Question Number	Scheme	Marks
1(a)(i)	$(2, 12)$	B1
(ii)	$(2, 15)$	B1
		(2)
(b)	$k \leqslant 3, \ \ k = 5$	M1 A1
		(2)
(c)	$(x =) -5$	B1
		(1)
		(5 marks)

<u>**Notes**</u>
Check for answers given in the body of the question. (There are lots!)
In the absence of any labelling for (a)(i) and (ii) the first answer is to be taken as (i)

(a)(i)
B1: $(2, 12)$ but allow coordinates given separately e.g. $x = 2, y = 12$

Condone missing brackets e.g. 2, 12 and condone $\begin{pmatrix} 2 \\ 12 \end{pmatrix}$

(a)(ii)
B1: $(2, 15)$ but allow coordinates given separately e.g. $x = 2, y = 15$

Condone missing brackets e.g. 2, 15 and condone $\begin{pmatrix} 2 \\ 15 \end{pmatrix}$

(b)
M1: One of $k \leqslant 3, \ \ k < 3$ or $k = 5$. Condone use of y for k but not x for k.
A1: $k \leqslant 3, \ \ k = 5$ or e.g. $k \leqslant 3$ or $k = 5$
Condone $k \leqslant 3$ and $k = 5$
Must be in terms of k. Allow $(-\infty, 3]$ for $k \leqslant 3$
An answer of $5 \leqslant k \leqslant 3$ scores M1A0

(c)
B1: -5 only and no other values.
But beware values of x that form part of their working which are clearly not part of their final answer e.g. $f(x) = 0$ when $x = -1$ so $f(x + 4) = 0$ when $x = -5$ scores B1
$x = ...$ is not required so just look for the correct value.
$(-5, 0)$ is B0

Question Number	Scheme	Marks
2(a)	$6 = 5a + b, \; 3.3 = 8a + b$	M1 A1
	$a = -0.9, b = 10.5$	dM1 A1
		(4)
(b)(i)	$10.5 \, \text{m}^3$	B1ft
(ii)	$-0.9\sqrt{t} + 10.5 = 0 \Rightarrow 0.9\sqrt{t} = 10.5 \Rightarrow t = \ldots$	M1
	$t = 136 \; (\text{min})$	A1
		(3)
		(7 marks)

Notes

(a)

M1: Sets up the two simultaneous equations, one of which must be correct.

Allow if square roots are present for this mark e.g. $6 = \sqrt{25}a + b$, $3.3 = \sqrt{64}a + b$

Condone e.g. $6 = \pm 5a + b$

A1: Two correct equations with the square roots processed. May be implied.

Allow e.g. $6 = \pm 5a + b$ and $3.3 = \pm 8a + b$ here.

dM1: Solves their equations by any means including calculator to obtain values for a and b.

A1: Correct values $a = -0.9, b = 10.5$ or the equivalents e.g. $a = -\dfrac{9}{10}, b = \dfrac{21}{2}$ only.

If they solve e.g. $6 = \pm 5a + b$ and $3.3 = \pm 8a + b$ and offer extra values score A0.

(b)(i)

B1ft: $10.5 \, \text{m}^3$ oe e.g. $\dfrac{21}{2} \, \text{m}^3$ <u>including the units</u>, ft on their b provided $b > 0$

(b)(ii)

M1: Solves their $-0.9\sqrt{t} + 10.5 = 0 \Rightarrow t = \ldots$ which must involve squaring, not square rooting.

You may need to check but may be implied by e.g. $t = \dfrac{1225}{9}$ (if a and b are correct)

Note that if a and b are correct, square rooting rather than squaring leads to 3.41565… and scores M0

A1: Awrt 136

Units are **not** required but if any are given they must be correct.

Condone "m" for "min"

Apply isw if necessary e.g. score A1 for $t = 136.1 = 137$ to the nearest minute.

Correct answer only of awrt 136 following a correct a and b scores M1A1

Question Number	Scheme	Marks
3(a)	$\left(f(x)=\right)-3\cos x$ or $\left(f(x)=\right)3\sin(x-90°)$	M1 A1
		(2)
(b)(i)	8	B1
(ii)	5	B1
		(2)
		(4 marks)

Notes
Check for answers given in the body of the question.

(a)

M1: $\left(f(x)=\right)\pm3\cos x$ or $\left(f(x)=\right)\pm3\cos(-x)$ or $\left(f(x)=\right)\pm3\sin(x\pm90°)$ or equivalent e.g.

$\left(f(x)=\right)\pm3\cos(x\pm180°)$

For this mark, their equation must be correct or be the equation of the given curve reflected in the x-axis.

Brackets are required if needed e.g. $\left(f(x)=\right)3\sin x-90°$ scores M0

The "$f(x)=$" is **not** required.

The degrees symbol is **not** required.

Condone use of another variable for this mark e.g. θ.

A1: $\left(f(x)=\right)-3\cos x$ or $\left(f(x)=\right)3\sin(x-90°)$ or $\left(f(x)=\right)-3\cos(-x)$ or equivalent.

If correct, the equations are likely to be one of the above but there are an infinite number of possibilities e.g. $\left(f(x)=\right)3\cos(x-180°)$, $\left(f(x)=\right)3\cos(x+180°)$, $\left(f(x)=\right)-3\sin(x-270°)$ etc.

The "$f(x)=$" is **not** required so ignore how they reference their expression.

The degrees symbol is **not** required.
Must be in terms of x not e.g. θ.

Note that just $\left(f(x)=\right)3\sin(x\pm90°)$ with no choice made scores M1A0

(b)(i)
B1: 8 cao. Must be 8 not just a list of roots.

(b)(ii)
B1: 5 cao. Must be 5 not just a list of roots.

Question Number	Scheme	Marks
4(i)	Uses a correct law of indices on 2^{4k-3} or 8^{1-k} The possibilities are endless but some more common examples are: For 2^{4k-3}: $2^{4k} \times 2^{-3}$, $\dfrac{2^{4k}}{8}$, $8^{\frac{4k-3}{3}}$, $4^{\frac{4k-3}{2}}$, $\left(\sqrt{2}\right)^{8k-6}$ For 8^{1-k}: 8×8^{-k}, $\dfrac{8}{8^k}$, $2^{3(1-k)}$, $4^{\frac{2(1-k)}{3}}$, $\left(\sqrt{2}\right)^{6-6k}$ These may be seen in isolation e.g. not in an equation. But not just e.g. $8 = 2^3$	M1
	e.g. $$2^{4k-3} = \dfrac{8^{1-k}}{4\sqrt{2}} \Rightarrow 2^{4k-3} = \dfrac{2^{3(1-k)}}{2^{\frac{5}{2}}} \Rightarrow 2^{4k-3} = 2^{3(1-k)-\frac{5}{2}}$$ $$\Rightarrow 4k-3 = 3(1-k) - \dfrac{5}{2} \Rightarrow k = \ldots$$	dM1
	$$k = \dfrac{1}{2}$$	A1
		(3)

Notes

Part (i) can be solved using many different approaches.
Please follow the general guidelines below and seek advice from your Team Leader if necessary.
Note that it is acceptable to use logs within their solution as long as the first mark is scored but
they should have a base that is a power of 2 (see note below*)

(i)

M1: This is effectively a B mark and is for applying a correct law of indices on 2^{4k-3} or 8^{1-k}
See scheme for some examples.

Note that this may be implied by e.g. $x = 2^k \Rightarrow 2^{4k-3} = \dfrac{x^4}{8}$

dM1: For this mark they must
 • reach a value for k
 • reach an equation of a form so that the indices in terms of k can be compared e.g.
  $\alpha^{f(k)} = \alpha^{g(k)} \Rightarrow f(k) = g(k)$ **or** of the form $\alpha^{f(k)} = \beta$ where $f(k)$ and $g(k)$ are linear

 Note that the second bullet point may be implied by e.g. $2^{4k-3} = \dfrac{2^{3(1-k)}}{2^{\frac{5}{2}}} \Rightarrow 4k-3 = 3(1-k) - \dfrac{5}{2}$

A1: Obtains $k = \dfrac{1}{2}$ from a correct equation of one of the forms above.

 Note that correct work leading to $128^k = 8\sqrt{2} \Rightarrow k = \dfrac{1}{2}$ or e.g. $k = \log_{128} 8\sqrt{2} = \dfrac{1}{2}$ is acceptable.

 *But note that e.g. $7k = \dfrac{\log_{10} 8\sqrt{2}}{\log_{10} 2} \Rightarrow k = \dfrac{1}{2}$ is not acceptable without justification and scores a

 maximum of M1dM1A0
 Correct answers with no working in (i) score no marks.

(ii)	$\dfrac{x\sqrt{3}+2}{\sqrt{3}-1} = x\sqrt{3}-4 \Rightarrow x\sqrt{3}+2 = \left(x\sqrt{3}-4\right)\left(\sqrt{3}-1\right)$	
	$\Rightarrow x\sqrt{3}+2 = 3x+4-4\sqrt{3}-x\sqrt{3}$	M1
	$\Rightarrow 2\sqrt{3}x-3x = 2-4\sqrt{3}$	
	$\Rightarrow x\left(2\sqrt{3}-3\right) = 2-4\sqrt{3}$	A1
	$\Rightarrow x = \dfrac{2-4\sqrt{3}}{2\sqrt{3}-3} \times \dfrac{2\sqrt{3}+3}{2\sqrt{3}+3}$	dM1
	$= -6-\dfrac{8}{3}\sqrt{3}$	A1
		(4)

	Alternative	
	$\dfrac{x\sqrt{3}+2}{\sqrt{3}-1} = x\sqrt{3}-4 \Rightarrow \dfrac{x\sqrt{3}+2}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = x\sqrt{3}-4$	
	$\Rightarrow \dfrac{3x+2\sqrt{3}+x\sqrt{3}+2}{3-1} = x\sqrt{3}-4$	M1
	$\Rightarrow 3x+2\sqrt{3}+x\sqrt{3}+2 = 2x\sqrt{3}-8 \Rightarrow \sqrt{3}x-3x = 10+2\sqrt{3}$	
	$\Rightarrow x\left(\sqrt{3}-3\right) = 10+2\sqrt{3}$	A1
	$x = \dfrac{10+2\sqrt{3}}{\sqrt{3}-3} \times \dfrac{\sqrt{3}+3}{\sqrt{3}+3}$	dM1
	$= \dfrac{10\sqrt{3}+30+6+6\sqrt{3}}{3-9} = -6-\dfrac{8}{3}\sqrt{3}$	A1

(ii)

M1: Attempts to form a linear equation in x with x terms on one side and constants on the other. The x does not need to be factorised out from the x terms for this mark.

A1: A **correct** linear equation in x with terms collected and x factorised out.

Possible correct equations are: $x\left(2\sqrt{3}-3\right) = 2-4\sqrt{3}$ or $x\left(\sqrt{3}-3\right) = 10+2\sqrt{3}$

May be unsimplified e.g. $x\left(\sqrt{3}+\sqrt{3}-3\right) = 4-2-4\sqrt{3}$

dM1: Makes x the subject and correctly rationalises the denominator of x of the form $\alpha+\beta\sqrt{3}$ where α and β are non-zero.

e.g. $x = \dfrac{...}{\alpha+\beta\sqrt{3}} \times \dfrac{\alpha-\beta\sqrt{3}}{\alpha-\beta\sqrt{3}}$ oe and proceeds to an answer of the form $a+b\sqrt{3}$ or $\dfrac{a+b\sqrt{3}}{c}$

Sight of e.g. $x = \dfrac{2-4\sqrt{3}}{2\sqrt{3}-3} \times \dfrac{2\sqrt{3}+3}{2\sqrt{3}+3}$ is sufficient but may be implied by e.g.

$x = \dfrac{2-4\sqrt{3}}{2\sqrt{3}-3} = \dfrac{4\sqrt{3}+6-24-12\sqrt{3}}{12-9}$

A1: Reaches $-6-\dfrac{8}{3}\sqrt{3}$ following dM1

Allow equivalents e.g. $-\dfrac{36}{6}-\dfrac{16}{6}\sqrt{3}$ but not e.g. $\dfrac{-18-8\sqrt{3}}{3}$

Apply isw once the correct answer is seen.

(ii) Alternative method: Attempts to rationalise first

M1: Multiplies the left hand side of the equation by $\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$ <u>leaving the rhs unchanged</u> and then

attempts to form a linear equation in x with x terms on one side and constants on the other. The x does not need to be factorised out from the x terms for this mark. Condone invisible brackets even if not recovered e.g.

$$\frac{x\sqrt{3}+2}{\sqrt{3}-1} = x\sqrt{3}-4 \Rightarrow \frac{x\sqrt{3}+2(\sqrt{3}+1)}{3-1} = x\sqrt{3}-4$$

A1: A **correct** linear equation in x with terms collected and x factorised out.

dM1: Makes x the subject and correctly rationalises the denominator of x of the form $\alpha + \beta\sqrt{3}$ where α and β are non-zero.

e.g. $x = \dfrac{...}{\alpha + \beta\sqrt{3}} \times \dfrac{\alpha - \beta\sqrt{3}}{\alpha - \beta\sqrt{3}}$ oe and proceeds to an answer of the form $a + b\sqrt{3}$.

Sight of e.g. $x = \dfrac{10+2\sqrt{3}}{\sqrt{3}-3} \times \dfrac{\sqrt{3}+3}{\sqrt{3}+3}$ is sufficient but may be implied by e.g.

$$x = \frac{10+2\sqrt{3}}{\sqrt{3}-3} = \frac{10\sqrt{3}+30+6+6\sqrt{3}}{3-9}$$

A1: Reaches $-6 - \dfrac{8}{3}\sqrt{3}$ following dM1

Allow equivalents e.g. $-\dfrac{36}{6} - \dfrac{16}{6}\sqrt{3}$ but not e.g. $\dfrac{-18-8\sqrt{3}}{3}$

Apply isw once the correct answer is seen.

Question Number	Scheme	Marks
	$$y = \frac{1}{2}x^4 - 3 + \frac{10}{x^2}$$	
5(a)	$$\left(\int \frac{1}{2}x^4 - 3 + \frac{10}{x^2}\, dx = \right) \frac{1}{10}x^5 - 3x - \frac{10}{x} + c$$	M1A1A1
		(3)
(b)(i)	$$\left(\frac{dy}{dx} = \right) 2x^3 - \frac{20}{x^3}$$	M1A1A1
		(3)
(b)(ii)	$$2x^3 - \frac{20}{x^3} = 3 \Rightarrow 2x^6 - 20 = 3x^3$$	M1
	$$2x^6 - 3x^3 - 20 = 0 \Rightarrow \left(2x^3 + 5 \right)\left(x^3 - 4 \right) = 0 \Rightarrow x^3 = \ldots \Rightarrow x = \ldots$$ or e.g. $$x^3 = a \Rightarrow (2a+5)(a-4) = 0 \Rightarrow a = \ldots \Rightarrow x^3 = \ldots \Rightarrow x = \ldots$$	dM1
	$$\Rightarrow x = -\sqrt[3]{\frac{5}{2}}, \quad \sqrt[3]{4}$$	A1A1
		(4)
		(10 marks)

Notes

(a)

M1: For increasing any power by one. E.g. $\frac{1}{2}x^4 \rightarrow \ldots x^5$ or $-3 \rightarrow \ldots x$ or $\frac{10}{x^2} \rightarrow \ldots x^{-1}$

Allow unprocessed indices for this mark e.g. $\frac{1}{2}x^4 \rightarrow \ldots x^{4+1}$

A1: For two correct and **simplified** terms of $\frac{1}{10}x^5 - 3x - \frac{10}{x}$.

Allow $0.1x^5$ for $\frac{1}{10}x^5$ and $-10x^{-1}$ for $-\frac{10}{x}$.

Condone $-3x^1$ for $-3x$ and $-\frac{10}{x^1}$ for $-\frac{10}{x}$ for <u>this</u> mark.

A1: $\frac{1}{10}x^5 - 3x - \frac{10}{x} + c$. Must include a constant of integration.

Ignore the lhs i.e. ignore what they call their integral.

Allow $0.1x^5$ for $\frac{1}{10}x^5$ and $10x^{-1}$ for $\frac{10}{x}$.

Condone spurious integral signs e.g. $\int \frac{1}{2}x^4 - 3 + \frac{10}{x^2}\, dx = \int \frac{1}{10}x^5 - 3x - \frac{10}{x} + c$

Do not allow e.g. $\frac{1}{10}x^5 - 3x + -\frac{10}{x} + c$.

Mark (b)(i) and (ii) together.

(b)(i)

M1: For decreasing any power by one. E.g. $\frac{1}{2}x^4 \to ...x^3$ or $-3 \to 0$ or $\frac{10}{x^2} \to ...x^{-3}$

Allow unprocessed indices for this mark e.g. $\frac{1}{2}x^4 \to ...x^{4-1}$

A1: For one correct and **simplified** term. $2x^3$ or $-\frac{20}{x^3}$. Allow $20x^{-3}$ for $\frac{20}{x^3}$.

A1: $2x^3 - \frac{20}{x^3}$. Allow $20x^{-3}$ for $\frac{20}{x^3}$.

(b)(ii)

M1: Sets their $2x^3 - \frac{20}{x^3}$ equal to 3 and multiplies to form a **polynomial equation** in x.

Allow copying slips but it must be their $\frac{dy}{dx}$ from part (b)(i) and it must be of the form

$Ax^m + \frac{B}{x^n}$ where m and n are integers and $m > 0$ and $n > 0$

It is for proceeding from $Ax^m + \frac{B}{x^n} = 3$ to $Ax^{m+n} + B = 3x^n$ or equivalent work.

dM1: Solves an equation of the form $px^6 + qx^3 + r = 0$, $p, q, r \neq 0$ via **non calculator** means to find at least one value for x. It is not for finding a value of x^3 so e.g.

$t = x^3 \Rightarrow 2x^6 - 3x^3 - 20 = 2t^2 - 3t - 20,\ 2t^2 - 3t - 20 = 0 \Rightarrow (2t+5)(t-4) = 0 \Rightarrow t = -\frac{5}{2},\ 4$

on its own scores M0 unless a value of x is found by cube rooting. Note we will condone the "cube rooting" done on a calculator for this mark but **not** the solving of their equation.

A1: Either solution of $x = -\sqrt[3]{\frac{5}{2}},\ \sqrt[3]{4}$ o.e. such as $-2.5^{\frac{1}{3}},\ 4^{\frac{1}{3}}$

Allow $(-2.5)^{\frac{1}{3}}$ for $-2.5^{\frac{1}{3}}$ and $2^{\frac{2}{3}}$ for $4^{\frac{1}{3}}$

Must come from correct work i.e. from the correct equation $2x^6 - 3x^3 - 20 = 0$ oe
Apply isw once a correct answer is seen.

A1: $x = -\sqrt[3]{\frac{5}{2}},\ \sqrt[3]{4}$ o.e. such as $-2.5^{\frac{1}{3}},\ -\frac{\sqrt[3]{20}}{2},\ 4^{\frac{1}{3}}$

Allow $(-2.5)^{\frac{1}{3}}$ for $-2.5^{\frac{1}{3}}$ and $2^{\frac{2}{3}}$ for $4^{\frac{1}{3}}$

Must come from correct work i.e. from the correct equation $2x^6 - 3x^3 - 20 = 0$ oe
Apply isw once correct answers are seen.

<u>Notes on evidence of use of a calculator in (b)(ii):</u>

e.g. $2x^6 - 3x^3 - 20 = 0 \Rightarrow (x^3 + 2.5)(x^3 - 4) \Rightarrow x^3 = -2.5,\ 4 \Rightarrow x = ...$

and

$x^3 = a \Rightarrow 2a^2 - 3a - 20 = 0 \Rightarrow (a + 2.5)(a - 4) \Rightarrow a = -2.5,\ 4 \Rightarrow x^3 = ... \Rightarrow x = ...$

Both score dM0A0A0

Question Number	Scheme	Marks
6.	$(x+5)(3x+2)(2x-5) = (3x^2+17x+10)(2x-5) = \ldots$	M1
	$= 6x^3 - 15x^2 + 34x^2 - 85x + 20x - 50$ $\left(= 6x^3 + 19x^2 - 65x - 50 \right)$	A1
	$6x^3 + 19x^2 - 65x - 50 = 3x^2 - 33x - 50$ $\Rightarrow 6x^3 + 16x^2 - 32x = 0$	dM1
	$2x(3x-4)(x+4) = 0$	ddM1
	$x = -4, \, 0, \, \dfrac{4}{3}$	A1
		(5)
		(5 marks)

Notes

M1: For an attempt to multiply out the $(x+5)(3x+2)(2x-5)$.
Must obtain terms in x^3, x^2, x and a constant.

A1: $6x^3 + 19x^2 - 65x - 50$ but may be left unsimplified e.g. $6x^3 - 15x^2 + 34x^2 - 85x + 20x - 50$
Award this mark if a correct unsimplified expansion is seen and errors are subsequently made when collecting terms.

dM1: Sets their $6x^3 + 19x^2 - 65x - 50 = 3x^2 - 33x - 50$, collects terms and proceeds to a cubic expression = 0. This may be implied if they cancel the 50's and then divide through by x to obtain a quadratic expression = 0. The "= 0" may be implied by subsequent work.
Depends on the previous method mark.

ddM1: Attempts to solve a cubic of the form $px^3 + qx^2 + rx = 0$, $p, q, r \neq 0$ by **non-calculator** methods This must involve dividing or factorising out x (or e.g. $2x$) followed by an attempt to solve a quadratic of the form $px^2 + qx + r = 0$, $p, q, r \neq 0$ by **non-calculator** methods – see General Guidance. Just quoting the quadratic formula followed by roots written down is M0
Depends on both previous method marks.

A1: $x = -4, \, 0, \, \dfrac{4}{3}$. Allow exact equivalents for $\dfrac{4}{3}$ including $1.\dot{3}$
Note that the $x = 0$ may appear earlier in their solution so look out for this.

Guidance for the ddM1A1 marks:

$$6x^3 + 16x^2 - 32x = 0 \Rightarrow 6x^2 + 16x - 32 = 0 \Rightarrow (3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$$

Scores ddM0A0

$$6x^3 + 16x^2 - 32x = 0 \Rightarrow 6x^2 + 16x - 32 = 0 \Rightarrow 2(3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$$
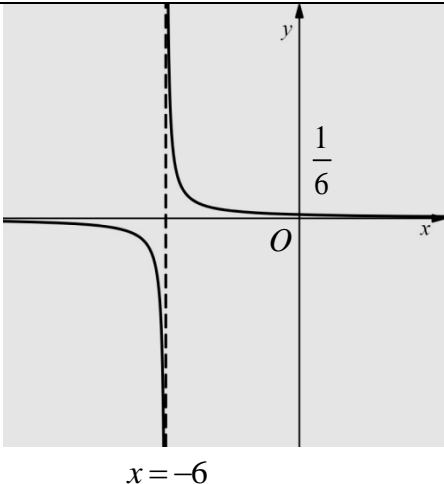
Scores ddM1A0

$$6x^3 + 16x^2 - 32x = 0 \Rightarrow x\left(6x^2 + 16x - 32\right) = 0 \Rightarrow x\left(x - \frac{4}{3}\right)(x+4) = 0 \Rightarrow x = 0, \, \frac{4}{3}, -4$$

Scores ddM0A0

$$3x^3 + 8x^2 - 16x = 0 \Rightarrow (3x-4)(x+4) = 0 \Rightarrow x = \frac{4}{3}, -4$$

Scores ddM0A0

Question Number	Scheme	Marks
7(a)	Correct shape and position	B1
	Correct **equation** of vertical asymptote. E.g. $x = -6$ oe e.g. $x + 6 = 0$	B1
	Correct y intercept $\left(0, \dfrac{1}{6}\right)$	B1
		(3)

Notes

There must be a sketch to score marks in (a)
Allow the equation of the asymptote and the y intersection to be identified away from the sketch but they must clearly be identified as the asymptote and/or the intersection and correspond to their sketch. If there is any ambiguity, the sketch takes precedence.

(a)
B1: Correct shape and position.
The right hand branch should
- be in quadrants 1 and 2 only (this includes just touching the y-axis)
- approach the x-axis from above on the right
- approach the vertical upwards on the left in quadrant 2

The left hand branch should
- be in quadrant 3 only
- approach the x-axis from below on the left
- approach the vertical downwards on the right

The branches must not clearly overlap.
The curve must not clearly turn back on itself at the vertical or horizontal asymptotes but condone "slips of the pen".

B1: Correct **equation** for vertical asymptote. The curve must be asymptotic here for their curve and for both branches if drawn.
The asymptote does not need to be "drawn" e.g. dotted or solid line.
Must be an equation (not just -6 marked on the x-axis)

B1: Correct y intercept. The curve must only cross the y-axis at this point.

Allow as $\dfrac{1}{6}$ marked on the y-axis or as $\left(0, \dfrac{1}{6}\right)$ and condone $\left(\dfrac{1}{6}, 0\right)$ as long as it is in the correct position on the positive y-axis.

Also allow $y = \dfrac{1}{6}$ if it is clearly indicating the point of intersection and not the equation of an asymptote.

See supplementary document for some examples.

(b)(i)	$mx - 4 = \dfrac{1}{x+6} \Rightarrow (mx-4)(x+6) = 1$	M1
	$\Rightarrow mx^2 + (6m-4)x - 25 = 0$ or $mx^2 + 6mx - 4x - 25 = 0$	A1
	$"b^2 - 4ac" = (6m-4)^2 - 4 \times m \times (-25)$	dM1
	$\Rightarrow (6m-4)^2 + 100m \ldots 0$ $\Rightarrow 36m^2 + 52m + 16 \ldots 0 \Rightarrow 9m^2 + 13m + 4 \ldots 0 *$	A1*
(ii)	$(m+1)(9m+4) \ldots 0 \Rightarrow m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$	M1 A1
		(6)

Mark (b)(i) and (ii) together.

(b)(i)

M1: Sets $mx - 4 = \dfrac{1}{x+6}$ and attempts to cross multiply

A1: Correct simplified quadratic equation with terms on one side which may be implied by subsequent work. The "= 0" may be implied.
The $(6m-4)x$ may be expanded.

dM1: Attempts the discriminant $"b^2 - 4ac"$ for their 3TQ. There must be no x's in their a, b and c.
May be seen embedded in an attempt to use the quadratic formula.

A1*: cso. Correct proof with an intermediate line after e.g. $(6m-4)^2 + 100m \ldots 0$
The $\ldots 0$ should appear before the final line and there must have been no contradictory
statements such as $(6m-4)^2 + 100m = 0$ **but** if the discriminant is found and a justification that
it must be $\ldots 0$ is given, this mark can be awarded.

(b)(ii)

M1: Uses a correct method to find the critical values of $9m^2 + 13m + 4 = 0$ **and** finds the outside
range for their critical values. Allow any method of solving $9m^2 + 13m + 4 = 0$ including use of
a calculator. Condone strict inequalities for this mark and condone use of x instead of m.

A1: $m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$ or e.g. $m \,,, \, -1$ or $m \ldots -\dfrac{4}{9}$ Condone $m \,,, \, -1$ and $m \ldots -\dfrac{4}{9}$ but **not**

$-\dfrac{4}{9} \,,, \, m \,,, \, -1$

Allow equivalents e.g. $(-\infty, -1], \left[-\dfrac{4}{9}, \infty\right)$ and allow exact equivalents for $-\dfrac{4}{9}$ including $-0.\dot{4}$

Note that they cannot go straight from $9m^2 + 13m + 4 \ldots 0$ to $m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$ without

finding or stating the roots first so that $9m^2 + 13m + 4 \ldots 0 \Rightarrow m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$ scores M0A0

But $9m^2 + 13m + 4 \ldots 0 \Rightarrow m = -1, \; m = -\dfrac{4}{9} \Rightarrow m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$ scores M1A1

$9m^2 + 13m + 4 \ldots 0 \Rightarrow (9m+4)(m+1) \ldots 0 \Rightarrow m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$ scores M1A1

Or e.g. a sketch of the quadratic with -1 and $-\dfrac{4}{9}$ indicated followed by $m \,,, \, -1, \quad m \ldots -\dfrac{4}{9}$

scores M1A1

Generally, there must be some intermediate work seen before the inequalities are written down.

Question Number	Scheme	Marks
8(a)	$$m = \frac{18-0}{2--3}$$	M1
	$$y-18 = \frac{18}{5}(x-2) \text{ or e.g. } y = \frac{18}{5}(x+3)$$ or $$18 = \frac{18}{5}(2)+c \Rightarrow c = ... \text{ or e.g. } 0 = \frac{18}{5}(-3)+c \Rightarrow c = ...$$	dM1
	$$y = \frac{18}{5}x + \frac{54}{5}$$	A1
		(3)
(b)	Examples: $$y = ax^2 + bx - 6$$ $$y = (x+3)(px+q)$$ $$y = ax^2 + bx + c, \ (0,-6),(2,18),(-3,0) \Rightarrow a = ... \text{ or } b = ... \text{ or } c = ...$$ NB $a = \frac{14}{5}, \ b = \frac{32}{5}, \ c = -6$	M1
	Full method to find all constants. E.g. Solves $9a-3b-6=0$ and $4a+2b-6=18 \Rightarrow a = ..., b = ...$	dM1
	Either $a = \frac{14}{5}$ or $b = \frac{32}{5}$	A1
	$$y = \frac{14}{5}x^2 + \frac{32}{5}x - 6$$	A1
		(4)
(c)	$$\frac{14}{5}x^2 + \frac{32}{5}x - 6 < y < \frac{18}{5}x + \frac{54}{5}, \quad x < 0$$	M1 A1ft
		(2)
		(9 marks)

Notes

(a)

M1: Attempts gradient $= \frac{18-0}{2--3}$.

Condone 1 sign/copying slip only if a correct formula is seen or implied.

Alternatively solves two simultaneous equations, e.g. $18 = 2m+c$ and $0 = -3m+c$ to find m or c.

dM1: Full attempt at equation of the line using $(-3, 0)$ or $(2, 18)$ with values correctly placed.

If they use $y = mx + c$ they must proceed as far as finding a value for c.

If they use e.g. $\frac{y}{18-0} = \frac{x+3}{2--3}$ both M's can be scored together.

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A1: $y = \frac{18}{5}x + \frac{54}{5}$ o.e.

(b)

M1: Uses a method that finds one of the unknowns leaving an equation in two unknowns.

- States equation is $y = ax^2 + bx - 6$ (the most common)
- States equation is $y = (x+3)(px+q), \; p \neq 1$ or e.g. $y = a(x+3)(x+b)$
- States equation is $y = ax^2 + bx + c$ and uses all 3 coordinates to find one unknown

dM1: Uses all three coordinates to find all three unknowns. Condone slips with substitution provided the intention is clear.

A1: Two correct unknowns found.

A1: Correct **equation**. E.g $y = \dfrac{14}{5}x^2 + \dfrac{32}{5}x - 6$ or $y = (x+3)\left(\dfrac{14}{5}x - 2\right)$ or $y = \dfrac{14}{5}(x+3)\left(x - \dfrac{5}{7}\right)$

but not e.g. $C = \ldots$

Note that attempts at e.g. $y = x^2 + bx - 6$ score no marks as there are insufficient unknowns.

(c)

M1: Uses two of the three required inequalities. Two may be combined as seen in the scheme. Accept „ $\leftrightarrow <$. Follow through on their equations for l and C.

$"\dfrac{14}{5}x^2 + \dfrac{32}{5}x - 6" < y < "\dfrac{18}{5}x + \dfrac{54}{5}"$ would count as 2 correct inequalities.

Do **not** allow inequalities in terms of R e.g. $\dfrac{14}{5}x^2 + \dfrac{32}{5}x - 6 < R < \dfrac{18}{5}x + \dfrac{54}{5}$

A1ft: Correct definition of R using their equations for l and C.

E.g. $y \ldots \dfrac{14}{5}x^2 + \dfrac{32}{5}x - 6, \; y \text{ „ } \dfrac{18}{5}x + \dfrac{54}{5}, \quad x \text{ „ } 0$ with inequalities used consistently.

If applying ft, their equations must be appropriate e.g. $y = mx + c, \; m, c \neq 0$ for l and a 3 term quadratic expression for C.

If they have a lower limit of -3 on the restriction on x e.g. $-3 \text{ „ } x \text{ „ } 0$ this is fine as long as it is consistent. If their lower limit is less than -3 then the left hand inequality can be strict or non-strict i.e. consistency is not required.

This mark should be withheld if any extra incorrect inequalities are given e.g. if they impose an extra restriction on y vertically then $-\dfrac{338}{35} \text{ „ } y \text{ „ } \dfrac{54}{5}$ would be correct (for correct (a) and (b))

but anything narrower than this would not.

Question Number	Scheme	Marks
9(a)	$\left(CQ^2 = \right) 0.5^2 + 1.84^2 - 2 \times 0.5 \times 1.84 \cos 0.8$	M1
	(Radius =) $CQ = 1.534\,\text{m}$	A1
		(2)
(b)	$\dfrac{\sin PCQ}{0.5} = \dfrac{\sin 0.8}{"1.534"}$ or $0.5^2 = "1.534"^2 + 1.84^2 - 2 \times "1.534" \times 1.84 \cos PCQ$	M1
	$\Rightarrow \sin PCQ = \dfrac{0.5 \sin 0.8}{"1.534"} \left(= 0.233....\right) \Rightarrow PCQ = 0.236\,*$ or $\cos PCQ = \dfrac{"1.534"^2 + 1.84^2 - 0.5^2}{2 \times "1.534" \times 1.84} \left(= 0.972...\right) \Rightarrow PCQ = 0.236\,*$	A1*
		(2)
(c)	$\dfrac{1}{2} r^2 \theta = \dfrac{1}{2} \times "1.534"^2 \times \left(2\pi - 0.236\right)$ oe e.g. $\pi "1.534"^2 - \dfrac{1}{2} \times "1.534"^2 \times 0.236$ $(= 7.114\ldots)$ OR $\dfrac{1}{2} ab \sin C = \dfrac{1}{2} \times 0.5 \times 1.84 \sin\left(0.8\right)$ or $\dfrac{1}{2} \times "1.534" \times 1.84 \sin\left(0.236\right) \left(= 0.3299...\right)$	M1
	Attempts $\dfrac{1}{2} r^2 \theta = \dfrac{1}{2} \times "1.534"^2 \times \left(2\pi - 0.236\right)$ **AND** $\dfrac{1}{2} ab \sin C = \dfrac{1}{2} \times 0.5 \times 1.84 \sin\left(0.8\right)$ **AND** adds	dM1
	(awrt) 7.4 (m^2)	A1
		(3)
(d)	Attempts $r\theta = "1.534" \times \left(2\pi - 0.236\right) \left(= 9.276...\right)$	M1
	Perimeter $= "1.534" \times \left(2\pi - 0.236\right) + 0.5 + \left(1.84 - 1.534\right) = 10.1$ (m)	A1
		(2)
		(9 marks)

Notes

It is acceptable in this question to work in degrees and convert if necessary.
NB 0.8 radians is 45.8366…° and angle *PCQ* is 13.5205…°
Note that there will be other valid methods – if in doubt use review.
See right angled triangle methods for (a) and (b) at the end of the MS

(a)

M1: Writes down the correct rhs for the cosine rule.

Allow e.g. $\cos 0.8 = \dfrac{0.5^2 + 1.84^2 - CQ^2}{2 \times 0.5 \times 1.84}$

A1: $CQ = 1.534$ This value only i.e. not awrt.

(b)

M1: A correct sine rule statement following through on their *CQ* which may appear as

$\sin PCQ = \dfrac{0.5 \sin 0.8}{1.534}$ and would also count as the intermediate step below.

A1*: Correct proof with an intermediate line such as $\sin PCQ = \dfrac{0.5\sin 0.8}{1.534}$ or $\sin PCQ = 0.233....$

If awarding for the value of $\sin PCQ$ then allow $\sin PCQ = $ awrt 0.23

There should be no incorrect statements in the proof e.g. $PCQ = \dfrac{0.5\sin 0.8}{1.534} = 0.236$

and no obvious incorrect work. E.g. although using CQ as 1.53 gives $PCQ = 0.237$ to 3 dp we will condone PCQ given as 0.236 if no incorrect work is seen as we assume greater accuracy for CQ has been used. If CQ is used to even less accuracy in their working e.g. 1.5 score M1A0 for otherwise correct work.
Must be 0.236 and not e.g. 0.2360

Alternative:

M1: A correct cosine rule statement following through on their CQ which may appear as

$\cos PCQ = \dfrac{1.534^2 + 1.84^2 - 0.5^2}{2\times 1.534\times 1.84}$ and would also count as the intermediate step below.

A1*: Correct proof with an intermediate line such as $\cos PCQ = \dfrac{1.534^2 + 1.84^2 - 0.5^2}{2\times 1.534\times 1.84}$ or

$\cos PCQ = 0.972....$

If awarding for the value of $\sin PCQ$ then allow $\cos PCQ = $ awrt 0.97

There should be no incorrect statements in the proof.
If they work in degrees and reach e.g. $PCQ = 13.5°$ it is acceptable to then state " $= 0.236$"
Must be 0.236 and not e.g. 0.2360

(c)

M1: Correct method for the area of major sector CQR **or** correct method for area of triangle PQC
May be seen embedded in a calculation. Condone the use of 1.8 for 1.84 as long as the intention is clear.

dM1: Fully correct method for the total area. E.g. finds area of major sector CQR and area triangle PQC and adds **or** finds (area of circle – minor sector CQR + area triangle PQC) or (area of circle + (area triangle PQC - minor sector CQR))

A1: (awrt) 7.4 (m^2)

Beware: The area of just the circle is 7.39… i.e. awrt 7.4 but on its own scores no marks.
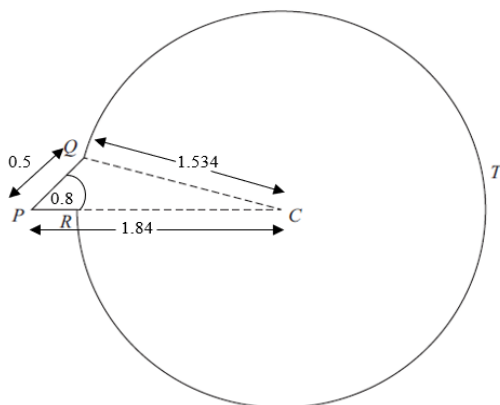**There are also other incorrect methods that give awrt 7.4 <u>so you need to check their working carefully!</u>**

(d)

M1: Attempts $r\theta = "1.534"\times\left(2\pi - 0.236\right)$ or e.g. $2\pi\times"1.534" - "1.534"\times 0.236$

A1: awrt 10.1 (m)

For reference:

Major sector area 7.114…

Minor sector area 0.277…

Triangle area 0.3299…

Angle $PQC = $ awrt 2.11

Question Number	Scheme	Marks
10(a)	$$m = \frac{4}{5}$$	B1
	$$y + 3 = \frac{4}{5}(x-4) \text{ or } -3 = \frac{4}{5} \times 4 + c \Rightarrow c = \ldots$$	M1
	$$y = \frac{4}{5}x - \frac{31}{5}$$	A1
		(3)
(b)	$$f'(4) = \frac{4}{5} \Rightarrow \frac{k\sqrt{4}(4-3)}{5} = \frac{4}{5} \Rightarrow k = \ldots$$	M1
	$$k = 2$$	A1
		(2)
(c)	$$\left(f'(x) = \right)\frac{2\sqrt{x}(x-3)}{5} = \frac{2x^{\frac{3}{2}}}{5} - \frac{6x^{\frac{1}{2}}}{5}$$	M1
	$$\left(f(x) = \right)\frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5}(+c)$$	M1 A1ft
	$$x = 4, y = -3 \Rightarrow -3 = \frac{4(4)^{\frac{5}{2}}}{25} - \frac{4(4)^{\frac{3}{2}}}{5} + c \Rightarrow c = \left(-\frac{43}{25}\right)$$	ddM1
	$$\left(f(x) = \right)\frac{4x^{\frac{5}{2}}}{25} - \frac{4x^{\frac{3}{2}}}{5} - \frac{43}{25}$$	A1
		(5)
		(10 marks)

Notes

Mark (a), (b) and (c) together

(a)

B1: States or implies that gradient of tangent (to the curve at *P*) is $\frac{4}{5}$

M1: Uses their **changed** gradient (i.e. not $-\frac{5}{4}$) with $x = 4$ and $y = -3$ where the -3 is their attempt at

finding *y* from $y = -\frac{5}{4}x + 2 = -\frac{5}{4} \times 4 + 2$ to form the equation of tangent.

E.g. $y + 3 = \frac{4}{5}(x-4)$ or $y = \frac{4}{5}x + c \Rightarrow -3 = \frac{4}{5} \times 4 + c \Rightarrow c = \ldots$

Note that some candidates are assuming *P* lies on the *x*-axis and use (4, 0) and this scores M0

A1: $y = \frac{4}{5}x - \frac{31}{5}$ or exact equivalent in this form e.g. $y = 0.8x - 6.2$ but not $y = \frac{(4x-31)}{5}$ unless

the correct form was seen previously then apply isw.

(b)

M1: Uses $f'(4)=$ their **changed** gradient (i.e. not $-\dfrac{5}{4}$) and solves for k.

Allow equivalent work e.g. uses $f'(4)\times-\dfrac{5}{4}=-1$ and solves for k.

A1: $k=2$

Correct answer with no working scores both marks.

(c)

M1: Attempts to expand and writes $f'(x)$ in the form $\alpha x^{\frac{3}{2}}+\beta x^{\frac{1}{2}}$

M1: Integrates with at least one index correct i.e. $...x^{\frac{3}{2}}\rightarrow...x^{\frac{5}{2}}$ or $...x^{\frac{1}{2}}\rightarrow...x^{\frac{3}{2}}$ which must come from correct work but indices may be unprocessed at this stage.

A1ft: Correct integration with or without the $+c$. Allow with constant k or follow through on an

incorrect k only i.e. the expression should be otherwise correct e.g. allow $\dfrac{2kx^{\frac{5}{2}}}{25}-\dfrac{2kx^{\frac{3}{2}}}{5}(+c)$ or

this expression with their k substituted. Need not be simplified at this stage so indices may be unprocessed.

The "f(x) =" is **not** required. Condone spurious integral signs.

ddM1: Must have a constant of integration now. It is for using $x=4$ and $y=-3$ in their

$\left(f(x)=\right)\dfrac{4x^{\frac{5}{2}}}{25}-\dfrac{4x^{\frac{3}{2}}}{5}+c$ in an attempt to find their c, where the -3 is their attempt at finding y

from $y=-\dfrac{5}{4}x+2$

Many candidates are assuming P lies on the x-axis and use (4, 0) and this scores ddM0
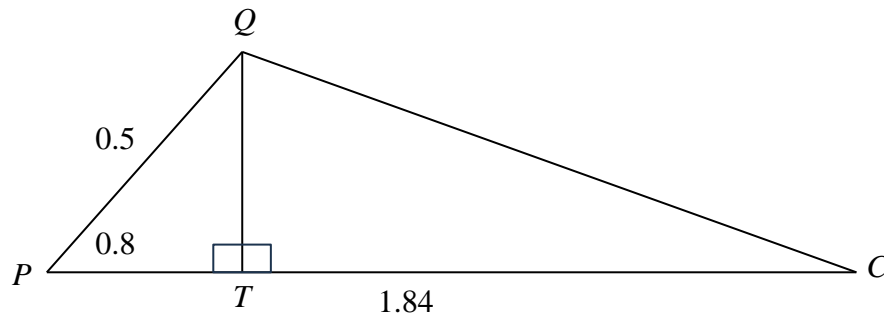
Depends on both previous M marks.

A1: $\left(f(x)=\right)\dfrac{4x^{\frac{5}{2}}}{25}-\dfrac{4x^{\frac{3}{2}}}{5}-\dfrac{43}{25}$

Correct simplified expression. The "f(x) =" is **not** required.

Condone spurious integral signs.

Apply isw if necessary once a correct expression is seen.

Question 9(a) and (b) using right angled triangles:

(a)

e.g. $QT = 0.5\sin 0.8 (= 0.3586...), \; PT = 0.5\cos 0.8 (= 0.3483...)$

$QC = \sqrt{QT^2 + CT^2} = \sqrt{0.3586...^2 + (1.84 - 0.3483...)^2} = 1.534$

Score M1 for a complete and correct method leading to a value for the length of QC and A1 as MS

(b)

e.g. $\sin PCQ = \dfrac{QT}{CQ} = \dfrac{0.5\sin 0.8}{1.534} = 0.23379... \Rightarrow PCQ = 0.236\,^*$

Score M1 for a complete and correct method leading to an expression for e.g. sin PCQ and A1 for a correct proof.