Question Number	Scheme	Marks		
1.(a)	(i) $a_2 = 1$	B1		
	(ii) $a_{107} = 3$	B1 (2)		
(b)	$\sum_{n=1}^{200} (2a_n - 1) = 5 + 1 + 5 + 1 + \dots + 5 + 1 = 100 \times (5 + 1)$	M1		
	n = 1 = 600	A1 (2)		
		(4 marks)		
	Notes	I		
$\begin{bmatrix} (a) (i) \\ B1 & a_2 \end{bmatrix}$	= 1 Accept the sight of 1. Ignore incorrect working			
(a)(ii) B1 a <sub>10</sub>				
If	If there are lots of 1's and 3's without reference to any suffices they need to choose 3.			
Lo				
	a,b = 1,5 (which are correct)			
_	a,b = 1,3 (which are the values for (a))			
	$a,b=3,7$ (which is using $2a_n+1$ )			
	a,b = 0,5 (which is a slip on the first value)			
Me	thods using AP (and GP) formulae are common and score 0 marks.			
A1 600 600	). ) should be awarded both marks as long as no incorrect working is seen			

Questior Number	Scheme	Marks
2.(a)	Attempts $(x \pm 2)^2 + (y \pm 5)^2 \dots = 0$	M1
	(i) Centre $(-2,5)$	A1
	(ii) Radius $\sqrt{50}$ or $5\sqrt{2}$	B1
		(3)
(b)	Gradient of radius = $\frac{(5)-4}{(-2)-5} = -\frac{1}{7}$ which needs to be in simplest form	B1ft
	Uses $m_2 = -\frac{1}{m_1}$ to find gradient of tangent	M1
	Equation of tangent $y-4 = "7"(x-5) \Rightarrow y = 7x-31$	M1 A1
		(4) (7 marks)
	Notes	
	<b>Tote that the epen set up here is M1 M1 B1</b> Attempts to complete the square on both terms or states the centre as $(\pm 2, \pm 5)$	
	For completing the square look for $(x \pm 2)^2 + (y \pm 5)^2 \dots = \dots$	
	Centre $(-2,5)$ Allow $x = -2, y = 5$ This alone can score both marks even fol	lowing incorrect
	ines eg $(x+2)^2 (y-5)^2 =$ where could be, for example a minus sign or	
	adius $\sqrt{50}$ or $5\sqrt{2}$ You may isw after a correct answer.	
If a cand	idate attempts to use $x^2 + y^2 + 2fx + 2gy + c = 0$ then M1 may be awarded for	a centre of
$(\pm 2, \pm 5)$		
	Note that the epen set up here is M1 M1 M1 A1	
	Correct answer for the gradient of the line joining $P(5,4)$ to their centre. You may ft on their centre but the value must be fully simplified.	
	warded for using $m_2 = -\frac{1}{m_1}$ to find gradient of tangent.	
	1	you may need to
	Do be aware that some good candidates may do the first two marks at once so yook at what value they are using for the gradient of the tangent.	you may need to
M1 F	or an attempt to find the equation of the tangent using $P(5,4)$ and a changed	gradient. Condone
I	racketing slips only. f the candidate uses the form $y = mx + c$ they must use x and y the correct way	v around and
1	roceed as far as $c =$ y = 7x - 31 stated. It must be written in this form.	
	It cannot be awarded from $y = mx + c$ by just stating $c = -31$ )	
-	s at (b) using differentiation.	
	$x^{2} + y^{2} + 4x - 10y - 21 = 0 \rightarrow 2x + 2y \frac{dy}{dx} + 4 - 10 \frac{dy}{dx} = 0.$	
M1 S	substitutes $P(5,4)$ into an expression of the form $ax + by \frac{dy}{dx} + c + d \frac{dy}{dx} = 0$ AN	D finds the
	alue of $\frac{dy}{dx} = (7)$ . The values of <i>a</i> , <i>b</i> , <i>c</i> and <i>d</i> must be non-zero.	
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M1	Uses $m = \frac{dy}{dx}\Big _{x=5}$ with $P(5,4)$ to find equation of tangent
A1	y = 7x - 31

3. (i) $ \begin{array}{c} (x-4)^2 \ge 2x-9 \Rightarrow x^2-10x+250 \\ \Rightarrow (x-5)^20 \\ \text{Explains that "square numbers are greater than or equal to zero" hence (as x \in \mathbb{R}), \Rightarrow (x-4)^2 \ge 2x-9 * \\ (ii) \\ \text{Shows that it is not true for a value of n} \\ \text{Fg. When } n=3, 2^n+1=8+1=9 \times \text{Not prime} \\ (i) \\ \text{Notes} \\ \hline \\ (i) \\ \text{A proof starting with the given statement} \\ \text{M1 Attempts to expand } (x-4)^2 and work from form (x-4)^2 \dots 2x-9 to form a 3TQ on one side of a equation or an inequality and work from form (x-4)^2 \dots 2x-9 to form a 3TQ on one side of a equation or an inequality and work from form (x-4)^2 \dots 2x-9 to form a 3TQ on one side of x = (x-4)^2 \ge 2x-9 This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers are greater than or equal to zero", hence (as x \in \mathbb{R}), (x-5)^2 \ge 0 \Rightarrow (x-4)^2 \ge 2x-9 This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement Answers via b^2 - 4acc are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x-axis, it does not show that it is always positive. The explanation could involve a sketch of y = (x-5)^2 but it must be accurate with a minimum on the +ve x axis with some statement alluding to why this shows (x-5)^2 \ge 0\frac{1}{2}(2m-1)-9 Alt to (i) via contradiction x acceptable and marks in a similar way M1 For setting up the contradiction x Assume that there is an s such that (x-4)^2 < 2x-9 \Rightarrow x^2 - 10x + 25 \dots 0A1 \Rightarrow (x-5)^2 \dots 0 \text{ or } (x-5)(x-5) \dots 0A1 \Rightarrow (x-5)^2 \ge 0 - 9 (x-4)^2 \ge 2x - 9\therefore Alt to part (i) States (x-5)^2 \ge 0$	Question Number	Scheme	Marks	
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(ii) $\begin{vmatrix} x \in \mathbb{R}, x \in (x-4)^2 \ge 2x-9 & * \\ E_g. When n = 3, 2^n + 1 = 8 + 1 = 9 \times \text{Not prime} \end{vmatrix} (i)Shows that it is not true for a value of nEg. When n = 3, 2^n + 1 = 8 + 1 = 9 \times \text{Not prime} (1)(4 marks)(1)(4 $		$\Rightarrow (x-5)^2 \dots 0$	A1	
(ii) $\begin{vmatrix} x \in \mathbb{R}, x = (x-4) \ge 2x-9^{-x} \\ B1 \\ (i) \\ Shows that it is not true for a value of n \\ Eg. When n = 3, 2^{n} + 1 = 8 + 1 = 9 \times Not prime \\ (1) \\ (4 marks) \\ B1 \\ (1) \\ (4 marks) \\ (2 marks) \\ (1) \\ (2 marks) \\ (1) \\ (2 marks) \\ (2 mar$		Explains that "square numbers are greater than or equal to zero" hence (as	A 1 4	
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'Assume that there is an x such that $(x-4)^2 < 2x-9 \Rightarrow x^2 - 10x + 25 \dots 0$ A1 $\Rightarrow (x-5)^2 \dots 0$ or $(x-5)(x-5) \dots 0$ A1* This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \ge 2x-9$  Alt to part (i) States $(x-5)^2 \ge 0$ $\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$	Pr	oof by contradiction is acceptable and marks in a similar way		
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A1 $\Rightarrow (x-5)^2 \dots 0$ or $(x-5)(x-5)\dots 0$ A1* This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \ge 2x-9$ Alt to part (i) States $(x-5)^2 \ge 0$ $\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$				
A1* This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \ge 2x-9$ Alt to part (i) States $(x-5)^2 \ge 0$ $\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$				
hence $(x-4)^2 \ge 2x-9$ Alt to part (i) States $(x-5)^2 \ge 0$ $\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$				
Alt to part (i) States $(x-5)^2 \ge 0$ $\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$				
$\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$				
$\Rightarrow x^2 - 10x + 25 \ge 0$ $\Rightarrow x^2 - 8x - 16 \ge 2x - 9$	Alt to part	(i) States $(x-5)^2 \ge 0$		
$\Rightarrow x^2 - 8x - 16 \geqslant 2x - 9$	-			
$\Rightarrow (x-4)^2 \ge 2x-9$				
	$\Rightarrow$ (	$(x-4)^2 \ge 2x-9$		

Question Number	Scheme	Marks
M1 St	ates $(x-5)^2 \ge 0$ and attempts to expand. There is no explanation required here	;
A1 Re	earranges to reach $x^2 - 8x - 16 \ge 2x - 9$	
A1* Re	eaches the given answer $(x-4)^2 \ge 2x-9$ with no errors	
Th E Co Co If	ows that it is not true for a value of <i>n</i> is requires a calculation (and value found) with a minimal statement that it is r g. ${}^{2^{6}}+1=65$ which is not prime' or ${}^{2^{5}}+1=33 \times$ ' ondone sloppily expressed proofs. Eg. ${}^{7}+1=\frac{129}{3}=43$ which is not prime' ondone implied proofs where candidates write $2^{5}+1=33$ which has a factor of there are lots of calculations mark positively. nly one value is required to be found (with the relevant statement) to score the he calculation cannot be incorrect. Eg. $2^{3}+1=10$ which is not prime	11

Quest Numb	Scheme	Marks		
4.(a	) $\left(2-\frac{1}{4}x\right)^6 = 2^6, + {}^6C_12^5\left(-\frac{1}{4}x\right)^1 + {}^6C_22^4\left(-\frac{1}{4}x\right)^2 + {}^6C_32^3\left(-\frac{1}{4}x\right)^3 + \dots$	B1, M1		
	$= 64 - 48x + 15x^2 - 2.5x^3$	A1 A1		
(b	(a) $\left(2 - \frac{1}{4}x\right)^{6} + \left(2 + \frac{1}{4}x\right)^{6} = \left(64 - 48x + 15x^{2} - 2.5x^{3}\right) + \left(64 + 48x + 15x^{2} + 2.5x^{3}\right)$	(4) M1		
	$\approx 128 + 30x^2$	B1ft A1		
		(3) (7 marks)		
	Notes			
(a) B1	For either 2 <sup>6</sup> or 64. Award for an unsimplified ${}^{6}C_{0}2^{6}\left(-\frac{1}{4}x\right)^{0}$			
M1	For an attempt at the binomial expansion. Score for a correct attempt at term 2,			
	Accept sight of ${}^{6}C_{1}2^{5}\left(\pm\frac{1}{4}x\right)^{1} {}^{6}C_{2}2^{4}\left(\pm\frac{1}{4}x\right)^{2} {}^{6}C_{3}2^{3}\left(\pm\frac{1}{4}x\right)^{3}$ condoning omis	sion of brackets.		
	Accept any coefficient appearing from Pascal's triangle. FYI 6, 15, 20			
A1	For any two simplified terms of $-48x + 15x^2 - 2.5x^3$			
A1		r $64-48x+15x^2-2.5x^3$ ignoring terms with greater powers. This may be awarded in (b) if it		
(b) M1	not fully simplified in (a). Allow the terms to be listed $64, -48x, 15x^2, -2.5x^3$ . Isw after sight f correct values. The expression written out without any method can be awarded all 4 marks. Note that this is now marked M1 B1 A1 or adding two sequences that must be of the correct form with the correct signs.			
	Look for $\left(A - Bx + Cx^2 - Dx^3\right) + \left(A + Bx + Cx^2 + Dx^3\right)$ but condone			
(A-B)	$Bx + Cx^2 + \left(A + Bx + Cx^2\right)$			
B1ft or	For this to be scored there must be some negative terms in (a) For one correct term (follow through). Usually $a = 128$ but accept either $a = 2 \times b = 2 \times '$ their' + ve 15	<'their'+ve 64		
A1	For $128 + 30x^2$ . CSO so must be from $(64 - 48x + 15x^2 - 2.5x^3) + (64 + 48x + 15x^2 - 2.5x^3)$	$x^{2} + 2.5x^{3}$		
	Allow $a = 128, b = 30$ following correct work.	/		
here.	This is a show that question so M1 must be awarded. It must be their final answ	er so do not isw		
Altern	ative method in (a):			
	$\frac{1}{4}x\Big)^{6} = 2^{6}\left(1 - \frac{1}{8}x\right)^{6} = 2^{6}\left(1 + 6\left(-\frac{1}{8}x\right) + \frac{6 \times 5}{2}\left(-\frac{1}{8}x\right)^{2} + \frac{6 \times 5 \times 4}{3!}\left(-\frac{1}{8}x\right)^{3} + \dots\right)$			
B1 M1	For sight of factor of either $2^6$ or 64 For an attempt at the binomial expansion seen in at least one term within the bra	ickets.		

Score for a correct attempt at term 2, 3 or 4.

Question Number	Scheme	Marks	
A	Accept sight of $6\left(\pm\frac{1}{8}x\right)^1 \frac{6\times5}{2}\left(\pm\frac{1}{8}x\right)^2 \frac{6\times5\times4}{3!}\left(\pm\frac{1}{8}x\right)^3$ condoning omission of brackets		
A1 F	or any two terms of $64 - 48x + 15x^2 - 2.5x^3$		
A1 F	A1 For all four terms $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers		
Attempts	to multiply out		
B1 F	or 64		
	Iultiplies out to form $a + bx + cx^2 + dx^3 +$ and gets b, c or d correct. As main scheme		

Question Number	Scheme	Marks	
5.(a)	$\frac{dP}{dx} = 12 - \frac{3}{2}x^{\frac{1}{2}}$	M1A1	
	Sets $\frac{dP}{dx} = 0 \rightarrow 12 - \frac{3}{2}x^{\frac{1}{2}} = 0 \rightarrow x^n =$	dM1	
	$x = 64$ When $x = 64 \implies P = 12 \times 64 - 64^{\frac{3}{2}} - 120 =$ Profit = (f) 136 000	A1	
	When $x = 64 \implies P = 12 \times 64 - 64^{\frac{3}{2}} - 120 =$	M1	
	Profit $=$ (£) 136 000	A1 (6)	
(b)	$\left(\frac{d^2 P}{dx^2}\right) = -\frac{3}{4}x^{-\frac{1}{2}}$ and substitutes in their $x = 64$ to find its value or state its sign At $x = 64$ $\frac{d^2 P}{dx^2} = -0.09375 < 0 \Rightarrow$ maximum	(6) M1	
	At $x = 64$ $\frac{d^2 P}{dx^2} = -0.09375 < 0 \Longrightarrow$ maximum	A1	
		(2) (8 marks)	
	Notes		
You should (a)	d mark parts a and b together. You may see work in (a) from (b)		
M1 Att	Attempts to differentiate $x^n \to x^{n-1}$ seen at least once. It must be an <i>x</i> term and <b>not</b> the $120 \to 0$		
C.A	$\frac{P}{x} = 12 - \frac{3}{2}x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{dy}{dx}$ all of the way through part (a).		
dM1 Set	ts their $\frac{dP}{dx} = 0$ and proceeds to $x^n = k, k > 0$ . Dependent upon the previous M. Don't be too		
cor	concerned with the mechanics of process. Condone an attempted solution of $\frac{dP}{dx}$ 0 where		
	build be an inequality = 64. Condone $x = \pm 64$ here		
	ostitutes their solution for $\frac{dP}{dr} = 0$ into P and attempts to find the value of P.		
	ne value of x must be positive. If two values of x are found, allow this mark for any attempt ng a positive value.		
A1 CS	O. Profit = (£) 136 000 or 136 thousand but not 136 or $P = 136$ . is cannot follow two values for x, eg $x = \pm 64$ Condone a lack of units or incorrect units such		
as \$ (b)			
	hieves $\frac{d^2 P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to find its value at $x = "64"$		
	ternatively achieves $\frac{d^2 P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to state its sign. Eg $\frac{d^2 P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$	2 < 0	
	low $\frac{d^2 P}{dx^2}$ appearing as $\frac{d^2 y}{dx^2}$ for the both marks.		
A1 Ac	hieves $x = 64$ , $\frac{d^2 P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states $\frac{d^2 P}{dx^2} = -\frac{3}{32} < 0$ (at $x = 64$ ) then the pro-	ofit is	
maximised	l.		

This requires the correct value of x, the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.

Alt: Achieves x = 64,  $\frac{d^2 P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$  and states as x > 0 or  $\sqrt{x} > 0$  means that  $\frac{d^2 P}{dx^2} < 0$  then the profit is maximised.

Part (b) merely requires the use of calculus so allow

M1 Attempting to find the value of  $\frac{dP}{dx}$  at two values either side, but close to their 64. Eg. For 64, allow the lower value to be  $63.5 \le x < 64$  and the upper value to be  $64 < x \le 64.5$ A1 Requires correct values, correct calculations with reason and conclusion

Question Number	Scheme	Marks
6.(a)	Sets $f(3) = 0 \rightarrow$ equation in k Eg. $27k - 135 - 96 - 12 = 0$	M1
	$\Rightarrow 27k = 243 \Rightarrow k = 9 * (= 0 \text{ must be seen})$	A1*
		(2)
<b>(b)</b>	$9x^{3} - 15x^{2} - 32x - 12 = (x - 3)(9x^{2} + 12x + 4)$	M1 A1
	$=(x-3)(3x+2)^{2}$	dM1 A1
		(4)
(c)	Attempts $\cos\theta = -\frac{2}{3}$	M1
	$\theta = 131.8^{\circ}, 228.2^{\circ} \text{ (awrt)}$	Al
		(2)
		(8 marks)
	Notes	
(a) M1 A <sup>2</sup>	ttempts to set $f(3) = 0 \rightarrow$ equation in k Eg. $27k - 135 - 96 - 12 = 0$ . Condone	slips
S	core when you see embedded values within the equation or two correct terms quation. It is implied by sight of $27k - 243 = 0$ or $27k = 135 + 96 + 12$ .	
A1* Co	Sompletes proof with at least one intermediate "solvable" line namely $27k = 24$ . $7k - 243 = 0 \implies k = 9$ . This is a given answer so there should be no errors.	$3 \Longrightarrow k = 9$ or
	is a "show that" question so expect to see	
	Either $f(3) = 0$ explicitly stated or implied by sight of $27k - 135 - 96 - 12 = 0$	) or
27 <i>k</i> -243 (ii	3=0 ) One solvable intermediate line followed by $k=9$	
		•••••
	candidate could use $k = 9$ and start with $f(x) = 9x^3 - 15x^2 - 32x - 12$	
	or attempting $f(3) = 9 \times 3^3 - 15 \times 3^2 - 32 \times 3 - 12$ .	
	It attempts to divide $f(x)$ by (x-3). See below on how to score such an attempt require that $f(2) = 0$ and maly a minimal statement to the effect that "so $k = 0$ "	
	hows that $f(3) = 0$ and makes a minimal statement to the effect that "so $k = 9$ " division is attempted it must be correct and a statement is required to the effect that there is no	
	mainder, "so $k = 9$ "	
If candida	ttes have divided (correctly) in part (a) they can be awarded the first two mark	s in (b) when
they start	factorsing the $9x^2 + 12x + 4$ term.	
(b)		
	ttempt to divide or factorise out $(x-3)$ . Condone students who use a different	value of <i>k</i> .
Fo	or factorisation look for first and last terms $9x^3 - 15x^2 - 32x - 12 = (x-3)(\pm 9x)^3$	$^{2}$ ±4)
		/
Fo	or division look for the following line $x-3\overline{\smash{\big)}9x^3-15x^2-32x-12}$ $9x^3-27x^2$	
	$9x^3 - 27x^2$	
	2	
	prrect quadratic factor $9x^2 + 12x + 4$ . You may condone division attempts that don't quite work as long as the correct	factor is seen
1		2019 06 MS

Question Number	Scheme	Marks
A1 (.	Attempt at factorising their $9x^2 + 12x + 4$ Apply the usual rules for factorising $(x-3)(3x+2)^2$ or $(x-3)(3x+2)(3x+2)$ on one line.	
	Accept $9(x-3)\left(x+\frac{2}{3}\right)^2$ oe. It must be seen as a product	
F	Remember to isw for candidates who go on to give roots $f(x) = (x-3)(3x+2)^2$	$\Rightarrow x = \dots$
	rt (b) is "Hence" so take care when students write down the answer to (b) with $2$	out method
If candid	ates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = \left(x + \frac{2}{3}\right)\left(x + \frac{2}{3}\right)\left(x - 3\right)$ score 0000	
	lates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = (3x+2)(3x+2)(x-3)$ they score SC 1010.	
If candid	ates state $x = -\frac{2}{3}, 3 \Rightarrow f(x) = 9\left(x + \frac{2}{3}\right)\left(x + \frac{2}{3}\right)\left(x - 3\right)$ they score SC 1010.	
	ate writes down $f(x) = (3x+2)(3x+2)(x-3)$ with no working they score SC 1	010.
If a candidate writes down $(x-3)(3x+2)$ are factors it is 0000		
(c)	~	
M1 A	correct attempt to find one value of $\theta$ in the given range for their $\cos \theta = -\frac{2}{3}$	
(N values.	You may have to use a calculator). So if (b) is factorised correctly the mark is fo	or one of the
	his can be implied by sight of awrt 132 or 228 in degrees or awrt 2.3 which is the	ne radian
	SO awrt $\theta = 131.8^{\circ}, 228.2^{\circ}$ with no additional solutions within the range $0 \le \theta$	< 360°
W	Vatch for correct solutions appearing from $3\cos\theta - 2 = 0 \implies \cos\theta = \frac{2}{3}$ . This is N	/10 A0
	without working are acceptable.	
	for two correct answers with no additional solutions within the range.	

Number	Scheme	Marks
7.(a)	Attempts to use $31500 = 16200 + 9d$ to find 'd'	M1
	For 16 200 + their $d = (1700)$ where d has been found by an allowable method	M1
	Year 2 salary is (£)17 900	A1
	9	(.
<b>(b)</b>	Attempts to use $31500 = 16200r^9$ to find 'r'	M1
	For 16 2000 × their $r = (1.077)$ where r has been found by an allowable method	M1
	Year 2 salary in the range $17440 \leq S \leq 17450$	A1
(c)		(.
(t)	Attempts $\frac{10}{2}$ {16200+31500} or $\frac{16200(1.077^{10}-1)}{1.077-1}$	M1
	Finds $\pm \left(\frac{10}{2} \{16200 + 31500\} - \frac{16200('1.077'^{10} - 1))}{'1.077' - 1}\right)$	dM1
	Difference = $\pounds7480$ cao	A1
		(.
		(9
	Notes	marks
Ac M1 A for A1 E	Scept an attempt at $31500 = 16200 + 9d$ resulting in a value for <i>d</i> . Scept the calculation $\frac{31500 - 16200}{9}$ condoning slips on the 31500 and 16200 correct attempt to find the second term by adding 16 200 to their ' <i>d</i> ' which must have und via an allowable method. Scept the found from an "incorrect" AP formula with 10 <i>d</i> being used instead of 9 <i>d</i> g 31500 = 16200 + 10 <i>d</i> or more likely $\frac{31500 - 16200}{10} = 1530$ usually leading to an a	•
17730 A1 Y	fear 2 salary is (£) 17 900	
(b)		
	ttempts to use the GP formula in an attempt to find 'r' $31500$	
Ac	ccept an attempt at $31500 = 16200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = \dots$ condoning numerical slip	ips.
Ac Ac	ccept an attempt at $31500 = 16200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = \dots$ condoning numerical slip ccept the calculation $\sqrt[9]{\frac{31500}{16200}}$ or $\sqrt[9]{\frac{35}{18}}$ condoning slips on the 31500 and 16200.	ips.
Ac Ac It	ccept an attempt at $31500 = 16200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = \dots$ condoning numerical slip	

M1	A correct attempt to find the second term by multiplying 16 200 by their 'r' which must been found via an allowable method. Allow r to be found from an "incorrect" GP formula with 10 being used instead of 9. Eg	have
	following $31500 = 16200r^{10}$ or $\sqrt[10]{\frac{31500}{16200}}$ . You may also award, condoning slips, for a	-
A1	attempt at 16200 × r where r is their solution of $31500 = 16200r^n$ where $n = 9$ or 10 For an answer in the range £17440 $\leq S \leq 17450$ Note that $r = 1.077 \Rightarrow 17447.40$	
(c) M1	A correct method to find the sum of either the AP or the GP For the AP accept an attempt at either $\frac{10}{2}$ {16200+31500} or $\frac{10}{2}$ {2×16200+9×'d'}	
	For the GP accept an attempt at either $\frac{16200(r''^{10}-1)}{r'-1}$ or $\frac{16200(1-r'^{10})}{1-r'}$	
dM1 around A1	Both formulae must be attempted "correctly" (see above) and the difference taken (eithed) FYI if d and r are correct, the sums are £238 500 and £231 019.(24) Difference = $\pounds7480$ CAO. Note that this answer is found using the unrounded van Note that using the rounded value will give $\pounds7130$ which is A0	-
If the	solutions for (a) and (b) are reversed, eg GP in (a) and AP in (b) then please send to revie	W.
(i)	General approach to marking part (i) This is now marked M1 A1 M1 A1 o	n epen
M1 A1	Takes log of both sides and uses the power law. Accept any base. Condone missing brack For a correct linear equation in x which only involve logs of base 2 usually $\log_2 6$ , $\log_2 \log_2 8$ but sometimes $\log_2 \frac{3}{4}$ and others so read each solution carefully	
M1 A1	Attempts to use a log law to create a linear equation in $\log_2 3$ Eg. $\log_2 6 = \log_2 2 + \log_2 3$ which is implied by $\log_2 6 = 1 + \log_2 3$ Eg. $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$ For $x = -\frac{1}{3} + \frac{\log_2 3}{6}$ oe in the form required by the question. Note that $x = \frac{\log_2 3 - \log_2 3}{6}$	$\frac{2}{2}$ is A0

Question Number	Please read notes for 8(i) before looking at scheme			Marks	
8.(i)	$8^{2x+1} = 6 \Longrightarrow 2x+1 = \log_8 6 \qquad N$	<b>M</b> 1	$2^{6x+3} = 6$		
	$\Rightarrow 2x + 1 = \frac{\log_2 6}{\log_2 8} \qquad A$	<b>A</b> 1	$\Rightarrow (6x+3)\log_2 2 = \log_2 6$	M1 A1	
	$\Rightarrow 2x + 1 = \frac{\log_2 2 + \log_2 3}{3}$	M1	$\Rightarrow (6x+3) = \log_2 2 + \log_2 3$	M1	
	$\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$	A1	$\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$	A1	
	$\log (7, 2y) = 2\log (y+1)$ 1		$2\log(n+1) \log(7-2n) = 1$	1	(4)
(ii)	$\log_{5}(7-2y) = 2\log_{5}(y+1) - 1$		$2\log_5(y+1) - \log_5(7-2y) = 1$	L	
	$\log_{5}(7-2y) = \log_{5}(y+1)^{2} - 1$		$\log_5(y+1)^2 - \log_5(7-2y) = 1$		M1
	$\log_5(7-2y) = \log_5(y+1)^2 - \log_5 5$		$\log_5 \frac{(y+1)^2}{(7-2y)} = 1$		dM1
	$\left(7-2y\right) = \frac{\left(y+1\right)^2}{5}$		$\frac{\left(y+1\right)^2}{\left(7-2y\right)} = 5$		A1
	$y^2 + 12y - 34 = 0 \Longrightarrow y =$		$y^2 + 12y - 34 = 0 \Longrightarrow y =$		ddM1
			$\sqrt{70}$ oe only		A1
					(5) (9 marks)
Notes					

There are many different ways to attempt this but essentially can be marked in a similar way. If index work is used marks are not scored until the log work is seen

Eg 1:  $8^{2x+1} = 6 \Longrightarrow 8^{2x} \times 8 = 6 \Longrightarrow 8^{2x} = \frac{3}{4}$ . 1<sup>ST</sup> M1 is scored for  $2x = \log_8 \frac{3}{4}$  and then 1<sup>ST</sup> A1 for  $2x = \frac{\log_2 \frac{3}{4}}{\log_2 8}$ 

but BOTH of these marks would be scored for  $2x \log_2 8 = \log_2 \frac{3}{4}$ 

 $2^{nd}$  M1 would then be awarded for  $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$  which may be implied by  $\log_2 3 - 2$ Two more examples where the candidate initially uses index work.

$8^{2x+1} = 6 \Longrightarrow 2^{3(2x+1)} = 6$	$8^{2x+1} = 6 \Longrightarrow 64^x = \frac{3}{4}$
$3(2x+1) = \log_2 6$ is M1 A1	$\Rightarrow x = \log_{64} \frac{3}{4}$ is M1
as it is a correct linear equation in $x$ involving a log $_2$ term	But $\Rightarrow x \log_2 64 = \log_2 \frac{3}{4}$ is M1 A1

Questi Numb	Plass read notes for XIII before looking at scheme	Marks
(ii) M1	Attempts a correct log law. This may include $2\log_5(y+1) \rightarrow \log_5(y+1)^2 \qquad 1 \rightarrow \log_5 5$ You may award this following incorrect work. Eg $1 = 2\log_5(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 2(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 \frac{2(y+1)}{(7-2y)}$	
dM1	Uses two correct log laws. It may not be awarded following errors (see above) It is awarded for $2\log_5(y+1)-1 = \log_5\frac{(y+1)^2}{5}$ , $2\log_5(y+1)-\log_5(7-2y) = \log_5\frac{(y+1)^2}{7}$ $1 + \log_5(7-2y) = \log_55(7-2y)$ or $2\log_5(y+1)-1 = \log_5(y+1)^2 - \log_55$	$\left(\frac{y+1}{-2y}\right)^2$
A1 ddM1 14.4 A1	A correct equation in 'y' not involving logs A correct attempt at finding at least one value of y from a 3TQ in y All previous M's must have been awarded. It can be awarded for decimal answer(s), 2.4 $y = -6 + \sqrt{70}$ or exact equivalent only. It cannot be the decimal equivalent but award if the candidate chooses 2.4 following the	
	answer. If $y = -6 \pm \sqrt{70}$ then the final A mark is withheld Special case: Candidates who write $\log_5(y+1)^2 - \log_5(7-2y) = 1 \Rightarrow \frac{\log_5(y+1)^2}{\log_5(7-2y)} = 1 \Rightarrow \frac{(y+1)^2}{(7-2y)} = 5$ can score M1 dM0 A0 ddM1 A1 if they find the correct answer.	

Question Number	Scheme	Marks	
9 (a)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow \qquad \cos\theta - 1 = 4\sin\theta \frac{\sin\theta}{\cos\theta}$	M1	
	$\cos^2\theta - \cos\theta = 4\sin^2\theta$ oe	A1	
	Uses $\sin^2 \theta = 1 - \cos^2 \theta \rightarrow \cos^2 \theta - \cos \theta = 4(1 - \cos^2 \theta)$	M1	
	$5\cos^2\theta - \cos\theta - 4 = 0$ *	A1 *	
(b)	$(5\cos 2x + 4)(\cos 2x - 1) = 0$	(4) M1	
	Critical values of $-\frac{4}{5}$ ,1	A1	
	Correct method to find x from their $\cos 2x = -\frac{4}{5}$	dM1	
	x = 0, 1.25	A1	
		(4) (8 marks)	
	Notes		
(a) M1 Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ oe in their $\cos \theta - 1 = 4\sin \theta \tan \theta$ . Condone slips in coefficients and the equation may have been adapted. This may be implied by candidates who multiply by $\cos \theta$ and reach $\cos \theta - 1 = 4\sin \theta \tan \theta \Rightarrow \cos^2 \theta - \cos \theta = 4\sin^2 \theta$ . This would be M1 A1 A1 Correct equation, without any fractional terms, in $\sin \theta$ and $\cos \theta$ If the identity $\sin^2 \theta = 1 - \cos^2 \theta$ is used before the multiplication by $\cos \theta$ then it will be for a correct equation, without any fractional terms, in $\cos \theta$ Condone incorrect notation $\cos^2 \theta^2$ for $\cos^2 \theta$ M1 Uses $\sin^2 \theta = 1 - \cos^2 \theta$ to produce an equation in just $\cos \theta$ A1* Proceeds to $5\cos^2 \theta - \cos \theta - 4 = 0$ with no arithmetical or notational errors. Both identities must be seen to have been applied. Candidates cannot just go from $\cos^2 \theta - \cos \theta = 4\sin^2 \theta$ to the answer without any evidence of the appropriate identity. No mixed variables within the lines of the "proof" Condone incomplete lines if it is part of their working. $\cos^2 \theta - \cos \theta = 4\sin^2 \theta$ Eg. $=4(1 - \cos^2 \theta)$ An example of a notational error is $\cos^2 \theta$ for $\cos^2 \theta$ (Note that this would only lose the A1*)			
Al Ci	Attempts to find the critical values of the given quadratic by a correct method. Critical values of $-\frac{4}{5}$ , 1. Allow this to be scored even if written as $\cos x =$ or even <i>x</i> . Allow these to be written down (from a calculator)		

Quest Numb		Scheme	
dM1		correct method to find one value of x from their $\cos 2x = -\frac{4}{5}$ Look for correct	order of
operat			
	It is	s dependent upon the previous mark.	
	This can be implied by awrt $1.5/71.6^{\circ}$ or awrt $1.24/1.25$ (rads)		
A1	Both $x = 0$ and awrt 1.25 with no other values in the range $0 \le x < \frac{\pi}{2}$ .		
	Co	ndone 1.25 written as $0.398\pi$ . Condone if written as $\theta =$	
Answers without working can score all marks:			
Score	Score M1 for one value and M1 A1 M1 A1 for both values and no others in the range.		

Question Number	Scheme	Marks	
10 (a)	$(f'(x)) = -\frac{72}{x^3} + 2$	M1 A1	
	Attempts to solve $f'(x) = 0 \Rightarrow x = \text{ via } x^{\pm n} = k,  k > 0$ $x > \sqrt[3]{36}$ oe	dM1 A1 (4)	
(b)	$\int \frac{36}{x^2} + 2x - 13  \mathrm{d}x = -\frac{36}{x} + x^2 - 13x  (+c)$	M1 A1	
	Uses limits 9 and 2 = $\left(-\frac{36}{9}+9^2-13\times9\right) - \left(-\frac{36}{2}+2^2-13\times2\right) = 0 *$	dM1 A1*	
(c)(i)	$\begin{cases} 8 \\ \int_{0}^{6} (36 + 2 + 4) 1 \\ 1 \\ 36 \\ 1 \\ 2 \\ 4 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1$	( <b>4</b> ) B1	
(11)	$\int_{2}^{6} \left(\frac{36}{x^{2}} + 2x + k\right) dx = 0 \Longrightarrow \left[-\frac{36}{x} + x^{2} + kx\right]_{2}^{6} = 0 \Longrightarrow (30 + 6k) - (-14 + 2k) = 0$ $44 + 4k = 0 \Longrightarrow k = -11$	N/1 A 1	
	$44 + 4k = 0 \Longrightarrow k = -11$	M1 A1 (3) (11 marks)	
	Notes		
M1 Attempts $f'(x)$ with one index correct. Allow for $x^{-2} \rightarrow x^{-3}$ or $2x \rightarrow 2$ A1 $f'(x) = -\frac{72}{x^3} + 2$ correct but may be unsimplified $f'(x) = 36 \times -2x^{-3} + 2$ dM1 Attempts to find where $f'(x) = 0$ . Score for $x^n = k$ where $k > 0$ and $n \neq \pm 1$ leading to $x =$ Do not allow this to be scored from an equation that is adapted incorrectly to get a positive k. Allow this to be scored from an attempt at solving $f'(x)0$ where can be any inequality A1 Achieves $x > \sqrt[3]{36}$ or $x > 6^{\frac{2}{3}}$ Allow $x \ge \sqrt[3]{36}$ or $x \ge 6^{\frac{2}{3}}$ but not $x > (\frac{1}{36})^{-\frac{1}{3}}$ We require an exact value but remember to isw. An answer of 3.302 usually implies the first 3 marks.			
(b) M1 For $x^n \to x^{n+1}$ seen on either $\frac{36}{x^2}$ or $2x$ . Indices must be processed. eg $x^{1+1} \to x^2$ A1 $\int \frac{36}{x^2} + 2x - 13  dx = -\frac{36}{x} + x^2 - 13x$ which may be unsimplified. Eg $x^2 \leftrightarrow \frac{2x^2}{2}$ Allow with $+ c$ dM1 Substitutes 9 and 2 into their integral and subtracts either way around. Condone missing brackets Dependent upon the previous M A1* Completely correct integration with either embedded values seen or calculated values (-40) - (-40) Note that this is a given answer and so the bracketing must be correct.			

Question Number	Scheme	Marks		
(c)(i) B1 F				
(c)(ii) M1 T				
•	<ul> <li>A restart (See scheme). For this to be awarded all terms must be integrated with k→kx, the limits 6 and 2 applied, the linear expression in k must be set equal to 0 and a solution attempted.</li> <li>An attempt at solving ∫<sub>2</sub><sup>6</sup> k+13 dx = 8 or equivalent. Look for the linear equation -8+4(13+k) = 0 or 4(13+k) = 8 and a solution attempted.</li> <li>Recognising that the curve needs to be moved up 2 units.</li> </ul>			
•	Sight of $\frac{8}{6-2}$ or $-13+2$			
A1 $k = -11$ . This alone can be awarded both marks as long as no incorrect working is seen.				