Question Number	Scheme	Marks
1(a)	$y = 6x^2 + 3\sqrt{x} + \frac{5}{8}$	
	$\left(\frac{dy}{dx} =\right) 12x + \frac{3}{2}x^{-\frac{1}{2}}$	M1 A1 A1
		(3)

Notes

(a)

M1: Attempts to differentiate achieving one correct index or for $\frac{5}{8} \to 0$

Award for $6x^2 \to ...x$ or $3\sqrt{x} \to ...x^{-\frac{1}{2}}$ (oe) or for $\frac{5}{8} \to 0$

Allow indices to be unprocessed for this mark e.g. $6x^2 \to ...x^{2-1}$ or $3\sqrt{x} \to ...x^{\frac{1}{2}-1}$

A1: One correct simplified algebraic term. E.g. $12x$ or $\frac{3}{2}x^{-\frac{1}{2}}$ oe e.g. $\frac{3}{2\sqrt{x}}, \frac{1.5}{x^{\frac{1}{2}}}$

Condone $12x^1$ for $12x$ for this mark.

Do not allow $3 \times \frac{1}{2}x^{-\frac{1}{2}}$ for $\frac{3}{2}x^{-\frac{1}{2}}$

Award as soon as a correct simplified term is seen.

A1: $\left(\frac{dy}{dx}\right) = 12x + \frac{3}{2}x^{-\frac{1}{2}}$ or simplified equivalent such as $12x + \frac{1.5}{\sqrt{x}}$.

Do **not** allow $12x^1$ for $12x$ for this mark.

There is no need for "$\frac{dy}{dx} =$" so just look for the correct expression and apply isw once a correct answer is seen.

$\left(\frac{dy}{dx}\right) = 12x + \frac{3}{2}x^{-\frac{1}{2}} + c$ scores A0 here as does $\left(\frac{dy}{dx}\right) = 12x + \frac{3}{2}x^{-\frac{1}{2}} + 0$

(b)	$x=\frac{1}{4}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=12\times\frac{1}{4}+\frac{3}{2}\times\sqrt{\left(\frac{4}{1}\right)}=...(6)$	M1
	$\left(\frac{1}{4},\frac{5}{2}\right)\Rightarrow y-\frac{5}{2}="6"\left(x-\frac{1}{4}\right)$ or e.g. $y="6"x+c\Rightarrow\frac{5}{2}=6\times\frac{1}{4}+c\Rightarrow c=...$	**d**M1
	$\Rightarrow y=6x+1$	A1
		(3)
		(6 marks)

Notes

(b)

M1: Substitutes $x=\frac{1}{4}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finds its value. Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ cannot be the same as "y".

Score for sight of $x=\frac{1}{4}$ substituted consistently into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ followed by a value **or** it may be implied by a correct value for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ if no substitution is seen.

Condone a miscopy of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ if the intention is clear.

Do **not** award this mark for work such as

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=12x+\frac{3}{2}x^{-\frac{1}{2}}+c=12\left(\frac{1}{4}\right)+\frac{3}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}+c=0\left(\text{or}\frac{5}{2}\right)\Rightarrow c=...$$

dM1: Correct method for finding the equation of the tangent (not the normal). Look for the correct use of their non-zero $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=\frac{1}{4}$ and the point $\left(\frac{1}{4},\frac{5}{2}\right)$ with the values correctly placed.

If the form $y=mx+c$ is used they must proceed as far as $c=...$ e.g. $\frac{5}{2}="6"\frac{1}{4}+c\Rightarrow c=...$

Depends on the first method mark.

A1: $y=6x+1$ Cso

Note if they do not use part (a) for part (b) e.g. just use a calculator they will need to obtain a gradient of 6 for the first mark, then apply the scheme.

Question Number	Scheme	Marks
2.	$\int (x-3)^2(2x+5)\,dx$	
	$(x-3)^2(2x+5) = \left(x^2-6x+9\right)(2x+5) = ...$ or e.g. $(x-3)^2(2x+5) = (x-3)\left(2x^2-x-15\right) = ...$	M1
	$= 2x^3-7x^2-12x+45$	A1
	$\int (x-3)^2(2x+5)\,dx = \frac{1}{2}x^4 - \frac{7}{3}x^3 - 6x^2 + 45x + c$	M1 A1ft A1
		(5 marks)

Notes

M1: Attempts to multiply out. Look for an attempt to multiply two brackets together to form a quadratic before the result is combined with the third bracket.
Condone slips e.g. $(x-3)^2 = x^2 \pm 9$ and it may be left unsimplified but it must lead to an expression that could be simplified to the form ax^3+bx^2+cx+d where $a,b,c,d \neq 0$
Condone an answer appearing without an intermediate step only if it is of the form (or can be expressed in the form) $2x^3+px^2+qx\pm 45$ where $p,q \neq 0$

A1: For $2x^3-7x^2-12x+45$ but allow this unsimplified with the terms uncollected e.g. $2x^3+5x^2-12x^2-30x+18x+45$ and award this mark as soon as a correct expression is seen. Inclusion of "+ c" here scores A0 but the final mark is still available.

So $(x-3)^2(2x+5) = 2x^3-7x^2-12x+45+c$ is A0 but

$\int (x-3)^2(2x+5)\,dx = \int\left(2x^3-7x^2-12x+45\right)dx + c$ is ok for A1

M1: For an attempt to integrate any 2 terms with **different powers** of their simplified or unsimplified expression by increasing the power of x by 1 following an attempt to expand all 3 brackets.
It is for any 2 of, $...x^3 \to ...x^4$, $...x^2 \to ...x^3$, $...x \to ...x^2$, $k \to kx$
Allow indices to be unprocessed e.g. $...x^3 \to ...x^{3+1}$
There must have been an attempt to **expand all 3 brackets** so do not award for e.g.

$(x-3)^2 = x^2-6x+9 \to \frac{x^3}{3} - 3x^2 + 9x$

or e.g. $(x-3)^2(2x+5) \to (x-3)^2(x^2+5x)$ or $(x-3)^2(2x+5) \to \frac{1}{3}(x-3)^3(2x+5)$

A1ft: For 2 **different** correctly integrated and simplified terms with **different powers** with indices processed.
Follow through on any **two different terms** with **different powers** of their simplified or unsimplified expansion.

A1:	$\frac{1}{2}x^4-\frac{7}{3}x^3-6x^2+45x+c$ or exact simplified equivalent including the + c all on one line. Apply isw once a correct expression, including "+ c" is seen. Condone the presence of spurious integral signs and/or dx's.

Alternative attempts to integrate such as integration by parts or by substitution – send to review.

Question Number	Scheme	Marks
3(a)	$5^2 = 2^2 + 4^2 - 2\times2\times4\cos"\theta"$ or e.g. $(5x)^2 = (2x)^2 + (4x)^2 - 2\times2x\times4x\cos"\theta"$ or $\cos"\theta" = \dfrac{2^2+4^2-5^2}{2\times2\times4}$ or e.g. $\cos"\theta" = \dfrac{(2x)^2+(4x)^2-(5x)^2}{2\times(2x)\times(4x)}$	M1
	$\Rightarrow \theta = 108.2°$	A1
		(2)
	Alternative using the cosine rule for a different angle and then the sine rule	
	e.g. $2^2 = 4^2 + 5^2 - 2\times4\times5\cos"A" \Rightarrow \cos"A" = \dfrac{4^2+5^2-2^2}{2\times4\times5}$ $\Rightarrow A = 22.33...°$ $\dfrac{2}{\sin 22.33...} = \dfrac{5}{\sin"\theta"}$	M1
	$\Rightarrow \sin"\theta" = \dfrac{5\sin 22.33...}{2} \Rightarrow \theta = 108.2°$	A1

(a) Notes

Note that some candidates may use equivalent correct ratios

e.g. $1:2:\frac{5}{2}$ **or** $x:2x:\frac{5}{2}x$ **or** $\frac{2}{11}:\frac{4}{11}:\frac{5}{11}$ **or** $\frac{2}{11}x:\frac{4}{11}x:\frac{5}{11}x$

M1: Attempts to use the cosine rule with the sides 2, 4 and 5 or e.g. $2x$, $4x$ and $5x$ or e.g. x, $2x$ and $2.5x$ **in the correct positions for the largest angle**.
Condone missing brackets e.g. $2x^2$ rather than $(2x)^2$ as long as the intention is clear.
If they start with a rearranged rule, the values or expressions must be in the correct positions.

A1: $\theta =$ awrt $108.2°$ (Degrees symbol not required)
Allow missing brackets to be recovered e.g. $\cos"\theta°" = \dfrac{2x^2+4x^2-5x^2}{2\times2x\times4x} \Rightarrow \theta = 108.2°$
Note that attempts to find any of the other angles, correct or incorrect, can be ignored if the largest angle is found correctly.

Alternative:

M1: Attempts to use the cosine rule with the sides 2, 4 and 5 or e.g. $2x$, $4x$ and $5x$ in the correct positions for their angle and then attempts to use the sine rule with their angle and the largest angle in the correct positions.
Note that they may also find the other smaller angle using the sine rule (or cosine rule) and then subtract the 2 smaller angles from 180°

A1: $\theta =$ awrt $108.2°$ (Degrees symbol not required)
Note that this method is likely to lead to the acute angle (71.79…°) and scores A0
Allow missing brackets to be recovered as above.

(b)	$A=\frac{1}{2}ab\sin C \Rightarrow 140=\frac{1}{2}2x\times 4x\sin"108.2°"$ or e.g. $A=\frac{1}{2}ab\sin C \Rightarrow 140=\frac{1}{2}2a\times\frac{5}{2}a\sin"22.33...°"$	B1ft
	$x^2=\frac{280}{8\sin("108.2°")}=36.8...\Rightarrow x=...$	M1
	Shortest side = $2\times\sqrt{36.8...}=12.1$ (cm)	A1
		(3)
		(5 marks)

(b) Notes

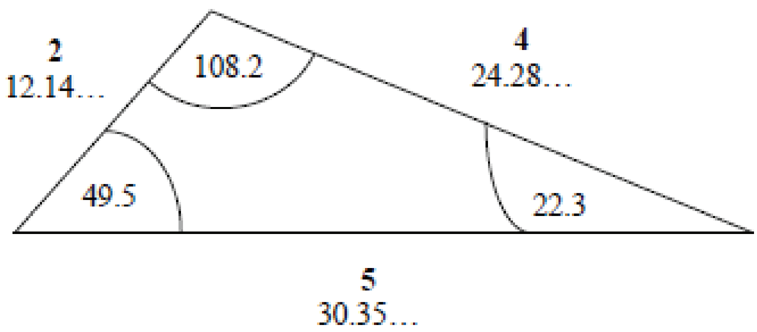
B1ft: Sets up a correct area equation in one variable e.g. x. using $A=\frac{1}{2}ab\sin C$ with A = 140 and with the sides and angle in the correct positions. Follow through on their 108.2° and they must have a value i.e. not just "C".
It is for e.g. $140=\frac{1}{2}\times 2x\times 4x\sin"108.2°"$ or with some letter other than "x" but note that $140=\frac{1}{2}\times x\times 2x\sin"108.2°"\Rightarrow x=...$ is also acceptable as the correct ratio is being used and leads directly to the length of the shortest side.
or if they didn't find the largest angle in part (a), it is for setting up the correct corresponding equation for their angle.

M1: Full use of $A=...x^2\sin C$ with A = 140 leading to a value for x (or any other letter) that involves taking the **square root** of their variable.
Note some candidates may use the exact value for $\sin C$ e.g. $\frac{\sqrt{231}}{16}$
You do not need to be concerned about the order of operations as long as it involves taking the square root to find "x".
Condone errors in sides and/or angles used.

A1: awrt 12.1 (cm). Units are not required but if they are given they must be correct.

<u>Q3 values for reference</u>

$$\text{NB scale factor } "x"=\sqrt{\frac{80\sqrt{231}}{33}}=\sqrt{36.845...}=6.0700...$$



Question Number	Scheme	Marks
4(i)	**Examples:** lhs: $3^{2p-5}=3^{2p}\times3^{-5}$ or $\frac{3^{2p}}{3^5}$ or $\frac{3^{2p}}{243}$ or $3^{2p}=\left(3^p\right)^2$ or $9^{\frac{1}{2}(2p-5)}$ or $9^{p-\frac{5}{2}}$ rhs: $\frac{1}{9}=\frac{1}{3^2}$ or 9^{-1}, $\sqrt{27}=3\sqrt{3}$ or $3^{\frac{3}{2}}$, $\frac{1}{9}\sqrt{27}=\frac{\sqrt{3}}{3}$	M1
	Examples:	dM1 A1
	$3^{2p-5}=\frac{1}{9}\sqrt{27}\Rightarrow3^{2p-5}=3^{-0.5}\Rightarrow2p-5=-0.5\Rightarrow p=2.25$	
	$3^{2p-5}=\frac{1}{9}\sqrt{27}\Rightarrow\frac{3^{2p}}{3^5}=3^{-0.5}\Rightarrow3^{2p}=3^{4.5}\Rightarrow2p=4.5\Rightarrow p=2.25$	
	$3^{2p-5}=\frac{1}{9}\sqrt{27}\Rightarrow3^{2p}\times3^{-5}=\frac{1}{9}\sqrt{27}\Rightarrow9^p=81\sqrt{3}\Rightarrow p=2.25$	
	$3^{2p-5}=\frac{1}{9}\sqrt{27}\Rightarrow3^{2p-5}=\frac{1}{3}\sqrt{3}\Rightarrow3\times3^{2p-5}=\sqrt{3}\Rightarrow3^{2p-4}=3^{0.5}$ $\Rightarrow2p-4=0.5\Rightarrow p=2.25$	
	$3^{2p-5}=\frac{1}{9}\sqrt{27}\Rightarrow9^{p-\frac{5}{2}}=9^{-1}\times9^{\frac{3}{4}}\Rightarrow p-\frac{5}{2}=-\frac{1}{4}\Rightarrow p=2.25$	
		(3)

(i) Notes

This part of the question can be solved in many different ways.
If you are unsure if a particular approach deserves credit, seek advice from your TL

M1: This is effectively a B mark and is for using a correct law of indices on the lhs or rhs of the equation. This may be seen in isolation i.e. not necessarily within the equation.
See scheme for some examples.

dM1: Reaches a value for p condoning arithmetic slips but with no incorrect index work e.g.
$\frac{1}{9}\times3^{\frac{3}{2}}=\left(\frac{1}{3}\right)^{\frac{3}{2}}$ or e.g. $3\times3^{\frac{1}{2}}=9^{\frac{1}{2}}$.
See main scheme for examples of acceptable work.
See also supplementary document for some example responses with marks.
Depends on the first method mark.

A1: Fully correct work leading to $p=2.25$ o.e. e.g. $\frac{9}{4}$, $2\frac{1}{4}$ following award of both Method marks.

Correct answer with no working scores no marks in (i)

(ii)	$x^2+1=\dfrac{36}{x^2-4} \Rightarrow \left(x^2+1\right)\left(x^2-4\right)=36 \Rightarrow x^4-4x^2+x^2-4=36$	M1
	$\Rightarrow x^4-3x^2-40=0$ or e.g. $\Rightarrow x^4-3x^2=40$	A1
	$\left(x^2-8\right)\left(x^2+5\right)=0 \Rightarrow x^2=...$	**d**M1
	$x=\pm 2\sqrt{2}$	A1
		(4)
		(7 marks)

Notes

M1: Attempts to cross multiply and expand to obtain a polynomial equation in x or another variable if a substitution is used e.g. $y=x^2 \Rightarrow y+1=\dfrac{36}{y-4} \Rightarrow (y+1)(y-4)=36 \Rightarrow y^2-4y+y-4=36$

A1: Correct 3 term quadratic equation in x^2. The terms do not have to all be on the same side but must be collected so e.g. $x^4-3x^2=40$ is acceptable.
If the terms are all one side, the "= 0" may be implied by their attempt to solve.
May also be implied by a substitution e.g.
$y=x^2 \Rightarrow y+1=\dfrac{36}{y-4} \Rightarrow (y+1)(y-4)=36 \Rightarrow y^2-4y+y-4=36 \Rightarrow y^2-3y=40$

dM1: Attempts to solve 3TQ in x^2 by any **non-calculator** means leading to values for x^2
This may be implied by substitution e.g. $y=x^2 \Rightarrow y^2-3y-40=0 \Rightarrow (y-8)(y+5)=0 \Rightarrow y=...$
This may also be implied by their values for x e.g. $\left(x^2-8\right)\left(x^2+5\right)=0 \Rightarrow x=\pm 2\sqrt{2}$
Note that e.g. $x^4-3x^2-40=0 \Rightarrow (x-8)(x+5)=0 \Rightarrow x=8, -5$ scores dM0
but $x^4-3x^2-40=0 \Rightarrow (x-8)(x+5)=0 \Rightarrow x^2=8, -5$ scores dM1 by implication.

A1: $x=\pm 2\sqrt{2}$ o.e. e.g. $\pm\sqrt{8}$ and no other values. Do **not** allow decimal values.
Do not isw if they obtain e.g. $x=\pm\sqrt{8}$ and then $x=\pm 4$ score A0
Similarly if they obtain $x=\pm 2\sqrt{2}$ and then declare e.g. therefore $x=2\sqrt{2}$ score A0
Must clearly be identified as values of x not any other variable e.g. y or t.

Question Number	Scheme	Marks
5(a)	$2x^2-16x+50\equiv 2\left(x^2-8x+25\right)\equiv 2(x\pm...)^2\pm...$	B1
	$\equiv 2(x-4)^2+18$	M1 A1
		(3)

Notes

(a)

B1: Writes $2x^2-16x+50$ in the form $2(x\pm...)^2\pm...$ or e.g. $2\left[(x\pm...)^2\pm...\right]$

M1: For $2(x-4)^2\pm...$ or e.g. $2\left[(x-4)^2\pm...\right]$. Allow e.g. $-\frac{16}{4}$ or $-\frac{8}{2}$ for -4

A1: For $2(x-4)^2+18$

Condone any spurious " = 0" e.g. $2(x-4)^2+18=0$

(b)	Eqn of l is $y = "\frac{18}{4}"x$ **or** $y = 2x^2 - 16x + 50 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 16$ At M $4x - 16 = 0 \Rightarrow x = 4,\ y = 18 \Rightarrow y = \frac{18}{4}x$	M1 B1 on ePEN
	$2x^2 - 16x + 50 = \frac{9}{2}x \Rightarrow 4x^2 - 41x + 100 = 0$ or e.g. $2x^2 + 50 = \frac{41}{2}x$	M1 A1
	$4x^2 - 41x + 100 = 0 \Rightarrow (x-4)(4x-25) = 0 \Rightarrow x = ...$	**dd**M1
	$x = \frac{25}{4}$ o.e.	A1
		(5)

Notes

(b)

M1: Deduces that equation of l is $y = "\frac{18}{4}"x$ o.e. Follow through on $y = "\frac{c}{-b}"x$ from (a)

Alternatively finds the coordinates of M (p, q) by using calculus and forms the equation $y = \frac{q}{p}x$

M1: Equates $y = 2x^2 - 16x + 50$ and their l of the form $y = \pm mx$ and collects terms to form a 3 term quadratic equation. Condone a miscopy of $y = 2x^2 - 16x + 50$ as long as the intention is clear.

A1: Correct 3 term quadratic equation. The terms do not have to all be on the same side but must be collected so e.g. $2x^2 + 50 = \frac{41}{2}x$ is acceptable.
If the terms are all one side, the "= 0" may be implied by their attempt to solve.

ddM1: Correct **non calculator** method of solving their 3TQ.
Depends on both previous method marks.
Do not allow for quoting the quadratic formula quoted followed by values e.g.

$$4x^2 - 41x + 100 = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{25}{4}, 4$$

But allow e.g. $4x^2 - 41x + 100 = 0 \Rightarrow x = \frac{41 \pm \sqrt{41^2 - 4 \times 4 \times 100}}{2 \times 4}$

Do not allow obvious use of a calculator where the factors do not correspond with their 3TQ

e.g. $4x^2 - 41x + 100 = 0 \Rightarrow \left(x - \frac{25}{4}\right)(x-4) = 0 \Rightarrow x = \frac{25}{4}, 4$

or e.g. $2x^2 - \frac{41}{2}x + 50 = 0 \Rightarrow (4x-25)(x-4) = 0 \Rightarrow x = \frac{25}{4}, 4$

or e.g. $4x^2 - 41x + 100 = 0 \Rightarrow (4x-25)(4x-16) = 0 \Rightarrow x = \frac{25}{4}, 4$

but allow e.g. $4x^2 - 41x + 100 = 0 \Rightarrow 4\left(x - \frac{25}{4}\right)(x-4) = 0 \Rightarrow x = \frac{25}{4}, 4$

and allow e.g. $4x^2-41x+100=0 \Rightarrow (4x-25)(4x-16)=(4x-25)(x-4)=0 \Rightarrow x=\frac{25}{4}, 4$

A1: $x=\frac{25}{4}$ only. If $x = 4$ is also seen, it must clearly be rejected or the $x=\frac{25}{4}$ clearly selected. Ignore any attempts to find y.

(c)	$y \le 2x^2-16x+50, \quad y \ge "\frac{9}{2}x"$	B1ft
	Fully defined region E.g. $\frac{9}{2}x \le y \le 2x^2-16x+50$ and $0 \le x \le 4$	B1ft
		(2)
		(10 marks)

Notes

Some candidates are answering (c) at the bottom of the first page of the question so please check there.

Allow use of $\mathrm{f}(x)$ for y.

Do not allow use of R for y e.g. $R \le 2x^2-16x+50$ or e.g. $y >$ line

B1ft: Uses C and l with inequalities in the correct directions and ignore any other inequalities for this mark.
Allow strict or non-strict inequalities and condone a mix of both for this mark.
Follow through on their equation for l and their possibly incorrect completed square form for C.

Examples:

$y < 2x^2-16x+50$ and $y \ge "\frac{9}{2}x"$ or e.g. $"\frac{9}{2}x" < y < 2x^2-16x+50$

B1ft: Correct and fully **consistently** defined region and no incorrect inequalities.
Follow through on their equation for l and their possibly incorrect completed square form for C.
For the restriction on x, allow e.g. $0 < x < 4$ or written separately as e.g. $x > 0$, $x < 4$ and allow $0 < x < a$ where $a < \frac{25}{4}$

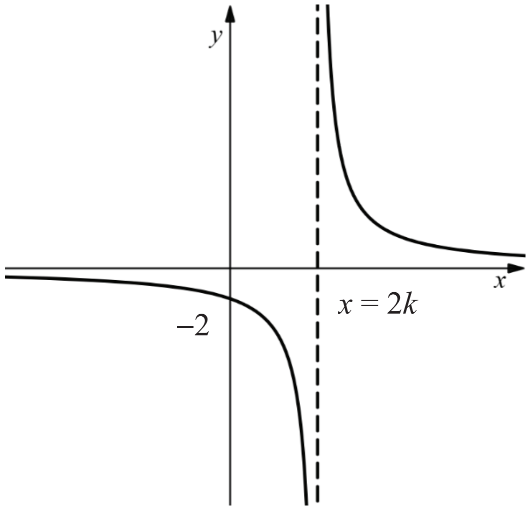
If extra inequalities are given that do not contradict then they can be ignored (see example below)

<u>Examples of full marks in (c):</u>

$$y \le 2x^2-16x+50, \quad y \ge "\frac{9}{2}x", \quad x \ge 0, \quad x \le 4$$

$$"\frac{9}{2}x" < y < 2x^2-16x+50, \quad 0 < x < 4$$

$$"\frac{9}{2}x" < y, \; y < 2x^2-16x+50, \quad 0 < x, \; x < 6.25, \; y > 0$$

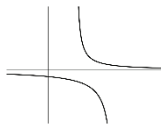
Question Number	Scheme		Marks
6(a)		Shape in quadrant 1	M1
		Fully correct shape and position	A1
		Cuts the y-axis at −2	B1
		Vertical asymptote at $x = 2k$	B1
			(4)

(a) Notes

Marks cannot be scored without a sketch.

M1: Correct shape in quadrant one for the right hand branch. It must not cross either axis but be tolerant of functions that don't go down as far as the x-axis and do extend as far left as the y-axis.
Condone curves that touch the x or y-axis as long as they do not cross the axis.
Ignore any asymptotes, correct or incorrect, for this mark.

A1: Correct shape and position (for whole curve). Be tolerant of slips of the pen but the intention should be clear. Look for the curve in quadrants 1, 3 and 4 with a vertical asymptote in quadrant 1 and 4. The left hand branch must extend into quadrant 3 and should not intentionally dip away from the x-axis to e.g. create an intentional maximum.
There should be no horizontal asymptotes other than the x-axis.

Do not allow this mark if the branches clearly overlap vertically. E.g. 

B1: Cuts or touches the negative y-axis **once** at −2
Must be in the correct position but allow as −2 or $y=-2$ and condone $(-2, 0)$ as long as it is in the correct place. Condone missing brackets e.g. $0,-2$
Allow to appear away from the sketch but it must be fully correct e.g. $(0,-2)$
If there is any ambiguity, the sketch takes precedence.

B1: For **one** vertical asymptote to the right of the y-axis marked $x=2k$
It must be labelled as an equation and not just a value on the y-axis and it must correspond with the sketch.
The asymptote does not need to be drawn as a dotted line but the curve must be asymptotic to the correct line for either branch.
Allow $x=2k$ to appear away from the sketch but it must be correct and correspond with the sketch. If there is any ambiguity, the sketch takes precedence.

6(b)	$\frac{4k}{x-2k}=6-2x \Rightarrow 4k=6x-12k-2x^2+4kx$ o.e.	M1 A1
	$2x^2-(4k+6)x+16k=0 \Rightarrow b^2-4ac=(4k+6)^2-4\times2\times16k$	**d**M1
	$4k^2-20k+9=0 \Rightarrow (2k-1)(2k-9)=0 \Rightarrow k=\frac{1}{2},\frac{9}{2}$ so $0<k\le\frac{1}{2}, k\ge\frac{9}{2}$	**dd**M1 A1
		(5)
		(9 marks)

(b) Notes

M1: Equates curve with given line and attempts to form a quadratic equation.
The terms do not need to be collected.
Look for $\frac{4k}{x-2k}=6-2x$ cross multiplies (condoning slips) and proceeds to a quadratic equation.

A1: Correct quadratic equation with brackets expanded which may be unsimplified e.g.
$\frac{4k}{x-2k}=6-2x \Rightarrow 4k=6x-12k-2x^2+4kx$

dM1: Correct attempt to find the discriminant b^2-4ac for their $ax^2+bx+c=0$ with both b and c expressions in terms of k. May be seen embedded in an attempt at the quadratic formula.
It is dependent upon the previous Method mark.

ddM1: Solves their 3 term quadratic equation/inequality in k resulting from $b^2-4ac...0$ **and** chooses the **outside region** for their critical values.
Allow calculator solutions for solving their 3TQ and condone factors that are inconsistent with their equation so just look for the correct or correct ft roots with an attempt at the outside regions e.g. condone $4k^2-20k+9=0 \Rightarrow \left(k-\frac{1}{2}\right)\left(k-\frac{9}{2}\right)=0 \Rightarrow k=\frac{1}{2},\frac{9}{2} \Rightarrow k<\frac{1}{2}, k>\frac{9}{2}$

The roots may be embedded so allow e.g. $4k^2-20k+9=0 \Rightarrow k\le\frac{1}{2},\ k\ge\frac{9}{2}$ to imply this mark.

Condone the boundaries not being included in the inequality and condone the omission of $k>0$
Condone the use of x rather than k.
It is dependent upon both previous Method marks.

A1: CSO $0<k\le\frac{1}{2},\ k\ge\frac{9}{2}$ o.e. Must be in terms of k.

The following are acceptable:

$0 < k \le \frac{1}{2},\ k \ge \frac{9}{2}$	$0 < k \le \frac{1}{2}$ and $k \ge \frac{9}{2}$	$0 < k \le \frac{1}{2}$ or $k \ge \frac{9}{2}$	$0 < k \le \frac{1}{2} \cup k \ge \frac{9}{2}$
$\left(0, \frac{1}{2}\right], \left[\frac{9}{2}, \infty\right)$	$\left\{k : 0 < k \le \frac{1}{2},\ k \ge \frac{9}{2}\right\}$		

But do **not** allow $0 < k \le \frac{1}{2} \cap k \ge \frac{9}{2}$.

Do not isw and mark their final answer.

Question Number	Scheme	Marks
7(a)	$\frac{1}{2}r^2 \times 1.65 = 30 \Rightarrow r^2 = \ldots$ or $r = \ldots$	M1
	$r = 6.03$	A1
	$(OQ =) 6.03 + 2.8 = 8.83\,(\text{m})*$	A1*
		(3)
	(a) Alternative	
	$\frac{1}{2}(OQ-2.8)^2 \times 1.65 = 30 \Rightarrow (OQ-2.8)^2 = \ldots$ or $(OQ-2.8) = \ldots$	M1
	$(OQ-2.8) = 6.030\ldots$	A1
	$(OQ =) 6.03 + 2.8 = 8.83\,(\text{m})*$	A1*

Notes

(a)

M1: Uses $\frac{1}{2}r^2 \times 1.65 = 30$ in an attempt to find a value for r^2 or r (where r is OP)

A1: $"r" = 6.03\ldots$ or exact $\sqrt{\frac{400}{11}}$ or $\frac{20}{\sqrt{11}}$ or $\frac{20\sqrt{11}}{11}$ or $\sqrt{36.36\ldots}$

A1*: $(OQ =) 6.03\left(\text{or e.g.} \frac{20\sqrt{11}}{11} \text{ or } \sqrt{36.36\ldots}\right) + 2.8 = 8.83\,(\text{m})$

Must be 8.83 not awrt 8.83
The "$OQ =$" is not required.

There must be some indication that they are adding $6.03\left(\text{or e.g.} \frac{20\sqrt{11}}{11}\right)$ to 2.8

There is no requirement for units but if any are given they must be correct.

Allow equivalent **correct** work in degrees e.g. $1.65\,\text{rads} = 1.65 \times \frac{180}{\pi}°$ etc.

Alternative:

M1: Uses $\frac{1}{2}(OQ-2.8)^2 \times 1.65 = 30$ in an attempt to find a value for $(OQ-2.8)^2$ or $(OQ-2.8)$

A1: $(OQ-2.8) = 6.03\ldots$ or exact $\sqrt{\frac{400}{11}}$ or $\frac{20}{\sqrt{11}}$ or $\frac{20\sqrt{11}}{11}$ or $\sqrt{36.36\ldots}$

A1*: $OQ = 6.03\left(\text{or e.g.} \frac{20\sqrt{11}}{11}\right) + 2.8 = 8.83\,(\text{m})$

Must be 8.83 not awrt 8.83
No requirement for units but if any are given they must be correct.

Note that any attempts to verify the result in (a) would need a complete argument e.g. attempts to find the area for 2 suitable values of OQ such as 8.835 and 8.825 for M1, A1 for both areas evaluated correctly and A1 for fully reasoning that OQ is 8.83 to 2dp.

(b)	Attempts $\frac{1}{2}\times 8.83^2\times(2\pi-1.65)=(180.\_\_)$	M1
	Total area = $\frac{1}{2}\times 8.83^2\times(2\pi-1.65)+30=...$	dM1
	awrt 210 (m^2) or 211 (m^2)	A1
		(3)
(c)	$"6.03"\times 1.65=(9.95)$ **or** $8.83\times(2\pi-1.65)=(40.91)$	M1
	Full method for perimeter = $'6.03"\times 1.65+8.83\times(2\pi-1.65)+2.8\times 2$	dM1
	= awrt 56 (m)	A1
		(3)
		(9 marks)

Notes

Allow equivalent correct work in degrees.

(b)

M1: Attempts $\frac{1}{2}\times 8.83^2\times(2\pi-1.65)$ or equivalent e.g. $\pi\times 8.83^2-\frac{1}{2}\times 8.83^2\times 1.65$
May be implied by awrt 180 or awrt 181 provided no incorrect working is seen.

dM1: Attempts $\frac{1}{2}\times 8.83^2\times(2\pi-1.65)+30=...$
Adds 30 to a correct attempt at the area of the major sector of the larger circle.

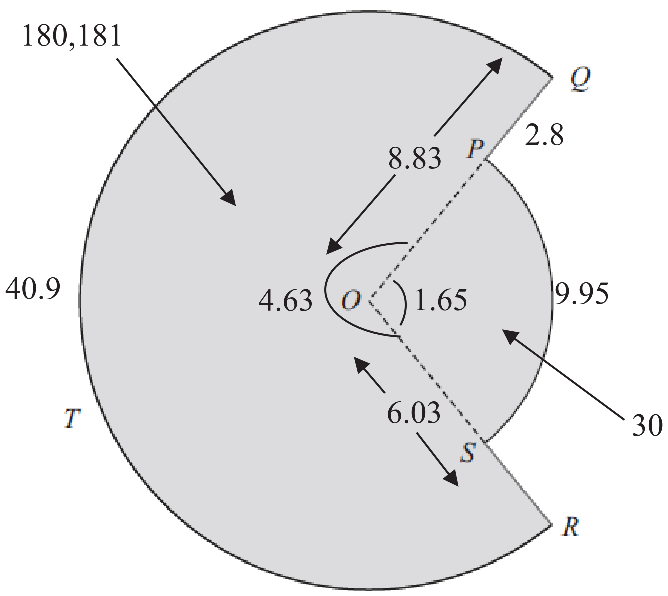
A1: awrt 210 (m^2) or awrt 211 (m^2)
No requirement for units but if any are given they must be correct.
Correct answer only scores no marks.

(c)

M1: Attempts $"6.03"\times 1.65$ which may be implied by awrt 9.95 or 3sf accuracy for their 6.03 provided no incorrect working is seen.
OR
Attempts $8.83\times(2\pi-1.65)$ which may be implied by awrt 40.9 provided no incorrect working is seen.

dM1: Full method for perimeter = $"6.03"\times 1.65+8.83\times(2\pi-1.65)+2.8\times 2$

A1: awrt 56 (m) . No requirement for units but if any are given they must be correct.
Correct answer only scores no marks.



Question Number	Scheme	Marks
8(a)	$f'\left(\sqrt{2}\right)=5 \Rightarrow 2\sqrt{2}+\frac{8}{2}+k=5 \Rightarrow k=...$	M1
	$1-2\sqrt{2}$	A1
		(2)
(b)	$x=\sqrt{2} \Rightarrow y=5\sqrt{2}-3\sqrt{2}\left(=2\sqrt{2}\right)$ So $y-"2\sqrt{2}"=-\frac{1}{5}\left(x-\sqrt{2}\right)$ **or** $y=-\frac{1}{5}x+c \Rightarrow "2\sqrt{2}"=-\frac{1}{5}\sqrt{2}+c \Rightarrow c=...$ $\Rightarrow y=-\frac{1}{5}x+\frac{11\sqrt{2}}{5}$	M1A1
		(2)

Notes

Mark all parts together.

(a)

M1: Substitutes $x=\sqrt{2}$ into $f'(x)$, sets = 5 and solves for *k*.

Condone slips in evaluating $f'\left(\sqrt{2}\right)$ as long as the intention is clear.

A1: Correct exact value $k=1-2\sqrt{2}$ oe e.g. $1-\sqrt{8}$

(b)

M1: Correct attempt at the equation of the normal using a gradient of $-\frac{1}{5}$, $x=\sqrt{2}$ and *y* from substituting $x=\sqrt{2}$ into $y=5x-3\sqrt{2}$

If they use $y = mx + c$ then they must reach as far as $c = \ldots$

A1: Correct equation in any form e.g. $y-2\sqrt{2}=-\frac{1}{5}\left(x-\sqrt{2}\right)$ or $5y+x=11\sqrt{2}$ or $y=-\frac{1}{5}x+\frac{11\sqrt{2}}{5}$

Apply isw as soon as a correct equation is seen.

(c)	$f'(x)=2x+\frac{8}{x^2}+k \Rightarrow \left(f(x)=\right)x^2-\frac{8}{x}+kx(+c)$	M1A1ft
	$(\sqrt{2}, 2\sqrt{2}) \Rightarrow 2\sqrt{2}=2-\frac{8}{\sqrt{2}}+(1-2\sqrt{2})\sqrt{2}+c \Rightarrow c=...(2+5\sqrt{2})$	dM1
	$\left(f(x)=\right)x^2-\frac{8}{x}+(1-2\sqrt{2})x+2+5\sqrt{2}$	A1
		(4)
		(8 marks)

Notes

Mark all parts together.

(c)

M1: Integrates with two terms having a correct index. Allow indices unprocessed.

It is for 2 from: $2x \to ...x^2\left(\text{or } x^{1+1}\right)$, $\frac{8}{x^2} \to \frac{...}{x}\left(\text{or } ...x^{-1} \text{ or } ...x^{-2+1}\right)$, $k \to ...x\left(\text{or } ...x^{0+1}\right)$

A1ft: Correct integration in any form with indices processed.

$\left(f(x)\right)=x^2-\frac{8}{x}+kx(+c)$ but follow through on their k which may be a decimal and allow the letter k. "$f(x)=$" is **not** required and "+ c" is **not** required.

Brackets are required around their $1-2\sqrt{2}$ but they may be implied by their attempt to find "c" so you may need to check their calculation for c.

dM1: Uses $x=\sqrt{2}$ and their attempt at y using $x=\sqrt{2}$ in $y=5x-3\sqrt{2}$ in their integrated function and finds the value of c.

A1: Correct simplified exact expression $\left(f(x)=\right)x^2-\frac{8}{x}+(1-2\sqrt{2})x+2+5\sqrt{2}$ or e.g.

$\left(f(x)=\right)x^2-\frac{8}{x}+x-2\sqrt{2}x+2+5\sqrt{2}$. "$f(x)=$" is **not** required.

Brackets are required around their $1-2\sqrt{2}$ if written as in the first example above.

Do not allow $1x^2$ or e.g. $\frac{2}{2}x^2$ for x^2 but allow $\sqrt{8}$ for $2\sqrt{2}$.

Apply isw once a correct function is seen.

Question Number	Scheme	Marks
9(a)	$\left(\mathrm{f}(x)=\right)-4\sin x$ or equivalent such as $\left(\mathrm{f}(x)=\right)4\cos\left(x+\frac{\pi}{2}\right)$	M1 A1
		(2)
(b)(i)	$\left(\frac{5\pi}{3}, 4\right)$	B1, B1
		(2)
(ii)	$\left(\frac{3\pi}{2}, -2\right)$	B1, B1
		(2)
		(6 marks)

Notes

Do not allow the use of degrees in this question.
If there is more than one answer for any part, score the final attempt.
Remember to check for answers written in the question.

(a)

M1: For any sine or cosine curve with an amplitude of 4 and a period of 2π and which passes through the origin. E.g. $\left(\mathrm{f}(x)=\right)\pm 4\sin(\pm x)$, $\left(\mathrm{f}(x)=\right)\pm 4\cos\left(x\pm\frac{\pi}{2}\right)$

Condone the use of θ for this mark and "f(x) =" (or "$\mathrm{f}(\theta)=$") is not required.

Do not allow ambiguous expressions e.g. $\left(\mathrm{f}(x)=\right)-4\mathrm{f}(\sin x)$

A1: Completely correct expression in terms of x e.g. $\left(\mathrm{f}(x)=\right)-4\sin x$ or equivalent such as $\left(\mathrm{f}(x)=\right)4\cos\left(x+\frac{\pi}{2}\right)$, $\left(\mathrm{f}(x)=\right)4\sin(x+\pi)$, $4\sin(-x)$

"f(x) =" is not required.

(b)

Allow answers to be written as coordinates or as $x = \ldots, y = \ldots$

If more than one answer is given, mark their final answer.

Condone missing brackets e.g. $\frac{5\pi}{3}, 4$ and condone punctuation other than a comma e.g. $\left(\frac{5\pi}{3}; 4\right)$

The x coordinates must be written as single fractions but allow equivalents e.g. $\frac{10\pi}{6}$ for $\frac{5\pi}{3}$

The y coordinate must be written as single number so do not allow 0 – 2 for –2

Do **not** allow coordinates the wrong way round e.g. $\left(4, \frac{5\pi}{3}\right)$ or $\left(-2, \frac{3\pi}{2}\right)$

(b) (i)

B1: One correct coordinate from $x=\frac{5\pi}{3}$ or $y=4$

B1: $\left(\frac{5\pi}{3}, 4\right)$ or $x=\frac{5\pi}{3}$ and $y=4$

(b) (ii)

B1: One correct coordinate from $x = \frac{3\pi}{2}$ or $y = -2$

B1: $\left(\frac{3\pi}{2}, -2\right)$ or $x = \frac{3\pi}{2}$ and $y = -2$

Question Number	Scheme	Marks
10(a)	$y=2x^3+\frac{1}{2}x^2-2x+5 \Rightarrow \frac{dy}{dx}=6x^2+x-2$ so at $x=0$, $\frac{dy}{dx}=-2$	M1
	Equation of normal at P is $y=\frac{1}{2}x+5$	A1
	Q is where $y=2x^3+\frac{1}{2}x^2-2x+5$ meets $y=\frac{1}{2}x+5$ $2x^3+\frac{1}{2}x^2-2x+5=\frac{1}{2}x+5 \Rightarrow 4x^3+x^2-5x=0$	**d**M1 A1
	$\Rightarrow x\left(4x^2+x-5\right)=0 \Rightarrow x(4x+5)(x-1)=0 \Rightarrow x=-\frac{5}{4}$	**dd**M1 A1
		(6)

Notes

(a)

M1: Attempts to find the gradient of the curve at P.

It is for attempting to find the value of $\frac{dy}{dx}$ when $x = 0$

There must be some evidence of differentiating e.g. $2x^3 \to ...x^2$ or $\frac{1}{2}x^2 \to ...x$ or $-2x \to -2$ or $5 \to 0$ followed by the substitution of $x = 0$ which may be implied.

Alternatively deduces that the gradient of the curve at P is -2

A1: Equation of normal at P is $y=\frac{1}{2}x+5$ oe e.g. $y-5=\frac{1}{2}(x-0)$

dM1: Sets $2x^3+\frac{1}{2}x^2-2x+5=kx+5$ where k is not their gradient of the tangent at P and collects terms to one side. Must be $kx + 5$ but condone a miscopy of the equation of C.

A1: Correct cubic equation with terms collected and all on one side e.g. $4x^3+x^2-5x=0$ or equivalent e.g. $2x^3+\frac{1}{2}x^2-\frac{5}{2}x=0$. The "= 0" may be implied by their attempt to solve.

ddM1: Cancels or factorises out an x term from an equation of the form $ax^3+bx^2+cx=0,\ a,b,c\neq 0$ and solves resulting 3TQ by any means including a calculator.

Condone factors that are inconsistent with their 3TQ so just look for the correct or correct ft roots.

Allow e.g. $4x^3+x^2-5x=0 \Rightarrow x(4x+5)(x-1)=0 \Rightarrow x=...$

Do **not** allow e.g. $4x^3+x^2-5x=0 \Rightarrow x=\frac{-1\pm\sqrt{1^2-4(4)(-5)}}{2\times 4}=...$

If roots of the 3TQ are just written down you may need to check.

They cannot just write down the roots of the cubic equation. Decimals should be correct to 2 sf or truncated to 2sf.

Depends on the previous method mark.

A1: $x=-\frac{5}{4}$ oe only. Ignore any attempts to find the y coordinate and ignore any incorrect attempts at finding the other x coordinate provided $x=-\frac{5}{4}$ is selected.

Question Number	Scheme	Marks
Note: full marks are available for those with an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ that still leads to a gradient at P of -2		

Beware in part (a) that it is possible to reach the correct answer fortuitously as follows:

$$y = 2x^3 + \tfrac{1}{2}x^2 - 2x + 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 + x - 2$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 6x^2 + x - 2 = 0 \Rightarrow x = \frac{1}{2}, -\frac{2}{3}$$

And then incorrectly assuming the gradient of the normal to C at P is $\frac{1}{2}$ and that l has the equation $y = \tfrac{1}{2}x + 5$

This will generally score no marks in (a) it is entirely fortuitous.

If you are unsure if a particular response deserves credit then use Review.

(b)	Sets $\frac{dy}{dx} = 6x^2 + x - 2 = \frac{1}{2}$	M1
	$12x^2 + 2x - 5 = 0$	A1
	$x = \frac{-2 \pm \sqrt{4+240}}{24}$	dM1
	$x = \frac{-1+\sqrt{61}}{12}$	A1
		(4)
		(10 marks)

Notes

(b)

M1: Sets their $\frac{dy}{dx}$ of the form $ax^2 + bx - 2,\ a, b \neq 0$ equal to $\frac{1}{2}$

A1: Correct 3 term equation $12x^2 + 2x - 5 = 0$ or equivalent e.g. $6x^2 + x - \frac{5}{2} = 0$

The constant terms must be collected but the terms do not need to be all on one side so allow e.g. $12x^2 + 2x = 5$

dM1: Correct attempt to solve a 3TQ resulting from $ax^2 + bx - 2 = \frac{1}{2}$.

Allow calculator solutions for solving their 3TQ and condone factors that are inconsistent with their equation so just look for the correct or correct ft roots.
Note that only the positive solution may be seen which is acceptable.
If values are just written down you may need to check. Decimals should be correct to 2 sf or truncated to 2sf.

A1: $x = \frac{-1+\sqrt{61}}{12}$ only.