Question Number	Scheme	Marks
	$y = 4x^3 + \frac{3}{2\sqrt{x}} - 8$	
1.(a)	$\left(\frac{dy}{dx}\right) = 12x^2 - \frac{3}{4}x^{-\frac{3}{2}}$	M1, A1, A1 **(3)**
(b)	$\int 4x^3 + \frac{3}{2\sqrt{x}} - 8\,dx = x^4 + 3x^{\frac{1}{2}} - 8x + c$	M1, A1, A1 **(3)** **(6 marks)**

Advice for examiners;

If labelled (a), (b) or even (i) and (ii) mark according to the scheme as below.

If it is obvious, i.e marked $\frac{dy}{dx} = ..$ and $\int$ = mark according to the scheme as given no matter what the order

If there are no labels, mark the first answer as their attempt at $\frac{dy}{dx}$ and the second as their attempt at $\int$

If you are not sure then please use review

(a)

M1: Correct attempt at differentiation.

Award for one correct index which must be processed ie $x^3 \to x^2$ (not x^{3-1}) including $-8 \to 0$

A1: One correct and simplified term in x. So, award for sight of $12x^2$ or $-\frac{3}{4}x^{-\frac{3}{2}}$ (or exact equivalent).

A1: $\left(\frac{dy}{dx}\right) = 12x^2 - \frac{3}{4}x^{-\frac{3}{2}}$ or exact simplified equivalent such as $12x^2 - \frac{3}{4x^{\frac{3}{2}}}$ Condone $12x^2 + -\frac{3}{4}x^{-\frac{3}{2}}$.

There must be no extra terms such as + c. ISW after correct answer. There is no need to have the $\frac{dy}{dx}$

(b)

M1: Correct attempt at integration.

Award for achieving one correct index which must be processed.

Look for one of $x^3 \to x^4$, $x^{-\frac{1}{2}} \to x^{\frac{1}{2}}$ or $-8 \to -8x$

A1: Two correct and simplified terms in x

A1: $x^4 + 3x^{\frac{1}{2}} - 8x + c$ or simplified equivalent such as $1x^4 + 3\sqrt{x} - 8x^1 + c$ including the + c

Condone extra spurious notation, e.g. $\int x^4 + 3x^{\frac{1}{2}} - 8x + c$. ISW after correct answer.

Question Number	Scheme	Marks
2.(a)	Attempts to substitute $y=x+4$ into $2x^2-xy=8 \Rightarrow 2x^2-x(x+4)=8$	M1
	$\Rightarrow 2x^2-x^2-4x=8 \Rightarrow x^2-4x-8=0$ *	A1*
		(2)
(b)	$x^2-4x-8=0 \Rightarrow (x-2)^2=12$	M1
	$\Rightarrow x=2\pm2\sqrt{3}$	A1
	Attempts y for either value $\Rightarrow y=2+2\sqrt{3}+4=...$	M1
	$\Rightarrow x=2+2\sqrt{3}, y=6+2\sqrt{3}$ AND $x=2-2\sqrt{3}, y=6-2\sqrt{3}$	A1
		(4)
		(6 marks)

(a)

M1: Awarded for an attempt to substitute $y=\pm x\pm4$ into $2x^2-xy=8 \Rightarrow 2x^2-x(\pm x\pm4)=8$

This may be seen with or without the bracket so look for either $2x^2-x(\pm x\pm4)=8$ or $2x^2\pm x^2\pm4x=8$

A1*: Proceeds to the given answer showing all necessary steps and no errors.
For example, look for intermediate working showing ALL of the following

- $y-x=4$ being adapted to $y=x+4$
- A line of $2x^2-x(x+4)=8$ or equivalent such as $2x^2-x^2-4x-8=0$
- The given answer $x^2-4x-8=0$

(b)

M1: Shows a non-calculator method of solving the given $x^2-4x-8=0$, not any incorrect version.
Do not accept factorisation.
For completing the square look for $x^2-4x-8=0 \Rightarrow (x-2)^2=k, k>0 \Rightarrow x=...$

For use of formula look, for example, for $x^2-4x-8=0 \Rightarrow x=\dfrac{4\pm\sqrt{16-(-32)}}{2}$ or $x=\dfrac{4\pm\sqrt{48}}{2}$

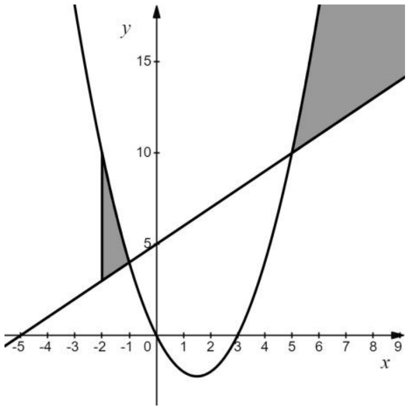
You cannot accept the correct simplified solutions following a quoted formula (with no additional working shown, i.e embedded values).

A correct quoted formula followed by an unsimplified solution with a slip E.g. $x=\dfrac{-4\pm\sqrt{(-4)^2-(-32)}}{2}$ or $x=\dfrac{-4\pm\sqrt{48}}{2}$ would score M1, A0 even if followed by the correct solutions and then potentially M1, A0

A1: Correctly finds both x values in an allowable form. Look for $2+2\sqrt{3}$ and $2-2\sqrt{3}$. Condone values written in the partially simplified form of $x=2\pm\sqrt{12}$ but not $x=\dfrac{4\pm\sqrt{48}}{2}$. Award this mark even if one value is subsequently rejected. **This can only be scored following the award of M1**

M1: Find a y value by adding 4 to an x value.
It is not dependent so 0,0,1,0 is possible for candidates who write down solutions for the equation in x. So $x^2-4x-8=0 \Rightarrow x=2\pm2\sqrt{3}$ without working followed by $y=6\pm2\sqrt{3}$ scores 0, 0, 1, 0

A1: Both pairs correctly matched up, and in the form requested by the question, i.e $p+q\sqrt{3}$
Allow $x=2\pm2\sqrt{3}, y=6\pm2\sqrt{3}$ Must follow M1, A1, M1
Note that a pair is matched up when you see $x=2+2\sqrt{3}, y=4+2+2\sqrt{3}=6+2\sqrt{3}$

Question Number	Scheme	Marks
3 (i)	$z<0$ or $z>\frac{1}{2}$	M1, A1
		(2)
(ii)		
	Correctly draws $x=-2$	B1
	Correct left or right-hand region shaded	B1
	Correct left and right-hand region shaded	B1
		(3)
		(5 marks)

(i)

M1: For an attempt, graphical, algebraic or otherwise that reaches either inequality.

Score for one of $z<0, z>\frac{1}{2}$ Allow non-strict inequalities here. **No working is necessary**

Award even in cases where the solution(s) seem contradictory. E.g. $z>\frac{1}{2}, z<\frac{1}{2}$

Condone an un-simplified version for this mark. E.g. $\frac{4}{z}<8 \Rightarrow z>\frac{4}{8}$

Alternatively score for an attempt that proceeds to **both** critical values

A1: Correct answer which must be in simplest form.

Condone the appearance of a comma or the word "and" between the two inequalities

If set notation is used accept versions such as $\{z\in\mathbb{R}: z<0\}\cup\left\{z\in\mathbb{R}: z>\frac{1}{2}\right\}$ but not with $\cap$

If they write $0>z>\frac{1}{2}$ would score M1, A0

(ii)

Advice for examiners:

You may see the representation of the inequalities on Figure 1 and/or Diagram 1.

- If only Figure 1 is used mark Figure 1
- If both Figure 1 and Diagram 1 are used Diagram 1 takes precedence.

B1: Correctly draws the line with equation $x=-2$ (at least drawn between the line and curve)

Allow this line to be dotted.

Alternatively has $x=-2$ as the boundary of a shaded region (at least drawn between the line and curve)

Be tolerant of slips of the pen here or gaps in the boundary of the shaded region

B1: Shading implies a correct and distinct left-hand **OR** right-hand region shaded.

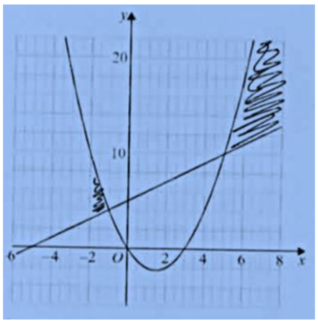
For the left-hand region, it is acceptable for the boundary line $x=-2$ to be 'dotted' or implied (by shading)

B1: Shading implies a correct and distinct left-hand and a correct and distinct right-hand region ONLY.

There must be an attempt to shade/include the whole of the left and right-hand region.

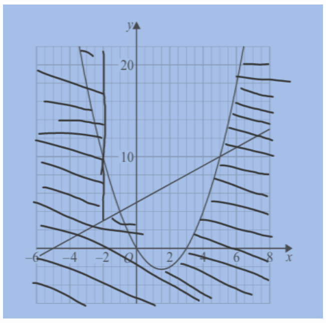
The line or the shading at the boundary at $x=-2$ must now be solid

Examples of how to apply the mark scheme;

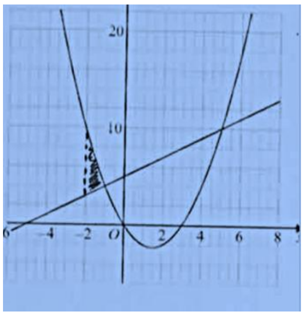


Award 1,1,0

B1: $x = -2$ is the boundary of a shaded region
B1: There is an acceptable attempt at shading either region.
Note that we could award this mark for either region
B0: There is not a solid line or completely shaded up to $x = -2$ boundary, so we have not got a fully correct diagram

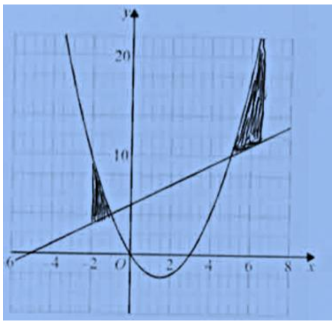


Award 1,0,0
B1: $x = -2$ is drawn (at least between the line and curve)
B0: No acceptable distinct region is indicated.
B0: Follows B0



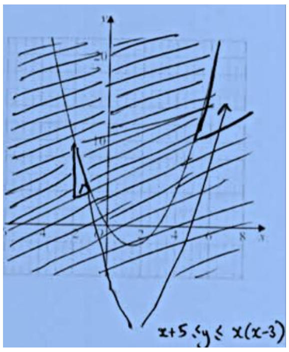
Award 1,1,0

B1: $x = -2$ drawn between the line and curve (accept dotted)
B1: There is an acceptable attempt at shading the left-hand region.
B0: There is no attempt to shade the right-hand region. Even if this right hand region had been shaded in, this mark would not be awarded due to the dotted line



award 1,1,0

B1: $x = -2$ is the boundary of a shaded region
B1: There is an acceptable attempt at shading the left-hand region.
B0: The shading on right-hand region is incomplete



Award 1,1,1
Candidates were not told to shade the required region so it is acceptable for them to indicate it in other ways as below
B1: $x = -2$ is drawn between the line and curve
B1: There is an acceptable attempt at indicating either region.
B1: There is an acceptable way of indicating the complete region

Alternatively, if they have drawn $x = -2$ with a solid line and write R in BOTH regions they can be awarded BOD 1,1,1

Question Number	Scheme	Marks
4 (a)	Grad $AB = \dfrac{8-2}{-1-7} = \left(-\dfrac{3}{4}\right)$	M1
	$y-2=-\dfrac{3}{4}(x-7)\Rightarrow 3x+4y-29=0$	dM1, A1
		(3)
(b)	Equation line L_2 $\quad y-8=\dfrac{4}{3}(x+1)$	M1
	Substitutes $y=0$ $\quad 0-8=\dfrac{4}{3}(x+1)\Rightarrow x=-7$ $\quad$ So $(-7,0)$	dM1, A1
		(3)
(c)	Attempts one required length. E.g. $AB^2=8^2+6^2\Rightarrow AB=10$	M1
	Area $ABC = \dfrac{1}{2}AB\times AC=\dfrac{1}{2}10\times 10=50$	dM1, A1
		(3)
		(9 marks)

(a)

M1: Correct attempt at gradient.

It must be an attempt at $\dfrac{\Delta y}{\Delta x}$ with attempt at subtraction at least once

dM1: Correct attempt at equation of line L_1 using their found gradient and either of the points.

It is dependent upon the previous M mark. Look for an attempted application of $y-y_1=\text{"}-\dfrac{3}{4}\text{"}(x-x_1)$.

You may well see $\dfrac{y-8}{x+1}=\dfrac{8-2}{-1-7}$ o.e. which would score M1, dM1

If they use the form $y=mx+c$ it must lead as far as $c=\ldots$

A1: $3x+4y-29=0$ but accept any integer multiple of this. Accept terms in a different order

……

Alt (a)

M1: Attempts to substitute BOTH $A(-1,8)$ and $B(7,2)$ into $ax+by+c=0$ or $y=mx+c$

dM1: And solves using non calculator methods to find values of a, b and c or m and c.

The solution is not unique if using $ax+by+c=0$ so they may start by stating a value for c, e.g. $c=2$

A1: $3x+4y-29=0$ but accept any integer multiple

……

(b)

M1: Uses the negative reciprocal of their gradient in (a) with $A(-1,8)$ to form equation of L_2

dM1: Substitutes $y=0$ in a correctly attempted L_2 and finds $x=\ldots$

It is dependent upon the previous M so L_2 must have been formed correctly

Condone slips where L_2 has been incorrectly adapted and $y=0$ is substituted into this to find x.

A1: $(-7,0)$ but condone $x=-7$ following substitution of $y=0$

This is a non calculator question so this must follow M1, dM1

……

Alternatives in (b) are possible: For example via similar triangles or methods without fully forming the equation of L_2

Note that it is possible to find the coordinates of C without fully forming the equation of L_2

For example, letting the coordinates of C to be $(k, 0)$ and using gradient AC leads to the equation $\frac{8}{-1-k} = "\frac{4}{3}" \Rightarrow k = -7$. Score M1 for using the negative reciprocal of their gradient in (a) with $A(-1, 8)$ to set up an equation in k. Score dM1 for solving it.

………..

(c)

M1: Attempts one required length. E.g. $AB^2 = 8^2 + 6^2 \Rightarrow AB = ...$ OR $AC^2 = '8'^2 + '6'^2 \Rightarrow AC = ...$

It is acceptable to award this from a diagram where you see the lengths 6 and 8 followed by AB or AC as 10. **There must be some evidence for this award**.

dM1: Correct attempt at area triangle ABC. It is dependent upon the previous M

A1: 50

……

Alternatives for part (c) exist so look carefully at what is attempted.

Alt I: Area of rectangle – area of three triangles

M1: Correct attempt at 3 of the 4 necessary shapes

Area rectangle = $14\times 8 = (112)$ Area triangles = $\frac{1}{2}\times 8\times 6 = (24)$, $\frac{1}{2}\times 8\times 6 = (24)$, $\frac{1}{2}\times 14\times 2 = (14)$

dM1: Complete area = $14\times 8 - \left\{\frac{1}{2}\times 8\times 6 + \frac{1}{2}\times 8\times 6 + \frac{1}{2}\times 14\times 2\right\}$

A1: 50

Alt II Let point D be the point at which L_1 crosses the x-axis (which is $\frac{29}{3}$ for a correctly found eqn)

M1: Correct attempt to find the area of triangle $ACD = \frac{1}{2}\times\left(7+\frac{29}{3}\right)\times 8$ or triangle $BCD = \frac{1}{2}\times\left(7+\frac{29}{3}\right)\times 2$

dM1: Attempts area triangle ACD – triangle $BCD = \frac{1}{2}\times\left(7+\frac{29}{3}\right)\times 8 - \frac{1}{2}\times\left(7+\frac{29}{3}\right)\times 2$

A1: 50

Alt III: If the coordinates of C are correct, triangle ABC is isosceles with AB and AC equal.

Therefore, only allow this method to score marks if they have a correct answer to part (b)

Let the mid- point of CB be M.

In a correct calculation $M = (0, 1)$ and area $ABC = \frac{1}{2}\times BC\times AM = \frac{1}{2}\times 10\sqrt{2}\times 5\sqrt{2}$

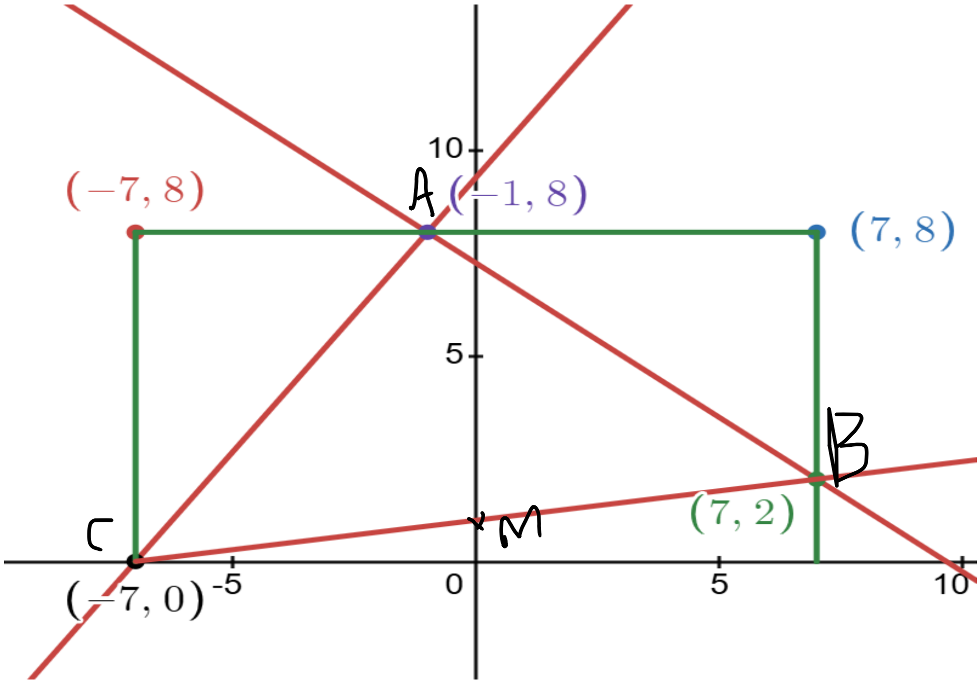
M1: Correct attempt at one length, e.g. $BC = \sqrt{14^2 + 2^2} = (10\sqrt{2})$ dM1: Complete attempt

Alt IV: The shoelace method

$$\begin{array}{cc} -7 & 0 \\ -1 & 8 \\ 7 & 2 \\ -7 & 0 \end{array}$$

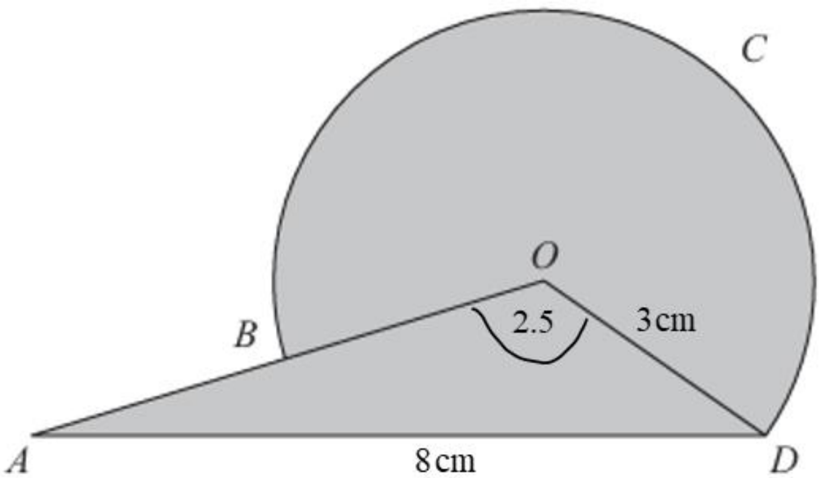
$$\frac{1}{2}\left|(-7\times 8 - 1\times 2 + 7\times 0) - (0\times -1 + 8\times 7 + 2\times -7)\right| = 50$$

If you are unaware of this method, please look it up. You require the coordinates of the vertices



(−7, 8)
A
(−1, 8)
(7, 8)
10
5
B
(7, 2)
C
M
(−7, 0)
-5
0
5
10

Question Number	Scheme	Marks
5 (a)	$\frac{\sin(OAD)}{3} = \frac{\sin 2.5}{8} \Rightarrow \sin(OAD) = 0.2244...$	M1
	(Angle (ODA)) = $\pi - 2.5 - \arcsin(0.2244...) = \pi - 2.5 - 0.226355... = 0.415$ *	dM1, A1*
		(3)
(b)	Attempts either $\frac{1}{2}\times 3^2 \times (2\pi - 2.5) = (17.02)$ or $\frac{1}{2}\times 3\times 8\times \sin 0.415 = (4.838...)$	M1
	Complete attempt at area = $\frac{1}{2}\times 3^2 \times (2\pi - 2.5) + \frac{1}{2}\times 3 \times 8 \times \sin 0.415$	dM1
	= awrt 21.9 cm^2	A1
		(3)
(c)	Attempts either $3(2\pi - 2.5) = (11.349..)$ or $AO^2 = 3^2 + 8^2 - 2\times 3\times 8\cos(0.415) = (29.07.)$	M1
	Complete attempt at perimeter = $3(2\pi - 2.5) + 8 + \sqrt{3^2 + 8^2 - 2\times 3 \times 8\cos(0.415)} - 3 = 11.35 + 8 + 5.39 - 3$	dM1
	= awrt 21.7 cm	A1
		(3)
		(9 marks)



Be aware that it is possible that the whole question could be done in degrees with 2.5 radians converted to $143.23°$, $143.24°$ **or better. Provided this accuracy is used full marks are possible, but the final conversion from** $23.79°$ **to 0.415 must be shown. If less accuracy is used, e.g** $143.2°$ **or** $143°$ **only the method marks are available in part (a), BUT full marks are possible in parts (b) and (c)**

FYI. Angle *AOD* = $143.23°$ **or** $143.24°$**, Angle *OAD* =** $12.96°$ **or** $12.97°$ **Angle *ODA* =** $23.79°$ **or** $23.80°$

(a) **This is a given answer and it is important that a full method is shown.**

Candidates will NOT gain credit in part (a) when starting with or using angle *ODA* = 0.415

M1: Attempts to use the sine rule with the angles and lengths in the correct position.

Allow $OAD = \theta$ or any other variable. Condone slips in the labelling of the angle.

The calculation must lead at least as far as $\sin OAD = ...$

E.g. $\frac{3}{\sin x} = \frac{8}{\sin 2.5} \Rightarrow \sin x = (0.2244...)$ OR $x = \arcsin(0.2244...) = (0.22635...)$

dM1: Full attempt at finding angle ODA. It is dependent upon the previous M1

Look for example, sight of $\pi - 2.5 - \arcsin("0.2244..")$ OR $3.14 - 2.5 - "0.226"$

A1*: Uses sufficient accuracy to achieve the given answer of 0.415 or the more accurate awrt 0.4152
Intermediate working must show or imply that the necessary accuracy has been achieved.
If a candidate achieves the correct answer of 0.4152… or better this is sufficient evidence
Additionally, a candidate could give intermediate answers to 4dp or better (rounded or truncated).
E.g. $\pi - 2.5 - \arcsin(0.2244)$, $\pi - 2.5 - 0.2263$, $\pi - 2.5 - 0.2264$.
If a decimal approximation of π is used it must be 3.142 or better
Note that the calculation $\pi - 2.5 - 0.226 = 0.415$ is M1, dM1, A0

……………………………………………………………………………………………………

Alt (a) using the cosine rule
M1: Uses the cosine rule to find *OA*.
Look for a correctly set up quadratic equation with an allowable attempt to solve
E.g $8^2 = 3^2 + y^2 - 2\times 3\times y\cos(2.5) \Rightarrow y^2 - 6y\cos 2.5 - 55 = 0 \Rightarrow y = 5.3924..$
dM1: Uses the sine rule with the sides and angles in the correct places. Dependent upon previous M.
$$\frac{\sin x}{"5.394"} = \frac{\sin 2.5}{8} \Rightarrow \sin x = (0.4034......)$$
A1*: Achieves awrt 0.415 following accuracy to 4dp throughout

……………………………………………………………………………………………………

(b) **Note that we are told that angle *ODA* = 0.415 radians so if used in (b) and (c) it must be correct or a more accurate value**

M1: Attempts either

- the area of the sector $\frac{1}{2}\times 3^2 \times (2\pi - 2.5) = (17.02)$ or decimal equivalent with accuracy to at least $\frac{1}{2}\times 3^2 \times (6.28 - 2.5)$
- or the area of the triangle E.g. $\frac{1}{2}\times 3\times 8\times \sin 0.415 = (4.838...)$ (other alternatives exist)

dM1: Complete attempt at area = $\frac{1}{2}\times 3^2 \times (2\pi - 2.5) + \frac{1}{2}\times 3\times 8\times \sin 0.415$ or its decimal equivalent
A1: awrt 21.9 cm^2. There is no requirement for the units, so they may be ignored even if incorrect

(c)
M1: Attempts either

- length of arc *BCD* = $3(2\pi - 2.5)$ or decimal equivalent of $3(6.28 - 2.5)$ or greater accuracy
- or length $AO^2 = 3^2 + 8^2 - 2\times 3\times 8\cos(0.415)$ or equivalent

Alternatives for length *OA* include a sine rule approach or an area approach.
Be aware that *AO* may have been found in part (a) and it is fine just to use this in part (c) for the M1
dM1: Complete attempt at perimeter
E.g. $3(2\pi - 2.5) + 8 + \sqrt{3^2 + 8^2 - 2\times 3\times 8\cos(0.415)} - 3$
or decimal equivalent such as $3\times 3.78 + 5 + \frac{8\sin(0.415)}{\sin(2.5)}$ where π is used as 3.14 or better

.

A1: awrt 21.7 cm. There is no requirement for the units, so they may be ignored even if incorrect

Question Number	Scheme	Marks
6 (a)	(i) -6 and 15	B1
	(ii) 1 and 8	B1
		(2)
(b)	$(\mathrm{f}(x)) = k(x+2)(x-5)^2$	M1
	$(0,10) \Rightarrow 10 = k(0+2)(-5)^2 \Rightarrow k = ...$	dM1
	$(\mathrm{f}(x)) = \frac{1}{5}(x+2)(x-5)^2$	A1
		(3)
(c)	Correctly deduces that $x = -2$	B1
	$\frac{1}{5}\cancel{(x+2)}(x-5)^2 = 10\cancel{(x+2)} \Rightarrow (x-5)^2 = 50$	M1
	$\Rightarrow x-5 = \pm\sqrt{50} \Rightarrow x = 5 \pm 5\sqrt{2}$	dM1, A1
		(4)
		(9 marks)

(a)(i)

B1: Requires both -6 and 15. May be seen within the question. Condone repetitions, e.g $-6, 15$ and 15

Condone roots being expressed as $(-6,0)$ and $(15,0)$ only if there are no other coordinates, e.g (0, 10)

(a)(ii)

B1: Requires both 1 and 8 May be seen within the question. Condone repetitions, e.g 1, 8 and 8

Condone roots being expressed as $(1,0)$ and $(8,0)$ only if there are no other coordinates, e.g (3, 10)

SC: If a candidate writes down three coordinates for (i) including $(-6,0)$ and $(15,0)$

and three coordinates for (ii) including $(1,0)$ and $(8,0)$

they should be awarded SC: B1, B0. This may be also awarded from the result of two sketches

(b)

M1: States or implies that $\mathrm{f}(x)$ or $y = k(x \pm 2)(x \pm 5)^2$. Allow with $k = 1$

dM1: Substitutes (0, 10) in $\mathrm{f}(x) = k(x \pm 2)(x \pm 5)^2$ and finds a value for k

A1: States or implies that $\mathrm{f}(x)$ or y is $\frac{1}{5}(x+2)(x-5)^2$ o.e. ISW after a correct answer

Some candidates may be able to write this down with little or no working which is fine for 3 marks

………

Alt (b) Note that the demand is that they should (**but not must**) leave $\mathrm{f}(x)$ in a factorised form

So, if a candidate attempts to use the form $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$ and

- substitutes $(-2,0)$, $(5,0)$ and $(0,10)$ in $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$
- substitutes $(5,0)$ in $\mathrm{f}'(x) = 3ax^2 + 2bx + c$

they can score M1, dM1 for an attempt at values for a, b, c and d and A1 for $\mathrm{f}(x) = \frac{1}{5}x^3 - \frac{8}{5}x^2 + x + 10$

……..

(c) M1, dM1, A1 can only be scored if they have a valid answer to part (b). That is $\mathrm{f}(x) = k(x+2)(x-5)^2$

B1: Correctly deduces that $x = -2$ is one of the solutions.

This may be scored at any point in the solution but must be from a non- calculator method

Examples of ways this may be deduced are

- sight of $x = -2$ being substituted into $y = 10(x+2)$ showing $y = 0$
- $(x+2)$ being divided or factored out of an equation
- sketching $y = 10(x+2)$ on the Figure and showing an intersection at (-2, 0)

M1: $k\cancel{(x+2)}(x-5)^2 = 10\cancel{(x+2)} \Rightarrow (x-5)^2 = \beta, \quad \beta > 0$

Factorises or cancels out the $(x+2)$ term leading to a quadratic factor/equation.

Allow with $k = 1$. Note that many will expand and form a 3TQ

dM1: Proceeds to at least one value for x using a correct NON- CALCULATOR method.

A1: $x = 5 \pm 5\sqrt{2}$ o.e such as $x = 5 \pm \sqrt{50}$

We may have candidates who attempt part (c) via WMA12 methods.

These can be scored in an identical way. They must start with an f(x) in a suitable form which must be

- either $\mathrm{f}(x) = k(x+2)(x-5)^2$
- or $\mathrm{f}(x) = \frac{1}{5}x^3 - \frac{8}{5}x^2 + x + 10$ or multiple thereof

Example of a correct solution

$x^3 - 8x^2 - 45x - 50 = 0$

$(x+2)(x^2 - 10x - 25) = 0$

$\Rightarrow x = -2,\ x = \dfrac{10 \pm \sqrt{100+100}}{2}$

$\Rightarrow x = -2,\ x = 5 \pm \sqrt{50}$

B1: Correctly deduces that $x = -2$ is one of the solutions.

This may be scored at any point in the solution but must be from a non- calculator method

M1: After reaching a cubic form, attempts to factorise out (or divide by) the $(x+2)$ term leading to a 3TQ equation. They cannot just write down the solutions of their cubic from the calculator

dM1: Proceeds to at least one value for x using a correct NON-CALCULATOR method.

A1: $x = 5 \pm 5\sqrt{2}$ o.e such as $x = 5 \pm \sqrt{50}$

Example of a solution that scores no marks

$x^3 - 8x^2 - 45x - 50 = 0$

$\Rightarrow x = -2,\ x = 5 \pm \sqrt{50}$

B1 (for $x = -2$) would only be awarded if they gave some additional evidence.

For example, they state $x = -2$ ($y = 0$) lies on both graphs.

It cannot be just awarded from their calculator solution to the cubic equation

Question Number	Scheme	Marks
7 (a)	$\left(\frac{dy}{dx}\right) = 2x^2 - 16x + 43$	M1, A1
	$= 2x^2 - 16x + 43 \equiv 2\left(x^2 - 8x\right) + ... \equiv 2(x-4)^2 ...$	B1ft, dM1
	$\equiv 2(x-4)^2 + 11$	A1
		(5)
(b)(i)	States minimum value of $\frac{dy}{dx}$ is 11.	B1 ft
(ii)	States that the x coordinate is 4	B1 ft
		(2)
(c)	Attempts $\frac{2}{3}\times"4"^3 - 8\times"4"^2 + 43\times"4" - \frac{20}{3}\left(=80\right)$	M1
	Attempts equation of tangent $y - 80 = 11(x-4)$	dM1
	$y = 11x + 36$	A1
		(3)
		(10 marks)

(a)

M1: Reduces the index on any term including $\frac{20}{3} \to 0$. The index must be processed for this award

A1: $\left(\frac{dy}{dx}\right) = 2x^2 - 16x + 43$ or exact equivalent. There should not be any extra terms such as + c

B1ft: Takes out a correct factor of '2' from their $'2'x^2 - 16x + 43$

This will be implied by the correct value of p for their $2x^2 - 16x + 43$

dM1: Achieves a correct value for q for their $2x^2 - 16x + 43$. Look for $"\frac{-16}{2p}"$

It is dependent upon having differentiated (i.e scoring the first M1)

A1: CAO $2(x-4)^2 + 11$

(b) The question states hence so ideally, answers in (b) should follow their part (a).
A method must be seen so don't allow answers in (b) following no working in (a).

Condone solutions in (b) arising from the second derivative. E.g. $\frac{d^2y}{dx^2} = 4x - 16 = 0 \Rightarrow x = 4$

(b)(i) Do not award marks in (b) for just writing down (4, 11)

B1ft: States 11 but follow through on their r from part (a) as long as $\frac{dy}{dx} = p(x+q)^2 + r$

Also allow candidates to substitute $x = 4$ in $\frac{dy}{dx}$ to reach 11. Ignore incorrect labelling such as $y = 11$

(b)(ii)

B1ft: States that the x coordinate is 4 but follow through on their $-q$ from part (a)

(c)

M1: Attempts to find the y coordinate of C using their $x = "4"$.

dM1: Correct attempt at equation of tangent using their 11 and a correctly found y coordinate

It is dependent upon the previous M1

A1: $y = 11x + 36$

Question Number	Scheme	Marks
	$f'(x)=4x^2+\dfrac{6}{x^2}-4$	
8 (a)(i)	Substitutes $x=\sqrt{3}$ in $f'(x)=4\times3+\dfrac{6}{3}-4$	M1
	$=10$ which is the same as the gradient of the tangent *	A1*
(a)(ii)	$4x^2+\dfrac{6}{x^2}-4=10\Rightarrow 2x^4-7x^2+3=0$	M1, A1
	$\Rightarrow\left(2x^2-1\right)\left(x^2-3\right)=0\Rightarrow x^2=\dfrac{1}{2}\Rightarrow x=\dfrac{\sqrt{2}}{2}$	dM1, A1
		(6)
(b)	States/implies that gradient of normal at P is $-\dfrac{1}{10}$ or y coordinate is $4\sqrt{3}$	M1
	Equation is $y-4\sqrt{3}=-\dfrac{1}{10}\left(x-\sqrt{3}\right)$	A1
		(2)
(c)	$f'(x)=4x^2+\dfrac{6}{x^2}-4\Rightarrow\quad f(x)=\dfrac{4}{3}x^3-\dfrac{6}{x}-4x+c$	M1, A1
	Uses $\left(\sqrt{3},4\sqrt{3}\right)\Rightarrow 4\sqrt{3}=\dfrac{4}{3}\times3\sqrt{3}-\dfrac{6}{\sqrt{3}}-4\times\sqrt{3}+c\Rightarrow c=...$	dM1
	$f(x)=\dfrac{4}{3}x^3-\dfrac{6}{x}-4x+6\sqrt{3}$	A1
		(4) **(12 marks)**

(a)(i)

M1: Substitutes $x=\sqrt{3}$ in $f'(x)$. There must be evidence of this so award for sight of

$f'(x)=4\times3+\dfrac{6}{3}-4$, $f'(x)=4\times\left(\sqrt{3}\right)^2+\dfrac{6}{\left(\sqrt{3}\right)^2}-4$ or $f'(x)=12+2-4$

A1*: Achieves $f'(\sqrt{3})=10$ and alludes to the fact that this is equal to the gradient of the tangent. Condone a minimal response here.

………

Note that it is possible to combine both parts of (a) even if it is labelled (i) and find the two possible coordinates of *P*.

If this is attempted $x=\sqrt{3}$ could be substituted into any of the following equations

$$4x^2+\frac{6}{x^2}-4=10\Rightarrow 4x^2+\frac{6}{x^2}-14=0\Rightarrow 4x^4-14x^2+6=0\Rightarrow 2x^4-7x^2+3=0$$

It is also possible to prove that $x=\sqrt{3}$ by solving $2x^4-7x^2+3=0$ via $\left(2x^2-1\right)\left(x^2-3\right)=0$

The M1 is scored when they attempt to solve their quartic using a non- calculator method and the A1 when they solve correctly and proceed to $x=\sqrt{3}$

………..

(a)(ii)

M1: Sets $4x^2+\frac{6}{x^2}-4=10$ and attempts to simplify.

Award for

- either collecting constant terms $4x^2+\frac{6}{x^2}-4=10 \Rightarrow 4x^2+\frac{6}{x^2}-14=0$
- or else attempting to multiply through by x^2 seen in at least 3 of the 4 terms

A1: $2x^4-7x^2+3=0$ o.e. such as $4x^4-14x^2+6=0$. The $=0$ may be implied by subsequent work

dM1: Makes suitable progress towards solving their quadratic in x^2 using non calculator methods. Allow

- factorisation $'2x^4-7x^2+3'=0 \Rightarrow \left(ax^2+b\right)\left(cx^2+d\right)=0$ with $|ac|=2$ and $|bd|=3$
- completion of square $'2x^4-7x^2+3'=0 \Rightarrow \left(x^2-\frac{7}{4}\right)^2=k,$
- formula $'2x^4-7x^2+3'=0 \Rightarrow x^2=\frac{7\pm\sqrt{49-24}}{4}$

Condone attempts where a factor of 2 seemingly disappears.

E.g. $4x^4-14x^2+6=0 \Rightarrow \left(2x^2-1\right)\left(x^2-3\right)=0$

Condone attempts in which the roots would not be real, E.g $k<0$, their discriminant <0

A1: Selects/finds $x=\frac{\sqrt{2}}{2}$ or equivalent $\frac{1}{\sqrt{2}}$ as the other possible x coordinate

Any other value found apart from $\sqrt{3}$, e.g $x=-\frac{\sqrt{2}}{2}$, must be either rejected or not selected

(b)

M1: States or implies that gradient at P is $-\frac{1}{10}$ **OR** the y coordinate of P is $4\sqrt{3}$

A1: $y-4\sqrt{3}=-\frac{1}{10}\left(x-\sqrt{3}\right)$ ISW after a correct answer

You may see $y=-\frac{1}{10}x+\frac{41}{10}\sqrt{3}$ if the form $y=mx+c$ is used

(c) The two marks for the integration may be awarded **anywhere** in Qu 8 as long as they state $\mathrm{f}(x)=...$ or $y=\ldots$ If it is clearly labelled or used in part (c) then the $\mathrm{f}(x)=...$ or $y=\ldots$ are not necessary

M1: Integrates with **TWO** terms having a correct index which must be processed.

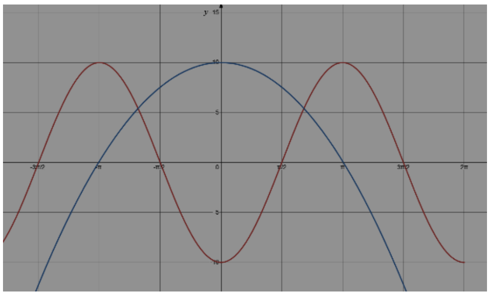
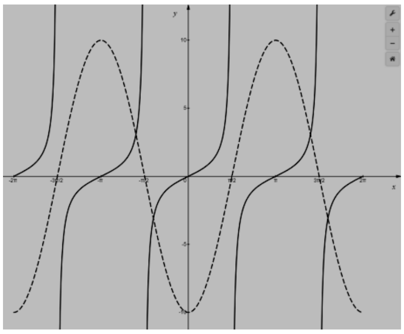
Condone the omission of the $+c$

A1: Integrates to $\frac{4}{3}x^3-\frac{6}{x}-4x+c$ with the constant of integration

dM1: Uses $x=\sqrt{3}, y=4\sqrt{3}$ in their integrated function and finds a value for c.

It is dependent upon the previous M. Condone the use of decimal values for this mark only.

A1: States $\frac{4}{3}x^3-\frac{6}{x}-4x+6\sqrt{3}$ It is not necessary to have the $\mathrm{f}(x)$ or the y

Question Number	Scheme	Marks
9. (a)	$-10\cos x$ or equivalent such as $10\sin\left(x-\frac{\pi}{2}\right)$	M1, A1 **(2)**
(b) (i)	 $\cap$ shaped curve symmetrical about y axis passing through $(0, 10)$	M1
	x intercepts at approximately $\pm\pi$	A1
(ii)	2 solutions	B1 **(3)**
(c)(i)	 Correct shape with intercepts every π rads	M1
	...and asymptotes at $(2p+1)\frac{\pi}{2}$	A1
(ii)	4 roots in the interval $-2\pi \leqslant x \leqslant 2\pi$ so $50\times4=200$	B1, B1 **(4)**
		(9 marks)

(a) **Candidates cannot write down multiple attempts at this sort of question.**
If there are two or three different answers mark their final full attempt only

M1: Writes down a suitable expression for $f(x)$, for example, $k\cos x,\quad k\in\mathbb{R}$ E.g $\cos x$

or equivalent such as $p\sin\left(x-\frac{\pi}{2}\right)$ or $p\cos(x\pm\pi)$ with k and p constants.

Score $10\cos(-x)$ M1, A0

A1: Completely correct expression for $f(x)$, for example, $-10\cos x$ which may be unsimplified $\cos x\times-10$

or equivalent such as $10\sin\left(x-\frac{\pi}{2}\right)$, $10\cos(x-\pi)$ or $10\cos(x+\pi)$

Please check the question space for possible answers to this

(b) **Scored on Diagram 1 or its Copy**

(b) (i)

M1: ∩ shaped curve symmetrical about y axis passing through $(0,10)$

Be tolerant of slips of the pen, slight asymmetry and a pointy maximum.
Condone a graph just meeting the x -axis rather than passing through it

A1: As above with x intercepts at approximately $\pm\pi$. Look for a tolerance of ½ a square
The graph must now pass through the x-axis

(b)(ii)

B1: States 2. No reason is required and no sketch is required.

(c) **Scored on Diagram 2 or its Copy. Take care as some students may use the Copy of Diagram 1.**
The shape of the graphs in (i) and (ii) are significantly different so mark positively.
If you feel that candidates have not understood the instructions and deserve credit use review.

(c)(i)

M1: Correct shape for a tangent graph with x intercepts at $-\pi$, 0 and π radians in the interval $-2\pi < x < 2\pi$.

Be tolerant of the exact position of the asymptotes (allow a tolerance of ½ a square) and allow if there are only 3 cycles. Ignore the graph outside the given domain

A1: Fully correct with asymptotes at $(2p+1)\frac{\pi}{2},\ p\in\mathbb{Z}$ Ignore the graph outside the given domain

(c)(ii) Full marks can only be awarded following M1 in (c)(i).
Candidates cannot score B0, B1 but M1, A0, B1, B1 is possible

B1: Either states 200 or gives a suitable explanation.
E.g. it is $50\times$ the number of their roots in the interval $-2\pi\leqslant x\leqslant 2\pi$

B1: Having scored M1 in (c)(i) and B1 in (c)(ii) candidates must

- State 200
- And give a suitable explanation. E.g. explains it is $50\times$ the number of roots in the interval $-2\pi\leqslant x\leqslant 2\pi$. There are other ways of explaining this so look carefully at what is written down. For example, candidates may state 2 roots every 2π radians so $100\times 2=200$