Question Number	Scheme		Marks
1.	<table><tr><td>a</td><td>b</td><td>c</td><td>abc</td></tr><tr><td>**1**</td><td>**10**</td><td>**4**</td><td>**40**</td></tr><tr><td>**2**</td><td>**6**</td><td>**7**</td><td>**84**</td></tr><tr><td>**3**</td><td>**2**</td><td>**10**</td><td>**60**</td></tr><tr><td colspan="4">All values of abc are multiples of 4</td></tr></table>	One correct set a, b & c Two correct rows Fully correct + statement	B1 M1 A1 **(3 marks)**

Main method: Numerical solution finding all known solutions

B1: Any correct set of values for a, b and c. You can ignore any attempt at the product abc

M1: Any two fully correct rows including the correct calculations for the products.
Condone extra/ incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for abc
- any extra (or incorrect) rows must be deleted or somehow excluded
- a minimal statement showing that they are aware that 40, 84 and 60 are multiples of 4. E.g they are all divisible by 4, use of a tick ✓, prove, QED, calculations such as $\frac{40}{4}=10$. They can be divided by 4 (on its own) is insufficient without further clarification such as "to give whole numbers"
- no incorrect calculations. E.g if their attempt to prove that 60 being a multiple of 4 includes $\frac{60}{4}=16$ then it is A0

Alt 1: Via deduction

B1: Either

Solves $a+b+c=15$ with $c=3a+1$ to conclude that $4a+b=14$ o.e. **and** states that b is even

Or

Uses $c=3a+1$ to state that if a is odd then c is even and if a is even then c is odd

M1: Attempts to generalise and offers one of the two possibilities for the product abc.

Either

states then when a is odd, b is even and c is even, therefore $abc = odd\times even\times even$ = multiple of 4

Or

states then when a is even, b is even and c is odd, therefore $abc = even\times even\times odd$ = multiple of 4

A1: Fully correct. The solution must have

- the two possible "calculations" for abc.
- no incorrect "calculations". E.g. if they state that $even\times even$ is a multiple of 4 because $2n\times 2n=4n^2$ it is A0. It must be along the lines of $2m\times 2n=4mn$ with different variables.

You may see a hybrid solution that can be marked along similar lines. Mark positively.
If you see a tabular solution and one via deduction, award marks for the better one.
Please use review if you are uncertain.

Question Number	Scheme	Marks
2.(a)	$u_2 = \frac{2}{3},\ u_3 = -4,\ u_4 = 3$	M1, A1, A1
		(3)
(b)	$\sum_{r=1}^{100} u_r = 33 \times \left(3 + \frac{2}{3} + -4\right) + 3$	M1
	$= -8$	A1
		(2)
		(5 marks)

(a)

M1: Attempts to use the iteration formula seen at least once. E.g. Award for $u_{1+1} = 2 - \frac{4}{3} = \ldots$

May be implied by

- either a correct value assigned to the correct term. Condone $\frac{2}{3}$ as 0.67 for u_2
- or a correct follow through value. So if $u_2 = -\frac{2}{3}$ then $u_3 = 2 - \frac{4}{-2/3}$ or 8 would score the mark

It must be a correct method. Watch for correct values attached to the wrong term

E.g $u_2 = 2 - \frac{4}{2},\ u_3 = 2 - \frac{4}{3}, u_4 = 2 - \frac{4}{4}$ does achieve $\frac{2}{3}$ but it is M0. It is assigned to u_3

A1: Any correct value assigned to the correct term. Either $u_2 = \frac{2}{3}$ (but condone as 0.67) $u_3 = -4$ or $u_4 = 3$

A1: All 3 values correct and labelled: $u_2 = \frac{2}{3},\ u_3 = -4$ AND $u_4 = 3$. May be awarded from part (b)

(b)

M1: Establishes a correct method to find the sum of 100 terms.

Amongst other correct methods are;

$33 \times \left(3 + "\frac{2}{3}" + "-4"\right) + 3$

$34 \times \left(3 + "\frac{2}{3}" + "-4"\right) - "\frac{2}{3}" - "-4"$

You may see listing but this cannot score the marks unless all terms are present or the correct result is given.

An incorrect part (a) is unlikely to score any marks as it is unlikely to be repeating every 3 terms

A1: -8. This alone scores both marks following a correct (a), as long as there is no incorrect method.

Question Number	Scheme	Marks
	$2\tan\theta+3\sin\theta=0$	
3 (a)	States or uses $\tan\theta=\dfrac{\sin\theta}{\cos\theta}\rightarrow 2\dfrac{\sin\theta}{\cos\theta}+3\sin\theta=0$	M1
	$\sin\theta(2+3\cos\theta)=0$	
	$2+3\cos\theta=0$	dM1
	$\cos\theta=-\dfrac{2}{3}\Rightarrow\theta=$ awrt $132°$ or awrt $228°$	A1
	$\cos\theta=-\dfrac{2}{3}\Rightarrow\theta=$ awrt $131.8°$ and awrt $228.2°$	A1
	$\sin\theta=0\Rightarrow\theta=180°,360°$	B1
		(5)
(b)	Sets $2x+40°=$ their $131.8°\Rightarrow x=...$	M1
	$x=$ awrt $45.9°$	A1
		(2)
		(7 marks)

(a) Condone a lack of a degrees symbol throughout.
Condone solutions that drop θ's in parts of the working

M1: States $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ or attempts to use $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the given equation. Condone slips in coefficients.

dM1: Attempts to use $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ o.e., cross multiplies and either factorises or cancels to form a linear equation in $\cos\theta$

A1: Achieves $\cos\theta=-\dfrac{2}{3}$ and finds one correct θ value to the nearest degree.

Condone radian solutions for this mark so allow awrt 2.3(0) or awrt 3.98 following $\cos\theta=-\dfrac{2}{3}$

A1: Achieves $\cos\theta=-\dfrac{2}{3}$ and finds both θ values in degrees correct to one dp.

Any extra values in the range (for $\cos\theta=-\dfrac{2}{3}$) is A0

B1: Achieves $\sin\theta=0$ followed by $\theta=180°,360°$ Condone 0 appearing as a solution

(b) Condone a lack of a degrees symbol

M1: Sets $2x+40°=$ their $131.8°$ and proceeds to a value for x.

Look for correct order of operations so $2x+40°="131.8°"\Rightarrow x=\dfrac{"131.8°"\pm 40°}{2}$

There may be multiple attempts, which can be ignored for this mark
Follow through on their smallest value (must be greater than $40°$), from part (a)

A1: $x=$ awrt $45.9°$ following a correct equation. Do not accept multiple solutions here

....

Note that there are many parallel solutions in part (a) that can be marked similarly.

Alt I

Using $\sin\theta = \tan\theta\cos\theta$ o.e

$2\tan\theta + 3\tan\theta\cos\theta = 0 \Rightarrow \tan\theta(2+3\cos\theta) = 0$

In this case the B1 mark is for achieving $\tan\theta = 0 \Rightarrow \theta = 180°, 360°$ Condone 0 appearing as a solution.

Alt II

Multiplying throughout by $\sin\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a cubic $\cos\theta$ which has roots of $\cos\theta = -\frac{2}{3}$ and $\cos\theta = 1$. Allow use of a calculator to solve cubic equations, etc.

$$2\frac{\sin\theta}{\cos\theta} + 3\sin\theta = 0 \overset{\times\sin\theta}{\rightarrow} 2\frac{\sin^2\theta}{\cos\theta} + 3\sin^2\theta = 0 \rightarrow 2\frac{(1-\cos^2\theta)}{\cos\theta} + 3(1-\cos^2\theta) = 0$$

$$\rightarrow 3\cos^3\theta + 2\cos^2\theta - 3\cos\theta - 2 = 0 \rightarrow \cos\theta = -\frac{2}{3}, \cos\theta = \pm 1$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: for $\cos\theta = k$, $|k| < 1, k \neq 0$

A1: For $\cos\theta = -\frac{2}{3}$ and a solution as in the main scheme

A1: : For $\cos\theta = -\frac{2}{3}$ and both solutions as in the main scheme

B1 Achieves $\cos\theta = \pm 1$ followed by $\theta = 180°, 360°$. Condone 0 appearing as a solution

Alt III

Multiplying throughout by $\cos\theta$ and using $\sin^2\theta + \cos^2\theta = 1$ eventually leads to a quartic in $\sin\theta$ which has roots $\sin\theta = \pm\frac{\sqrt{5}}{3}$ and $\sin\theta = 0$

$$2\sin\theta = -3\sin\theta\cos\theta \overset{SQUARING}{\rightarrow} 4\sin^2\theta = 9\sin^2\theta\cos^2\theta$$

$$\rightarrow 4\sin^2\theta = 9\sin^2\theta(1-\sin^2\theta) \rightarrow 9\sin^4\theta - 5\sin^2\theta = 0$$

$$\rightarrow \sin^2\theta = \frac{5}{9}, \sin\theta = 0$$

M1: For $\tan\theta = \frac{\sin\theta}{\cos\theta}$

dM1: For $\sin^2\theta = k$ or $\sin\theta = k$, $|k| < 1, k \neq 0$

A1: For $\sin\theta = \frac{\sqrt{5}}{3}$, $\sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and a correct solution as in the main scheme

A1: For $\sin\theta = \frac{\sqrt{5}}{3}$, $\sin\theta = -\frac{\sqrt{5}}{3}$ or $\sin^2\theta = \frac{5}{9}$ and both solutions (only) in the main scheme

B1: Achieves $\sin\theta = 0$ followed by $\theta = 180°, 360°$ Condone 0 appearing as a solution

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The solution must be obtained from correct working.

Calculator solutions like $2\tan\theta + 3\sin\theta = 0 \Rightarrow \theta = 131.8°$ and $228.2°$ score no marks in (a)

Even the solution $\theta = 180°, 360°$ cannot score the B1 mark if it is not preceded by $\sin\theta = 0$ or $\tan\theta = 0$

(b) is hence or otherwise so must follow their solution in (a) or be a restart.

Question Number	Scheme	Marks
	$f(x)=4x^3+ax^2-29x+b$	
4.(a)	Sets $f\left(-\frac{1}{2}\right)=0\rightarrow\ 4\times\left(-\frac{1}{2}\right)^3+a\times\left(-\frac{1}{2}\right)^2-29\times\left(-\frac{1}{2}\right)+b=0$	M1
	$\Rightarrow\frac{1}{4}a+b+14=0\Rightarrow\ a+4b=-56$ *	A1*
		(2)
(b)	Sets $f(2)=-25\rightarrow\ 4\times2^3+a\times2^2-29\times2+b=-25$	M1
	$4a+b=1$	A1
		(2)
(c)	(i) Solves $a+4b=-56$ with their $4a+b=1\Rightarrow a=...,b=...$	M1
	$a=4,b=-15$	A1
	(ii) $4x^3+4x^2-29x-15=(2x+1)(2x^2+x-15)$	M1, A1
	$=(2x+1)(2x-5)(x+3)$ cso	A1
		(5)
		(9 marks)

(a)

M1: Attempts to set $f\left(-\frac{1}{2}\right)=0\rightarrow$ equation in a and b Condone slips (e.g. on signs) but $f\left(\frac{1}{2}\right)=0$ is M0.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation.

A1*: Completes proof with at least one intermediate simplified and correct line such as $\frac{1}{4}a+b+14=0$. Simplifying each term is sufficient $-\frac{1}{2}+\frac{1}{4}a+\frac{29}{2}+b=0$. Any incorrect lines written after the start of their 'proof' should be scored A0*

....

Solutions where the = 0 is implied but not stated should be awarded M1 A0*

E.g. $f\left(-\frac{1}{2}\right)=4\times\left(-\frac{1}{2}\right)^3+a\times\left(-\frac{1}{2}\right)^2-29\times\left(-\frac{1}{2}\right)+b=\frac{1}{4}a+b+14$. So $\frac{1}{4}a+b=-14\Rightarrow a+4b=-56$

....

(b)

M1: Attempts to set $f(2)=-25\rightarrow$ equation in a and b Condone slips, e.g sign slips or setting $f(2)=25$

Setting $f(-2)=-25$ is not a slip and should be marked M0.

A1: $4a+b=1$ or other simplified equivalent such as $a+\frac{1}{4}b-\frac{1}{4}=0$. The constant terms must be collected.

Note that this is not a given answer so allow recovery.

(c)(i)

M1: Attempts to solve $a+4b=-56$ (allowing for slips) with their $4a+b=1\Rightarrow a=...,b=...$

Award as long as values appear for both a and b, so allow these just written down (say from a calculator) There is no need their calculations

A1: Both values correct: $a=4,b=-15$. Both just written down without working scores M1 A1 provided (b) is correct (c)(ii)

M1: Attempt to divide or factorise out $(2x+1)$ from their $4x^3+4x^2-29x-15$ or multiple thereof

For factorisation look for correct first and last terms E.g. $4x^3+"a"x^2-29x+"b"=(2x+1)(2x^2+kx\pm"b")$

For division look for a quadratic quotient of the form $2x^2+px+q$ with some "correct" work to find the term in x for example. Condone sign slip when attempting to find p

A1: Correct quadratic factor $2x^2+x-15$

A1: cso $(2x+1)(2x-5)(x+3)$. Note that M1, A1 must have been awarded and the full product must be seen. It is not acceptable to see just the factors

....

The question states "Hence" so there is an expectation that they use the factor of $(2x+1)$ which is given.

Solutions that just state $4x^3+4x^2-29x-15=(2x+1)(2x-5)(x+3)$ score M0 and therefore A0 A0

If they wish to start with another factor they must prove it is a factor first.

E.g. $f(-3)=4\times-27+4\times9-29\times-3-15=0\Rightarrow(x+3)$ is a factor followed by division by $(x+3)$

....

Tabular or methods of division may be seen in (a) and (b). The question does not demand use of the factor or remainder theorem so these methods should be rewarded.

The M1 would be scored for a full attempt and A1 for a correct answer or proof.

If you cannot see what they are doing and find it hard to award, then put in review please

Example of a tabular version for (a):

$(2x+1)$ is a factor of $f(x)=4x^3+ax^2-29x+b$

	$2x^2$	$(-29-2b)x$	b
$2x$	$4x^3$	$2(-29-2b)x^2$	$2bx$
1	$2x^2$	$(-29-2b)x$	b

Then states that $2(-29-2b)+2=a\Rightarrow$

Example of a long division method for (b).

When $f(x)=4x^3+ax^2-29x+b$ is divided by $(x-2)$ the remainder is -25

$$\begin{array}{r l}
 & 4x^2 \quad +(a+8)x \quad +(2a-13) \\
x-2 \,\big) & 4x^3+ax^2-29x+b \\
 & \underline{4x^3-8x^2} \\
 & (a+8)x^2-29x \\
 & \underline{(a+8)x^2-(2a+16)x} \\
 & (2a-13)x+b \\
 & \underline{(2a-13)x+(-4a+26)} \\
 & b+4a-26
\end{array}$$

Remainder is -25 so $b+4a-26=-25\Rightarrow b+4a=1$

Question Number	Scheme	Marks
5. (i)	$3^a = 70 \Rightarrow a\log 3 = \log 70$	M1
	$\Rightarrow a = \dfrac{\log 70}{\log 3} = 3.867$	A1
		(2)
(ii)	$4 = \log_3 81$ or $3\log_3 b = \log_3 b^3$ or $\log_3 5b = \log_3 5 + \log_3 b$	B1
	$4 + 3\log_3 b = \log_3 5b \Rightarrow \log_3 81b^3 = \log_3 5b$	M1
	$\Rightarrow 81b^{\cancel{3}2} = 5\cancel{b} \Rightarrow b = \ldots$	dM1
	$\Rightarrow b = \sqrt{\dfrac{5}{81}}$	A1
		(4)
		(6 marks)

This question demands working from the candidates so answers cannot appear without appropriate methods

(i)

M1: Takes (the same) log of both sides and uses the power law.

Alternatively writes $a = \log_3 70$ with the correct base. But $a = \log 70$ is M0

A1: CSO. 3.867 following a suitable intermediate line. This is not awrt but isw after sight of 3.867

Cannot be achieved via Trial and Improvement

(ii) Note that this was marked **M1 A1 dM1 A1**. It is now being marked **B1 M1 dM1 A1**

B1: One correct log law used on a term **of the initial equation**.

Can be achieved without the bases being present

Look for $4 = \log_3 81$ or $3\log_3 b = \log_3 b^3$ or $\log_3 5b = \log_3 5 + \log_3 b$

M1: Correctly combines two terms **of the initial equation** to form a log equation of the correct form

$\log_3 \ldots = \log_3 \ldots$ or $\log_3 \ldots = \pm 4$.

Possible answers are; $\log_3 81b^3 = \log_3 5b$, $\log_3 81b^2 = \log_3 5$, $\log_3\left(\dfrac{b^3}{5b}\right) = -4$

For this mark you can condone a slip on the sign of the 4.

E.g. $4 + 3\log_3 b = \log_3 5b \Rightarrow \log_3 b^3 - \log_3 5b = 4 \Rightarrow \log_3 \dfrac{b^3}{5b} = 4$

dM1: Removes the logs correctly and proceeds to a value for b using a correct method.

Must follow M1 and an equation of the form $\log_3 \ldots = \log_3 \ldots$ or $\log_3 \ldots = \pm 4$

A1: $b = \sqrt{\dfrac{5}{81}}$ or exact equivalent such as $b = \dfrac{1}{9}\sqrt{5}$. The negative (and 0) solution, if found, must be discarded.

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Correct answer from incorrect working: Watch for solutions like the following

$$4 + 3\log_3 b = \log_3 5b \Rightarrow 4 = \log_3 5b - \log_3 b^3 \Rightarrow 4 = \frac{\log_3 5b}{\log_3 b^3} \Rightarrow 81 = \frac{5b}{b^3} \Rightarrow b = \sqrt{\frac{5}{81}}$$

Score B1 M0 dM0 A0

....

Question Number	Scheme	Marks
6. (a)	Identifies $h = 3$	B1
	Area = $\frac{3}{2}\{0+0+2(1.52+2.74+3.12+3.08)\}$	M1
	$= 31.38\text{ m}^2$	A1
		(3)
(b)	Calculates $"31.38"\times 1.5\times 60$	M1
	= awrt 2800 m^3	A1
		(2)
(c)	Underestimate with valid reason, e.g. area of trapezia are smaller than cross sectional area	B1
		(1)
		(6 marks)

(a)

B1: Identifies $h = 3$. This may be stated or implied by the sight of $\frac{\mathbf{3}}{2}\{....+2(...+...+...)\}$

M1: Correct method of finding area using the trapezium rule. The form of the rule is given so look for the form $\frac{3}{2}\{0+0+2(1.52+2.74+3.12+3.08)\}$ but you can condone slips on the digits of the heights, e.g 3.21 for 3.12. You may not see the zeros or the trailing $\}$ which is fine. The bracketing may be hard to decipher but the correct answer implies the method. If they state another value of h e.g. $\frac{15}{6}=2.5$ then look for $\frac{2.5}{2}\{0+0+2(1.52+2.74+3.12+3.08)\}$ condoning slips. If h is not stated, then this mark can be implied by $\frac{...}{2}\{0+0+2(1.52+2.74+3.12+3.08)\}$ condoning slips.

Condone an attempt using two triangles and three trapezia. Use the same criteria for this award. The area of any triangle and the area of any trapezium must be found from a correct formula.

A1: Accept 31.38 or awrt 31.4 (m^2) following correct expression for area.

Units are not necessary. The exact fractional answer is $\frac{1569}{50}$ is also acceptable

As the area is below the axis, a minority of candidates are taking h or the y values to be negative. This is fine for the first two marks and all three marks provided the modulus is taken.

(b)

M1: Attempts their (a) $\times 1.5 \times 60$ or equivalent such as their (a) $\times 90$

A1: AWRT 2800 m^3 Units are not necessary

(c)

B1: States **underestim**ate AND gives a **valid reason**.

The reason, in almost all cases, should explicitly compare the area of the trapezia with the shaded area.
All reasons must make some reference to 'trapezia' or 'area of a trapezium'
The Figure could be used to explain their reasoning so look carefully at what the candidate states
Condone 'the trapezia are smaller than the cross section'
'the bottom lines of the trapezia are above the curve'
'the curve is above the lines of the trapezia' (remember the y axis is pointing down)
Do not accept any reference to convex or concave without further clarification.
Merely stating " the area we get from the trapezium rule is less than the shaded area" is too vague. There needs to be some explanation of why it is an underestimate, not just the fact that it is.

Question Number	Scheme	Marks
7(a)	States or implies that gradient $XN = -\frac{2}{5}$	B1
	Uses $(4,-3)$ with a gradient of $\pm\frac{2}{5}$ to form equation of XN $y+3=-\frac{2}{5}(x-4)$	M1
	$5y+15=-2x+8 \Rightarrow 2x+5y+7=0$ *	A1*
		(3)
(b)	(i) Solves $2x+5y+7=0$ and $y=\frac{5}{2}x-\frac{55}{2} \Rightarrow x=..., y=...$	M1
	$N=(9,-5)$	A1
	(ii) Attempts $("9"-4)^2+("-5"+3)^2$	M1
	Correct form of equation E.g. $(x-4)^2+(y+3)^2=("9"-4)^2+("-5"+3)^2$	M1
	$(x-4)^2+(y+3)^2=29$	A1
		(5)
		(8 marks)

(a) **This is a given answer so be careful of students who work backwards.**

B1: States or implies that gradient $XN = -\frac{2}{5}$ o.e such as -0.4

This cannot be scored from candidates who show you they are just working back from the given answer.

M1: Uses $(4,-3)$ with a gradient of $\pm\frac{2}{5}$ o.e. to form equation of XN

E.g $\pm\frac{2}{5}=\frac{y-(-3)}{x-4}$. If the form $y=mx+c$ is used look for $-3=\left(\pm\frac{2}{5}\right)\times 4+c$ leading to $c=....$

A1*: Shows sufficient working and proceeds to $2x+5y+7=0$. There should be one correct line showing working between $y+3=-\frac{2}{5}(x-4)$ and $2x+5y+7=0$, for example $5y+15=-2x+8$

If $y=mx+c$ is used expect to see $c=-\frac{7}{5}$ and $y=-\frac{2}{5}x-\frac{7}{5}$ before $2x+5y+7=0$.

Note that this is a given answer so there cannot be incorrect work used leading to this answer.

(b)(i)

M1: Attempts to solve $2x+5y+7=0$ and $y=\frac{5}{2}x-\frac{55}{2}$ leading to $x=..., y=...$

Allow the values to be just written down, for example via use of a calculator

A1: $N=(9,-5)$ which may be written $x=9, y=-5$

(b)(ii)

M1: Attempts radius or radius ^{2}. Condone slips on lhs, for example $r \text{ or } d = (\text{"9"}-4)^2 + (\text{"}-5\text{"}+3)^2$

You must see an attempt to take the difference for both components before each are squared.

M1: Correct form of equation E.g. $(x-4)^2 + (y+3)^2 = \text{"}r\text{"}^2$

Look for $(x-4)^2 + (y+3)^2 = (\text{"9"}-4)^2 + (\text{"}-5\text{"}+3)^2$

Condone for this mark a slip on one of the components when finding their r^2

A1: $(x-4)^2 + (y+3)^2 = 29$ or exact equivalent. The constant terms must be collected and not $25+4$
ISW after a correct answer.

Alt (b)(ii)

Note that an alternative version is $x^2 + y^2 - 8x + 6y - 4 = 0$. It is possible to proceed using this alternative version of the formula for a circle

M1: States equation of circle is $x^2 + y^2 - 8x + 6y + c = 0$

M1: Substitutes their $N = (9, -5)$ into $x^2 + y^2 - 8x + 6y + c = 0$ o.e. and finds "c"

A1: $x^2 + y^2 - 8x + 6y - 4 = 0$ o.e.

Question Number	Scheme	Marks
8. (a)	Uses $S_{10}=360 \Rightarrow \frac{10}{2}\{2\times a+9\times 4\}=360$	M1
	$\Rightarrow 2a+36=72 \Rightarrow a=18$	A1
		(2)
(b)	Uses $S_n=2146 \Rightarrow \frac{n}{2}\{2\times "18"+(n-1)\times 4\}=2146$	M1
	$\Rightarrow n\{16+2n\}=2146$	
	$\Rightarrow n^2+8n-1073=0$ *	A1*
		(2)
(c)	(i) States 29	B1
	(ii) Attempts $"18"+("29"-1)\times 4=130$	M1, A1
		(3)
		(7 marks)

(a)

M1: Uses $S_{10}=360$ with a correct formula to set up a linear equation in "*a*". E.g. $\frac{10}{2}\{2\times a+9\times 4\}=360$

Alternatively uses the sum of all 10 terms

$a+(a+4)+(a+8)+(a+12)+(a+16)+(a+20)+(a+24)+(a+28)+(a+32)+(a+36)=360$

A1: $a=18$ following a correct equation

(b)

M1: Attempts to use $S_n=2146 \Rightarrow \frac{n}{2}\{2\times "18"+(n-1)\times 4\}=2146$ following through on their value of "*a*"

A1*: Proceeds to $n^2+8n-1073=0$ showing sufficient working, which in most circumstances should be regarded as one correct simplified intermediate line. (See main scheme)

(c)(i)

B1: States 29.

The -37 if written down must be deleted/crossed out, or the 29 chosen by being "ringed" or underlined

(c)(ii)

M1: Attempts $a+(n-1)\times 4$ with their values of *a* and *n*. E.g. $"18"+("29"-1)\times 4$

Alternatively uses $2146 \Rightarrow \frac{29}{2}\{"18"+l\} \Rightarrow l=...$

A1: Maximum number of seats $=130$ which may be labelled $l=130$

This alone scores both marks (but only if they had a = 18 and n =29 earlier)

Question Number	Scheme	Marks
	$y=\frac{2}{3}x^2-9\sqrt{x}+13$	
9 (a)	$\frac{dy}{dx}=\frac{4}{3}x-\frac{9}{2\sqrt{x}}$	M1, A1
	Attempts to solve $\frac{dy}{dx}>0\Rightarrow x^{\frac{3}{2}}>\frac{27}{8}\Rightarrow x>\frac{9}{4}$	dM1, A1
		(4)
(b)	$\int \frac{2}{3}x^2-9\sqrt{x}+13\,dx=\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\quad(+c)$	M1
	Area $=\left[\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\right]_0^9=117$	dM1, A1
	Substitutes $x=9$ into $\frac{dy}{dx}=\frac{4}{3}x-\frac{9}{2\sqrt{x}}\Rightarrow\frac{dy}{dx}=\left(\frac{21}{2}\right)$	M1
	Finds where tangent meets x-axis $0-40=\frac{21}{2}(x-9)\Rightarrow x=\frac{109}{21}$	dM1 A1
	Area of R $=\left[\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\right]_0^9-\frac{1}{2}\times40\times\left(9-\frac{109}{21}\right)$	ddM1
	$=117-\frac{1600}{21}=\frac{857}{21}$	A1
		(8) **(12 marks)**

There are no marks for integrating in part (a) or differentiating in (b). This is a problem-solving question and candidates should be awarded these marks only for work in the correct part.

(a)

M1: Attempts $\frac{dy}{dx}$ with one index correct. Allow for $x^2\to x^1$ or $\sqrt{x}\to x^{-\frac{1}{2}}$

A1: $\frac{dy}{dx}=\frac{4}{3}x-\frac{9}{2\sqrt{x}}$ o.e which may be left un simplified. The LHS is not required

dM1: Attempts to find where $\frac{dy}{dx}>0$ and proceeds to $x^{\frac{3}{2}}>k$ or $x^3>k$ following correct squaring

Also allow $\frac{dy}{dx}\ldots0$ and proceeds to $x^{\frac{3}{2}}\ldots k$ but also condone $\frac{dy}{dx}=0$ leading to $x^{\frac{3}{2}}=k$

If a candidate has a correct derivative, then condone $\frac{4}{3}x-\frac{9}{2\sqrt{x}}\ldots0\Rightarrow x\ldots\frac{9}{4}$ where ... is =, > or …

A1: States $x>\frac{9}{4}$ or $x\ldots\frac{9}{4}$ o.e. following the award of both M's. Do not accept other ranges such as $x<0$

(b)

M1: Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$ scored for increasing a correct index by 1 which could include $13 \to 13x$

dM1: Uses limits of 9 (and 0) either way around and finds area under curve between 0 and 9.
It is dependent upon the previous M mark. Award when values are embedded

A1: Achieves 117 as the area under the curve between 0 and 9.
Be careful when awarding this mark as sight of 117 does not necessarily merit this award

You must see M1 and dM1 and have correct integration. Allow for $\left[\frac{2}{9}x^3 - 6x^{\frac{3}{2}} + 13x\right]_0^9 = 117$

The use of a calculator to integrate is not allowed

M1: Attempts the gradient of the tangent.
Substitutes $x = 9$ into their $\frac{dy}{dx}$ (from part a or restarted) which must have one index correct.

A correct value (or follow through value) can imply this method only if there is a $\frac{dy}{dx}$. It cannot be awarded if there is not a calculation for $\frac{dy}{dx}$ in the question

dM1: Correct method of finding where tangent meets the x-axis. Dependent upon previous M
For this to be awarded you must see all elements or their equivalent. E.g.

- Substitution of $x = 9$ into their $\frac{dy}{dx}$ (from part a or restarted) which must have one index correct
- use of (9, 40) with their $\frac{dy}{dx}$ to form the equation of the tangent at P
- substitution of $y = 0$ into their correctly found tangent to find where tangent crosses x-axis

A1: Correct value $x = \frac{109}{21}$

ddM1: Complete method of finding the area of R. This is dependent upon all previous M's
It is usually scored for the area under the curve between 0 and 9 – the area of the triangle.
The area of the triangle may be attempted by integration. Look for a correct attempt at the method for the equation of a tangent, integrated to a form $\alpha x^2 + \beta x$ with limits of the x intercept of their tangent and 9

A1: Area $R = \frac{857}{21}$

....

Note that there are other ways in which the area could be attempted. Look carefully at their attempt please.

ALT I: (Area under curve between 0 and x intercept) +(Area between curve and line between x intercept and 9)

E.g. $\int_0^{\frac{109}{21}} \frac{2}{3}x^2 - 9\sqrt{x} + 13\,dx + \int_{\frac{109}{21}}^{9} \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13\right) - \left(\frac{21}{2}x - \frac{109}{2}\right) dx$

M1: As the main method. Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$, scored for increasing a correct index by 1

dM1: As main method with same dependency. In this case it would be for applying the limits 0 to k and k to 9 to the integrated $\frac{2}{3}x^2 - 9\sqrt{x} + 13$

A1: Implied by correct final answer

M1, dM1, A1: As main method

ddM1: $\int_0^{"\frac{109}{21}"} \frac{2}{3}x^2 - 9\sqrt{x} + 13\,dx + \int_{"\frac{109}{21}"}^{9} \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13\right) - "\left(\frac{21}{2}x - \frac{109}{2}\right)"\,dx$

Complete method of finding the area of R. This is dependent upon all previous M's. See main scheme for general principles for awarding this mark.

A1: Area $R = \frac{857}{21}$

ALT II: (Area between curve and line between 0 and 9) - (Area of triangle under axis)

E.g. $\int_0^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13\right) - \left(\frac{21}{2}x - \frac{109}{2}\right) dx - \frac{1}{2} \times \frac{109}{2} \times \frac{109}{21}$

M1: As the main method. Attempts to integrate $\frac{2}{3}x^2 - 9\sqrt{x} + 13$, scored for increasing a correct index by 1

dM1: As main method with same dependency. Look for applying the limits 0 and 9 to the integrated $\frac{2}{3}x^2 - 9\sqrt{x} + 13$

A1: Area between curve and line, between 0 and 9, is $\frac{729}{4}$

M1, dM1, A1: As main method

ddM1: $\int_0^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13\right) - "\left(\frac{21}{2}x - \frac{109}{2}\right)"\,dx - \frac{1}{2} \times "\frac{109}{2}" \times "\frac{109}{21}"$

Complete method of finding the area of R. This is dependent upon all previous M's. See main scheme for general principles for awarding this mark.

A1: Area $R = \frac{857}{21}$

Question Number	Scheme	Marks
10.(i)(a)	$(3+2x)^6$	
	Any correct form for any term of ${}^6C_1 3^5(2x)^1$ or ${}^6C_2 3^4(2x)^2$ or ${}^6C_4 3^2(2x)^4$	M1
	Any two correct: t(2) = $6\times3^5\times(2x)^1$, t(3) $=15\times3^4\times(2x)^2$ t(5) = $15\times3^2\times(2x)^4$	A1
	All 3 terms correct 2nd term = $2916x$, 3rd term= $4860x^2$, 5th term=$2160x^4$	A1
		(3)
(b)	Method using consecutive terms of a GP e.g $\frac{2160x^4}{4860x^2}=\frac{4860x^2}{2916x}$	M1
	$\Rightarrow ...x=...$	dM1
	$\Rightarrow x=\frac{15}{4}$	A1
		(3)
(ii) (a)	Attempts to use $S_\infty=\frac{a}{1-r}\Rightarrow\frac{8}{5}=\frac{\sin^2\theta}{1-2\cos\theta}$	M1
	$\Rightarrow\frac{8}{5}=\frac{1-\cos^2\theta}{1-2\cos\theta}$	dM1
	$\Rightarrow 8-16\cos\theta=5-5\cos^2\theta$	
	$\Rightarrow 5\cos^2\theta-16\cos\theta+3=0$	A1*
		(3)
(b)	Solves $5\cos^2\theta-16\cos\theta+3=0\Rightarrow\cos\theta=\frac{1}{5}$	B1
	Attempts $ar=\sin^2\theta\times2\cos\theta=\left(1-\left(\frac{1}{5}\right)^2\right)\times2\times\frac{1}{5}$	M1
	$=\frac{48}{125}$	A1
		(3) **(12 marks)**

(i) (a)

M1: For an attempt at using the binomial expansion to find **one** of the required terms.
Condone the omission of the brackets on the $2x$ terms. Allow using combination notation.
Don't be concerned over any labelling (2nd term). May be awarded from a full expansion

A1: Any **two** correct terms, simplified or not. The ${}^6C_2$ must now be numerical.
Don't be concerned by labelling and accept as part of a large list or expansion

A1: All three terms correct and simplified.
They may be part of a larger or complete expansion, which is fine. Look for the 2nd , 3rd and 5th terms
Accept the 3 terms with +'s between. So accept for example $2916x+4860x^2+2160x^4$

Note that the expansion can be done by $(3+2x)^6=3^6\left(1+\frac{2}{3}x\right)^6$ The unsimplified versions of the terms are:

t (2) = $3^6\times6\left(\frac{2}{3}x\right)$ t(3) = $3^6\times\frac{6\times5}{2}\left(\frac{2}{3}x\right)^2$ and t(5) = $3^6\times\frac{6\times5\times4\times3}{4\times3\times2\times1}\left(\frac{2}{3}x\right)^4$

(i)(b)

M1: Uses the fact that the 2nd, 3rd and 5th terms of their binomial expansion in (a) are consecutive terms of a GP to set up an equation in x.

There are a few ways in which the equation could be set up so look carefully at what is written.

The most common ways are $\frac{"2160x^4"}{"4860x^2"} = \frac{"4860x^2"}{"2916x"}$, $\frac{"4860x^2"}{"2160x^4"} = \frac{"2916x"}{"4860x^2"}$ and

$"2160x^4" = \frac{"4860x^2"}{"2916x"} \times "4860x^2"$. Condone slips e.g. 2196 for 2916

The candidates **must** be using what they believe are the 2nd, 3rd and 5th terms of the binomial expansion and not the 2nd, 3rd or 4th or any other combination.

If a list is given it must be the 2nd, 3rd and 5th terms in $a + bx + cx^2 + dx^3 + ex^4 + ...$ where $a, b, c, d \neq 0$

dM1: It is dependent upon the first M and having the correct index on the powers of x.

It is for proceeding to an equation of the form $...x = ...$

A1: $x = \frac{15}{4}$ or exact equivalent. Condone an extra $x = 0$

....

If there are unlabelled/mislabelled terms in (a) that are used in (b) and you feel that they deserve more credit than that afforded by the scheme, please use review. A common error is described below.

It seems that some candidates are finding the terms in x^2, x^3 and x^5 in (i) (a) and (b). We will treat as a SC **only when appearing in both parts**. FYI: These are $4860x^2, 4320x^3$ and $576x^5$. These values would score M1 in (i)(a) for the x^2 term and potentially M1, dM1 (when proceeding to ...x =...) in (i)(b)

....

(ii)(a) Allow use of a different parameter, e.g. $\theta \leftrightarrow x$

M1: Attempts to use $S_\infty = \frac{a}{1-r} \Rightarrow \frac{8}{5} = \frac{\sin^2\theta}{1-2\cos\theta}$ Condone slips only. The formula must be correct

dM1: Uses the identity $\sin^2\theta = 1 - \cos^2\theta$ to form an equation in just $\cos\theta$

A1*: Completes proof with sufficient working.

Condone one slip in notation. E.g. $\sin\theta^2 \leftrightarrow \sin^2\theta$, $\cos \leftrightarrow \cos\theta$ but the final line must be correct and include the $= 0$

(ii)(b)

B1: States or uses $\cos\theta = \frac{1}{5}$

M1: Attempts $ar = \sin^2\theta \times 2\cos\theta = \left(1 - "\left(\frac{1}{5}\right)"^2\right) \times 2 \times "\frac{1}{5}"$

Alternatively finds the value of θ from their $\cos\theta = \frac{1}{5}$ and uses this in $ar = \sin^2\theta \times 2\cos\theta$

A1: $\frac{48}{125}$ or exact equivalent. Eg. 0.384.

It cannot be scored by rounding a non exact answer or from a decimal value of θ

So, for example, answers of $\frac{48}{125}$ achieved using a value of $\theta =$ 78.46….. don't score the final A1