

\begin{tabular}{|c|c|c|}
\hline Question Number \& Scheme \& Marks \\
\hline 2.(a)

(b) \& \begin{tabular}{l}
Attempts $(x \pm 2)^{2}+(y \pm 5)^{2} \ldots \ldots=0$ \\
(i) Centre $(-2,5)$ \\
(ii) Radius $\sqrt{50}$ or $5 \sqrt{2}$ \\
Gradient of radius $=\frac{(5)-4}{(-2)-5}=-\frac{1}{7}$ which needs to be in simplest form Uses $m_{2}=-\frac{1}{m_{1}}$ to find gradient of tangent \\
Equation of tangent $y-4=" 7 "(x-5) \Rightarrow y=7 x-31$

 \& 

M1 \\
A1 \\
B1 \\
(3) \\
B1ft \\
M1 \\
M1 A1 \\
(4) \\
(7 marks)
\end{tabular} \\

\hline \multicolumn{3}{|c|}{Notes} \\
\hline  \& that the epen set up here is M1 M1 B1
empts to complete the square on both terms or states the centre as $( \pm 2, \pm 5)$
r completing the square look for $(x \pm 2)^{2}+(y \pm 5)^{2} \ldots \ldots . . \ldots$ \& \\
\hline \multicolumn{3}{|l|}{A1 Centre $(-2,5)$ Allow $x=-2, y=5$ This alone can score both marks even following incorrect lines eg $(x+2)^{2} . .(y-5)^{2}=\ldots$ where $\ldots$ could be , for example a minus sign or blank} \\
\hline \multicolumn{3}{|l|}{If a candidate attempts to use $x^{2}+y^{2}+2 f x+2 g y+c=0$ then M1 may be awarded for a centre of $( \pm 2, \pm 5)$} \\
\hline \& that the epen set up here is M1 M1 M1 A1 \& \\
\hline \multicolumn{3}{|l|}{$\begin{array}{ll}\text { B1 ft } & \text { Correct answer for the gradient of the line joining } P(5,4) \text { to the } \\ \text { You may } \mathrm{ft} \text { on their centre but the value must be fully simplifie }\end{array}$} \\

\hline \& | arded for using $m_{2}=-\frac{1}{m_{1}}$ to find gradient of tangent. |
| :--- |
| be aware that some good candidates may do the first two marks at once $k$ at what value they are using for the gradient of the tangent. | \& u may need to \\


\hline | M1 F |
| :--- |
| b |
| If |
| pr | \& If the candidate uses the form $y=m x+c$ they must use $x$ and $y$ the correct way around and proceed as far as $c=\ldots$ \& adient. Condone round and \\

\hline \multicolumn{3}{|r|}{(It cannot be awarded from $y=m x+c$ by just stating $c=-31$ )} \\
\hline
\end{tabular}

Attempts at (b) using differentiation.
B1 $\quad x^{2}+y^{2}+4 x-10 y-21=0 \rightarrow 2 x+2 y \frac{\mathrm{~d} y}{\mathrm{~d} x}+4-10 \frac{\mathrm{~d} y}{\mathrm{~d} x}=0$.
M1 Substitutes $P(5,4)$ into an expression of the form $a x+b y \frac{\mathrm{~d} y}{\mathrm{~d} x}+c+d \frac{\mathrm{~d} y}{\mathrm{~d} x}=0$ AND finds the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}=(7)$. The values of $a, b, c$ and $d$ must be non-zero.

M1 Uses $m=\left.\frac{\mathrm{d} y}{\mathrm{~d} x}\right|_{x=5}$ with $P(5,4)$ to find equation of tangent
A1 $y=7 x-31$

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3. (i) | $\begin{array}{r} (x-4)^{2} \geqslant 2 x-9 \Rightarrow x^{2}-10 x+25 \ldots 0 \\ \Rightarrow(x-5)^{2} \ldots 0 \end{array}$ <br> Explains that "square numbers are greater than or equal to zero" hence (as $x \in \mathbb{R}), \Rightarrow(x-4)^{2} \geqslant 2 x-9 *$ <br> Shows that it is not true for a value of $n$ <br> Eg. When $n=3,2^{n}+1=8+1=9 \times$ Not prime | M1 <br> A1 <br> A1* <br> (3) <br> B1 <br> (1) <br> (4 marks) |
| Notes |  |  |
| (i) A proof starting with the given statement <br> M1 Attempts to expand $(x-4)^{2}$ and work from form $(x-4)^{2} \ldots 2 x-9$ to form a 3TQ on one side an equation or an inequality <br> A1 Achieves both $x^{2}-10 x+25$ and $(x-5)^{2}$. Allow $(x-5)^{2}$ written as $(x-5)(x-5)$ <br> A1* For a correct proof. Eg <br> "square numbers are greater than or equal to zero", hence (as $x \in \mathbb{R}$ ), $(x-5)^{2} \geqslant 0$ $\Rightarrow(x-4)^{2} \geqslant 2 x-9$ <br> This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement <br> Answers via $b^{2}-4 a c$ are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x -axis, it does not show that it is always positive. The explanation could involve a sketch of $y=(x-5)^{2}$ but it must be accurate with a minimum on the $+\mathrm{ve} x$ axis with some statement alluding to why this shows $(x-5)^{2} \geqslant 0$ <br> Approaches via odd and even numbers will usually not score anything. They would need to proceed using the main scheme via $(2 m-4)^{2} \geqslant 4 m-9$ and $(2 m-1-4)^{2} \geqslant 2(2 m-1)-9$ |  |  |
|  |  |  |
| Alt to (i) via contradiction |  |  |
| Proof by contradiction is acceptable and marks in a similar way |  |  |
| M1 For setting up the contradiction <br> 'Assume that there is an $x$ such that $(x-4)^{2}<2 x-9 \Rightarrow x^{2}-10 x+25 \ldots 0$ <br> A1 $\Rightarrow(x-5)^{2} \ldots 0$ or $(x-5)(x-5) \ldots 0$ <br> A1* This is not true as square numbers are always greater than or equal to 0 , hence $(x-4)^{2} \geqslant 2 x-9$ |  |  |
|  |  |  |
| $\begin{aligned} & \Rightarrow x^{2}-10 x+25 \geqslant 0 \\ & \Rightarrow x^{2}-8 x-16 \geqslant 2 x-9 \\ & \Rightarrow(x-4)^{2} \geqslant 2 x-9 \end{aligned}$ |  |  |


| Question <br> Number | Scheme | Marks |
| :--- | :--- | :---: |
| M1 $\quad$ States $(x-5)^{2} \geqslant 0$ and attempts to expand. There is no explanation required here |  |  |
| A1 | Rearranges to reach $x^{2}-8 x-16 \geqslant 2 x-9$ |  |
| A1* | Reaches the given answer $(x-4)^{2} \geqslant 2 x-9$ with no errors |  |
| ............................................................................................................................................................... |  |  |

(ii)

B1 Shows that it is not true for a value of $n$
This requires a calculation (and value found) with a minimal statement that it is not true
Eg. ' $2^{6}+1=65$ which is not prime' or ' $2^{5}+1=33 \times$ '
Condone sloppily expressed proofs. Eg. ' $2^{7}+1=\frac{\mathbf{1 2 9}}{3}=43$ which is not prime'
Condone implied proofs where candidates write $2^{5}+1=33$ which has a factor of 11
If there are lots of calculations mark positively.
Only one value is required to be found (with the relevant statement) to score the B1
The calculation cannot be incorrect. Eg. $2^{3}+1=10$ which is not prime

\begin{tabular}{|c|c|c|}
\hline Question Number \& Scheme \& Marks \\
\hline 4.(a)

(b) \& \[
$$
\begin{gathered}
\left(2-\frac{1}{4} x\right)^{6}=2^{6},+{ }^{6} \mathrm{C}_{1} 2^{5}\left(-\frac{1}{4} x\right)^{1}+{ }^{6} \mathrm{C}_{2} 2^{4}\left(-\frac{1}{4} x\right)^{2}+{ }^{6} \mathrm{C}_{3} 2^{3}\left(-\frac{1}{4} x\right)^{3}+\ldots \\
=64-48 x+15 x^{2}-2.5 x^{3} \\
\left(2-\frac{1}{4} x\right)^{6}+\left(2+\frac{1}{4} x\right)^{6}=\left(64-48 x+15 x^{2}-2.5 x^{3}\right)+\left(64+48 x+15 x^{2}+2.5 x^{3}\right) \\
\approx 128+30 x^{2}
\end{gathered}
$$

\] \& | B1, M1 |
| :--- |
| A1 A1 |
| (4) |
| M1 |
| B1ft A1 |
| (3) |
| (7 marks) | \\

\hline \multicolumn{3}{|c|}{Notes} \\
\hline \multicolumn{3}{|l|}{(a)} \\
\hline B1 \& ither $2^{6}$ or 64 . Award for an unsimplified ${ }^{6} \mathrm{C}_{0} 2^{6}\left(-\frac{1}{4} x\right)^{0}$ \& \\

\hline M1 \& an attempt at the binomial expansion. Score for a correct attempt at term 2,3 cept sight of ${ }^{6} \mathrm{C}_{1} 2^{5}\left( \pm \frac{1}{4} x\right)^{1}{ }^{6} \mathrm{C}_{2} 2^{4}\left( \pm \frac{1}{4} x\right)^{2}{ }^{6} \mathrm{C}_{3} 2^{3}\left( \pm \frac{1}{4} x\right)^{3}$ condoning om ccept any coefficient appearing from Pascal's triangle. FYI 6, 15, 20 \& | r 4. |
| :--- |
| n of brackets. | \\

\hline \& any two simplified terms of $-48 x+15 x^{2}-2.5 x^{3}$ \& \\

\hline | A1 |
| :--- |
| (b) | \& $64-48 x+15 x^{2}-2.5 x^{3}$ ignoring terms with greater powers. This may be a t fully simplified in (a). Allow the terms to be listed $64,-48 x, 15 x^{2},-2.5 x^{3}$ orrect values. The expression written out without any method can be awar e that this is now marked M1 B1 A1 \& | rded in (b) if it |
| :--- |
| Isw after sight all 4 marks. | \\

\hline M1 \& adding two sequences that must be of the correct form with the correct sign for $\left(A-B x+C x^{2}-D x^{3}\right)+\left(A+B x+C x^{2}+D x^{3}\right)$ but condone \& \\
\hline \multicolumn{3}{|l|}{$\left(A-B x+C x^{2}\right)+\left(A+B x+C x^{2}\right)$} \\
\hline \multicolumn{3}{|l|}{B1ft For one correct term (follow through). Usually $a=128$ but accept either $a=2 \times$ 'their' + ve 64 or $\quad b=2 \times$ 'their' $+v e 15$} \\

\hline  \& \multicolumn{2}{|l|}{\multirow[t]{2}{*}{| For $128+30 x^{2}$. CSO so must be from $\left(64-48 x+15 x^{2}-2.5 x^{3}\right)+\left(64+48 x+15 x^{2}+2.5 x^{3}\right)$ |
| :--- |
| Allow $a=128, b=30$ following correct work. |
| This is a show that question so M1 must be awarded. It must be their final answer so do not isw |}} \\

\hline here. \& \& \\
\hline
\end{tabular}

Alternative method in (a):

$$
\left(2-\frac{1}{4} x\right)^{6}=2^{6}\left(1-\frac{1}{8} x\right)^{6}=2^{6}\left(1+6\left(-\frac{1}{8} x\right)+\frac{6 \times 5}{2}\left(-\frac{1}{8} x\right)^{2}+\frac{6 \times 5 \times 4}{3!}\left(-\frac{1}{8} x\right)^{3}+\ldots\right)
$$

B1 For sight of factor of either $2^{6}$ or 64
M1 For an attempt at the binomial expansion seen in at least one term within the brackets.
Score for a correct attempt at term 2, 3 or 4.

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| Accept sight of $6\left( \pm \frac{1}{8} x\right)^{1} \frac{6 \times 5}{2}\left( \pm \frac{1}{8} x\right)^{2} \frac{6 \times 5 \times 4}{3!}\left( \pm \frac{1}{8} x\right)^{3}$ condoning omission of brackets |  |  |

A1 For any two terms of $64-48 x+15 x^{2}-2.5 x^{3}$
A1 For all four terms $64-48 x+15 x^{2}-2.5 x^{3}$ ignoring terms with greater powers

Attempts to multiply out
B1 For 64
M1 Multiplies out to form $a+b x+c x^{2}+d x^{3}+\ldots$ and gets $b, c$ or $d$ correct.
A1A1 As main scheme

| Question Number | Scheme | Mark |
| :---: | :---: | :---: |
| 5.(a) | $\frac{\mathrm{d} P}{\mathrm{~d} x}=12-\frac{3}{2} x^{\frac{1}{2}}$ <br> Sets $\frac{\mathrm{d} P}{\mathrm{~d} x}=0 \rightarrow 12-\frac{3}{2} x^{\frac{1}{2}}=0 \rightarrow x^{n}=\ldots$ $x=64$ <br> When $x=64 \Rightarrow P=12 \times 64-64^{\frac{3}{2}}-120=\ldots$ <br> Profit $=(£) 136000$ <br> $\left(\frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}\right)=-\frac{3}{4} x^{-\frac{1}{2}}$ and substitutes in their $x=64$ to find its value or state its sign <br> At $x=64 \quad \frac{\mathrm{~d}^{2} P}{\mathrm{~d} x^{2}}=-0.09375<0 \Rightarrow$ maximum | M1A1 <br> dM1 <br> A1 <br> M1 <br> A1 <br> (6) <br> M1 <br> A1 <br> (2) |
| Notes |  |  |
| You should mark parts $a$ and $b$ together. You may see work in (a) from (b) <br> (a) <br> M1 Attempts to differentiate $x^{n} \rightarrow x^{n-1}$ seen at least once. It must be an $x$ term and not the $120 \rightarrow 0$ <br> A1 $\frac{\mathrm{d} P}{\mathrm{~d} x}=12-\frac{3}{2} x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{\mathrm{d} y}{\mathrm{~d} x}$ all of the way through part (a). <br> $\mathrm{dM1}$ Sets their $\frac{\mathrm{d} P}{\mathrm{~d} x}=0$ and proceeds to $x^{n}=k, k>0$. Dependent upon the previous M. Don't be too concerned with the mechanics of process. Condone an attempted solution of $\frac{\mathrm{d} P}{\mathrm{~d} x} \ldots 0$ where ... could be an inequality <br> A1 $x=64$. Condone $x= \pm 64$ here |  |  |
| M1 Substitutes their solution for $\frac{\mathrm{d} P}{\mathrm{~d} x}=0$ into $P$ and attempts to find the value of $P$. <br> The value of $x$ must be positive. If two values of $x$ are found, allow this mark for any attempt using a positive value. |  |  |
| A1 as \$ (b) |  |  |
| M1 <br> A1 <br> maximis | eves $\frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}=k x^{-\frac{1}{2}}$ and attempts to find its value at $x=" 64 "$ rnatively achieves $\frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}=k x^{-\frac{1}{2}}$ and attempts to state its sign. $\operatorname{Eg} \frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}=-\frac{3}{4} x$ $\frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}$ appearing as $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ for the both marks. <br> eves $x=64, \frac{\mathrm{~d}^{2} P}{\mathrm{~d} x^{2}}=-\frac{3}{4} x^{-\frac{1}{2}}$ and states $\frac{\mathrm{d}^{2} P}{\mathrm{~d}^{2}}=-\frac{3}{32}<0($ at $x=64)$ then the | is |

This requires the correct value of $x$, the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.
Alt: Achieves $x=64, \frac{\mathrm{~d}^{2} P}{\mathrm{~d} x^{2}}=-\frac{3}{4} x^{-\frac{1}{2}}$ and states as $x>0$ or $\sqrt{x}>0$ means that $\frac{\mathrm{d}^{2} P}{\mathrm{~d} x^{2}}<0$ then the profit is maximised.

Part (b) merely requires the use of calculus so allow
M1 Attempting to find the value of $\frac{\mathrm{d} P}{\mathrm{~d} x}$ at two values either side, but close to their 64 . Eg. For 64 , allow the lower value to be $63.5 \leqslant x<64$ and the upper value to be $64<x \leqslant 64.5$
A1 Requires correct values, correct calculations with reason and conclusion


A1 Correct quadratic factor $9 x^{2}+12 x+4$.
You may condone division attempts that don't quite work as long as the correct factor is seen.

## Question

Number
Scheme
dM1 Attempt at factorising their $9 x^{2}+12 x+4$ Apply the usual rules for factorising
A1 $(x-3)(3 x+2)^{2}$ or $(x-3)(3 x+2)(3 x+2)$ on one line.
Accept $9(x-3)\left(x+\frac{2}{3}\right)^{2}$ oe. It must be seen as a product
Remember to isw for candidates who go on to give roots $\mathrm{f}(x)=(x-3)(3 x+2)^{2} \Rightarrow x=$

Note: Part (b) is "Hence" so take care when students write down the answer to (b) without method If candidates state $x=-\frac{2}{3}, 3 \Rightarrow \mathrm{f}(x)=\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ score 0000
If candidates state $x=-\frac{2}{3}, 3 \Rightarrow \mathrm{f}(x)=(3 x+2)(3 x+2)(x-3)$ they score SC 1010.
If candidates state $x=-\frac{2}{3}, 3 \Rightarrow \mathrm{f}(x)=9\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ they score SC 1010.
If candidate writes down $\mathrm{f}(x)=(3 x+2)(3 x+2)(x-3)$ with no working they score SC 1010 .
If a candidate writes down $(x-3)(3 x+2)$ are factors it is 0000
(c)

M1 A correct attempt to find one value of $\theta$ in the given range for their $\cos \theta=-\frac{2}{3}$
(You may have to use a calculator). So if (b) is factorised correctly the mark is for one of the values.

This can be implied by sight of awrt 132 or 228 in degrees or awrt 2.3 which is the radian solution.
A1 CSO awrt $\theta=131.8^{\circ}, 228.2^{\circ}$ with no additional solutions within the range $0 \leqslant \theta<360^{\circ}$
Watch for correct solutions appearing from $3 \cos \theta-2=0 \Rightarrow \cos \theta=\frac{2}{3}$. This is M0 A 0
Answers without working are acceptable.
M1 For one correct answer
M1 A1 For two correct answers with no additional solutions within the range.


M1 A correct attempt to find the second term by multiplying 16200 by their ' $r$ ' which must have been found via an allowable method.
Allow $r$ to be found from an "incorrect" GP formula with 10 being used instead of 9. Eg following $31500=16200 r^{10}$ or $\sqrt[10]{\frac{31500}{16200}}$. You may also award, condoning slips, for an attempt at $16200 \times r$ where $r$ is their solution of $31500=16200 r^{n}$ where $n=9$ or 10

A1 For an answer in the range $£ 17440 \leqslant \mathbb{S} \leqslant 17450$
Note that $r=1.077 \Rightarrow 17447.40$
(c)

M1 A correct method to find the sum of either the AP or the GP
For the AP accept an attempt at either $\frac{10}{2}\{16200+31500\}$ or $\frac{10}{2}\left\{2 \times 16200+9 \times^{\prime} d^{\prime}\right\}$
For the GP accept an attempt at either $\frac{16200\left('^{\prime} '^{10}-1\right)}{' r '-1}$ or $\frac{16200\left(1-r^{\prime} '^{10}\right)}{1-r^{\prime}}$
dM1 Both formulae must be attempted "correctly" (see above) and the difference taken (either way around)

FYI if $d$ and $r$ are correct, the sums are $£ 238500$ and $£ 231019$.(24)
A1 Difference $=£ 7480 \quad$ CAO. Note that this answer is found using the unrounded value for $r$. Note that using the rounded value will give $£ 7130$ which is A0

If the solutions for (a) and (b) are reversed, eg GP in (a) and AP in (b) then please send to review.

## (i) General approach to marking part (i) This is now marked M1 A1 M1 A1 on epen

M1 Takes log of both sides and uses the power law. Accept any base. Condone missing brackets
A1 For a correct linear equation in $x$ which only involve logs of base 2 usually $\log _{2} 6, \log _{2} 2$ or $\log _{2} 8$ but sometimes $\log _{2} \frac{3}{4}$ and others so read each solution carefully
M1 Attempts to use a log law to create a linear equation in $\log _{2} 3$
Eg. $\log _{2} 6=\log _{2} 2+\log _{2} 3$ which is implied by $\log _{2} 6=1+\log _{2} 3$
Eg. $\log _{2} \frac{3}{4}=\log _{2} 3-\log _{2} 4$ which may be implied by $\log _{2} 3-2$
A1 For $x=-\frac{1}{3}+\frac{\log _{2} 3}{6}$ oe in the form required by the question. Note that $x=\frac{\log _{2} 3-2}{6}$ is A0


## Question

Number
(ii)

M1 Attempts a correct log law. This may include
$2 \log _{5}(y+1) \rightarrow \log _{5}(y+1)^{2} \quad 1 \rightarrow \log _{5} 5$
You may award this following incorrect work. Eg
$1=2 \log _{5}(y+1)-\log _{5}(7-2 y) \Rightarrow 1=\log _{5} 2(y+1)-\log _{5}(7-2 y) \Rightarrow 1=\log _{5} \frac{2(y+1)}{(7-2 y)}$
dM1 Uses two correct log laws. It may not be awarded following errors (see above)
It is awarded for $2 \log _{5}(y+1)-1=\log _{5} \frac{(y+1)^{2}}{5}, 2 \log _{5}(y+1)-\log _{5}(7-2 y)=\log _{5} \frac{(y+1)^{2}}{(7-2 y)}$
$1+\log _{5}(7-2 y)=\log _{5} 5(7-2 y) \quad$ or $2 \log _{5}(y+1)-1=\log _{5}(y+1)^{2}-\log _{5} 5$
A1 A correct equation in ' $y$ ' not involving logs
ddM1 A correct attempt at finding at least one value of $y$ from a 3 TQ in $y$
All previous M's must have been awarded. It can be awarded for decimal answer(s), 2.4 and -
14.4

A1 $y=-6+\sqrt{70}$ or exact equivalent only.
It cannot be the decimal equivalent but award if the candidate chooses 2.4 following the exact answer. If $y=-6 \pm \sqrt{70}$ then the final A mark is withheld

Special case:
Candidates who write
$\log _{5}(y+1)^{2}-\log _{5}(7-2 y)=1 \Rightarrow \frac{\log _{5}(y+1)^{2}}{\log _{5}(7-2 y)}=1 \Rightarrow \frac{(y+1)^{2}}{(7-2 y)}=5$
can score M1 dM0 A0 ddM1 A1 if they find the correct answer.




| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |

(c)(i)

B1 For sight of 8. Allow this to be scored from a restart, from a calculator or $\ldots=8$
(c)(ii)

M1 This may be awarded in a variety of ways

- A restart (See scheme). For this to be awarded all terms must be integrated with $k \rightarrow k x$ the limits 6 and 2 applied, the linear expression in $k$ must be set equal to 0 and a solution attempted.
- An attempt at solving $\int_{2}^{6} k+13 \mathrm{~d} x=8$ or equivalent. Look for the linear equation $-8+4(13+k)=0$ or $4(13+k)=8$ and a solution attempted.
- Recognising that the curve needs to be moved up 2 units.
- Sight of $\frac{8}{6-2}$ or $-13+2$


A1 $k=-11$. This alone can be awarded both marks as long as no incorrect working is seen.

