Question Number	Scheme	Marks
1.	$\int \left(3x^2-2\right)\left(\dfrac{2}{x^2}+4\right)\mathrm{d}x$	
	$\left(3x^2-2\right)\left(\dfrac{2}{x^2}+4\right)=6+12x^2-4x^{-2}-8$	M1, A1
	$\int \left(3x^2-2\right)\left(\dfrac{2}{x^2}+4\right)\mathrm{d}x=\int\left(12x^2-4x^{-2}-2\right)\mathrm{d}x=4x^3+\dfrac{4}{x}-2x+c$	M1, A1ft, A1
		(5 marks)

Note:Attempts to integrate by parts send to review

M1: Attempts to multiply out and achieves the correct form of $Ax^2+\dfrac{B}{x^2}+\text{constant}\quad A,B\neq 0,$ Condone slips and it may be left unsimplified but with the powers correctly processed. May be implied by further work.

A1: For $12x^2-4x^{-2}-2$ but allow this unsimplified with the terms uncollected. May be implied by further work.

M1: For an attempt to integrate scored for $\int x^n\mathrm{d}x\to x^{n+1},\ n\neq -1,$ on any term in x following an attempt to expand. The index must be processed, i.e. $x^2\to x^3$ and not x^{2+1}.

A1ft: Correct follow through on any **two terms** from $ax^2\to\dfrac{ax^3}{3}\qquad b\to bx\qquad cx^{-2}\to -cx^{-1}$.

The terms **do not have to be collected** but the coefficient for each term must be simplified so $-4\times\dfrac{x^{-1}}{-1}$ would not be one of the valid terms for this mark.
Accept e.g. $"-2"x^1$ for $"-2"x$
Does not need to be an expression for this mark i.e. terms may be on separate lines.
Ignore other incorrect terms for this mark.

A1: $4x^3+\dfrac{4}{x}-2x+c$ or exact simplified equivalent **including the + *c***. So accept $\dfrac{4}{x}\leftrightarrow 4x^{-1}$
Accept e.g. $-2x^1$
Condone spurious notation such as an integral sign at the start or a dx at the end.
Do not ISW if candidates proceed say to multiply by x to get rid of the fractional terms.

Question Number	Scheme	Marks
2.(a)	Uses $t=2,\ H=2.7 \Rightarrow \dfrac{2a+b}{6}=2.7$ $t=7,\ H=4.5 \Rightarrow \dfrac{7a+b}{11}=4.5$ (may be seen as $2a+b=16.2$ AND $7a+b=49.5$)	M1A1
	Solves $7a+b=49.5$, $2a+b=16.2 \Rightarrow a=6.66$ OR $b=2.88$	dM1
	$H=\dfrac{6.66t+2.88}{t+4}$	A1
		(4)
(b)	Sets $t=0,\ H=\dfrac{2.88}{4}=0.72\text{ m}$	M1, A1ft
		(2)
		(6 marks)

Mark (a) and (b) together

(a)

M1: Attempts to use $t=2,\ H=2.7$ **OR** $t=7,\ H=4.5$ in the given model. Does not need to be simplified.

A1: Achieves two correct simultaneous equations in *a* and *b*

$2a+b=16.2$ oe **AND** $7a+b=49.5$ oe

They do not need to be simplified i.e. $\dfrac{2a+b}{6}=2.7$ and $\dfrac{7a+b}{11}=4.5$ scores M1A1

Look out for attempts where one correct equation is formed and they use the other information to directly substitute into this equation which can imply this mark.

e.g. $\dfrac{2a+(49.5-7a)}{6}=2.7$ would score M1A1

dM1: Solves their simultaneous equations leading to values of *a* and *b*. It is dependent on the previous method mark.

Allow a solution via a calculator – you do not need to check this but **do not** allow $\dfrac{2a+b}{6}=2.7,\ \dfrac{7a+b}{11}=4.5 \Rightarrow a=...,\ b=...$ without rearranging to simultaneous equations of the form $Pa+Qb=r$ first.

Note they can proceed from an equation in one variable $\dfrac{2a+(49.5-7a)}{6}=2.7 \Rightarrow a=...,\ b=...$

A1: $H=\dfrac{6.66t+2.88}{t+4}$ Condone *a* and *b* to be written as fractions e.g. $H=\dfrac{\frac{333}{50}t+\frac{72}{25}}{t+4}$.

Can only be scored provided all previous marks have been scored.

(b)

M1: Sets $t=0$ leading to a value for *H*. You may need to check this if just a value is stated. i.e. $\dfrac{"2.88"}{4}$

A1ft: States 0.72m or 72 cm **including units** (condone a correct fraction for 0.72)

Follow through on their *b* provided $0<H<2.7$

Question Number	Scheme	Marks
3(a)	Uses $\frac{\sin\theta°}{12}=\frac{\sin 32°}{8} \Rightarrow \sin\theta° = ...$	M1
	θ=awrt 52.64, awrt 127.36	A1, A1
		(3)
(b)	$\left(BC^2\right)=12^2+8^2-2\times12\times8\times\cos\left(180°-32°-"127.36"°\right)$	M1
	$\Rightarrow BC =$ awrt 5.32 (cm)	A1
		(2)
(c)	$A=\frac{1}{2}ab\sin C \Rightarrow A=\frac{1}{2}\times12\times"5.32"\times\sin 32° = \text{awrt}\,16.9\,(\text{cm}^2)$	M1, A1
		(2)
		(7 marks)

Note that attempts working in radians can score a maximum of M1A0A0 M1A1 M1A1 provided there is not a mixture of angle units within the same equation or expression

(a)

M1: Correct application of the sine rule $\frac{\sin\theta°}{12}=\frac{\sin 32°}{8} \Rightarrow \sin\theta° = ...$ Look for the angles and lengths to be in the correct positions within the formula but condone slips in rearranging. It cannot be implied by a correct angle (M0A0A0) but allow a correct angle to imply this mark following a correct equation.

A1: awrt 53 or awrt 127 Must be in degrees. M1 must have been scored.

A1: θ=awrt 52.64, awrt 127.36 Must be in degrees and no other angles. Must be seen in (a).

In parts (b) and (c) if their obtuse angle for angle *CBA* is (180-32-their acute angle from (a)) (=“95.36”) then (b)M0A0 (c) M0A0

(b) If they use angle *CBA* in part (b) it must be obtuse, which must be 180-their acute angle from (a) whether it is found in part (a) or part (b)

M1: For a full attempt to find BC or BC^2 which may be achieved by:

- an attempt to use the cosine rule (condone the omission of the lhs) to find BC or BC^2 with the sides and angle in the correct places.
- an attempt to use the sine rule with the sides and angles in the correct place.
  For example, $\frac{BC}{\sin(180-32-"127.4")°}=\frac{8}{\sin 32°} \Rightarrow BC = ...$
- an attempt to form a 3TQ in BC and solves to find a value for BC
  Look for e.g. $8^2 = BC^2+12^2-2\times12\times BC\times\cos 32 \Rightarrow BC^2-(24\cos 32)BC+80=0 \Rightarrow BC = ...$
  with the sides and angle in the correct places.
  You do not need to check the mechanics of the rearrangement provided a 3TQ is achieved and they proceed to find a root. You do not need to check this.

A1: $BC =$ awrt 5.32 (cm) Ignore units (Note via the quadratic in BC they would need to reject the other root if found)

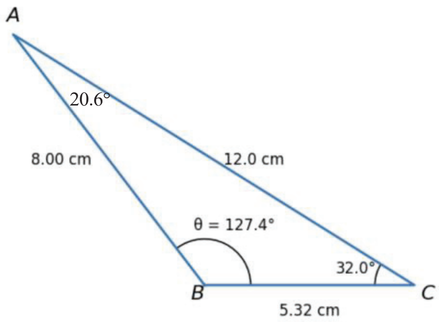
(c) Condone using their acute angle for angle *CBA* in this part

M1: Full attempt to find the area using a correct combination of sides and angles

Score for $A=\frac{1}{2}\times12\times'5.32'\times\sin32°$ or $A=\frac{1}{2}\times12\times8\times\sin\left(180°-32°-'127.4'\right)$

A1: awrt 16.9 (cm^2) Ignore units. Allow this to be scored if their length *BC* is awrt 5.3

The following diagram may help



Question Number	Scheme	Marks
4.(i)	Writes $\sqrt{128}$ as $2^{\frac{7}{2}}$ or 16 as 2^4	B1
	Attempts both powers of 2 and uses subtraction law $\Rightarrow \left(2^{3p-5}=\right) 2^{\frac{7}{2}-4}$	M1
	$\Rightarrow 3p-5=-\frac{1}{2} \Rightarrow p=\frac{3}{2}$	A1
		(3)
(ii)	$\left(x^2-2\right)^2 = x^2+18 \Rightarrow x^4-4x^2+4=x^2+18$	
	$\Rightarrow x^4-5x^2-14=0$	M1, A1
	$\Rightarrow \left(x^2-7\right)\left(x^2+2\right)=0 \Rightarrow x^2=...$	dM1
	$\Rightarrow x=\pm\sqrt{7}$	A1
		(4)
		(7 marks)

(i) Note B0M1A1 is possible
Note that they may rearrange the equation first. In such cases both the lhs and rhs will need to be written as powers of 2 (or e.g. 4)

B1: Correctly writes (not implied) either

- $\sqrt{128}$ as $2^{\frac{7}{2}}$ or $8\sqrt{2}$
- 16 as 2^4 (or may see $\frac{1}{16}$ written as 2^{-4})

Note they may write these numbers as e.g. $4^{\cdots}$ which is an alternative method

M1: Proceeds to write $\frac{1}{16}\times\sqrt{128}$ as 2^n. The index does not need to be evaluated for this mark but must be correct e.g. $2^{\frac{7}{2}-4}$. This cannot be implied by $(3p-5=)-\frac{1}{2}$ but condone e.g. $3p-5=-4+\frac{7}{2}$ which shows they have added the powers and not relied on the use of a calculator.
May alternatively attempt to write both sides as powers of another number e.g. 4 or 8 which is acceptable.
Ignore the use of logs provided M1 is scored.

A1: $p=\frac{3}{2}$ oe provided M1 has been scored. If a calculator has clearly been relied on to solve then send to review.

(ii) Note they may work in another variable which is acceptable

M1: Multiplies out and simplifies to a 3TQ equation in x^2. The constant may be left on the other side of the equation. If terms are collected on one side then condone the omission of = 0

A1: Correct 3TQ equation in x^2 The constant may be left on the other side of the equation. The = 0 may be implied by further work.

dM1: Attempts to solve their 3TQ equation in x^2 by either factorisation, the formula or completing the square. See usual rules for solving a quadratic. It cannot be for just stating the roots via a calculator. It is dependent on the previous method mark.

A1: $x = \pm\sqrt{7}$ and no other solutions

Question Number	Scheme	Marks
5 (a)	$y = x^3 + 2x + 3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2$ Gradient at $x = 2$ is $3 \times 2^2 + 2 = 14$	M1, A1 **(2)**
(b)	$y_Q = (2+h)^3 + 2(2+h) + 3 = 15 + 14h + 6h^2 + h^3$ Gradient $PQ =$ $\frac{y_Q - y_P}{x_Q - x_P} = \frac{(15 + 14h + 6h^2 + h^3) - 15}{2 + h - 2} = \frac{h^3 + 6h^2 + 14h}{h} = h^2 + 6h + 14$	M1, A1 dM1, A1 **(4)**
(c)	States as $h \to 0$ gradient of PQ tends to gradient of tangent	B1 **(1)** **(7 marks)**

(a) Note that no algebraic differentiation seen scores M0A0

M1: Attempts to substitute $x = 2$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = ax^2 + b$. It can only be implied by 14 provided a correct derivative is seen.

A1: 14 provided M1 is scored. Minimum acceptable e.g. $3x^2 + 2 = 14$ Do not isw if e.g. they change 14 to $-\frac{1}{14}$

(b)

M1: Attempts to substitute $2+h$ into $x^3 + 2x + 3$ and expand so that all brackets have been removed. Condone slips in expanding and any miscopying slips provided the intention is clear.

A1: $15 + 14h + 6h^2 + h^3$ but allow if the terms are not collected. May be implied by further work.

dM1: Dependent on the previous method mark.

Scored for attempting $\frac{y_Q - y_P}{x_Q - x_P} = \frac{('15 + 14h + 6h^2 + h^3') - 15}{2 + h - 2}$ oe i.e. they must be subtracting the correct way round consistently for both the numerator and denominator.

The expression is sufficient and condone invisible brackets provided the intention is clear. May be implied by further work i.e. if all terms are divided by h already.

A1: $h^2 + 6h + 14$

Alt (b) Simultaneous equations

M1: Attempts the straight line equation $(2+h)^3+2(2+h)+3=(2+h)m+c$ and expands at least the left hand side so that all of the brackets have been removed. Condone slips in expanding and any miscopying slips provided the intention is clear.

A1: $15+14h+6h^2+h^3\left(=(2+h)m+c\right)$ (may be implied)

dM1: Attempts to solve simultaneously with the equation $15=2m+c$ and proceeds to find an expression for m

Look for $15+14h+6h^2+h^3=2m+hm+c \Rightarrow 15+14h+6h^2+h^3-2m-hm=15-2m$
$\Rightarrow 14h+6h^2+h^3=hm \Rightarrow m=...$

A1: $h^2+6h+14$

(c)

B1: States as $h \to 0$ oe (e.g. as h tends to 0) gradient of PQ tends to gradient of tangent oe
May say e.g. "as h tends to 0 the gradients will be the same".

Condone any sort of reference of tending towards 0 e.g. close to / near to / approaching / almost etc

Do not allow this mark for a reference of h being equal to 0 but condone use of equal/same for the relationship between the answers to part (a) and part (b) instead of referring to the gradient of PQ tending towards the gradient of the tangent at P.
May use other notation or words for gradient which is acceptable.

Question Number	Scheme	Marks
6 (a)	$\sin BOA = \dfrac{7.3}{9} \Rightarrow BOA = 0.946$	M1, A1
		(2)
(b)	Attempts e.g. $\frac{1}{2}\times 9^2 \times(\pi + '0.946') = ...$ OR e.g. $\frac{1}{2}\times 7.3\times\sqrt{9^2-7.3^2}$	M1
	Attempts e.g. $\frac{1}{2}\times 9^2 \times(\pi + '0.946') - \frac{1}{2}\times 7.3\times\sqrt{9^2-7.3^2} = ...$	dM1
	$= 146\ (\text{cm}^2)$	A1
		(3)
(c)	Attempts e.g. $9\times(\pi + '0.946') = ...$ OR $9+7.3+\sqrt{9^2-7.3^2}$	M1
	Attempts e.g. $9\times(\pi + '0.946') + 9 + 7.3 + \sqrt{9^2-7.3^2} = ...$	dM1
	$= 58.4\ (\text{cm})$	A1
		(3)
		(8 marks)

Work must be seen or used in the correct part to score

(a)

M1: A correct method to find the size of angle BOA. This mark can be scored for a correct equation which does not need to be rearranged. e.g. $\sin BOA = \dfrac{7.3}{9}$ or $\dfrac{9}{\sin\left(\frac{\pi}{2}\right)} = \dfrac{7.3}{\sin BOA}$ scores M1 Allow to work in degrees for this mark i.e. may see $\sin 90°$ instead.

Alternatively uses e.g. Pythagoras' theorem with the lengths in the correct places (but condone slips in rearranging) and forms a different correct equation using another trigonometric identity

A1: awrt 0.946

Note in (b) and (c) they may attempt to find areas or lengths and subtract from the area of the circle / the circumference of the circle.

(b)

M1: Attempts to find the area of sector $DOBCD$ **OR** the area of triangle OAB

For sector $DOBCD$ look for e.g. $\frac{1}{2}\times 9^2 \times(\pi + "0.946") = ...$ May be implied by awrt 166

May split the sector into sections e.g. a semi-circle and a smaller sector. They need to combine these together correctly for this mark. If just a value is stated you may need to check this on your calculator.

For area of triangle OAB look for e.g. $\frac{1}{2}\times 7.3\times\sqrt{9^2-7.3^2}$, $\frac{1}{2}\times 7.3\times 9\times\sin\left(\frac{\pi}{2} - "0.946"\right)$ May be implied by awrt 19.2

dM1: Attempts both areas **AND** subtracts the correct way round. It is dependent on the previous method mark. If just a value is stated you may need to check this on your calculator.

Note there are a variety of different approaches to finding these areas so the overall method should be correct but allow for slips in processing.

A1: awrt 146

(c)

M1: Attempts to find the length of arc *DCB* **OR** the combined length of *DA*+*AB*

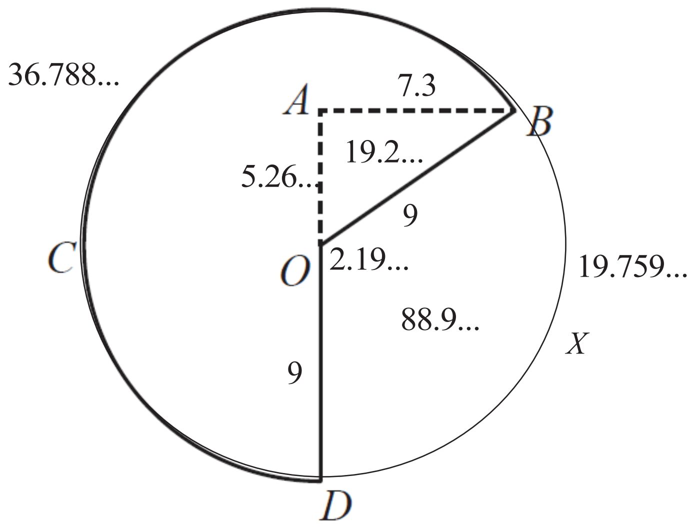
For length of arc *DCB* look for e.g. $9\times(\pi+'0.946')=...$ May be implied by awrt 37.

May split the arc into sections e.g. a semi-circle and a smaller arc. They need to combine these together correctly for this mark. e.g. may use the length of arc *DXB* and subtracting from the circumference. If just a value is stated you may need to check this on your calculator.

For the length of *DA*+*AB* look for e.g. $9+7.3+\sqrt{9^2-7.3^2}$

dM1: Attempts all the required lengths and adds together. It is dependent on the previous method mark. Note there are a variety of different approaches to finding these lengths so the overall method should be correct but allowing for slips in processing. If just a value is stated you may need to check this on your calculator.

A1: awrt 58.4 (**Note that the perimeter of Figure 2 is 54.8**)



Question Number	Scheme	Marks
7 (a) (i)	States that gradient normal is -2 or gradient tangent is $\frac{1}{2}$	B1
	Substitutes $x=4$ into $\frac{1}{2}=\frac{k-\sqrt{x}}{2x^2} \Rightarrow \frac{k-2}{32}=\frac{1}{2} \Rightarrow k = 18$ *	M1, A1*
(ii)	(Substitutes $x=4$ in $y+2x-5=0 \Rightarrow$) $y=-3$	B1
	Method of tangent $y+3=\frac{1}{2}(x-4) \Rightarrow y=\frac{1}{2}x-5$	M1, A1
		(6)
(b)	$\mathrm{f}'(x)=\frac{18-\sqrt{x}}{2x^2}=9x^{-2}-\frac{1}{2}x^{-\frac{3}{2}} \Rightarrow \quad \mathrm{f}(x)=-\frac{9}{x}+x^{-\frac{1}{2}}(+c)$	M1, A1, A1
	Uses $P(4,\text{'}-3\text{'}) \Rightarrow \text{'}-3\text{'}=-\frac{9}{4}+\frac{1}{2}+c \Rightarrow c=\ldots$	dM1
	$(\mathrm{f}(x)=)-\frac{9}{x}+x^{-\frac{1}{2}}-\frac{5}{4}$	A1
		(5) **(11 marks)**

(a)(i) Attempts to substitute $k=18$ and $x=4$ into $\mathrm{f}'(x)$ send to review

B1: States that gradient normal is -2 or gradient tangent is $\frac{1}{2}$. Alternatively, may be seen in a **correct** equation provided it is explicitly seen e.g. $\frac{1}{2}=\frac{k-2}{2\times 4^2}$ or $-2=\frac{32}{\sqrt{4}-k}$

It is insufficient to just state e.g. gradient / $\frac{dy}{dx}=-2$ or hence gradient $=\frac{1}{2}$ as these are too vague.

M1: Substitutes $x=4$ into $\frac{1}{2}=\frac{k-\sqrt{x}}{2x^2}$ oe and proceeds to find a value for k. Condone slips in the rearrangement or slips when substituting in/evaluating provided the overall method was correct. Accept $16=k-2 \Rightarrow k=\ldots$ to score this mark.

A1*: $k=18$ with both previous marks scored and no errors seen. **Must be seen in (a)(i)**
Minimum acceptable:
e.g. $\frac{1}{2}=\frac{k-2}{32} \Rightarrow k=18$ scores B1M1A1* ($\frac{1}{2}$ seen within a correct equation proceeding to $k=18$)
e.g. $m=\frac{1}{2}$ $\quad k-2=16 \Rightarrow k=18$ scores B1M1A1* (separately states $\frac{1}{2}$ and forms a valid equation proceeding to $k=18$)
But $16=k-2 \Rightarrow k=18$ scores B0M1A0* (never states $\frac{1}{2}$ or -2)

(a)(ii)

B1: $y=-3$ (may be seen or used. Allow if seen in (a)(i)) May be implied if they equate the equation of the normal and the equation of the tangent and substitute in $x=4$ to find c.

M1: Attempts to find the equation of the tangent using a gradient of $\pm\frac{1}{2}$ and point $(4, ...)$

E.g. $y+3=\frac{1}{2}(x-4)$. If the form $y=mx+c$ is used the method must proceed as far as $c = \ldots$

Do not condone sign slips when substituting in the coordinates

A1: $y=\frac{1}{2}x-5$ oe in the form $y=mx+c$

(b) Note that if they only work in terms of *k* instead of 18 then max score M1A1A0dM0A0
If creditworthy work is seen in (a) it must be used in (b) to score.

M1: Attempts to split the fraction into two terms, and attempts to integrate at least one of the terms achieving a correct index. Indices do not need to be processed for this mark. Condone the omission of the $+c$

Look for $\left(\mathrm{f}'(x)=\right)\frac{18-\sqrt{x}}{2x^2}=$ at least one of $...x^{-2}\pm...x^{-\frac{3}{2}}$ integrating to at least one of

$\Rightarrow \left(\mathrm{f}(x)=\right)...x^{-2+1}\pm...x^{-\frac{1}{2}+1}\ (+c)$

Terms may be listed separately which is fine.

A1: One correct term of $\left(\mathrm{f}(x)=\right)-\frac{9}{x}+x^{-\frac{1}{2}}$ (which may be unsimplified). Terms may be listed separately which is fine as you just need to look for one correct term. The index must be processed.

A1: $\left(\mathrm{f}(x)=\right)-\frac{9}{x}+x^{-\frac{1}{2}}\ (+c)$ (which may be unsimplified) but condone the omission of $+c$ Must be an expression for this mark. Indices must be processed.

dM1: Uses $P(4, '-3')$ in their integrated function (it is dependent on the previous method mark) and proceeds to find a value for c. You do not need to check the mechanics of the rearrangement for this mark. Alternatively states e.g. $x=4,\ y='-3'$ and states a value for c. You may only have to check the value of c if neither the method is shown substituting in the values nor the values stated.

A1: $\left(\mathrm{f}(x)=\right)-\frac{9}{x}+x^{-\frac{1}{2}}-\frac{5}{4}$ or exact equivalent in simplest form. (fractions must be cancelled down). Do not isw if they e.g. proceed to multiply all terms by 4

Question Number	Scheme	Marks
8 (a)	$x \geqslant 5,\ x = -4$	M1, A1 **(2)**
(b)	States or implies that $\mathrm{f}(x) = k(\pm x \pm 4)^2(\pm x \pm 5)$ oe	M1
	e.g. substitutes $(0, 20)$ into $\mathrm{f}(x) = k(x+4)^2(x-5) \Rightarrow k = \ldots$	dM1
	$(\mathrm{f}(x) =) -\frac{1}{4}(x+4)^2(x-5)$ or $(\mathrm{f}(x) =)\ \frac{1}{4}(x+4)^2(5-x)$	A1 **(3)**
(c)	$x = 5$	B1
	$-\frac{1}{4}(x+4)^2(x-5) = 3x(x-5) \Rightarrow (x-5)\left\{3x + \frac{1}{4}(x+4)^2\right\} = 0$	
	$\Rightarrow (x-5)(x^2 + 20x + 16) = 0$	M1
	$\Rightarrow (x+10)^2 = 84 \Rightarrow x = \ldots$	dM1
	$x = -10 \pm 2\sqrt{21}$	A1 **(4)**
		(9 marks)

(a) Allow equivalent notation including set or interval notation

M1: For either $x \geqslant 5$ or $x = -4$. May be seen as e.g. $[5, \infty)$ or $\{x \in \mathbb{R} : x \geqslant 5\}$ or condone $x > 5$, or $(5, \infty)$ Allow variations on these as long as the intention is clear.

A1: $x \geqslant 5,\ x = -4$ oe Do not withhold this mark for use of 'and' oe and again allow variations on notation as long as the intention is clear and the inequality $x \geqslant 5$ is correct

(b)

M1: States or implies that $\mathrm{f}(x) = k(\pm x \pm 4)^2(\pm x \pm 5)$ even allowing $k = \pm 1$

dM1: Substitutes $(0, 20)$ into $\mathrm{f}(x) = k(x+4)^2(x-5) \Rightarrow k = \ldots$ or $\mathrm{f}(x) = k(x+4)^2(5-x) \Rightarrow k = \ldots$ It is dependent on the previous method mark. May use a different letter to k

A1: $(\mathrm{f}(x) =) -\frac{1}{4}(x+4)^2(x-5)$ oe such as $(\mathrm{f}(x) =) \frac{1}{4}(x+4)^2(5-x)$

Alt(b) Simultaneous equations $y = ax^3 + bx^2 + cx + d$

M1: Attempts to form 3 equations from

$(-4, 0) \Rightarrow 0 = -64a + 16b - 4c + d$

$(5, 0) \Rightarrow 0 = 125a + 25b + 5c + d$

$(0, 20) \Rightarrow d = 20$

$(-4, 0) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3ax^2 + 2bx + c \Rightarrow 48a - 8b + c = 0$

dM1: Forms all 4 equations and solves simultaneously to achieve values for *a*, *b*, *c* and *d*. It cannot be scored for just stating the values via a calculator.

A1: As above in main scheme and notes

(c) Note that B0M1dM1A1 is possible

B1: $x=5$ seen in part (c)

M1dM1A1 can only be scored if they have a valid answer to part (b) which is of the form $f(x)=k(x\pm4)^2(x-5)$

M1: Sets their $-\frac{1}{4}(x+4)^2(x-5)=3x(x-5)$ and cancels by $(x-5)$ to set up a 3TQ equation or factorises out $(x-5)$ proceeding to e.g. $(x-5)(x^2+20x+16)=0$ You do not need to be concerned with the mechanics of the rearrangement but the brackets should be expanded and terms collected.

dM1: Correct method of solving their quadratic factor via factorisation, completing the square or the formula (usual rules apply). If the quadratic formula is correctly stated there still has to be evidence of substituting in the values for *a*, *b* and *c* before stating the roots. It is dependent on the previous method mark.
This mark cannot be scored for just stating the roots via a calculator which is dM0A0.

A1: $-10\pm2\sqrt{21}$ following M1dM1 isw if they achieve $\frac{-20\pm\sqrt{336}}{2}$ and then make errors simplifying.

Alternative method – using the factor theorem

e.g. $-\frac{1}{4}(x+4)^2(x-5)=3x(x-5)\Rightarrow x^3+15x^2-84x-80=0\Rightarrow(x-5)(x^2+20x+16)=0$

$$\Rightarrow x=5,\ x=\frac{-20\pm\sqrt{20^2-4\times1\times16}}{2\times1}=-10\pm2\sqrt{21}$$

B1: $x=5$ seen in part (c)

M1: After reaching a cubic form, attempts to factorise out (or divide by) the $(x-5)$ term leading to a 3TQ equation. They cannot just write down the solutions of their cubic from the calculator.

dM1: As above in main scheme notes

A1: As above in main scheme notes

Question Number	Scheme	Marks
9. (a)	20	B1 **(1)**
(b)	$(160°, -1)$	B1 **(1)**
(c)	$x = 3670°, 3850°$	B1, dB1 **(2)**
(d)(i)	$(160°, 4)$	B1ft
(ii)	$(240°, -1)$	B1ft **(2)**
		(6 marks)

Must be in degrees but condone lack of degree symbols throughout.

(a)
B1: 20 (allow if embedded in the equation)

(b)
B1: $(160°, -1)$ Condone missing brackets (one or both). May be written separately $x = 160°,\ y = -1$

(c)

B1: One of $3670°, 3850°$

dB1: Both $3670°, 3850°$ and no others. Cannot be scored without the previous mark being scored.

(d) (i)
B1ft: $(160°, 4)$ OR follow through on their answer to (b). So, accept $(a, b) \to (a, -4b)$ even where a may appear to be an angle in radians as we are looking for y-coordinate to change. Condone missing brackets (one or both). May be written separately e.g. $x = 160°,\ y = 4$

(d)(ii)
B1ft: $(240°, -1)$ OR follow through on their answer to (a). So, accept $(180 + 3k, -1)$ with their k substituted in. Condone missing brackets (one or both).
May be written separately e.g. $x = 240°,\ y = -1$

Special Case: All 3 of (b) $(-1,\ 160°)$ (d)(i) $(4, 160°)$ (d)(ii) $(-1, 240°)$ score (b) B0 (d)(i) B1 (d)(ii) B1

Question Number	Scheme	Marks
10 (a)	Substitute $y=2x+k$ fully into $x^2+2y^2-6x+y-1=0$	
	$\Rightarrow x^2+2(2x+k)^2-6x+2x+k-1=0$	M1
	$\Rightarrow 9x^2+(8k-4)x+2k^2+k-1=0$	A1
	l touches C when $'b^2-4ac' \Rightarrow (8k-4)^2-4\times9\times(2k^2+k-1)$	dM1
	$8k^2+100k-52=0 \Rightarrow 2k^2+25k-13=0$ *	A1*
		(4)
(b)	$2k^2+25k-13=0 \Rightarrow k=\left(\frac{1}{2},\right)-13$	B1
	When $k=-13 \Rightarrow 9x^2-108x+324=0$	M1
	$\Rightarrow x^2-12x+36=0 \Rightarrow (x-6)^2=0 \Rightarrow x=6$	A1
	Substitutes $k=-13, x=6$ in $y=2x+k \Rightarrow y=...$	dM1
	$P(6,-1)$	A1
		(5)
		(9 marks)

(a)

M1: Substitutes $y=2x+k$ fully into $x^2+2y^2-6x+y-1=0$

Alternatively substitutes $x=\dfrac{y-k}{2}$ fully into $x^2+2y^2-6x+y-1=0$

Condone slips provided an equation in x or y and k only is achieved. Condone the omission of $=0$

e.g. $x^2+2(2x+k)^2-6x+2x+k-1=0$

A1: Correct simplified equation in x or y and k only. The two terms in x (or the two terms in y) must be collected. The $=0$ and the equation may be implied by further work i.e. their attempt at the discriminant.

$9x^2+(8k-4)x+2k^2+k-1=0$ or or $9y^2+(-2k-8)y+k^2+12k-4=0$

For the x or y term they may take a factor out e.g. $9x^2+2(4k-2)x+2k^2+k-1=0$ which is fine.

dM1: Attempts $'b^2-4ac'$ for their quadratic in x or y with b and c expressions in k. It is dependent on the previous method mark. Ignore any inequality or the presence or absence of $=0$ for this mark.

A1*: Proceeds to the given answer of $2k^2+25k-13=0$ with all previous marks scored and no errors seen. They must have multiplied out all the brackets after attempting the discriminant before proceeding to the given quadratic. Condone invisible brackets to be recovered. The $=0$ must have appeared before they usually divide through by 4.

(b)

B1: Achieves at least $k=-13$ Ignore any other solutions/ do not withhold this mark but if it is rejected then B0

M1: Substitutes their negative k into their equation in (a) (may restart) to form a 3TQ in x only or y only. Condone the omission of $=0$. Condone miscopying the equation provided the intention is clear.

s

A1: Achieves $x=6$ or $y=-1$ following a correct 3TQ in x ($x^2-12x+36=0$ oe) or y $9y^2+18y+9=0$ oe

dM1: Substitutes $k=-13, x="6"$ in $y=2x+k \Rightarrow y=...$ It is dependent on the previous method mark.

Alternatively substitutes $k=-13, y="-1"$ in $x=\frac{y-k}{2} \Rightarrow x=...$ You may need to check on your calculator if just a value is stated.

A1: $(6,-1)$ Condone omission of brackets. May be written separately $x=6,\ y=-1$
Additional solutions is A0