Question	Scheme	Marks
1	$4x^2-3x+7\geqslant 4x+9$ $\Rightarrow 4x^2-7x-2...0 \Rightarrow (4x+1)(x-2)...0 \Rightarrow x=...$ or $\Rightarrow 4x^2-7x-2...0 \Rightarrow x=\dfrac{-(-7)\pm\sqrt{(-7)^2-4(4)(-2)}}{2\times 4} \Rightarrow x=...$ or $\Rightarrow 4x^2-7x-2...0 \Rightarrow 4\left(x^2-\frac{7}{4}x-\frac{1}{2}\right)...0 \Rightarrow 4\left(\left(x-\frac{7}{8}\right)^2-\left(\frac{7}{8}\right)^2-\frac{1}{2}\right)...0 \Rightarrow x=...$	M1
	$x=-\frac{1}{4},2$	A1
	$x\leqslant "-\frac{1}{4}",\quad x\geqslant "2"$	M1
	$x\leqslant -\frac{1}{4}$ or $x\geqslant 2$ oe	A1
		(4)
		(4 marks)

Notes:

M1: Gathers terms to one side and solves their 3TQ which is not $4x^2-3x+7=0$ to obtain at least one critical value for x. Allow by factorisation, the quadratic formula or completing the square. Do not be concerned about the inequality symbol for the first two marks and the "…0" can be implied.

Note that $4x^2-7x-2...0 \Rightarrow \left(x+\frac{1}{4}\right)(x-2)...0 \Rightarrow x=...$ scores M0

For completing the square, the conditions for their $4x^2-7x-2...0$ are as follows:

Either $4x^2-7x-2...0 \Rightarrow 4\left(x^2+bx+c\right)...0$ then as general guidance

Or $4x^2-7x-2...0 \Rightarrow \left(2x-\frac{7}{4}\right)^2 \pm q \pm 2...0 \Rightarrow x=...$

A1: Both correct critical values. Allow unsimplified but must be single fractions e.g. allow $-\frac{2}{8},\frac{16}{8}$

These values may be seen embedded in their final answer but the **first M1 must have been scored**.

M1: Chooses the <u>outside</u> region for their 2 critical values. Allow with < and > for this mark.

Condone poor notation e.g. $"-\frac{1}{4}"\geqslant x\geqslant "2"$ or e.g. $"-\frac{1}{4}" > x > "2"$ or e.g. $"2" < x < "-\frac{1}{4}"$

A1: Correct solution only – **depends on all previous marks**. Allow e.g. $x\leqslant -\frac{1}{4},\ x\geqslant 2$;

$x\leqslant -\frac{1}{4}$ and $x\geqslant 2$; $\left(-\infty,-\frac{1}{4}\right],[2,\infty)$; $\left(-\infty,-\frac{1}{4}\right]$ or $[2,\infty)$; $\left(-\infty,-\frac{1}{4}\right]$ and $[2,\infty)$

Do **not** allow $-\frac{1}{4}\geqslant x\geqslant 2$; $x\leqslant -\frac{1}{4}\cap x\geqslant 2$; $\left(-\infty,-\frac{1}{4}\right]\cap[2,\infty)$

Allow unsimplified constants but must be single fractions e.g. allow $-\frac{2}{8},\frac{16}{8}$ as above.

Apply isw if possible if there is no contradiction so e.g. correct work leading to $x \leqslant -\frac{1}{4},\ x \geqslant 2$ followed by $-\frac{1}{4} \geqslant x \geqslant 2$ can score full marks. If several different answers are offered, mark the final answer.

Particular cases and how to mark them:

Case 1

$$4x^2 - 3x + 7 \geqslant 4x + 9 \Rightarrow 4x^2 - 7x - 2 \geqslant 0$$

$$x = -\frac{1}{4},\ 2 \Rightarrow x \leqslant -\frac{1}{4},\ x \geqslant 2$$

Scores M0A0 M1A0

Case 2

$$4x^2 - 3x + 7 \geqslant 4x + 9 \Rightarrow 4x^2 - 7x - 2 \geqslant 0$$

$$\Rightarrow x \leqslant -\frac{1}{4},\ x \geqslant 2$$

Scores M0A0M0A0

Question	Scheme	Marks
2(a)	$2x+2y=350$	B1
		(1)
(b)	E.g. $xy=7350,\quad x\times y=7350$	B1
		(1)
(c)	$x(175-x)=7350$ or $(175-y)y=7350$	M1
	E.g. $x^2-175x+7350=0\Rightarrow(x-70)(x-105)=0\Rightarrow x=...$	dM1
	$x=70$ or 105	A1
	$(x>y\Rightarrow)x=105, y=70$	A1
		(4)
		(6 marks)

Notes:

Ignore labelling and mark all parts together.
Ignore any units given whether correct or incorrect.

(a)

B1: Correct equation. Accept any correct equation e.g. $2(x+y)=350,\ x+y=175$

(b)

B1: Correct equation. Accept any correct equation e.g. $y=\dfrac{7350}{x}$

(c)

M1: Substitutes for x or y into either of their equations to produce a 2 or 3 term quadratic equation in one variable. Note that terms do not need to be collected for this mark so e.g. $x(175-x)=7350$ or $(175-y)y=7350$ oe is acceptable for this mark.

Condone slips when substituting if the intention is clear e.g. substituting for $2x$ rather than x: $x(350-2x)=7350$

(Note, if substituting $x=\dfrac{7350}{y}$ into $2x+2y=350$ they will need to multiply through by y to score this mark. Condone slips but must produce a 2 or 3 term quadratic equation.)

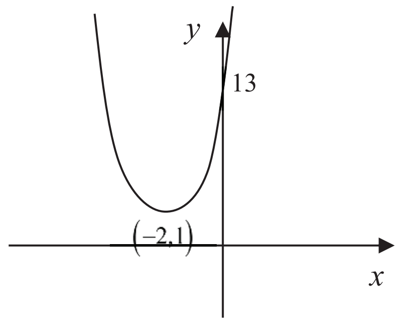
dM1: Depends on the first method mark. Solves their 3 term quadratic equation, any suitable means such as formula or by calculator (may be implied by correct values for their quadratic).

A1: Correct **simplified** roots of the quadratic. Do not be concerned about which is x and y for this mark so just look for the values 70 and 105. It is possible that candidates may only select the larger value for x (105) or the smaller value for y (70) **but** if both roots are found they must both be correct.

A1: Both x and y correct and correctly assigned and all previous marks scored.

Special Case:

Candidates who use Trial and Improvement in part (c) or guess the correct answers with no quadratic formed allow a special case of M1M1A0A0. If the values are the wrong way round or both sets of answers are offered then score special case of M1M0A0A0.

Question	Scheme	Marks
3(a)	$a = 3$	B1
	$b = \pm 2$	M1
	$3x^2 + 12x + 13 = 3(x+2)^2 + 1$ or $a = 3,\ b = 2,\ c = 1$	A1
		(3)
(b)	 Correct U shape with minimum in second quadrant	B1
	Intercept 13 on y-axis.	B1
	Vertex at $(-2, 1)$	B1ft
		(3)
		(6 marks)

Notes:

(a)

B1: Correct value for a stated or shown by working.

M1: Obtains $b = \pm 2$. Allow unsimplified e.g. $b = \pm\frac{4}{2}, \pm\frac{12}{6}$. May be implied by e.g. $3(x \pm 2)^2 + ...$

A1: Fully correct expression or correct values.

Note that there are various methods e.g. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \Rightarrow 3x^2 + 12x + 13 = ...$

or $a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c \Rightarrow a = 3,\ 2ab = 12, ab^2 + c = 13 \Rightarrow b, c = ...$

(b)

<u>There must be a sketch to score marks in (b)</u>

<u>Labelling on the sketch takes precedence</u>

Treat part (b) as Hence, or otherwise, i.e. part (a) may be incorrect but full marks are available in part (b) for a correct sketch of $y = 3x^2 + 12x + 13$

B1: Correct U shape with minimum point in the second quadrant but **not** on the x-axis. Do not allow a clear V shape.

B1: Correct y intercept labelled or stated. Allow as just 13 or (0, 13) or (13, 0) as long as it is in the correct position. If stated away from the sketch it must be as (0, 13) and correspond to the sketch. Their curve must at least touch at (0, 13). Any other intercepts can be ignored.

B1ft: Correct vertex labelled in some way, or follow through their b and c i.e. $x = -b$ and $y = c$. Must be a turning point but could be a maximum or a minimum. Allow the labelling as shown or as $x =..,\ y = …$or as the ordinates shown on the axes. It must correspond to the sketch. If stated away from the sketch it must be correct or correct follow through and correspond to the sketch.

Question	Scheme	Marks
4(a)	$y=\dfrac{5x^2+\sqrt{x^3}}{\sqrt[3]{8x}}=\dfrac{5x^2+x^{\frac{3}{2}}}{...x^{\frac{1}{3}}}$	M1
	$=\dfrac{5x^{2-\frac{1}{3}}}{2}+\dfrac{x^{\frac{3}{2}-\frac{1}{3}}}{2}$	M1
	$=\dfrac{5}{2}x^{\frac{5}{3}}+\dfrac{1}{2}x^{\frac{7}{6}}$	A1A1
		(4)
(b)	$\left(\dfrac{dy}{dx}=\right)...x^{\frac{2}{3}}+...x^{\frac{1}{6}}$	M1
	$\left(\dfrac{dy}{dx}=\right)\dfrac{25}{6}x^{\frac{2}{3}}+\dfrac{7}{12}x^{\frac{1}{6}}$	A1ftA1
		(3)
		(7 marks)

Notes:

(a)

M1: Converts the radicals into powers of x, with at least one power correct. E.g. $\sqrt{x^3}\rightarrow x^{\frac{3}{2}}$ or $\sqrt[3]{8x}\rightarrow ...x^{\frac{1}{3}}$

M1: Correct subtraction index law seen used at least once. May be implied.

It is for $\dfrac{5x^2}{\sqrt[3]{8x}}\rightarrow\dfrac{...x^2}{...x^p}\rightarrow ...x^{2-p}$ or $\dfrac{\sqrt{x^3}}{\sqrt[3]{8x}}\rightarrow\dfrac{...x^q}{...x^p}\rightarrow ...x^{q-p}$

Indices do **not** need to be processed for this mark.

A1: One term correct (coefficient and index) or both indices correct. E.g. $\dfrac{5}{2}x^{\frac{5}{3}}$ or $\dfrac{1}{2}x^{\frac{7}{6}}$ or $...x^{\frac{5}{3}}+...x^{\frac{7}{6}}$. **Depends on both M marks.**

A1: Fully correct expression in the required form. **Depends on all previous marks.**
$y=...$ is not required. Apply isw once a correct expression is seen.

(b) **The $\dfrac{dy}{dx}=$ is not required throughout in part (b)**

M1: $x^n\rightarrow ...x^{n-1}$ for at least one term where $n\neq 1$.

A1ft: Correct differentiation of one of their terms where $n\neq 1$.
Follow through their part (a). But not for $kx\rightarrow k$
May be unsimplified.
Depends on both M marks in part (a).

A1: Correct derivative, with terms in simplest form. Apply isw once a correct expression is seen.
Score A0 if they include "+ c"

Question	Scheme	Marks
5(a)	$\angle AOB = \cos^{-1}\dfrac{1.5}{4} = ...$ or $\angle AOB = \tan^{-1}\dfrac{\sqrt{4^2-1.5^2}}{1.5} = ...$ or $\angle AOB = \dfrac{\pi}{2} - \sin^{-1}\dfrac{1.5}{4} = \ldots$ or $\angle AOB = \dfrac{\pi}{2} - \tan^{-1}\dfrac{1.5}{\sqrt{4^2-1.5^2}} = ...$	M1
	$= 1.186$ *	A1*
		(2)
(b)	$l = 4\times 1.186\left(=4.744\right)$	M1
	$BC = 6-4\sin 1.186\left(=6-3.708... = 2.29...\right)$ or $BC = 6-4\cos\left(\dfrac{\pi}{2}-1.186\right)\left(=6-3.708... = 2.29...\right)$ or $BC = 6-\sqrt{4^2-1.5^2}\left(=6-3.708... = 2.29...\right)\left(=\dfrac{12-\sqrt{55}}{2}\right)$ or $BC = 6-\dfrac{1.5}{\tan"0.384"}\left(=6-3.708... = 2.29...\right)$	M1
	Perimeter is $4\times 1.186...+\left(6-4\cos 0.384...\right)+1.5+6+4=..$	ddM1
	= awrt 18.5 (m)	A1
		(4)
(c)	Area sector = $\dfrac{1}{2}\times 4^2\times 1.186...$	M1
	Area *OBCD* Examples: $6\times 1.5-\dfrac{1}{2}\times"3.708"\times 1.5$ or $\left(6-"3.708"\right)\times 1.5+\dfrac{1}{2}\times"3.708"\times 1.5$ or $\dfrac{1}{2}\left(6+6-"3.708"\right)\times 1.5$ or $6\times 1.5-\dfrac{1}{2}\times 1.5\times 4\sin 1.186$ or $\dfrac{1}{2}\times 6\times 1.5+\dfrac{1}{2}\left(6-"3.708"\right)\times 1.5$ $(= 6.219\ldots)$	M1 A1 on EPEN
	$\dfrac{1}{2}\times 4^2\times 1.186+6.219... =$	ddM1
	= awrt 15.7 (m^2)	A1
		(4)
		(10 marks)

Notes:

Allow equivalent correct work in degrees.
Ignore any presence or absence of units throughout.

(a)

M1: For a complete and correct method to find angle *AOB* in radians or degrees. May attempt angle *AOB* directly using cosine, or angle *BOD* using sine then subtract angle *BOD* from $\frac{\pi}{2}$ or equivalent work using tan.

A1*: Obtains angle *AOB* = 1.186 with no errors.

Example in degrees:

$$\sin \angle BOD = \frac{1.5}{4} \Rightarrow \angle BOD = \sin^{-1}\frac{3}{8} = 22.0243...$$

$$\angle AOB = 90 - 22.0243... = 67.9756...$$

$$= \frac{\pi \times 67.9756...}{180} = 1.186 *$$

(b)

M1: Uses the correct arc length formula with $r = 4$ and 1.186. May be implied by awrt 4.74 or 4.75.

M1: Attempts the length of *BC* using an appropriate trig function and angle, or Pythagoras. May see $6 - 4\sin 1.186$ or $6 - 4\cos"0.384"$ or may use $6 - \sqrt{4^2 - 1.5^2}$. May be implied by awrt 2.3 provided no incorrect working is seen.

ddM1: Depends on both previous marks. Adds the **five** lengths of the sides (or equivalent work) to obtain the perimeter.

A1: Correct answer, accept awrt 18.5. No units required and ignore any given.

(c)

M1: Correct attempt at the sector area. Look for $\frac{1}{2} \times 4^2 \times 1.186...$ but may be implied by e.g. $\frac{1186}{125}$ or awrt 9.5. Allow equivalent expressions in degrees e.g. $\frac{1}{2} \times 4^2 \times 67.9756... \times \frac{\pi}{180}$ but **not** e.g. $\frac{1}{2} \times 4^2 \times 67.9756$

M1: Correct attempt at the area of *OBCD* – which may be found as the difference of a rectangle and triangle or sum of rectangle and triangle or a trapezium. See scheme for details.
The attempt to find *BC*, if used, must have involved the use of trigonometry or e.g. Pythagoras.

ddM1: Depends on both previous marks. Correct attempt at the full area, so adds sector area to area of *OBCD*

A1: Correct answer, accept awrt 15.7. No units required and ignore any given.

Note that some candidates assume *BC* = 2 (i.e. 6 – 4) which greatly simplifies the question.
The maximum possible marks in (b) and (c) are: (b) M1M0ddM0A0 (c) M1M0ddM0A0

Question	Scheme	Marks
6(a)	$\left(r-\frac{1}{r}\right)^2 = r^2 - r\times\frac{1}{r} - r\times\frac{1}{r} + \frac{1}{r^2}$	M1
	$= r^2 + \frac{1}{r^2} - 2$	A1
		(2)
(b)	$\frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$	M1
	$= \frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2} = 3-2\sqrt{2}$	A1
		(2)
(b) ALT	$\frac{1}{3+2\sqrt{2}} = p+q\sqrt{2} \Rightarrow 1 = \left(p+q\sqrt{2}\right)\left(3+2\sqrt{2}\right) = 3p+4q+2p\sqrt{2}+3q\sqrt{2}$ $\Rightarrow \begin{matrix} 3p+4q=1 \\ 2p+3q=0 \end{matrix}$	M1
	$\Rightarrow \begin{matrix} 3p+4q=1 \\ 2p+3q=0 \end{matrix} \Rightarrow \begin{matrix} 6p+8q=2 \\ 6p+9q=0 \end{matrix} \Rightarrow q=-2 \Rightarrow 3p-8=1 \Rightarrow p=3$ $\frac{1}{3+2\sqrt{2}} = 3-2\sqrt{2}$	A1
		(2)
(c)	$\left(\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2 = 3+2\sqrt{2} + \frac{1}{3+2\sqrt{2}} - 2$	M1
	$= 3+2\sqrt{2}+3-2\sqrt{2}-2 = ...(=4)$	dM1
	so $\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}} = 2$	A1
		(3)
(c) Alt	$\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}} = 2 \Rightarrow 3+2\sqrt{2}-1 = 2\sqrt{3+2\sqrt{2}}$	M1
	$\Rightarrow \left(2+2\sqrt{2}\right)^2 = 4\left(3+2\sqrt{2}\right)$	dM1
	$\Rightarrow 4+8\sqrt{2}+8 = 12+8\sqrt{2}$ ✓ Hence true	A1
		(3)
		(7 marks)

Notes:

(a)

M1: Expands the bracket to obtain 3 or 4 terms with at least 2 correct which may be unsimplified. Allow use of a different variable e.g. x for this mark.

A1: Correct simplified expansion in terms of r. Accept terms in any order.

Accept e.g. $r^2 + r^{-2} - 2$ and accept $r^2 + \left(\frac{1}{r}\right)^2 - 2$. Do **not** isw and mark their final answer.

Note that if a correct simplified expression is seen and they then re-write the expression correctly in a different way then this mark should be awarded provided their expression is correct.

(b)

M1: Correct process to rationalise the denominator, so look for multiplying numerator and denominator by $3-2\sqrt{2}$ or any multiple of this expression. No processing is required so just look for the statement as shown above.

A1: Shows an intermediate line before obtaining $3-2\sqrt{2}$

Examples of an acceptable intermediate line are:

$$\frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-8},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-8},\ \frac{3-2\sqrt{2}}{1}$$

Do **not** allow $\frac{3-2\sqrt{2}}{1}$ as the final answer.

(b) ALT

M1: Sets $\frac{1}{3+2\sqrt{2}} = p + q\sqrt{2}$, multiplies up, compares rational and irrational parts to produce 2 equations in p and q.

A1: Solves simultaneously **showing working** and obtains $3-2\sqrt{2}$.

(c)

M1: Applies their result from (a) or the correct result to $\left(\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2$

dM1: Depends on the first mark. Applies their result from (b) and cancels $\sqrt{2}$ terms to achieve a constant.

A1: Reaches the correct answer from correct work. **There is no requirement to justify the positive square root.**

(c) ALT

M1: Multiplies equation through by $\sqrt{3+2\sqrt{2}}$ and applies $\sqrt{3+2\sqrt{2}} \times \sqrt{3+2\sqrt{2}} = 3+2\sqrt{2}$

dM1: Depends on the first mark. Squares both sides. May or may not have cancelled the 2 before squaring.

A1: Achieves similar expression for both sides and gives a (minimal) conclusion.

Note that there will be other methods for 6(c) e.g.:

$$\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=\frac{3+2\sqrt{2}-1}{\sqrt{3+2\sqrt{2}}}$$

M1: Attempts common denominator and applies $\sqrt{3+2\sqrt{2}}\times\sqrt{3+2\sqrt{2}}=3+2\sqrt{2}$

$$\left(\frac{2+2\sqrt{2}}{\sqrt{3+2\sqrt{2}}}\right)^2=\frac{12+8\sqrt{2}}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=\frac{36-32}{9-8}=4$$

dM1: Squares and attempts to rationalise the denominator

So $\sqrt{3+2\sqrt{2}}-\dfrac{1}{\sqrt{3+2\sqrt{2}}}=2$ *

A1*: Fully correct work

Question	Scheme	Marks
7(a)	$0=10-2x \Rightarrow x=5$ or $y=2, y=10-2x \Rightarrow x=4$	B1
	Examples: $\frac{1}{2}\times 2(5+4-a)=\frac{27}{4}$ or $\frac{1}{2}\times 2\left(5+4-\frac{2}{k}\right)=\frac{27}{4}$ Trapezium or $\frac{1}{2}\times 2a+\frac{1}{2}\times 2(5-a+4-a)=\frac{27}{4}$ or $\frac{1}{2}\times 2\times\frac{2}{k}+\frac{1}{2}\times 2\left(5-\frac{2}{k}+4-\frac{2}{k}\right)=\frac{27}{4}$ Triangle + Trapezium or $\frac{1}{2}\times 2a+2(4-a)+\frac{1}{2}\times 1\times 2=\frac{27}{4}$ or $\frac{1}{2}\times 2\times\frac{2}{k}+2\left(4-\frac{2}{k}\right)+\frac{1}{2}\times 1\times 2=\frac{27}{4}$ Triangle + Rectangle + Triangle or $\frac{1}{2}\times 5\times 2+\frac{1}{2}(4-a)\times 2=\frac{27}{4}$ or $\frac{1}{2}\times 5\times 2+\frac{1}{2}\left(4-\frac{2}{k}\right)\times 2=\frac{27}{4}$ 2 Triangles	M1
	$\Rightarrow k=\frac{8}{9}, a=\frac{9}{4}$	A1 A1ft
		(4)
(b)	Two of $y \geqslant "\frac{8}{9}"x,\ y \leqslant 10-2x, x > "\frac{9}{4}"$	M1
	All three of $y \geqslant \frac{8}{9}x,\ y \leqslant 10-2x, x > \frac{9}{4}$	A1
		(2)
		(6 marks)

Notes:

(a)

B1: Correct x intercept for $y=10-2x$ or correct x coordinate for intersection of $y=2$ with $y=10-2x$. May be seen on the diagram or in a sketch or implied by their working.

M1: Correct method for area and sets equal to $\frac{27}{4}$ to form an equation in a or k. The $2\times\frac{1}{2}$ may be implied. May use e.g. trapezium, triangle + trapezium, triangle + rectangle + triangle or 2 triangles. See above for some examples. Note that with a correct "5" and "4" the area is $9-a$ or $9-\frac{2}{k}$.

A1: Correct value for a or k as a single value not a calculation.

A1ft: Correct values for a and k. Correct values or follow through using $ak=2$ from their first value to the second **provided the M1 is scored**. Allow exact equivalents or exact ft equivalents so if correct allow e.g. $k=0.\dot{8}$, $a=2.25$ or $2\frac{1}{4}$

(b)
M1: Any two of the three correct inequalities with their k and/or their a or the letters a and k. Note inequalities in y may be combined in to one so $"\frac{8}{9}"x < y < 10-2x$ would count as two correct inequalities. Inequalities must be in the correct direction but accept with $<$ or $\leqslant$ for the M. Do not allow incorrect notation such as $R_2 \geqslant "\frac{8}{9}"x,\ R_2 \leqslant 10-2x$

A1: All three correct with the correct values of a and k – **there is no follow though for this mark**.
Note inequalities in y may be combined in to one as above.
Note that there may be an upper limit on the x inequality which is fine provided it is in an acceptable range and consistent.
Note that $y = \frac{8}{9}x$ and $y = 10 - 2x$ intersect when $x = \frac{45}{13}$ so the upper limit may be this value or greater. In general the following are acceptable:

$$y \geqslant \frac{8}{9}x,\ y \leqslant 10-2x,\ \frac{9}{4} < x \leqslant \alpha \text{ if } \alpha \geqslant \frac{45}{13} \text{ or } y \geqslant \frac{8}{9}x,\ y \leqslant 10-2x,\ \frac{9}{4} < x < \alpha \text{ if } \alpha > \frac{45}{13}$$

The following examples would score both marks,

$$y \geqslant \frac{8}{9}x,\ y \leqslant 10-2x,\ \frac{9}{4} < x \leqslant \frac{45}{13}$$

$$y \geqslant \frac{8}{9}x,\ y \leqslant 10-2x,\ \frac{9}{4} < x < 5$$

Note that some candidates are also including limits on y and this is acceptable provided the values are in the correct range. Allow strict or non strict inequalities e.g.

$y > 2, y \geqslant 2,\ 2 < y < \frac{11}{2},\ 2 \leqslant y \leqslant \frac{11}{2}$ but e.g. $y < 2, y \leqslant 2$ scores A0

<u>Question 8 – General Guidance for marking</u>

Note that some candidates are misreading the $x^{-\frac{1}{2}}$ as $x^{\frac{1}{2}}$ in part (a) and/or part (b)

In the majority of cases it will be clear if this is the case as candidates often write down the expression before they differentiate in part (a) or integrate in part (b)

If it is clear that a candidate thinks the expression is $\frac{1}{4}x^3 - 8x^{\frac{1}{2}}$ in either or both parts then <u>FULL</u> marks should be awarded for correct work – details are below. We are condoning the fact that $P(4,12)$ does not lie on $y = \frac{1}{4}x^3 - 8x^{\frac{1}{2}}$

If candidates do not write down the original expression (in (a) or (b)) then evidence of misreading or not can be taken from their derivative or integral.

<u>Note:</u>

If candidates integrate in (a) and differentiate in (b) then the maximum marks available are:

(a) M0A0M1dM1A0 (b) M0A0dM0A0

If parts are not labelled then you should assume that the first attempt is part (a) and the second attempt is part (b).

8. Scheme - No Misread

Question	Scheme	Marks
8(a)	$y=\frac{1}{4}x^3-8x^{-\frac{1}{2}} \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{1}{4}\times 3x^2-8\times -\frac{1}{2}x^{-\frac{3}{2}}$	M1 A1
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{x=4}=...\left(\frac{25}{2}\right)$	M1
	$y-12="\frac{25}{2}"(x-4)$	dM1
	$25x-2y-76=0$ oe e.g. $-25x+2y+76=0$	A1
		(5)
(b)	$(\mathrm{f}(x)=)\int \frac{1}{4}x^3-8x^{-\frac{1}{2}}\,\mathrm{d}x=\frac{1}{4}\frac{x^4}{4}-\frac{8x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1 A1
	$\mathrm{f}(4)=12 \Rightarrow 16-32+c=12 \Rightarrow c=...(28)$	dM1 A1
	So $(\mathrm{f}(x)=)\frac{x^4}{16}-16\sqrt{x}+"28"$	A1ft
		(5)
		(10 marks)

Notes:

(a)

M1: Correct method of differentiation, at least one power reduced by 1.

Award for $\frac{1}{4}x^3 \rightarrow ...x^2$ or $-8x^{-\frac{1}{2}} \rightarrow ...x^{-\frac{3}{2}}$

A1: Correct differentiation, need not be simplified.

M1: Finds $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 4$. Requires the substitution of $x = 4$ into a "changed" function to find a value.

Note that some candidates are using the "12" in some way to find the gradient e.g.

$$12=\frac{1}{4}\times 3(4)^2-8\times -\frac{1}{2}(4)^{-\frac{3}{2}}+c \Rightarrow c=2 \Rightarrow m=2 \text{ and this scores M0}$$

dM1: Depends on previous M. Correct method for the tangent **not** the normal. Uses their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 4$ with $y = 12$ and $x = 4$ correctly placed. If using $y = mx + c$ they must proceed as far as finding a value for c.

A1: Correct equation in the required form including "= 0" or any non-zero integer multiple of it.

(b)

M1: Attempts to integrate f(x), look for power increased by 1 on at least one term.

Award for $\frac{1}{4}x^3 \rightarrow ...x^4$ or $-\frac{1}{8}x^{-\frac{1}{2}} \rightarrow ...x^{\frac{1}{2}}$

A1: Correct integration, need not be simplified. (no need for constant for this mark).

dM1: Depends on first M. Sets f (4) = 12 and proceeds to find a value for c - must have a constant of integration to score this mark.

A1: Correct value for c found.

A1ft: Correct answer following through their c only i.e. the algebraic part must be correct.

Accept with fractional index rather than square root. Allow $0.0625x^4$ for $\frac{x^4}{16}$. **Depends on the previous method mark.**

The "f (x) =" is **not** required so just look for the correct expression. Apply isw if necessary, e.g. correct work leading to $\left(\text{f}(x)=\right)\frac{x^4}{16}-16x^{\frac{1}{2}}+28$ followed by $\left(\text{f}(x)=\right)x^4-256x^{\frac{1}{2}}+448$ can score full marks in (b).

Condone poor notation e.g. leaving the final answer as $\left(\text{f}(x)=\right)\int\frac{x^4}{16}-16x^{\frac{1}{2}}+28$

8. Scheme – Misread in (a) and/or (b)

Question	Scheme	Marks
8(a)	$y=\frac{1}{4}x^3-8x^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx}=\right)\frac{1}{4}\times 3x^2-8\times\frac{1}{2}x^{-\frac{1}{2}}$	M1 A1
	$\left.\frac{dy}{dx}\right\|_{x=4}=\ldots(10)$	M1
	$y-12="10"(x-4)$	dM1
	$10x-y-28=0$ oe e.g. $-10x+y+28=0$	A1
		(5)
(b)	$(\mathrm{f}(x)=)\int \frac{1}{4}x^3-8x^{\frac{1}{2}}\,dx=\frac{1}{4}\frac{x^4}{4}-\frac{8x^{\frac{3}{2}}}{\frac{3}{2}}(+c)$	M1 A1
	$\mathrm{f}(4)=12\Rightarrow 16-\frac{128}{3}+c=12\Rightarrow c=\ldots\left(\frac{116}{3}\right)$	dM1 A1
	So $(\mathrm{f}(x)=)\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+"\frac{116}{3}"$	A1ft
		(5)
		(10 marks)

Notes:

(a)

M1: Correct method of differentiation, at least one power reduced by 1.

Award for $\frac{1}{4}x^3\to\ldots x^2$ or $-8x^{\frac{1}{2}}\to\ldots x^{-\frac{1}{2}}$

A1: Correct differentiation, need not be simplified. Ignore any spurious "= 0".

M1: Finds $\frac{dy}{dx}$ at $x = 4$. Requires the substitution of $x = 4$ into a "changed" function to find a value.

Note that some candidates are using the "12" in some way to find the gradient e.g.

$$12=\frac{1}{4}\times 3(4)^2-8\times-\frac{1}{2}(4)^{-\frac{1}{2}}+c\Rightarrow c=2\Rightarrow m=2 \text{ and this scores M0}$$

dM1: Depends on previous M. Correct method for the tangent **not** the normal. Uses their $\frac{dy}{dx}$ at $x = 4$ with $y = 12$ and $x = 4$ correctly placed. If using $y = mx + c$ they must proceed as far as finding a value for c.

A1: Correct equation in the required form including "= 0" or any non-zero integer multiple of it.

(b)

M1: Attempts to integrate f (x), look for power increased by 1 on at least one term.

Award for $\frac{1}{4}x^3\to\ldots x^4$ or $-\frac{1}{8}x^{\frac{1}{2}}\to\ldots x^{\frac{3}{2}}$

A1: Correct integration, need not be simplified. (no need for constant for this mark).

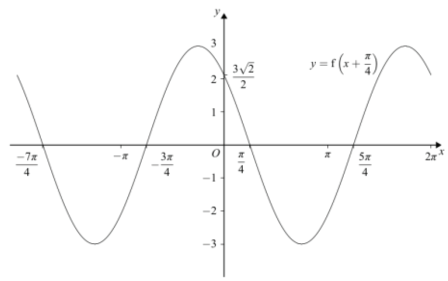
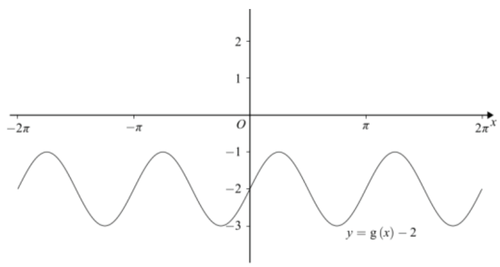
dM1: Depends on first M. Sets f (4) = 12 and proceeds to find a value for c - must have a constant of integration to score this mark.

A1: Correct value for c found.

A1ft: Correct answer following through their c only i.e. the algebraic part must be correct. Accept with fractional index rather than square root. Allow $0.0625x^4$ for $\frac{x^4}{16}$ and equivalent mixed fractions for $\frac{16}{3}$ and $\frac{116}{3}$. **Depends on the previous method mark.**

The "f (x) =" is **not** required so just look for the correct (or correct ft) expression. Apply isw if necessary e.g. correct work leading to $\left(\mathrm{f}(x)=\right)\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+\frac{116}{3}$ followed by $\left(\mathrm{f}(x)=\right)3x^4-256x^{\frac{3}{2}}+1856$ can score full marks in (b).

Condone poor notation e.g. leaving the final answer as $\left(\mathrm{f}(x)=\right)\int\frac{x^4}{16}-\frac{16}{3}x^{\frac{3}{2}}+\frac{116}{3}$

Question	Scheme		Marks
9(i)(a)	$(y=)3\cos(x)$		M1 A1
			(2)
(b)		Same shape translated left or right	B1
		All x intercepts labelled correctly.	B1
		Correct y intercept $\frac{3\sqrt{2}}{2}$	B1
			(3)
(ii)(a)	$(y=)\sin(2x)$		M1 A1
			(2)
(b)		Same shape translated down below the x-axis.	B1
		Correct y intercept -2 labelled.	B1
			(2)
			(9 marks)

Notes:

(i)(a) Remember to check the first page – (i)(a) is often attempted against the question.

M1: Identifies the curve as a cosine function of the form $\alpha\cos(\beta x)$.

Also allow for a sine function of the form $\alpha\sin\left(\beta x+\frac{\pi}{2}\right)$ oe.

A1: For a fully correct expression as shown or any equivalent. There is no requirement for $y=\ldots$ or $\mathrm{f}(x)=\ldots$ Allow e.g. $3\cos\theta$

(b) There must be a sketch to score the marks in (i)(b) which may be done on Figure 3

B1: Applies a horizontal translation $y=\mathrm{f}(x)$, either direction. The maximum at (0, 3) must have moved to the left or to the right of the y-axis. There should also be one minimum to the left of the y-axis and one minimum to the right of the y-axis. Ignore any scale on the y-axis.

B1: All x intercepts labelled correctly. These must be the only x intercepts and the curve must pass through these points. Allow even if there is still a maximum on the y-axis. Allow as shown or as coordinate pairs e.g. $\left(-\frac{7\pi}{4},0\right)$ and allow e.g. $\left(0,-\frac{7\pi}{4}\right)$ as long as the intercepts are in the correct positions. Condone 3sf decimal answers: Awrt –5.50(or –5.5), –2.36, 0.785, 3.93

Condone use of degrees: –315°, –135°, 45°, 225° (with or without the degrees symbol)

B1: Correct exact y intercept labelled. Allow as shown or as e.g. $\left(0,\frac{3\sqrt{2}}{2}\right)$ and allow e.g. $\left(\frac{3\sqrt{2}}{2},0\right)$ as long as it is in the correct position. It must not be a clear maximum.

(ii)(a) Remember to check the first page – (ii)(a) is often attempted against the question **M1:** Identifies the curve as a sine function of the form $\alpha\sin(\beta x)$. Allow also for a cosine function of the form $\alpha\cos\left(\beta x \pm \frac{\pi}{2}\right)$ oe **A1:** Any correct expression. There is no requirement for $y = \ldots$ or $\mathrm{g}(x) = \ldots$ Allow e.g. $\sin 2\theta$ **(b) There must be a sketch to score the marks in (ii)(b) which may be done on Figure 4** **B1:** Applies a vertical translation down to $y = \mathrm{g}(x)$. It must have the same number of cycles as $\mathrm{g}(x)$ and look the same as $\mathrm{g}(x)$ and must lie entirely below the x-axis. Do not allow a clear "zig-zag" shape with cusps. Ignore any scale on the x-axis, correct or otherwise. **B1:** Correct y intercept. Allow as shown or as e.g. $(0,-2)$ and allow e.g. $(-2,0)$ as long as it is in the correct position and allow it to be shown by the intercept passing through the line with equation $y = -2$.

Question	Scheme	Marks
10(a)(i)	Equation is $y=\frac{1}{2}(x+2)$	B1
		(1)
(ii)	l_1 intersect parabola $\Rightarrow -\frac{1}{4}(x+2)(x-b)=\frac{1}{2}(x+2) \Rightarrow x=...$	M1
	$x=b-2$	A1
	$y=\frac{1}{2}b$	A1
		(3)
(b)	l_2 passes through $(b, 0)$ and has gradient $-2 \Rightarrow y=...$	M1
	$y-0=-2(x-b)$	A1
		(2)
(c)	So equation is $y-"\frac{1}{2}b"="-2"(x-"b-2")$	M1
	$y-\frac{1}{2}b=-2x+2b-4 \Rightarrow y=-2x+\frac{5}{2}b-4$ *	A1*
		(2)
(d)	$y=-2x+2b=-2x+\frac{5}{2}b-4 \Rightarrow 2b=\frac{5}{2}b-4 \Rightarrow b=...$ or $y=-2x+\frac{5}{2}b-4,\ x=b,\ y=0 \Rightarrow 0=-2b+\frac{5}{2}b-4 \Rightarrow b=...$ or $-\frac{1}{4}(x+2)(x-b)=-2(x-b) \Rightarrow x=...(6) \Rightarrow b-2=6 \Rightarrow b=...$	M1
	$b=8$	A1
		(2)
		(10 marks)

Notes:

(a)(i)

B1: Correct equation in any form e.g. $y-0=\frac{1}{2}(x+2)$, $y=\frac{1}{2}x+1$, $y=\frac{1}{2}(x--2)$. Do not allow e.g. $l_1=\frac{1}{2}x+1$. Apply isw if necessary, so allow e.g. $y=\frac{1}{2}(x+2)$ if followed by $y=\frac{1}{2}x+2$

(a)(ii) Allow the marks for (a)(ii) to appear anywhere in their solution provided they are solving their l_1 with the given parabola.

Do not allow methods in (a)(ii) that assume that the *y* coordinate of *P* is the same as the *y* intercept of *C* unless this is justified by further work.

M1: Sets $-\frac{1}{4}(x+2)(x-b)=$ their $\frac{1}{2}(x+2)$ and makes some attempt to solve for *x* (or *y*) however poor – e.g. allow if there is a slip in the use of the quadratic formula. Condone copying errors provided the intention is clear. Note that some candidates do not spot the cancelling of $(x + 2)$ and end up solving a quadratic equation.

A1: Correct simplified *x*

A1: Correct simplified y

(b) Allow the work for (b) to appear anywhere in their solution

M1: Uses a gradient of -2 and the point $(b, 0)$ in an attempt to find the equation of l_2.

If using $y = mx + c$ they must proceed as far as finding c in terms of b. The coordinates $(b, 0)$ must be correctly placed.

A1: Any correct equation. E.g. $y - 0 = -2(x - b)$, $y = -2(x - b)$, $y = -2x + 2b$. Apply isw if necessary.

(c) Allow the work for (c) to appear anywhere in their solution including in part (b) provided they have attempted part (a)(ii) and have coordinates for *P*.

M1: Uses a gradient of -2 and their coordinates of P in terms of b from (a) in an attempt to find the equation of l_2. If using $y = mx + c$ they must proceed as far as finding c in terms of b. The coordinates of P must be correctly placed.

A1*: Correct equation reached from fully correct working with full marks in (a)(ii).

(d)

M1: Equates intercepts in the two equations for the line l_2 and solves for b. They must use the **<u>given equation</u>** in (c) and their attempt at the equation using $(b, 0)$ (however labelled) or uses the equation from (c) with $x = b$ when $y = 0$ to find b or equates the parabola to the line $y = -2(x - b)$, solves for x, equates to their x coordinate for P and then solves for b.

A1: $b = 8$

Correct answer only in (d) scores both marks.