x+1)^2=\log_4\dfrac{12-2x}{(x+1)^2}$	M1
	$\dfrac{12-2x}{(x+1)^2}=16$	A1
	$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$	M1
	$x=-\dfrac{1}{8}$ oe e.g. -0.125	A1
		(5)
		Total 5

B1: For $2\log_4(x+1)=\log_4(x+1)^2$ seen or implied.

M1: **Correct** attempt to combine two log terms.

E.g. $2+\log_4(x+1)^2=\log_4 16+\log_4(x+1)^2=\log_4 16(x+1)^2$

Condone if they make a slip **before** combining as long as they are combining two log terms correctly.

e.g.

$$\log_4(12-2x)=2+\log_4(x+1)^2\Rightarrow\log_4(12-2x)+\log_4(x+1)^2=2\Rightarrow\log_4(12-2x)(x+1)^2=...$$

or e.g.

$$\log_4(12-2x)=2+\log_4(x+1)^2=\log_4 2+\log_4(x+1)^2=\log_4 2(x+1)^2$$

A1: Obtains this equation in any form not involving logs **from correct work so must follow B1M1**.

M1: Solves a 3TQ by any acceptable method including a calculator to obtain at least one **real** value for *x*. May be implied by their value(s) and may follow incorrect log work **but the B1 must have been scored**. You may need to check.

Note that we are allowing the use of a calculator to solve the 3TQ here. This means that correct work leading to e.g.:

$$\frac{12-2x}{(x+1)^2}=16\Rightarrow 16x^2+34x+4=0$$

Followed by

$$\Rightarrow(8x+1)(x+2)=0\Rightarrow x=-\frac{1}{8}$$

Is acceptable for full marks as long as it is preceded by correct log work.

A1: This value only from correct work and all previous marks scored.
The other value (–2) must clearly be rejected or omitted.

Special Case: Beware incorrect log work which leads to the correct answer:

$$2\log_4(x+1)=\log_4(x+1)^2$$

$$\log_4(12-2x)-\log_4(x+1)^2=\frac{\log_4(12-2x)}{\log_4(x+1)^2}$$

$$\frac{12-2x}{(x+1)^2}=16$$

$$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$$

$$x=-\frac{1}{8}$$

This scores B1M0A0M1A0

Allow this SC if no combination of logs is shown at all, but the correct quadratic is produced.

e.g. $\log_4(12-2x)=2+2\log_4(x+1)\Rightarrow\frac{12-2x}{(x+1)^2}=16$ etc.

or e.g.

$\log_4(12-2x)=\log_4 16+\log_4(x+1)^2\Rightarrow 12-2x=16(x+1)^2$ etc.

Could both score B1M0A0M1A0

Special Case: Beware incorrect work removing logs which leads to the correct answer:

$$2\log_4(x+1)=\log_4(x+1)^2$$

$$\log_4(12-2x)-\log_4(x+1)^2=\log_4\frac{12-2x}{(x+1)^2}=2$$

$$\frac{12-2x}{(x+1)^2}=2^4=16$$

$$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$$

$$x=-\frac{1}{8}$$

This scores B1M1A0M1A0

Alternative:

$$\log_4(12-2x)=2+2\log_4(x+1)$$

$$\Rightarrow 4^{\log_4(12-2x)}=4^{2+2\log_4(x+1)}=4^{2+\log_4(x+1)^2}$$

$$=4^2\times 4^{\log_4(x+1)^2}$$

$$\Rightarrow 12-2x=16(x+1)^2\text{ etc.}$$

Score as:

B1: $2\log_4(x+1)=\log_4(x+1)^2$

M1: Uses correct index law e.g. $4^{a+b}=4^a\times 4^b$

Then follow main scheme.

Question Number	Scheme	Marks
7(a)	$(u_{100} =) 20+99(0.5) = (?)\ 9.50*$	B1*
		(1)
(b)	$S_{300} = \frac{1}{2}(300)\{2\times 20+299(0.5)\} = ...$ **or** $S_{300} = \frac{1}{2}(300)\{20+169.50\} = ...$	M1
	$= (?)\ 8425$	A1
		(2)
(c)	$20\times r^{299} = 250 \Rightarrow r = \sqrt[299]{\frac{250}{20}} (= 1.008483032...)$	M1
	$S_{300} = \frac{20(1-r^{300})}{1-r} = (27362.948...)$ $28425 - 27362.948...$	M1
	(£)1060	A1
		(3)
		Total 6

(a)

B1*: Correct method shown e.g. $20+99(0.5)=69.50$ is sufficient.

Note that the £ symbol is not required but $20+99(0.5)=69.5$ scores B0

Condone attempts to work backwards e.g.

$69.5 = 20+(n-1)\times 0.5 \Rightarrow 49.5 = (n-1)\times 0.5 \Rightarrow n = 100$ hence true

Note that 69.5 can be used in this case but a (minimal) conclusion is required.

(b)

M1: Attempts $\frac{1}{2}n(2\times a+(n-1)d)$ with $n = 300$, $a = 20$ and $d = 0.5$

or e.g. $\frac{1}{2}n(a+l)$ with $n = 300$, $a = 20$ and $l = 169.5(0)$ (or an attempt at $20+299\times 0.5$)

A1: Correct value. The £ symbol is not required.

(c)

M1: Correct strategy to find r e.g. $\sqrt[299]{\frac{250}{20}}$. Award for e.g. $(r=)\sqrt[299]{\frac{250}{20}}$ or e.g. $(r=)\left(\frac{250}{20}\right)^{\frac{1}{299}}$. May be implied by awrt 1.01 if no incorrect work is seen e.g. clear use of $\sqrt[300]{\frac{250}{20}}$ which gives 1.008454636… is M0

May be done via logs e.g. $r = 10^{\frac{1}{299}\log 12.5}$

M1: Uses their r with a correct GP sum formula with a = 20 and n = 300 and subtracts from the answer to part (b) either way round.
Depends on having attempted the **sum** of an **AP** in (b).
May be implied by their working/value.

Note this has been seen more than once for the GP sum following $\sqrt[299]{\frac{250}{20}}$:

$$S_{300} = \frac{20(1-12.5)}{1-1.008483032}$$

and scores M0 as it suggests $S_n = \frac{a(1-r^{n-1})}{1-r}$ has been used.

A1: Cao **not awrt** with or without the "£" symbol.

Note that the accuracy being used for "r" varies considerably.

Some rounded values for reference are:

r	S_{300}	$\lvert$Correct (b) – $S_{300}\rvert$
1.01	37576.93252…	9151.932524…
1.008	24796.03972…	3628.960276…
1.0085	27458.42195…	966.5780531…
1.00848	27345.9295…	1079.070499…

Question Number	Scheme	Marks
8(a)	$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$ $\Rightarrow x^4-10x^2+9=0$	M1A1
	$x^4-10x^2+9=0 \Rightarrow (x^2-1)(x^2-9)=0 \Rightarrow x^2=...$ $\Rightarrow x=...$	M1
	$x=1,\quad x=3$	A1
		(4)

(a) **Note we are now marking part (a) as M1A1M1A1 not M1M1A1A1**

M1: Equates the 2 curves and multiplies through by x^2 to obtain a quadratic equation in x^2

A1(M1 on EPEN): Correct 3TQ in x^2 with terms collected and all on one side.

Note that the "= 0" may be implied by their attempt to solve.

M1(A1 on EPEN): Solves a 3TQ in x^2 by a suitable method e.g. factorising, completing the square or quadratic formula **and takes the square root** to obtain at least one value for x. They may use a calculator, **but** the solution must be via x^2.

E.g. $x^4-10x^2+9=0 \Rightarrow x^2=1,\ 9 \Rightarrow x=1,\ 3$ is acceptable for this mark as is

$x^4-10x^2+9=0 \Rightarrow (x^2-1)(x^2-9)=0 \Rightarrow x=1,\ 3$

They cannot go from e.g. $x^4-10x^2+9=0$ to $x=1,\ 3$

Note $x^4-10x^2+9=0 \Rightarrow (x-1)(x+1)(x-3)(x+3)=0 \Rightarrow x=1,\ 3$ scores M0

A1: Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P=1, Q=3$

Examples:

$$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$$
$$\Rightarrow x^4-10x^2+9=0 \Rightarrow x=1,\ 3$$

Scores M1A1M0A0

$$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$$
$$\Rightarrow x^4-10x^2+9=0 \Rightarrow x^2=1,\ 9 \Rightarrow x=1,\ 9$$

Scores M1A1M1A0 (BOD)

Attempts to solve $x^4-10x^2+9=0$ by identifying roots/factors and then using e.g. long division should be sent to review.

Part (a) may also be done without obtaining a quartic:

$$y = x^2 + 3,\ y = 13 - \frac{9}{x^2} \Rightarrow x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^2 - 10 + \frac{9}{x^2} = 0$$

$$\Rightarrow \left(x - \frac{9}{x}\right)\left(x - \frac{1}{x}\right) = 0$$

$$x - \frac{9}{x} = 0 \Rightarrow x^2 = 9 \Rightarrow x = 3 \quad \text{or} \quad x - \frac{1}{x} = 0 \Rightarrow x^2 = 1 \Rightarrow x = 1$$

M1: Solves simultaneously and attempts to factorise to $\left(x - \frac{\alpha}{x}\right)\left(x - \frac{\beta}{x}\right) = 0$

A1: Correct factorisation

M1: Attempts to solve via x^2

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1$, $Q = 3$

Part (a) may also be done via y e.g.:

$$y = x^2 + 3,\ y = 13 - \frac{9}{x^2} \Rightarrow y = 13 - \frac{9}{y - 3} \Rightarrow y^2 - 16y + 48 = 0$$

$$y^2 - 16y + 48 = 0 \Rightarrow y = 4,\ 12$$

$$y = 4 \Rightarrow x = 1,\ y = 12 \Rightarrow x = 3$$

M1: Solves simultaneously to obtain a 3TQ in y

A1: Correct 3TQ in y

M1: Solves their 3TQ in y and uses at least one value of y to find a value for x

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1$, $Q = 3$

(b)	$\int\left\{13-\frac{9}{x^2}-\left(x^2+3\right)\right\}\mathrm{d}x=13x+\frac{9}{x}-\frac{x^3}{3}-3x(+c)$ or $\int\left(13-\frac{9}{x^2}\right)\mathrm{d}x=13x+\frac{9}{x}(+c),\ \int\left(x^2+3\right)\mathrm{d}x=\frac{x^3}{3}+3x(+c)$	M1A1
	$\left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3=10(3)+\frac{9}{3}-\frac{3^3}{3}-\left(10+9-\frac{1}{3}\right)=...$ or $\left[13x+\frac{9}{x}\right]_1^3-\left[\frac{x^3}{3}+3x\right]_1^3=39+3-(13+9)-\left\{9+9-\left(\frac{1}{3}+3\right)\right\}=...$	**d**M1
	$=\frac{16}{3}$	A1
		(4)
		Total 8

Note that this question has the calculator warning so attempts that do not use algebraic integration score no marks.

(b)

M1: Evidence of integration $x^n \to x^{n+1}$ for at least one term for either **curve** or for the difference between the **2 curves** e.g. it is not for integrating x^4-10x^2+9

A1: Correct integration for both curves either as a difference (either way round) or separately. E.g. $\pm\left(13x+\frac{9}{x}-\frac{x^3}{3}-3x(+c)\right)$ or $13x+\frac{9}{x}$ and $\frac{x^3}{3}+3x$

Allow simplified or unsimplified but indices must be processed.

dM1: Depends on the first method mark.

Substitutes their positive values from part (a) and subtracts if attempted as a difference or substitutes their values into the separate integrations, subtracts and then subtracts the 2 results either way round to obtain a value.

Condone poor bracketing as long as the intention is clear.

May be implied by the correct final answer following correct integration.

If the integration and/or limits are incorrect **and** the substitution is not shown explicitly **then** you may need to check.

A1: $\frac{16}{3}$ or **exact** equivalent. $-\frac{16}{3}$ scores A0 unless it gets corrected to $+\frac{16}{3}$

Question Number	Scheme	Marks
9(a)	$\left(2\tan\theta = 3\cos\theta \Rightarrow\right)\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta = 3\left(1-\sin^2\theta\right)$	M1
	$2\sin\theta = 3\left(1-\sin^2\theta\right) \Rightarrow 3\sin^2\theta + 2\sin\theta - 3 = 0\,*$	A1*
		(3)
(b)	$\left(\sin\left(2x+\dfrac{\pi}{3}\right)=\right)\dfrac{-1\pm\sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: −1.387…, 0.7207…	M1
	$2x+\dfrac{\pi}{3} = \sin^{-1}(0.7207...) \Rightarrow x = ...$	M1
	−0.121, −2.50, 0.645, 3.02	A1A1
		(4)
		Total 7

(a)

For full marks condone a complete proof entirely in x (or another variable) instead of θ

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to write the equation in terms of sine and cosine only.

Must be the correct identity so e.g. $2\tan\theta = \dfrac{2\sin\theta}{2\cos\theta}$ is M0

May be implied by e.g. $2\tan\theta = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta$

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to obtain a quadratic equation in sine only.

Must be the correct identity so e.g. $3\cos^2\theta = 1-3\sin^2\theta$ is M0

A1*: Correct work with all necessary steps shown leading to the given answer.

Condone e.g. $2\sin\theta + 3\sin^2\theta - 3 = 0$

There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider

- $\cos^2\theta$ written as $\cos\theta^2$ a notational error
- $\tan\theta = \dfrac{\sin}{\cos}\theta$ as a missing variable

Working backwards:

$$\left(3\sin^2\theta + 2\sin\theta - 3 = 0 \Rightarrow\right)3\left(1-\cos^2\theta\right) + 2\sin\theta - 3 = 0$$
$$\Rightarrow 2\sin\theta - 3\cos^2\theta = 0$$
$$\Rightarrow 2\sin\theta = 3\cos^2\theta$$
$$\Rightarrow 2\tan\theta = 3\cos\theta\,* \text{ Hence proven}$$

M1: Uses $\sin^2\theta = 1-\cos^2\theta$ to obtain an equation in sine and cosine^2 only.

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ which may be implied.

A1*: Reaches $2\tan\theta = 3\cos\theta$ with the same conditions as above but with a (minimal) conclusion.

(b)

M1: Attempts to solve the quadratic $3\sin^2 x + 2\sin x - 3 = 0$ to obtain a value for $\sin x$ where x is any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the root(s) must be correct but condone premature rounding e.g. $0.72,\ -1.3$

M1: Attempts to find one angle within the range by finding the inverse sine of one of their roots, subtracting $\frac{\pi}{3}$ and dividing by 2.

For this mark allow to work in degrees if done correctly. E.g. they would need to change $\frac{\pi}{3}$ to 60° and then find the inverse sine of one of their roots in degrees, subtract 60° and divide by 2.

May be implied by a correct value of x in degrees or radians.

NB the answers in degrees are: $-143,\ -6.94,\ 36.9,\ 173$ (3sf)

Do **not** allow the mixing of degrees and radians for this mark e.g. $x = \dfrac{46.1... - \frac{\pi}{3}}{2}$

A1: Any two of awrt $-0.12,\ -2.5,\ 0.64$ or $0.65,\ 3.0$ (Must be in radians)

A1: All four of awrt $-0.121,\ -2.50,\ 0.645,\ 3.02$ and no others in the range. (Must be in radians)

Condone -2.5 for -2.50 but the others must be awrt as shown for this final mark.

Question Number	Scheme	Marks
10(a)	$x^2+y^2+4x-30y+209=0$ $\Rightarrow (x\pm 2)^2+(y\pm 15)^2...=0$	M1
(i)	Centre (–2, 15)	A1
(ii)	Radius $\sqrt{20}$	A1
		(3)
(b)	$y=mx+1\Rightarrow (x+2)^2+(mx+1-15)^2=20$ or $y=mx+1\Rightarrow x^2+(mx+1)^2+4x-30(mx+1)+209=0$	M1
	$x^2+m^2x^2+4x-28mx+180=0$ $b^2-4ac=0\Rightarrow (4-28m)^2-4(1+m^2)\times 180=0$	**d**M1
	$(4-28m)^2-4(1+m^2)180=0\Rightarrow 16-224m+784m^2-720-720m^2=0$ $\Rightarrow 2m^2-7m-22=0\,*$	A1*
		(3)
(c)	$2m^2-7m-22=0\Rightarrow m=\frac{11}{2},\,-2$	M1
	$m=\frac{11}{2}\Rightarrow \frac{125}{4}x^2-150x+180=0\Rightarrow x=\frac{12}{5}\Rightarrow y=\frac{71}{5}$ or $m=-2\Rightarrow 5x^2+60x+180=0\Rightarrow x=-6\Rightarrow y=13$	M1
	$\left(\frac{12}{5},\frac{71}{5}\right)$ **or** $(-6,13)$ oe	A1
	$\left(\frac{12}{5},\frac{71}{5}\right)$ **and** $(-6,13)$ oe	A1
		(4)
		Total 10

(a) **Look out for answers to part (a) written in the body of the question.**

M1: Attempts to complete the square for **both** variables or states a centre of (±2, ±15)
For completing the square allow $(x\pm 2)^2...(y\pm 15)^2...=...$

A1: Centre (–2, 15) or e.g. $x=-2,\ y=15$. Condone e.g. –2, 15 or [–2, 15]

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x\pm 2)^2+(y\pm 15)^2=20$
Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) **Mark (b) and (c) together.**

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2-4ac...0$ where … is "=" or ">" or "<" etc. or equivalent e.g. $b^2=4ac$ where $a=A+Bm^2$, $b=C+Dm$, $c=E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2-4ac=0$ or equivalent throughout and before the final line e.g. not $b^2-4ac>0$ and brackets expanded. **Note this is a given answer so must follow correct work.**

(b) Alternative: Perpendicular distance from a point to a line:

$$(-2, 15),\ \ mx - y + 1 = 0 \ \text{ with } d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

$$\sqrt{20} = \frac{|-2m-15+1|}{\sqrt{m^2+1}} \Rightarrow 20(m^2+1) = (2m+14)^2$$

$$\Rightarrow 16m^2 - 56m - 176 = 0 \Rightarrow 2m^2 - 7m - 22 = 0^*$$

M1: Substitutes into a correct distance formula with their centre and the given L and sets equal to their radius.

dM1: Squares, multiplies up to obtain $\alpha(Am^2+B) = \beta(Cm+D)^2$ oe, where A, B, C and D are non-zero.

A1*: Obtains the printed answer with no errors

(c)

M1: Solves the **given** quadratic equation by any valid means, including calculator, to obtain at least one value for m. May be implied by their value(s).
One correct value **only** with no incorrect work is sufficient for M1
Condone $x = \ldots$ for this mark.

M1: Uses at least one of their values of m to attempt one position for P.
This must be a **complete and correct method to find a position for *P*** i.e. finds at least one value for x **and** then the corresponding value for y correctly.
E.g. substitutes at least one value for x into their $(x+2)^2 + (mx+1-15)^2 = 20$, solves the resulting 3TQ by any method including a calculator and then uses the correct $y = mx + 1$ to find the y value. Condone slips when e.g. simplifying their 3TQ providing the complete method is correct.

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots, y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots, y = \ldots$

(c) Alternative 1:

Finds the intersections of the possible tangents with the perpendiculars passing through the centre of the circle:

M1: $\left(m=\frac{11}{2}, -2\right)$ As above

$$\text{For } m=\frac{11}{2} \text{ perpendicular is } y-15=-\frac{2}{11}(x+2)\left(y=-\frac{2}{11}x+\frac{161}{11}\right)$$

$$-\frac{2}{11}x+\frac{161}{11}=\frac{11}{2}x+1 \Rightarrow x=\frac{12}{5},\ y=\frac{71}{5}$$

or

$$\text{For } m=-2 \text{ perpendicular is } y-15=\frac{1}{2}(x+2)\left(y=\frac{1}{2}x+16\right)$$

$$\frac{1}{2}x+16=-2x+1 \Rightarrow x=-6,\ y=13$$

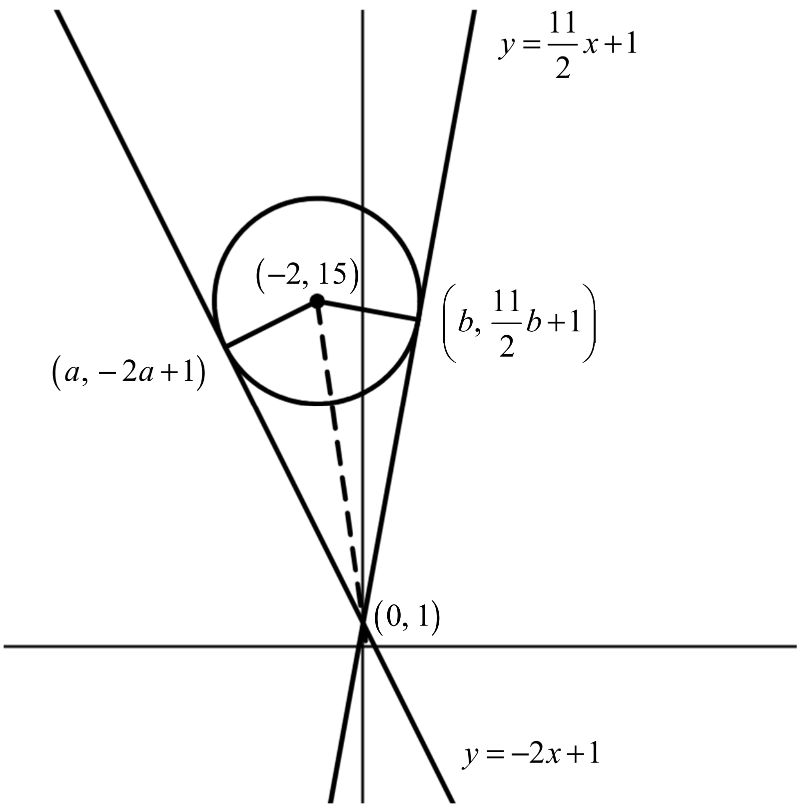
M1: For a complete method by

- forming the equation of at least one of the perpendiculars, with the negative reciprocal gradient and the coordinates of their centre correctly placed
- solving simultaneously with the corresponding tangent to find x or y
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots, y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots, y = \ldots$

(c) Alternative 2:



$$(15-1)^2+2^2=a^2+(2a)^2+\left(\sqrt{20}\right)^2$$

or

$$(15-1)^2+2^2=b^2+\left(\frac{11}{2}b\right)^2+\left(\sqrt{20}\right)^2$$

Then

$$a^2=36 \Rightarrow a=-6 \Rightarrow -2a+1=13$$

or

$$b^2=\frac{144}{25} \Rightarrow b=\frac{12}{5} \Rightarrow \frac{11}{2}b+1=\frac{71}{5}$$

Score as:

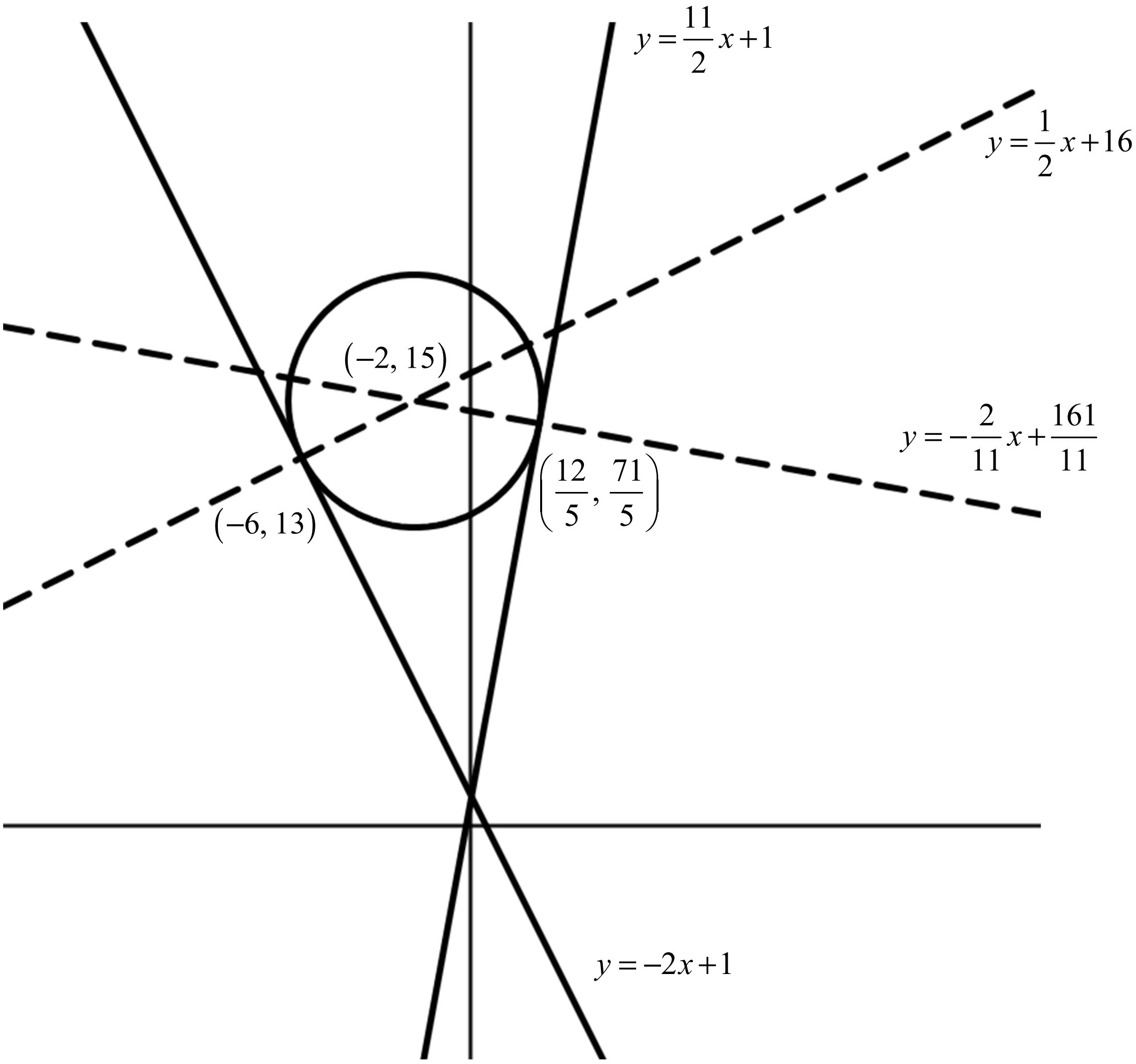
M1: As above

M1: For a complete method by
- using Pythagoras correctly to find the distance or distance^2 between (0, 1) and their centre
- applying Pythagoras correctly with a general point on either tangent and their radius to obtain an equation in one variable and solves
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots, y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots, y = \ldots$

General diagram for reference:



Question Number	Scheme	Marks
11(i)	E.g. $n=5 \Rightarrow 3^5+2=245$	M1
	245 is not a prime number or e.g. 245 is divisible by 5 so not true	A1
		(2)

M1: Attempts to evaluate 3^n+2 with n a prime number.
May be seen as a list of attempts with various numbers even if some are non-prime e.g. n = 1 as long as **at least one prime number** is attempted.
Condone a slip in evaluating provided the intention was to substitute in a valid value for n.

A1: A correct calculation for the value and a conclusion.
There must be some reference to it not being prime **or** they show that the number is divisible by e.g. 5 and state that it is false/not true.
The value they have chosen must be clearly identified as their counter example and other considerations of n can be ignored.

Examples:

$3^5+2=245$ which is not prime is M1A1

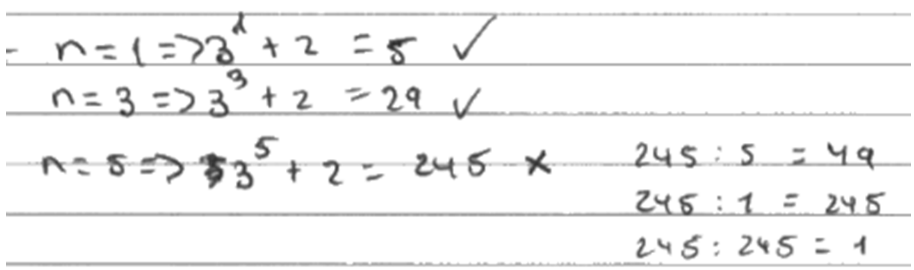
when $n=11$ then $3^n+2=177149$ and $177149 \div 7 = 25307$ so false is M1A1

$3^5+2=245$ so false is M1A0

$3^5+2=245$ which is divisible by 25 so not prime is M1A0

Most candidates are likely to choose n = 5 as their counter example but other numbers are possible e.g.:

n	3^n+2
7	2189
11	177149
13	1594325
17	129140165



Scores: M1A0 (we would need e.g. “not prime” or 245/5 = 49 so false) but note that we would have ignored the consideration of any non-primes e.g. n = 1

(ii)	$m = 3k+1 \Rightarrow m^2 - 1 = (3k+1)^2 - 1 = 9k^2 + 6k + 1 - 1$ **or** $m = 3k+2 \Rightarrow m^2 - 1 = (3k+2)^2 - 1 = 9k^2 + 12k + 4 - 1$	M1
	$m^2 - 1 = 9k^2 + 6k = 3(3k^2 + 2k)$ **or** $m^2 - 1 = 9k^2 + 12k + 3 = 3(3k^2 + 4k + 1)$	A1
	$m = 3k+1 \Rightarrow m^2 - 1 = (3k+1)^2 - 1 = 9k^2 + 6k + 1 - 1$ **and** $m = 3k+2 \Rightarrow m^2 - 1 = (3k+2)^2 - 1 = 9k^2 + 12k + 4 - 1$	dM1
	$3(3k^2 + 2k)$ and $3(3k^2 + 4k + 1)$ are both multiples of 3 so $m^2 - 1$ must be divisible by 3 when *m* is not divisible by 3	A1
		(4)
		Total 6

For the accuracy marks there should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if *m* is used instead of their *k* if the work is otherwise correct.

M1: Starts the proof by considering at least one algebraic expression not divisible by 3 e.g. $m = 3k + 1$ and attempts $m^2 - 1$ by expanding the brackets.
Alternatively, writes $m^2 - 1 = (m-1)(m+1)$ and substitutes at least one algebraic expression not divisible by 3.
Allow equivalent representations of numbers not divisible by 3 e.g. $m = 3k + 2$ or $m = 3k - 1$
Condone arithmetical slips and condone the use of e.g. $m = 3m + 2$ or $m = 3m - 1$

A1: They must

- obtain at least one <u>correct</u> expression
- show that their expression is a multiple of 3 by factoring out 3 or e.g. $3k$ e.g. $3(3k^2 + 2k)$ or e.g. $3k(3k+2)$ or e.g. $3(k+1)(3k+1)$

  or

  show that their expression is divisible by 3 e.g. $\frac{9k^2 + 6k}{3} = 3k^2 + 2k$.

  or

  make a reasoned argument as to why the expression is divisible by 3 e.g. obtains $9k^2 + 6k$ and states e.g. is divisible by 3 as both 9 and 6 are divisible by 3 (or equivalent reasoning)
  Note that this may be seen in the final conclusion.

Condone a spurious "= 0" e.g. $3(3k^2 + 2k) = 0$ **for this mark**.

dM1: Considers both **distinct** cases where *m* is not divisible by 3 (see above)

See table below – they must have one of case *A* and one of case *B*.

A1: Fully correct proof. They must

- achieve correct expressions for $m^2 - 1$ for both cases
- show that each expression is a multiple of 3 as above
- make a concluding overall statement. "Hence is a multiple of 3". Accept "hence proven", "statement proved", "QED" if they have shown for each separate case that the expression is a multiple of 3. Ignore any reference to what *m* is e.g. "for all integers *m*"

Do **not** condone any spurious "= 0" e.g. $3\left(3k^2 + 2k\right) = 0$ **for this mark**.

You can ignore any cases considered where *m* is divisible by 3 e.g. $m = 3k$ provided this does not clearly form part of their proof.

For reference:

Case	m	$m^2 - 1$
A	$3k+1$	$9k^2 + 6k$
B	$3k+2$	$9k^2 + 12k + 3$
B	$3k-1$	$9k^2 - 6k$
A	$3k-2$	$9k^2 - 12k + 3$