**WMA11 October 2020 Mark Scheme**

| Question | Scheme | Marks |
|---|---|---|
| 1 | $a = 162$ | B1 |
| | $b = 5$ | B1 |
| | $c = 12$ | B1 |
| | | **(3 marks)** |

**Notes**

Make sure you mark in this order on epen.

**B1** $a = 162$

**B1** $b = 5$ condone $p = 5$

**B1** $c = 12$ condone $q = 12$

**Note:** The values may be implied by their expression. If there is a contradiction between what appears to be their final expression and values for $a$, $b$ or $c$ which are incorrectly stated afterwards then treat this as isw.

| Question | Scheme | Marks |
|---|---|---|
| 2(a) | $b=2$ | B1 |
| | $\ldots\pm\ldots(x\pm3)^2$ | M1 |
| | $(\mathrm{f}(x)=)\ 21-2(x-3)^2$ | A1 |
| | | (3) |
| (b) | $R$ is $(0,-4)$ or "$h$" $=4$ | B1 |
| | $\mathrm{f}(x)-7=14-2(x-3)^2 \Rightarrow x=\ldots$ or $\mathrm{f}(x)-7=-4+12x-2x^2 \Rightarrow x=\ldots$ $\left(\text{NB } x=3\pm\sqrt{7}\right)$ | M1 |
| | $\text{Area}=\frac{1}{2}\times\left(\text{"}3+\sqrt{7}\text{"}-\left(\text{"}3-\sqrt{7}\text{"}\right)\right)\times\text{"}4\text{"}$ | dM1 |
| | $=4\sqrt{7}$ | A1 |
| | | (4) |
| | | (7 marks) |

**Notes**

**(a)**

**B1** $b=2$ implied from their expression or stated. Eg look for $-2(x\pm\ldots)^2$

Beware of expressions such as $21+2(x-3)^2$ which would be B0

**M1** Sight of $(x\pm3)^2$

**A1** $21-2(x-3)^2$. Condone $21-2(x+-3)^2$ or they may state the values for $a,b$ and $c$. If there is a contradiction between what appears to be their final expression and values for $a$, $b$ or $c$ which are incorrectly stated afterwards then treat this as isw.

**(b)**

**B1** $R$ is $(0,-4)$ or "$h$" $=4$ (The coordinate may be seen on a diagram as a point of their triangle or quadratic so allow just $-4$ rather than the full coordinate to be indicted on the $y$-axis ). It may also be implied by their working.

**M1** Attempts to solve $\mathrm{f}(x)-7=0$

Score for an attempt to solve a 3TQ of the form:

$-2x^2+12x+C=0$ where $C\neq3$ or $A-2(x-3)^2=0$ where $A\neq21$

or if they use their part (a) $\text{"}a\text{"}-\text{"}b\text{"}(x+\text{"}c\text{"})^2=0 \Rightarrow d-\text{"}b\text{"}(x+\text{"}c\text{"})^2=0$ where $d\neq a$

(It must have a different $y$ intercept to the given $\mathrm{f}(x)$ or their $\mathrm{f}(x)$ from part (a) but the coefficients of $x$ and $x^2$ must remain the same.)
Condone slips in their working. They should be solving their quadratic using either the formula or completing the square. (usual rules for solving). Allow to be solved directly from a calculator. If only the roots are written, then these will need to be checked.

**dM1** Fully correct method for the area of their triangle. It is dependent on the previous method mark.
Score for values embedded in $\frac{1}{2}\times(\beta-\alpha)\times\text{"}y-\text{intercept"}$ where $\alpha, \beta$ are the roots of their 3TQ.

This should be $\frac{1}{2}\times(2\times\sqrt{...})\times\text{"}y-\text{intercept"}$ oe

Alternatively, if their quadratic has a negative $y$-intercept they may find the area of a large right-angled triangle and subtract the area of the small right-angled triangle. Look for expressions of the form:

$$\frac{1}{2}\times\beta\times\text{"}y-\text{intercept"}-\frac{1}{2}\times\alpha\times\text{"}y-\text{intercept"}$$

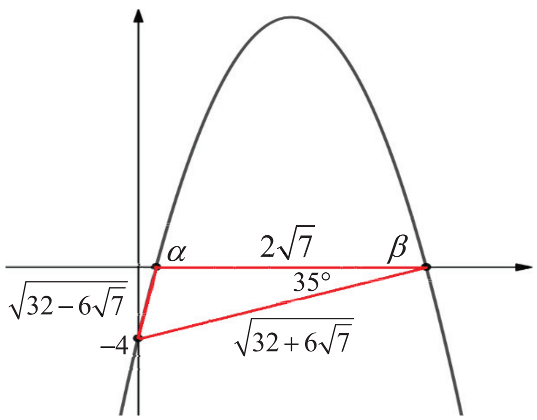
They may even attempt finding all the lengths of the triangle, applying the cosine rule to find an angle and then applying the area sine rule. (See diagrams below).
Alternatively, they may apply the shoelace method:
$\frac{1}{2}\left|\left((3+\sqrt{7})\times 0+(3-\sqrt{7})\times(-4)+0\times 0\right)-\left(0\times(3-\sqrt{7})+0\times 0+(-4)\times(3+\sqrt{7})\right)\right|=4\sqrt{7}$ oe
You may see other credit worthy methods so if in doubt send to review.

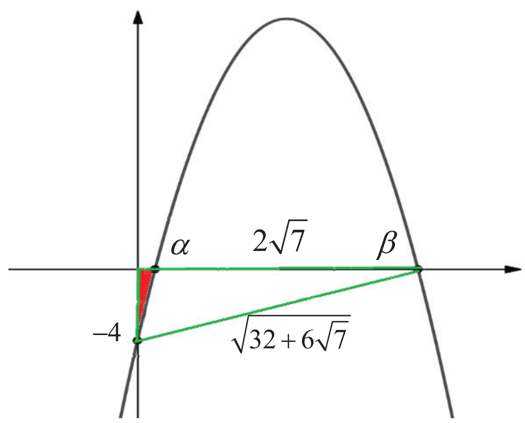
**A1** $4\sqrt{7}$ Cao (Allow e.g. $=2\sqrt{28}$, $\sqrt{112}$). They may work in decimals and state the correct exact answer of $4\sqrt{7}$ which can score full marks **if it follows from a correct method**.

**Note**

$\mathrm{f}(x)+7=0 \Rightarrow 28-2(x-3)^2$ or $10+12x-2x^2 \Rightarrow x=3\pm\sqrt{14} \Rightarrow$ Area $=\frac{1}{2}\times\left(2\sqrt{14}\right)\times 10=10\sqrt{14}$

would score B0M1M1A0




Large triangle $=6+2\sqrt{7}=11.29...$
Small triangle $=6-2\sqrt{7}=0.708...$

| Question | Scheme | Marks |
|---|---|---|
| 3(a) | $\frac{1}{2}\times 3^2\times\alpha = 7.2 \Rightarrow \alpha = ...$ or $\frac{1}{2}\times 3^2\times 1.6 = 7.2 \Rightarrow \alpha = 1.6$ | M1 |
| | $\alpha = 1.6$ * | A1* |
| | | **(2)** |
| (b)(i) | Angle $COA = \frac{1}{2}(2\pi - 1.6)(= 2.34...)$ $(\approx 134°)$ | M1 |
| | Area $COA = \frac{1}{2}\times 5\times 3\sin("2.34")$ $(= 5.38...)$ | M1 |
| | Total Area $= 2\times\frac{1}{2}\times 5\times 3\sin("2.34") + 7.2$ | dM1 |
| | = 18 ($cm^2$) Awrt 18 ($cm^2$) (Ans = 17.96) | A1 |
| (ii) | Arc $AB = 3\times 1.6(= 4.8)$ | B1 |
| | $(AC^2 =)\ 5^2 + 3^2 - 2\times 5\times 3\cos("2.34")$ | M1 |
| | Total perimeter $= 2\times\sqrt{5^2+3^2-2\times 5\times 3\cos("2.34")} + 3\times 1.6$ | dM1 |
| | = Awrt 19.6 (cm) | A1 |
| | | **(8)** |
| Alt (b)(i) | $AB = 2\times 3\sin 0.8$ | M1 |
| | $ON = 3\cos 0.8$ | M1 |
| | Total Area $= \frac{1}{2}(5 + ON)\times AB + 7.2 - \frac{1}{2}\times 3\cos 0.8\times 2\times 3\sin 0.8$ | dM1 |
| | = 18 ($cm^2$) Awrt 18 ($cm^2$) (Ans = 17.96) | A1 |
| | | **(10 marks)** |

**Notes**

**(a)**

**M1** Uses a correct sector area formula and 7.2 to find the value for $\alpha$. They should show the values embedded in the equation and proceed to find a value for $\alpha$.
Alternatively, substitutes in $\alpha = 1.6$ into the area of a sector formula and achieves 7.2.

**A1*** Correct proof starting with $\frac{1}{2}\times 3^2\times\alpha = 7.2$ and at least one intermediate line of working and no errors. Eg $\frac{1}{2}\times 3^2\times\alpha = 7.2 \Rightarrow \alpha = \frac{7.2}{4.5} = 1.6$ scores M1A1
Alternatively, they must conclude that $\alpha = 1.6$ or if there is a preamble then there should be some form of completion which could be a tick, QED etc.
If they use a different variable such as $\theta$ they must state/link somewhere that $\alpha = 1.6$

**(b)(i)** **Mark both (i) and (ii) together. If no angle calculation is seen then use what they think is their angle *COA* in bi and bii. Beware of values on the diagram that may imply a method.**

**M1** $\frac{1}{2}(2\pi-1.6)$ Correct method for angle $COA$. Sight of awrt 2.34 is sufficient to score this mark and may be on the diagram. (May also be implied by $134°$)

**M1** Uses a correct method for the area of triangle $COA$ or $COB$. It is sufficient to see the values embedded in the expression such as $\frac{1}{2}\times5\times3\sin(\text{"}2.34\text{"})$ $(=\text{awrt } 5.38/5.39)$. Angle may be in degrees. If they state $\frac{1}{2}ab\sin C$ oe but embed values as $\frac{1}{2}abC$ condone as a slip for M1.

**dM1** Fully correct strategy for the area. It is dependent on the previous method mark so allow if their angle $COA$ is incorrect. Look for $2\times$area of triangle $COA+7.2$. Embedded values are sufficient.

**A1** awrt 18 (cm$^2$) **Must come from a correct method**

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Alt b(i)

M1 Find the length $AB$ $=2\times3\sin0.8$ (awrt 4.30)

M1 Finds the length $ON$ where $N$ is the midpoint of $AB$ (awrt 2.09)

dM1 Fully correct strategy for the area. Look for $7.2+\text{area of triangle } ABC-\text{area of triangle } AOB$

A1 awrt 18 (cm$^2$) **Must come from a correct method**

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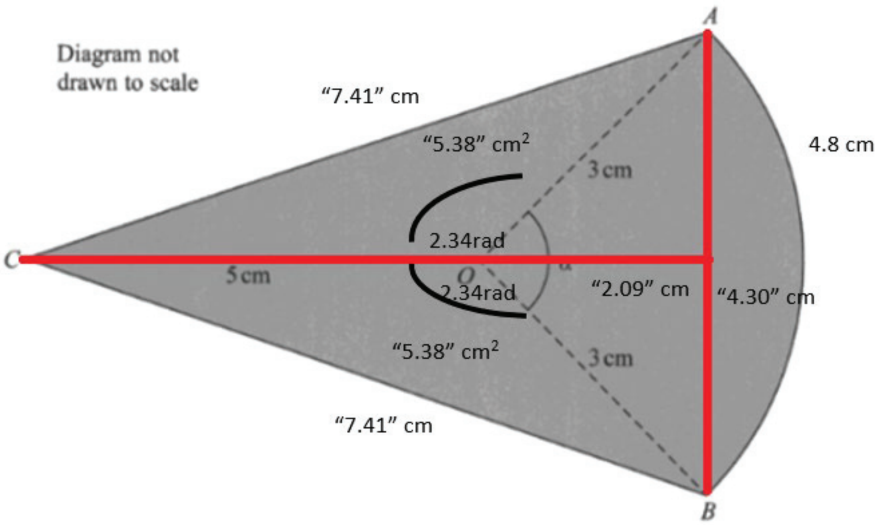
**(ii)**

**B1** Correct expression or value for the arc length (4.8)

**M1** Uses a correct method for $AC^2$, $AC$, $CB^2$ or $CB$. Embedded values in the associated formula is sufficient or sight of awrt 54.9 or awrt 7.41 would imply this mark. (Angle in degrees $\approx134°$) For this mark condone candidates confusing $AC^2/CB^2$ and $AC/CB$.

**dM1** Total perimeter $=2\times\sqrt{5^2+3^2-2\times5\times3\cos(\text{"}2.34\text{"})}+3\times1.6$. It is dependent on the B and the M marks and they must have remembered to square root $AC^2$ or $CB^2$.

**A1** awrt 19.6 (cm) (Ans = 19.619) **Must come from a correct method**



| Question | Scheme | Marks |
|---|---|---|
| 4 | $y = 3x+4 \Rightarrow x^2 + (3x+4)^2 + 6x - 4(3x+4) = 4$<br>**or**<br>$x = \dfrac{y-4}{3} \Rightarrow \left(\dfrac{y-4}{3}\right)^2 + y^2 + 6\left(\dfrac{y-4}{3}\right) - 4y = 4$ | M1 |
| | $5x^2 + 9x - 2\ (=0)$ or $5y^2 - 13y - 46\ (=0)$ | M1A1 |
| | $(5x-1)(x+2) = 0 \Rightarrow x = \ldots$ or $(5y-23)(y+2) = 0 \Rightarrow y = \ldots$ | dM1 |
| | $x = 0.2,\ x = -2$ or $y = 4.6,\ y = -2$ | B1 |
| | Substitutes their $x$ into their $y = 3x + 4$ / Substitutes their $y$ into their $x = \dfrac{y-4}{3}$ | M1 |
| | $x = 0.2\left(\text{or } \dfrac{1}{5}\right),\ y = 4.6\left(\text{or } 4\dfrac{3}{5} \text{ or } \dfrac{23}{5}\right)$<br>**and**<br>$x = -2,\ y = -2$ | A1 |
| | | **(7 marks)** |

**Notes**

**M1** rearrange the linear equation to $y = \ldots$ or $x = \ldots$ and attempts to fully substitute into the second equation.

**M1** Collect terms together to produce a 2 or 3 term quadratic expression = 0. The '= 0' may be implied by later work. Condone slips in their rearrangement of the equation.

**A1** Correct quadratic equation in $x$ or $y$. Condone the absence of "$=0$"
They may be multiples of the main scheme. Eg $10x^2 + 18x - 4 = 0$ or $10y^2 - 26y - 92 = 0$

**dM1** Attempt to factorise and solve or complete the square and solve or uses a correct quadratic formula for a 3 term quadratic and obtains at least one value of $x$ or $y$. (see general guidance for solving a quadratic). If factorising then their **seen** factorised expression must equal their **seen** 3TQ quadratic expression. If they use the quadratic formula, we must see the values embedded in a correct formula.
Dependent on both previous method marks.
**They cannot just write down their calculator values for this mark**
Guidance on use of calculators for factorising quadratics and how to score:

$\ldots. \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow (5x-1)(x+2) = 0 \Rightarrow x = \dfrac{1}{5}, -2 \Rightarrow \ldots$ max score M1M1A1 dM0 B1M1 A0

$\ldots. \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow (x-\tfrac{1}{5})(x+2) = 0 \Rightarrow x = \dfrac{1}{5}, -2 \Rightarrow \ldots$ max score M1M1A1 dM0 B1M1 A0

$\ldots \Rightarrow 10x^2 + 18x - 4 = 0 \Rightarrow x = \dfrac{1}{5}, -2 \Rightarrow \ldots$ max score M1M1A1 dM0 B1M1 A0

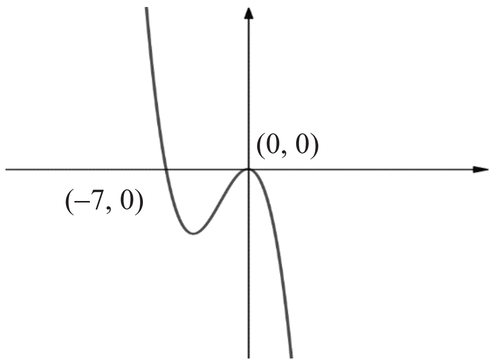
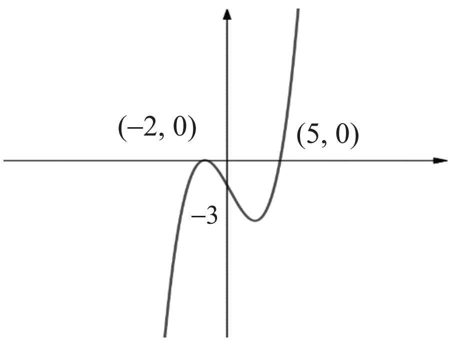
**B1** Correct answers for either both values of $x$ or both values of $y$ (possibly unsimplified) which can only be scored if they have achieved a correct 3TQ.

**M1** Substitute at least one value of $x$ to find $y$ or vice versa. This may be implied by their final answers.

**A1** Fully correct solution **with all previous marks awarded** such that

- working is shown to produce a correct quadratic equation
- working is shown to solve the quadratic equation
- both pairs of coordinates found and simplified. Only withhold if the wrong $x$ coordinate is clearly paired with the other $y$ coordinate.

Condone recovery of invisible brackets.

| Question | Scheme | | Marks |
|---|---|---|---|
| 5(i)(a) |  | B1 Horizontal translation $\leftarrow$<br>B1 Maximum at origin<br>B1 $(-7,0)$ | B1B1B1 |
| | | | **(3)** |
| (b) |  | B1 Reflection in $y$-axis<br>B1 Touches at $(-2,0)$ and passes through $(5,0)$<br>B1 Passes through $(0,-3)$ | B1B1B1 |
| | | | **(3)** |
| (ii)a | $x=0 \Rightarrow y=k\cos\left(\frac{\pi}{6}\right)=\sqrt{3}$<br>$k\frac{\sqrt{3}}{2}=\sqrt{3}\Rightarrow k=2$ | | B1 |
| (b) | $(p=)\frac{\pi}{3}$ **or** $(q=)\frac{4\pi}{3}$<br>$(p=)\frac{\pi}{3}$ **and** $(q=)\frac{4\pi}{3}$ | | B1<br>B1 |
| | | | **(3)** |
| | | | **(9 marks)** |

**Notes**

**(i)(a)**

**B1** Horizontal translation $\leftarrow$
The negative cubic should appear in quadrants 2, 3 and 4 only and the local maximum should be on the $x$ axis where $x \le 0$. The local minimum must be in the third quadrant. Condone any part of the graph which may look linear but withhold this mark if the graph curves back on itself.

**B1** Local maximum at the origin (does not need to be labelled)

**B1** Passes through (–7, 0). Allow just the $x$ value instead of both coordinates marked on the axis or written in the text and condone a slip of $x$ and $y$ the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets. Do not allow 7 instead of $-7$ but condone transcription errors if the correct coordinate is stated in the text.

**(b)**

**B1** Reflection in the $y$-axis. A positive cubic should be drawn appearing in quadrants 1, 3 and 4 only. The local maximum point should be on the $x$ axis where $x \le 0$ and the local minimum point should be in quadrant 4. Do not accept the local minimum on the $y$-axis.

**B1** Touches at (–2, 0) and passes through (5, 0). Allow just the $x$ values instead of both coordinates marked on the axis or written in the text and condone a slip of $x$ and $y$ the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets. Do not allow 2 instead of $-2$ or $-5$ instead of 5 but condone transcription errors if the correct coordinates are stated in the text.

**B1** Passes through (0, –3). Allow just $-3$ instead of both coordinates marked on the axis or written in the text and condone a slip of $x$ and $y$ the wrong way round as long as the sketch would give the correct coordinates. May be listed but cannot be awarded without a sketch. Condone lack of brackets.
Do not allow 3 instead of $-3$ but condone transcription errors if the correct coordinate is stated in the text.

**(ii)(a)**

**B1** $k = 2$

**B1** $(p =)\,\frac{\pi}{3}$ **or** $(q =)\,\frac{4\pi}{3}$ Award for sight of either of these values or equivalent. Ignore labelling.
Allow awrt 1.05 or awrt 4.19 for this mark and allow in degrees (60 or 240).

**B1** $(p =)\,\frac{\pi}{3}$ **and** $(q =)\,\frac{4\pi}{3}$ ignore labelling and condone $\left(0, \frac{\pi}{3}\right), \left(0, \frac{4\pi}{3}\right)$. Allow exact equivalents.

| Question | Scheme | Marks |
|---|---|---|
| **6(a)** | E.g. $m=\dfrac{2-11}{8+4}$ or $m=\dfrac{11-2}{-4-8}$ | M1 |
| | $m=-\dfrac{3}{4}$ | A1 |
| | | **(2)** |
| **(b)** | $M$ is $\left(2,\ \dfrac{13}{2}\right)$ | B1 |
| | $m_N=-1\div"-\dfrac{3}{4}"$ | M1 |
| | $y-"\dfrac{13}{2}"="\dfrac{4}{3}"\left(x-"2"\right)$ | M1 |
| | $8x-6y+23=0$ | A1 |
| | | **(4)** |
| **(c)** | $AB=\sqrt{(-4-8)^2+(11-2)^2}\ (=15)$ or $AB^2=(-4-8)^2+(11-2)^2\ (=225)$ | M1 |
| | $\dfrac{1}{2}\times MC\times AB=37.5\Rightarrow MC=\dfrac{75}{15}\ (=5)$ or $MC^2=25$ | M1 |
| | $m_N=\dfrac{4}{3}, MC=5\Rightarrow\ C$ is $\left("2"-3,\ "\dfrac{13}{2}"-4\right)$ or $\left("2"+3,\ "\dfrac{13}{2}"+4\right)$ | dM1 |
| | $(-1,\ 2.5)$ **or** $(5,\ 10.5)$ **or** $x=-1,\ x=5$ **or** $y=2.5,\ y=10.5$ | A1 |
| | $(-1,\ 2.5)$ **and** $(5,\ 10.5)$ | A1 |
| | | **(5)** |
| | | **(11 marks)** |

**Notes**

**(a)**

**M1** Correct gradient method. Method must be correct for **both** the numerator and denominator. Could also be solved by setting up two simultaneous equations and solving correctly.
ie $11=-4m+c \quad 2=8m+c \quad \Rightarrow 9=-12m\Rightarrow m=...$ (M1)

**A1** Correct fraction or decimal (allow $\dfrac{-3}{4}$ or $\dfrac{3}{-4}$ or $-0.75$). If they find the equation of the line then they must identify $-\dfrac{3}{4}$ as the gradient.

**(b)**

**B1** Correct midpoint. Allow unsimplified e.g. $\left(\dfrac{-4+8}{2}, \dfrac{11+2}{2}\right)$. Coordinates can be just stated, seen in their working or they may appear on the diagram. Condone lack of brackets.

**M1** Applies the perpendicular gradient rule to their gradient from part (a)

**M1** Correct straight line method using their midpoint and a "changed" gradient. If using $y = mx + c$, must reach as far as $c = \ldots$.

**A1** Allow any integer multiple

**(c)**

**M1** Correct application of Pythagoras using the points $A$ and $B$ (or $\frac{1}{2}AB$ using $M$)

**M1** Uses their $AB$ and 37.5 correctly to find $MC$ or $MC^2$. This may be implied by their working to find eg $AM$ or $BM$

**dM1** **Correct strategy** to find both $x$ coordinates, or both $y$ coordinates or a coordinate pair for the possible point $C$**.** To score this mark, look for sight of a linear or quadratic equation in one variable that would find either an $x$ or a $y$ coordinate for one of the possible pairs of coordinates for $C$. Condone slips in their rearrangements and they do not need to reach a value for either $x$ or $y$ for this mark.

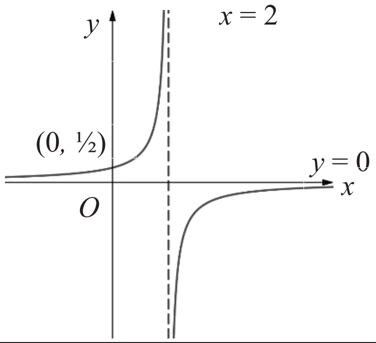
Usually look for:

- Recognising that a distance of "5" for $MC$ means by Pythagoras that $x = "2" \pm 3$ or $y = "\dfrac{13}{2}" \pm 4$. Alternatively, $C$ is $\left("2"-3, "\dfrac{13}{2}"-4\right)$ or $\left("2"+3, "\dfrac{13}{2}"+4\right)$
  (If $MC$ is not 5 look for application of Pythagoras.)
- Forming an equation of a circle with radius "5" from a point $C$ and solving simultaneously with their $l$ eg
  $(x-"2")^2 + (y-"\tfrac{13}{2}")^2 = "25"$ and $"8x-6y+23=0" \Rightarrow (x-2)^2 + (\tfrac{4}{3}x+\tfrac{23}{6}-\tfrac{13}{2})^2 = 25$ oe
  (they may form an equation in $y$).
- Shoelace method:
  $$\begin{vmatrix} -4 & 11 \\ x & \tfrac{4}{3}x+\tfrac{23}{6} \\ 8 & 2 \\ -4 & 11 \end{vmatrix} = \tfrac{1}{2}\left(-4(\tfrac{4}{3}x+\tfrac{23}{6})+2x+88)-(11x+8(\tfrac{4}{3}x+\tfrac{23}{6})-8)\right) = 37.5$$

It is dependent on **both of the previous method marks**. There may be other credit worthy methods so if unsure then send to review.

**A1** $(-1, 2.5)$ **or** $(5, 10.5)$ (does not have to be written as coordinates)
**or** $x=-1,\ x=5$ **or** $y=2.5,\ y=10.5$

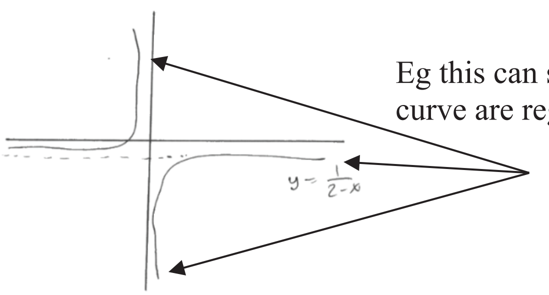
**A1** $(-1, 2.5)$ **and** $(5, 10.5)$. Condone lack of brackets.
They may state the values for $x$ and $y$ but the coordinates must be paired correctly.

| Question | Scheme | Marks |
|---|---|---|
| **7(a)** | <br>B1 Negative reciprocal shape<br>B1 Intercept at $\left(0, \frac{1}{2}\right)$<br>B1 $x=2,\ y=0$ | B1B1B1 |
| | | **(3)** |
| **(b)** | $4x+k=\dfrac{1}{2-x} \Rightarrow (4x+k)(2-x)=1 \Rightarrow 8x+2k-4x^2-kx-1(=0)$ oe | M1 |
| | $4x^2+(k-8)x+1-2k=0$<br>$a=4,\ b=k-8,\ c=1-2k$ or $a=-4,\ b=8-k,\ c=2k-1$ | A1 |
| | $(k-8)^2-4\times4(1-2k)(>0)$ oe | M1 |
| | $k^2-16k+64-16+32k>0 \Rightarrow k^2+16k+48>0$* | A1* |
| | | **(4)** |
| **(c)** | $k^2+16k+48=0 \Rightarrow (k+12)(k+4)=0 \Rightarrow k=...$ | M1 |
| | $k=-12,\ -4$ | A1 |
| | $k<-12$ or $k>-4$ | M1A1 |
| | | **(4)** |
| | | **(11 marks)** |

**Notes**

**(a)**

**B1** Negative reciprocal shape (top left/bottom right sections) in any position on a set of axes with no clear vertical or horizontal 'overlaps'.
Condone slips of the pen as long as there is not a clear turning point, but it should not curve back on itself. If there are several attempts, then mark the last one. If there are contradictions between points stated in the text and any on the graph, then the graph takes precedence.




Eg this can score B1 for the correct shape. The ends of their curve are regarded as a slip of the pen

**B1** Intercept at $\left(0, \frac{1}{2}\right)$ and no others. Allow $\frac{1}{2}$ to be marked on the $y$-axis and condone $\left(\frac{1}{2}, 0\right)$ if the intercept is in the correct place. Cannot be awarded without a sketch.

**B1** $x = 2$ **and** $y = 0$ and no other asymptotes. Do not allow "$x$-axis" as an asymptote. Do not award without a sketch.

**(b)** **Mark (b) and (c) together**

**M1** Sets line = curve, multiplies by $(2-x)$, attempts to expand and collects terms on one side of the equation. Condone the absence of =0.

**A1** Correct equation unsimplified with all terms on one side, condone the absence of =0. This may be implied by later work such as their values of $a$, $b$ and $c$ substituted in to $b^2 - 4ac$ and full marks can be awarded in this question if =0 is never seen.

**M1** Attempts discriminant in terms of $k$ using their $a$, $b$ and $c$ from their 3TQ. Sight of these terms embedded in $b^2 - 4ac$ is sufficient for this mark. Condone invisible brackets and ignore the use of $>, <$ or $=$ for this mark.

**A1*** Correctly multiplies out their expression for the discriminant, sets $> 0$ and proceeds to the given answer with no errors including any omission of brackets in their work.
They must have used a correct inequality on at least the previous line before their final answer or stated $b^2 - 4ac > 0$ somewhere in their working.

**(c)**

**M1** Solves the given quadratic to obtain 2 values for $k$. See guidance on solving quadratics. They may just write the roots down from their calculator which is acceptable.

**A1** $-12, -4$

**M1** Attempts 'outside regions' so this may include $\le$ or $\ge$ are condoned for this mark.
$-4 < k < -12$ would imply this mark. Ignore if in terms of $x$ for this mark. Cannot be scored from a diagram.

**A1** Accept any of '$k < -12$, $k > -4$', '$k < -12$ or $k > -4$', '$\{k : k < -12 \cup k > -4\}$',
'$\{k : -\infty < k < -12 \cup -4 < k < \infty\}$' or their equivalent expressions
Must be in terms of $k$. (Not $x$ or any other variable)

DO NOT ALLOW '$k < -12$ and $k > -4$' '$-12 > k > -4$', '$-4 < k < -12$'

**Note:** These inequalities without any working score full marks in (c)

| Question | Scheme | Marks |
|---|---|---|
| 8(a) | $y=(x-2)(x^2-8x+16)\Rightarrow y=x^3-8x^2+16x-2x^2+16x-32\Rightarrow$ <br> $y=x^3\pm...x^2\pm...x\pm32$ | M1 |
| | $=x^3-10x^2+32x-32$ | A1 |
| | $\frac{dy}{dx}=3x^2-20x+32$ * | M1A1* |
| | | **(4)** |
| (b) | $x=6\Rightarrow y=(6-2)(6-4)^2=16$ | B1 |
| | $\frac{dy}{dx}=3(6)^2-20(6)+32=20$ | B1 |
| | $y-"16"="20"(x-6)$ | M1 |
| | $y=20x-104$ | A1 |
| | | **(4)** |
| (c) | $3x^2-20x+32="20"\Rightarrow 3x^2-20x+12=0$ | M1 |
| | $3x^2-20x+12=0\Rightarrow(3x-2)(x-6)=0\Rightarrow x=...$ | dM1 |
| | $\alpha=\frac{2}{3}$ | A1 |
| | | **(3)** |
| | | **(11 marks)** |

**Notes**

**(a)**

**M1** Attempts to multiply out the three brackets, condoning slips in their working.
Usually $y=(x-2)(x^2-8x+16)\Rightarrow y=x^3-8x^2+16x-2x^2+16x-32\Rightarrow x^3\pm...x^2\pm...x\pm32$
Score for expressions of the form $x^3\pm...x^2\pm...x\pm32$. Middle terms do not need to be collected.

**A1** $x^3-10x^2+32x-32$ If they do not collect terms together until after differentiating, A1 can be awarded by subsequent work. You would have to see the individual differentiated terms collected rather than implied by the final answer.
**They must have attempted to multiply out the brackets for this mark.**

**M1** $x^n\to x^{n-1}$ correct on one term so either $...x^3\to...x^2$ $\quad ...x^2\to...x$ $\quad Ax\to A$ $\quad B\to0$

**A1*** Correct proof with no errors including omission of brackets. At some point they should have had $y=...$ and their final line should finish with $\frac{dy}{dx}=3x^2-20x+32$ including the $\frac{dy}{dx}$ but the terms on the rhs can be in any order.

Alternative method: Product rule - Note the order of marking

2nd M1 $\left(\frac{dy}{dx}=\right)$ $(x-2)\times A(x-4)\pm B(x-4)^2$ applies the product rule. Look for this form or equivalent.

1st A1 $\left(\frac{dy}{dx}=\right)$ $(x-2)\times 2(x-4)+(x-4)^2$

1st M1 $\left(\frac{dy}{dx}=\right)$ $2x^2-8x-4x+16+x^2-8x+16 \Rightarrow 3x^2-20x+32$ attempts to multiply out and collect terms to form a 3TQ

2nd A1* $\frac{dy}{dx}=3x^2-20x+32$ * with no errors

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M1 $y=(x-2)(x^2-8x+16) \Rightarrow y=x^3\pm...x^2\pm...x\pm32$ (does not require middle terms to score M1)

A1 $y=x^3-10x^2+32x-32$ oe

M1 $\int(3x^2-20x+32)\,dx=x^3-10x^2+32x+C$ look for correct index on one term

A1* deduce that $C=-32$ and conclude $\frac{dy}{dx}=3x^2-20x+32$ with no errors seen

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**(b)**

**B1** 16 is identified as the $y$ coordinate. Beware that $\frac{d^2y}{dx^2}=16$ when $x=6$

**B1** 20 is identified as the gradient. Eg $\frac{dy}{dx}=20$, $m=20$, $g=20$ or may be used within their equation for the tangent.

**M1** Correct straight line method $y-"16"="20"(x-6)$ using:

- their value of $y$ from substituting in $x=6$ into $y=(x-2)(x^2-8x+16)$ or $y=...$ from (a)
- their gradient found from substituting $x=6$ into $\frac{dy}{dx}=3x^2-20x+32$. This cannot be a changed gradient (eg gradient of a normal)

If they use $y=mx+c$ they must proceed as far as $c=...$

**A1** $y=20x-104$ cao

**(c)**

**M1** Equates $3x^2-20x+32$ with their 20 and collects terms to obtain a 3TQ. Condone slips in their rearrangement.

**dM1** Attempts to solve their 3TQ (see general guidance for solving quadratics) . If they just state the roots then you may need to check these on a calculator. It is dependent on the previous method mark.

**A1** $\alpha=\frac{2}{3}$ (allow $x=\ldots$) Ignore sight of 6. Answer on its own scores full marks.

(Note that values of 4, $\frac{8}{3}$ imply they have solved $3x^2-20x+32=0$ which is 0 marks)

| Question | Scheme | Marks |
|---|---|---|
| 9 | $\dfrac{21x^3-5x}{2\sqrt{x}}=\alpha x^{\frac{5}{2}}+\ldots$ or $\dfrac{21x^3-5x}{2\sqrt{x}}=\ldots+\beta x^{\frac{1}{2}}$ | M1 |
| | $\mathrm{f}(x)=\dfrac{27}{3}x^3-\dfrac{21}{2}\times\dfrac{2}{7}x^{\frac{7}{2}}+\dfrac{5}{2}\times\dfrac{2}{3}x^{\frac{3}{2}}(+c)\quad\left(=9x^3-3x^{\frac{7}{2}}+\dfrac{5}{3}x^{\frac{3}{2}}(+c)\right)$ | M1A1A1 |
| | $\mathrm{f}(9)=10\Rightarrow 9(9)^3-3(9)^{\frac{7}{2}}+\dfrac{5}{3}(9)^{\frac{3}{2}}+c=10\Rightarrow c=\ldots$ | dM1 |
| | $(\mathrm{f}(x)=)\ 9x^3-3x^{\frac{7}{2}}+\dfrac{5}{3}x^{\frac{3}{2}}-35$ | A1 |
| | | **(6 marks)** |

## Notes

**On EPEN it is B1M1A1A1dM1A1. We are marking this as M1M1A1A1dM1A1**

**M1** Uses correct index laws to obtain at least one correct index from splitting the fraction.
Award for $\alpha x^{\frac{5}{2}}+\ldots$ or $\ldots+\beta x^{\frac{1}{2}}$

**M1** $x^n\to x^{n+1}$ correctly seen on one term (usually the $27x^2\to\ldots x^3$). The indices do not need to be processed. This mark can also be awarded for integrating terms from incorrect attempts to split the fraction. Eg $\dfrac{21x^3-5x}{2x^{\frac{1}{2}}}=42x^{\frac{7}{2}}-10x^{\frac{3}{2}}\quad 42x^{\frac{7}{2}}\to\ldots x^{\frac{9}{2}}$ or $-10x^{\frac{3}{2}}\to\ldots x^{\frac{5}{2}}$

**This mark cannot be awarded for only seeing:**

$$\frac{21x^3-5x}{2x^{\frac{1}{2}}}\to\frac{\ldots x^4\pm\ldots x^2}{\ldots x^{\frac{3}{2}}}$$

**A1** Two correct terms simplified or unsimplified, but the indices must have been processed.

**A1** All correct simplified or unsimplified (+ $c$ not required)

**dM1** Uses f (9) = 10 and attempts to find $c$. Do not be too concerned by the mechanics of their arrangement. It is dependent on the previous method mark.

**A1** $9x^3-3x^{\frac{7}{2}}+\dfrac{5}{3}x^{\frac{3}{2}}-35$ All correct and simplified. Accept other simplified equivalent expressions for $\mathrm{f}(x)$ such as $\dfrac{1}{3}(27x^3-9x^{\frac{7}{2}}+5x^{\frac{3}{2}}-105)$