Pearson Edexcel International A Level Mathematics

Mechanics 3

Past Paper Collection (from 2020)

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Last updated: July 1, 2024

Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Thursday 9 January 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

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1. A rough disc is rotating at a constant angular speed of 5 revolutions per minute about a vertical axis. The axis is perpendicular to the plane of the disc and passes through the centre of the disc. A particle, P, of mass m kg is placed on the disc at distance 0.2 m from the axis. The particle does not move relative to the disc. The coefficient of friction between P and the disc is μ.

Find the smallest possible value of μ.

(6)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

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2. A particle, P, of mass 0.5 kg is moving along the positive x-axis. At time t seconds, $t \geqslant 0$, P is x metres from the origin O and is moving away from O with velocity v m s^{-1}, where $v = \dfrac{1}{(4x + 3)}$

When $t = 0$, P is at O.

(a) Find the distance of P from O when $t = 2$

(5)

(b) Find the magnitude of the resultant force acting on P when $t = 2$

(5)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 10 marks)

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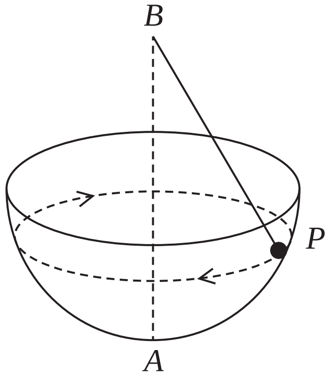

Figure 1

Figure 1 shows a hemispherical bowl of internal radius a fixed with its open plane face uppermost and horizontal. The lowest point of the bowl is A. A light inextensible string of length $a\sqrt{3}$ has one end fixed to the point B, where B is vertically above A and $AB = 2a$. A particle, P, of mass m is attached to the other end of the string.

The particle moves in a horizontal circle on the smooth inner surface of the bowl with constant angular speed ω. The string remains taut and the particle remains in contact with the bowl throughout the motion.

(a) Find, in terms of m, a, ω and g, the tension in the string.

(7)

(b) Show that $\omega \geqslant \sqrt{\dfrac{2g}{3a}}$

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 11 marks)

Q3

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4. A light elastic string has modulus of elasticity $2mg$ and natural length l. One end of the string is fixed to a point A on a rough plane inclined to the horizontal at angle α, where $\sin\alpha = \frac{3}{5}$. A particle, P, of mass m is attached to the other end of the string. Initially P is held at rest on the plane at the point B, where B is above A and $AB = \frac{1}{2}l$. The string lies along a line of greatest slope of the plane.

The particle P is released from rest and moves down the plane along the line of greatest slope. The coefficient of friction between P and the plane is μ, where $\mu < \tan\alpha$.

Given that P comes to instantaneous rest at the point C, where $AC = l + e$,

(a) show that

$$\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)}$$

(6)

Given that $e = l$

(b) find the magnitude of the instantaneous change in the acceleration of P at C.

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 11 marks)

Q4

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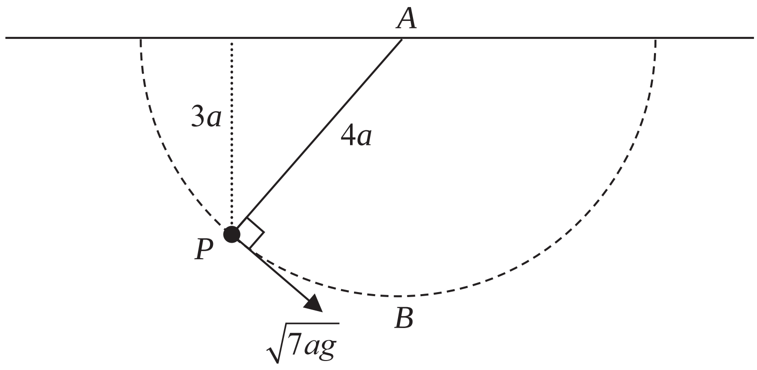

Figure 2

One end of a light inextensible string of length $4a$ is attached to a fixed point A on a horizontal ceiling. A particle, P, of mass m is attached to the other end of the string. The particle is held in equilibrium at a vertical distance $3a$ below the ceiling, with the string taut. The particle is then projected with speed $\sqrt{7ag}$, in the direction perpendicular to the string, in the vertical plane containing A and the string, as shown in Figure 2. In the subsequent motion the string remains taut.

(a) Find the speed of P at the instant before it hits the ceiling.

(4)

The point B is the lowest point of the path of P. The first time P passes through B the tension in the string is T_1 and the second time P passes through B the tension in the string is T_2

Given that the coefficient of restitution between P and the ceiling is $\frac{1}{2}$

(b) find the ratio $T_1 : T_2$ in its simplest form.

(7)

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Question 5 continued

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Question 5 continued

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Question 5 continued

(Total 11 marks)

Q5

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6. A particle, P, of mass 0.4 kg is attached to the midpoint of a light elastic spring of natural length 0.8 m and modulus of elasticity 20 N. The ends of the spring are attached to the fixed points A and B on a smooth horizontal table, where $AB = 1.2$ m. Initially P is at rest at the midpoint O of AB where AOB is a straight line. The particle P now receives an impulse of magnitude 2 Ns so that P starts to move directly towards B.

(a) Prove that P moves with simple harmonic motion.

(4)

(b) Write down, in terms of π, the period of the motion.

(1)

(c) Find the amplitude of the motion.

(3)

(d) Find the length of time in each complete oscillation for which AP is greater than 0.5 m.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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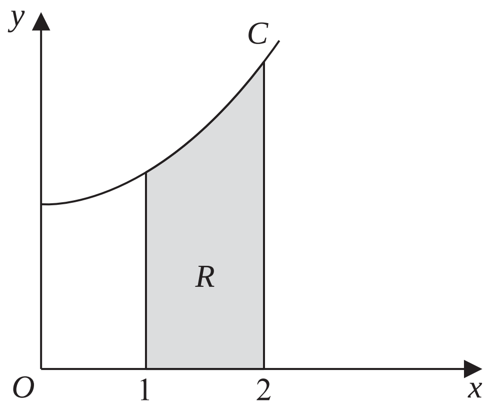

Figure 3

Figure 3 shows part of the curve C with equation $y = x^2 + 4$. The shaded region R is bounded by C, the line with equation $x = 1$, the x-axis and the line with equation $x = 2$

The unit of length on each axis is one centimetre.

A uniform wooden solid, S, is made in the shape formed by rotating the region R through 360° about the x-axis.

(a) Using algebraic integration,

(i) show that the volume of S is $\frac{613\pi}{15}$ cm^3

(ii) find, to 3 significant figures, the distance of the centre of mass of S from O.

(8)

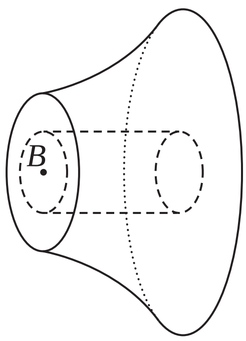

Figure 4

A solid, S_1, is formed by removing a solid cylinder of radius 3 cm and length 1 cm from S. A metal cylinder, of radius 3 cm and length 1 cm is placed in the resulting hole to form a new solid T, as shown in Figure 4. The axis of the metal cylinder coincides with the axis of symmetry of S_1. The point B is the centre of the smaller plane face of T. The mass per unit volume of S_1 is M and the mass per unit volume of the metal cylinder is $5M$.

(b) Find the distance of the centre of mass of T from B.

(5)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Advanced Level

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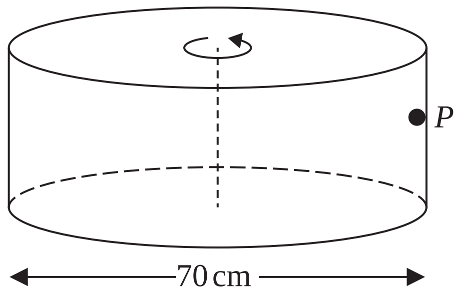

Figure 1

Figure 1 shows a hollow cylinder with diameter 70 cm. The cylinder is rotating at a constant angular speed about its axis, which is vertical. As the cylinder rotates, a particle *P* remains in contact with the same point on the rough inside surface of the cylinder. The particle is moving in a horizontal circle of diameter 70 cm.

The cylinder makes 2 complete revolutions every second.

The coefficient of friction between *P* and the cylinder is μ.

Find the range of possible values of μ.

(6)

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Question 1 continued

Q1

(Total 6 marks)

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2.

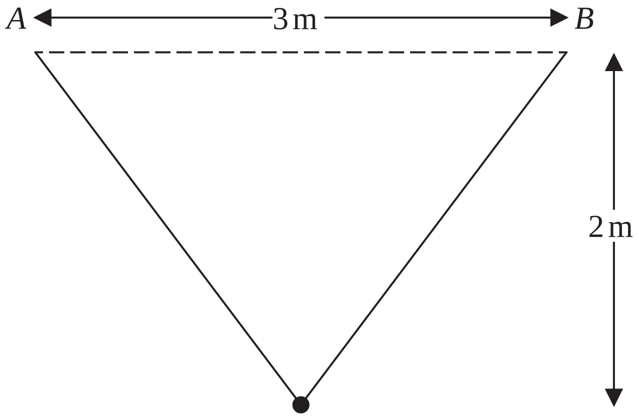

Figure 2

A smooth bead of weight 12 N is threaded onto a light elastic string of natural length 3 m. The points *A* and *B* are on a horizontal ceiling, with *AB* = 3 m. One end of the string is attached to *A* and the other end of the string is attached to *B*.

The bead hangs freely in equilibrium, 2 m below the ceiling, as shown in Figure 2.

(a) Find the tension in the string.

(4)

(b) Show that the modulus of elasticity of the string is 11.25 N.

(2)

The bead is now pulled down to a point vertically below its equilibrium position and released from rest.

(c) Find the elastic energy stored in the string at the instant when the bead is moving at its maximum speed.

(2)

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Question 2 continued

Q2

(Total 8 marks)

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3.

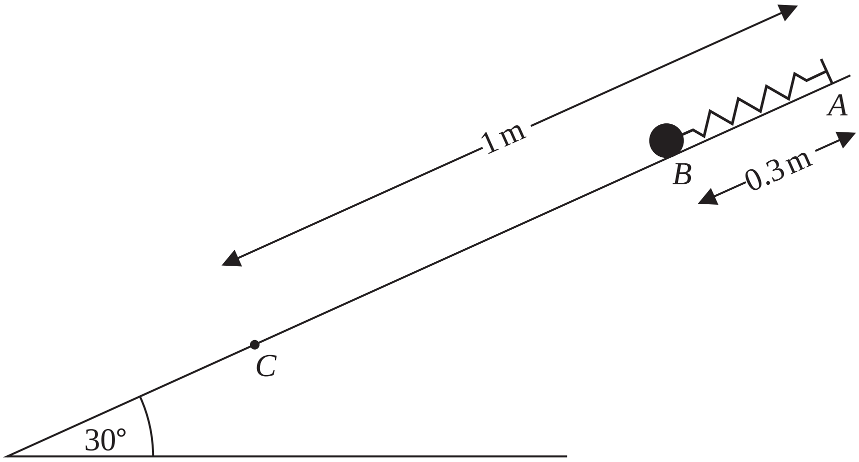

Figure 3

A particle P of mass 2 kg is attached to one end of a light elastic spring, of natural length 0.8 m and modulus of elasticity 12 N. The other end of the spring is attached to a fixed point A on a rough plane. The plane is inclined at 30° to the horizontal. Initially P is held at rest on the plane at the point B, where B is below A, with AB = 0.3 m and AB lies along a line of greatest slope of the plane. The point C lies on the plane with AC = 1 m, as shown in Figure 3.

The coefficient of friction between P and the plane is 0.3

After being released P passes through the point C.

Find the speed of P at the instant it passes through C.

(7)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. (a) Use algebraic integration to show that the centre of mass of a uniform solid hemisphere of radius a is a distance $\frac{3}{8}a$ from the centre of its plane face.

[*You may assume that the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$]

(5)

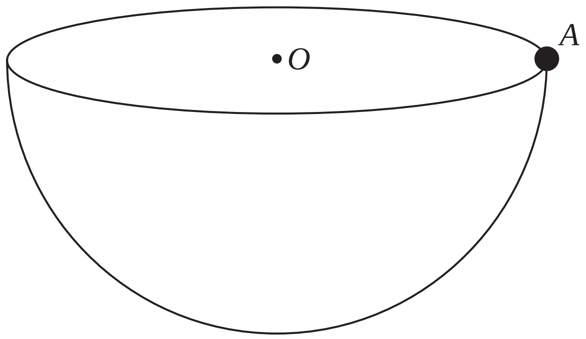

Figure 4

A uniform solid hemisphere has mass m and radius a. A particle of mass km is attached to a point A on the circumference of the plane face of the hemisphere to form the loaded solid S. The centre of the plane face of the hemisphere is the point O, as shown in Figure 4.

The loaded solid S is placed on a horizontal plane. The curved surface of S is in contact with the plane and S rests in equilibrium with OA making an angle α with the horizontal, where $\tan\alpha = \sqrt{3}$

(b) Find the exact value of k.

(5)

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Question 4 continued

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Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 10 marks)

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5. A particle P of mass 0.5 kg moves along the positive x-axis in the positive x direction.

At time t seconds, $t \geqslant 1$, P is x metres from the origin O and is moving with speed v m s^{-1}.

The resultant force acting on P has magnitude $\dfrac{2}{x^3}$ N and is directed towards O.

When $t = 1$, $x = 1$ and $v = 3$

Show that

(a) $v^2 = \dfrac{4}{x^2} + 5$

(5)

(b) $t = \dfrac{a + \sqrt{bx^2 + c}}{d}$, where a, b, c and d are integers to be found.

(7)

Leave blank

Question 5 continued

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Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 12 marks)

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6. A light elastic string has natural length a and modulus of elasticity $\frac{3}{4}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium at the point O, vertically below A.

(a) Find the distance OA.

(2)

The particle P is now pulled vertically down to a point B, where $AB = 3a$, and released from rest.

(b) Show that, throughout the subsequent motion, P performs **only** simple harmonic motion, justifying your answer.

(6)

The point C is vertically below A, where $AC = 2a$.

Find, in terms of a and g,

(c) the speed of P at the instant that it passes through C,

(3)

(d) the time taken for P to move directly from B to C.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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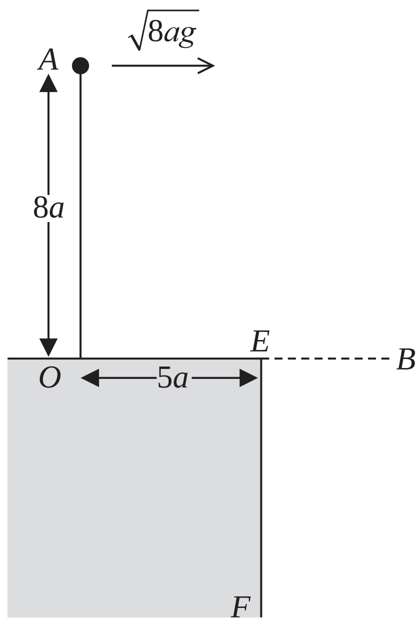

Figure 5

A particle of mass m is attached to one end of a light inextensible string of length $8a$. The other end of the string is fixed to the point O on the smooth horizontal surface of a desk. The point E is on the edge of the desk, where $OE = 5a$ and OE is perpendicular to the edge of the desk. The particle is held at the point A, vertically above O, with the string taut.

The particle is projected horizontally from A with speed $\sqrt{8ag}$ in the direction OE, as shown in Figure 5.

When the particle is above the level of OE the particle is moving in a vertical circle with radius $8a$.

Given that, when the string makes an angle θ with the upward vertical through O, the tension in the string is T,

(a) show that $T = 3mg(1 - \cos\theta)$

(7)

At the instant when the string is horizontal, the particle passes through the point B.

(b) Find the instantaneous change in the tension in the string as the particle passes through B.

(3)

The particle hits the vertical side EF of the desk and rebounds. As a result of the impact, the particle loses one third of the kinetic energy it had immediately before the impact.

In the subsequent motion the string becomes slack when it makes an angle α with the upward vertical through O.

(c) Show that $\cos\alpha = \dfrac{7}{12}$

(7)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 17 marks)

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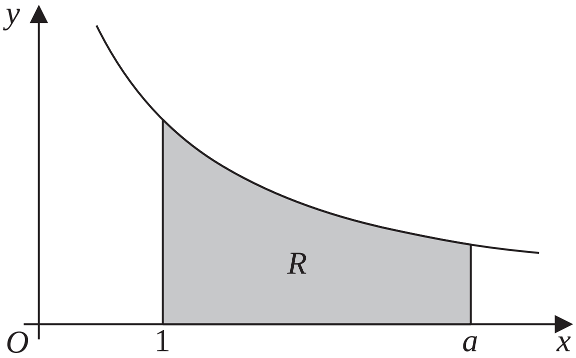

Figure 1

The region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \dfrac{1}{x}$, the line with equation $x = 1$, the positive x-axis and the line with equation $x = a$ where $a > 1$

A uniform solid S is formed by rotating R through 2π radians about the x-axis.

(a) Show that the volume of S is

$$\pi\left(1 - \frac{1}{a}\right)$$

(3)

(b) Find the x coordinate of the centre of mass of S.

(5)

Leave blank

Question 1 continued

Q1

(Total 8 marks)

Leave blank

2. A particle P of mass m is at a distance x above the surface of the Earth. The Earth exerts a gravitational force on P. This force is directed towards the centre of the Earth. The magnitude of this force is inversely proportional to the square of the distance of P from the centre of the Earth.

At the surface of the Earth the acceleration due to gravity is g.

The Earth is modelled as a fixed sphere of radius R.

(a) Show that the magnitude of the gravitational force on P is $\dfrac{mgR^2}{(x+R)^2}$

(3)

A particle is released from rest from a point above the surface of the Earth. When the particle is at a distance R above the surface of the Earth, the particle has speed U.

Air resistance is modelled as being negligible.

(b) Find, in terms of U, g and R, the speed of the particle when it strikes the surface of the Earth.

(7)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 10 marks)

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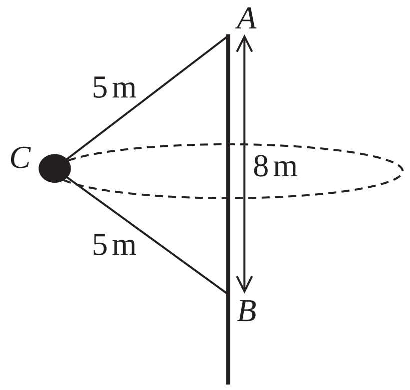

Figure 2

A fairground ride consists of a cabin C that travels in a horizontal circle with a constant angular speed about a fixed vertical central axis. The cabin is attached to one end of each of two rigid arms, each of length 5 m. The other end of the top arm is attached to the fixed point A at the top of the central axis of the ride. The other end of the lower arm is attached to the fixed point B on the central axis, where AB is 8 m, as shown in Figure 2.

Both arms are free to rotate about the central axis.

The arms are modelled as light inextensible rods.

The cabin, together with the people inside, is modelled as a particle.

The cabin completes one revolution every 2 seconds.

Given that the combined mass of the cabin and the people is 600 kg,

(a) find

(i) the tension in the upper arm of the ride,

(ii) the tension in the lower arm of the ride.

(9)

In a refined model, it is assumed that both arms stretch to a length of 5.1 m.

(b) State how this would affect the sum of the tensions in the two arms, justifying your answer.

(2)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

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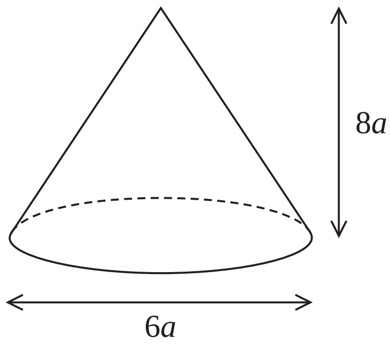

Figure 3

A uniform right solid cone C has diameter $6a$ and height $8a$, as shown in Figure 3.

The solid S is formed by removing a cone of height $4a$ from the top of C and then removing an identical, inverted cone. The vertex of the removed cone is at the point O in the centre of the base of C, as shown in Figure 4.

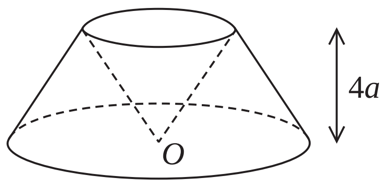

Figure 4

(a) Find the distance of the centre of mass of S from O.

(5)

The point A lies on the circumference of the base of S and the point B lies on the circumference of the top of S. The points O, A and B all lie in the same vertical plane, as shown in Figure 5.

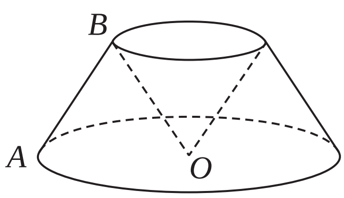

Figure 5

The solid S is freely suspended from the point B and hangs in equilibrium.

(b) Find the size of the angle that AB makes with the downward vertical.

(4)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 9 marks)

Q4

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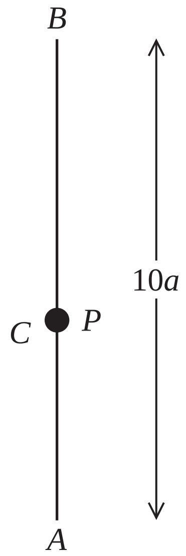

Figure 6

The fixed points, A and B, are a distance $10a$ apart, with B vertically above A.

One end of a light elastic string, of natural length $2a$ and modulus of elasticity $2mg$, is attached to a particle P of mass m and the other end is attached to A.

One end of another light elastic string, of natural length $4a$ and modulus of elasticity $6mg$, is attached to P and the other end is attached to B.

The particle P rests in equilibrium at the point C, as shown in Figure 6.

(a) Show that each string has an extension of $2a$.

(5)

The particle P is now pulled down vertically, so that it is a distance a below C and then released from rest.

(b) Show that in the subsequent motion, P performs simple harmonic motion.

(4)

(c) Find, in terms of a and g, the speed of P when it is a distance $\frac{7}{2}a$ above A.

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 13 marks)

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6.

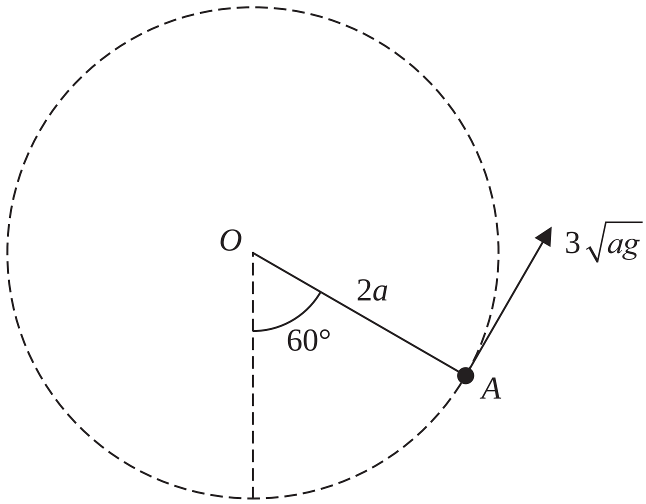

Figure 7

A particle of mass m is attached to one end of a light inextensible string of length $2a$. The other end of the string is attached to a fixed point O. The particle is initially held at the point A with the string taut and OA making an angle of 60° with the downward vertical.

The particle is then projected upwards with a speed of $3\sqrt{ag}$, perpendicular to OA, in the vertical plane containing OA, as shown in Figure 7.

In an initial model of the motion of the particle, it is assumed that the string does not break.

Using this model,

(a) show that the particle performs complete vertical circles.

(8)

In a refined model it is assumed that the string will break if the tension in it exceeds $7mg$.

Using this refined model,

(b) show that the particle still performs complete vertical circles.

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 13 marks)

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7.

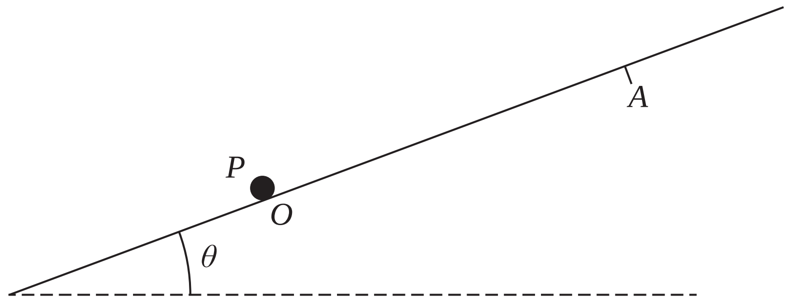

Figure 8

A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 3 N. The other end of the string is attached to a fixed point O on a rough plane. The plane is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{7}$

The coefficient of friction between P and the plane is $\frac{\sqrt{5}}{5}$

The particle P is initially at rest at the point O, as shown in Figure 8.

The particle P then receives an impulse of magnitude 4 Ns, directed up a line of greatest slope of the plane.

The particle P moves up the plane and comes to rest at the point A.

(a) Find the extension of the elastic string when P is at A.

(8)

(b) Show that the particle does not remain at rest at A.

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over

1.

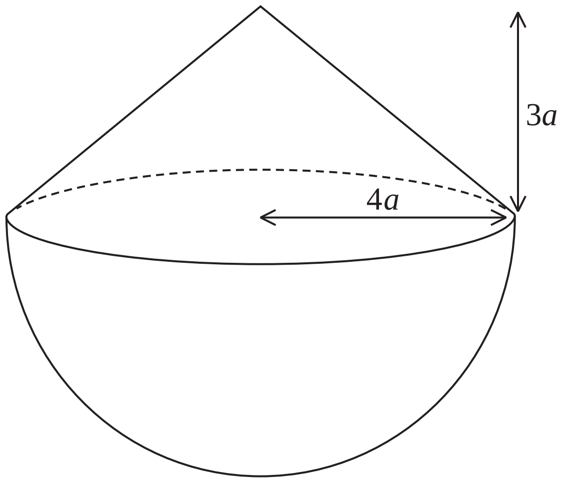

Figure 1

A hollow toy is formed by joining a uniform right circular conical shell C, with radius $4a$ and height $3a$, to a uniform hemispherical shell H, with radius $4a$. The circular edge of C coincides with the circular edge of H, as shown in Figure 1.

The mass per unit area of C is λ and the mass per unit area of H is $k\lambda$ where k is a constant.

Given that the centre of mass of the toy is a distance $4a$ from the vertex of the cone, find the value of k.

(6)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

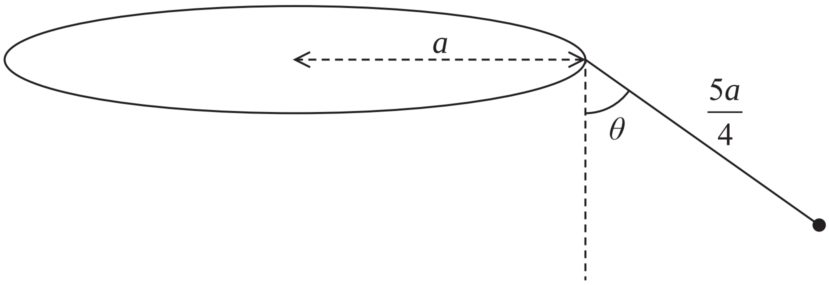

Figure 2

Figure 2 shows a fairground ride that consists of a chair of mass m attached to one end of a rigid arm of length $\frac{5a}{4}$. The other end of the arm is freely hinged to the rim of a thin horizontal circular disc of radius a. The disc rotates with constant angular speed ω about a vertical axis through the centre of the disc. As the ride rotates the arm remains in a vertical plane through the centre of the disc. The arm makes a constant angle θ with the vertical, where $\tan\theta = \frac{3}{4}$

The chair is modelled as a particle and the arm is modelled as a light rod.

(a) Find the tension in the arm in terms of m and g

(3)

(b) Find ω in terms of a and g

(6)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. The finite region enclosed by the curve with equation $y = 3 - \sqrt{x}$ and the lines $x = 0$ and $y = 0$ is rotated through 2π radians about the x-axis, to form a uniform solid S.

Use algebraic integration to

(a) show that the volume of S is $\frac{27}{2}\pi$

(4)

(b) find the x coordinate of the centre of mass of S.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

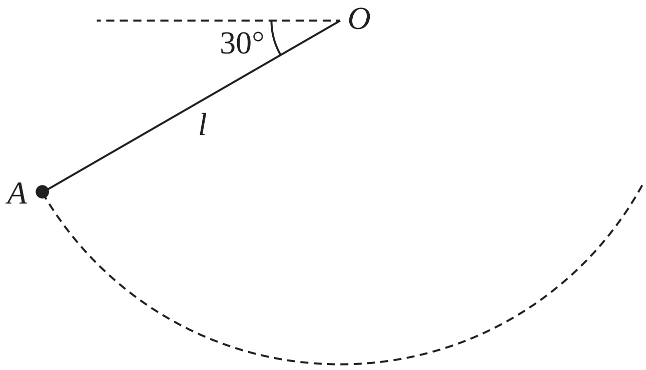

Figure 3

A circus performer has mass m. She is attached to one end of a cable of length l.
The other end of the cable is attached to a fixed point O

Initially she is held at rest at point A with the cable taut and at an angle of 30° below the horizontal, as shown in Figure 3.

The circus performer is released from A and she moves on a vertical circular path with centre O

The circus performer is modelled as a particle and the cable is modelled as light and inextensible.

(a) Find, in terms of m and g, the tension in the cable at the instant immediately after the circus performer is released.

(2)

(b) Show that, during the motion following her release, the greatest tension in the cable is 4 times the least tension in the cable.

(7)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A particle P of mass 0.5 kg moves on the x-axis under the action of a single force.

At time t seconds, $t \geqslant 0$

- $OP = x$ metres, $0 \leqslant x < \frac{\pi}{2}$
- the force has magnitude $\sin 2x$ N and is directed towards the origin O
- P is moving in the positive x direction with speed $v\,\text{m s}^{-1}$

At time $t = 0$, P passes through the origin with speed $2\,\text{m s}^{-1}$

(a) Show that $v = 2\cos x$

(6)

(b) Show that $t = \frac{1}{2}\ln\left(\sqrt{2} + 1\right)$ when $x = \frac{\pi}{4}$

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. A particle P of mass 0.4 kg is attached to one end of a light elastic string, of natural length 0.8 m and modulus of elasticity 0.6 N. The other end of the string is fixed to a point A on a rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{7}$

The particle P is projected from A, with speed $1.8\,\text{m s}^{-1}$, along the surface of the table.

After travelling 0.8 m from A, the particle passes through the point B on the table.

(a) Find the speed of P at the instant it passes through B.

(5)

The particle P comes to rest at the point C on the table, where ABC is a straight line.

(b) Find the total distance travelled by P as it moves directly from A to C.

(6)

(c) Show that P remains at rest at C.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 14 marks)

7.

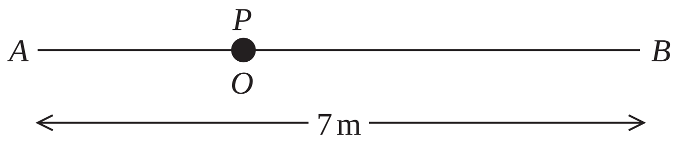

Figure 4

The fixed points A and B are 7 m apart on a smooth horizontal surface.

A light elastic string has natural length 2 m and modulus of elasticity 4 N. One end of the string is attached to a particle P of mass 2 kg and the other end is attached to A

Another light elastic string has natural length 3 m and modulus of elasticity 2 N. One end of this string is attached to P and the other end is attached to B

The particle P rests in equilibrium at the point O, where AOB is a straight line, as shown in Figure 4.

(a) Show that $OA = 2.5$ m.

(4)

The particle P now receives an impulse of magnitude 6 N s in the direction OB

(b) (i) Show that P initially moves with simple harmonic motion with centre O

(ii) Determine the amplitude of this simple harmonic motion.

(8)

The point C lies on OB. As P passes through C the string attached to B becomes slack.

(c) Find the speed of P as it passes through C

(2)

(d) Find the time taken for P to travel directly from O to C

(3)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

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Total Marks

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Turn over

Leave blank

1. A particle P is moving in a straight line with simple harmonic motion of period 4 s. The centre of the motion is the point O

At time $t = 0$, P passes through O

At time $t = 0.5$ s, P is moving with speed $2\,\text{m}\,\text{s}^{-1}$

(a) Show that the amplitude of the motion is $\dfrac{4\sqrt{2}}{\pi}$ m

(4)

(b) Find the maximum speed of P

(2)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. In this question solutions relying on calculator technology are not acceptable.

A particle P of mass 2 kg is moving along the positive x-axis.

At time t seconds, where $t \geqslant 0$, P is x metres from the origin O and is moving away from O with speed $v\,\text{m s}^{-1}$ where $v = \dfrac{1}{\sqrt{(2x+1)}}$

(a) Find the magnitude of the resultant force acting on P when its speed is $\dfrac{1}{3}\,\text{m s}^{-1}$

(6)

When $t = 0$, P is at O

(b) Find the value of t when P is 7.5 m from O

(5)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 11 marks)

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3.

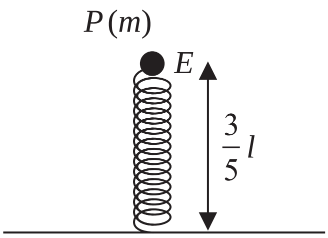

Figure 1

A particle P of mass m is attached to one end of a light elastic spring of natural length l and modulus of elasticity kmg, where k is a constant. The other end of the spring is fixed to horizontal ground.

The particle P rests in equilibrium, with the spring vertical, at the point E.

The point E is at a height $\frac{3}{5}l$ above the ground, as shown in Figure 1.

(a) Show that $k = \frac{5}{2}$

(2)

The particle P is now moved a distance $\frac{1}{4}l$ vertically downwards from E and released from rest. Air resistance is modelled as being negligible.

(b) Show that P moves with simple harmonic motion.

(4)

(c) Find the speed of P as it passes through E.

(4)

(d) Find the time from the instant P is released to the first instant it passes through E.

(2)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 12 marks)

Leave blank

4. A light elastic string has natural length $2a$ and modulus of elasticity $2mg$.

One end of the elastic string is attached to a fixed point O. A particle P of mass $\frac{1}{2}m$ is attached to the other end of the elastic string.

The point A is vertically below O with $OA = 4a$.

Particle P is held at A and released from rest. The speed of P at the instant when it has moved a distance a upwards is $\sqrt{3ag}$

Air resistance to the motion of P is modelled as having magnitude kmg, where k is a constant.

Using the model and the work-energy principle,

(a) show that $k = \frac{1}{4}$

(7)

Particle P is now held at O and released from rest. As P moves downwards, it reaches its maximum speed as it passes through the point B.

(b) Find the distance OB.

(4)

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Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

Leave blank

5.

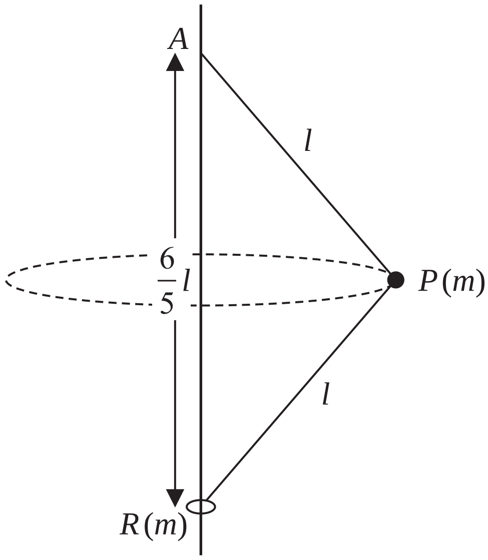

Figure 2

A small smooth ring R of mass m is threaded on to a thin smooth fixed vertical pole. One end of a light inextensible string of length $2l$ is attached to a point A on the pole. The other end of the string is attached to R. A particle P of mass m is attached to the midpoint of the string. The particle P moves with constant angular speed in a horizontal circle, with both halves of the string taut, and $AR = \frac{6l}{5}$, as shown in Figure 2.

It may be assumed that in this motion the string does not wrap itself around the pole and that at any instant, the triangle APR lies in a vertical plane.

(a) Show that the tension in the lower half of the string is $\frac{5mg}{3}$

(3)

(b) Find, in terms of l and g, the time for P to complete one revolution.

(8)

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Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 11 marks)

Leave blank

6.

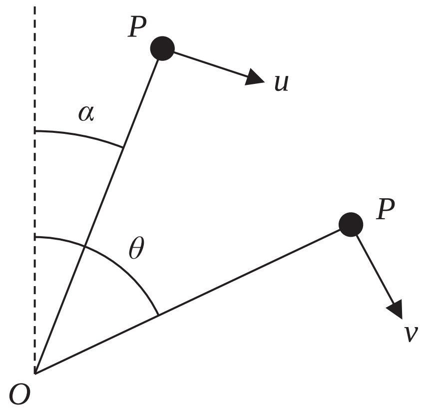

Figure 3

A light rod of length a is free to rotate in a vertical plane about a horizontal axis through one end O. A particle P of mass m is attached to the other end of the rod. The particle P is held at rest with the rod making an angle α with the upward vertical through O, where $\tan\alpha = \frac{3}{4}$

The particle P is then projected with speed u in a direction which is perpendicular to the rod. At the instant when the rod makes an angle θ with the upward vertical through O, the speed of P is v, as shown in Figure 3.

Air resistance is assumed to be negligible.

(a) Show that $v^2 = u^2 + \frac{2ag}{5}(4 - 5\cos\theta)$

(4)

It is given that $u^2 = \frac{6ag}{5}$ and P moves in complete vertical circles.

When $\theta = \beta$, the force exerted on P by the rod is zero.

(b) Find the value of $\cos\beta$

(6)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. [*You may assume that the volume of a cone of height h and base radius r is* $\frac{1}{3}\pi r^2 h$.]

A uniform solid right circular cone C, with vertex V, has base radius r and height h.

(a) Show that the centre of mass of C is $\frac{3}{4}h$ from V

(4)

A solid F, shown below in Figure 4, is formed by removing the solid right circular cone C' from C, where cone C' has height $\frac{1}{3}h$ and vertex V

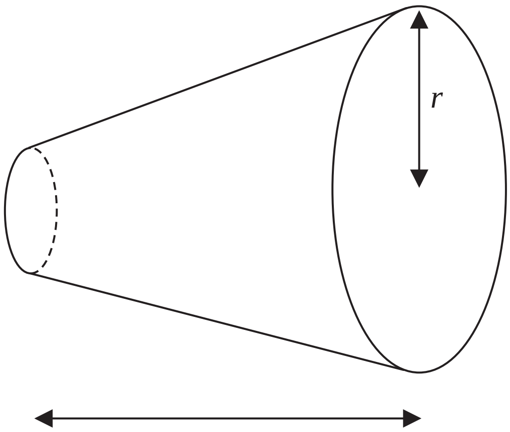

Figure 4

(b) Show that the distance of the centre of mass of F from its larger plane face is $\frac{3}{13}h$

(5)

The solid F rests in equilibrium with its curved surface in contact with a horizontal plane.

(c) Show that $13r^2 \leqslant 17h^2$

(5)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

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Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 14 marks)

TOTAL FOR PAPER = 75 MARKS

END

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Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
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Total Marks

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Turn over ▶

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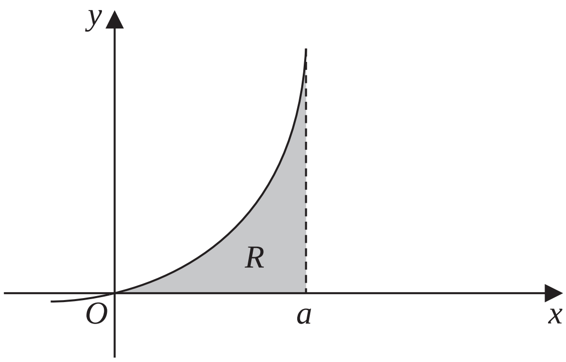

Figure 1

A uniform lamina is in the shape of the region R.

Region R is bounded by the curve with equation $y = x(x + a)$ where a is a positive constant, the positive x-axis and the line with equation $x = a$, as shown shaded in Figure 1.

Find the ***y* coordinate** of the centre of mass of the lamina.

(7)

Question 1 continued

(Total for Question 1 is 7 marks)

2.

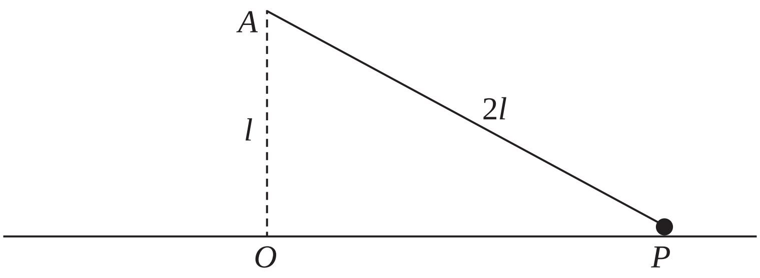

Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length $2l$. The other end of the string is attached to a fixed point A above a smooth horizontal floor. The particle moves in a horizontal circle on the floor with the string taut. The centre O of the circle is vertically below A with $OA = l$, as shown in Figure 2.

The particle moves with constant angular speed ω and remains in contact with the floor.

Show that

$$\omega \leqslant \sqrt{\frac{g}{l}}$$

(8)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P of mass m kg is initially held at rest at the point O on a smooth inclined plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{2}{5}$

The particle is released from rest and slides down the plane against a force which acts towards O. The force has magnitude $\frac{1}{3}mx^2$ N, where x metres is the distance of P from O.

(a) Find the speed of P when $x = 2$

(6)

The particle first comes to instantaneous rest at the point A.

(b) Find the distance OA.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4.

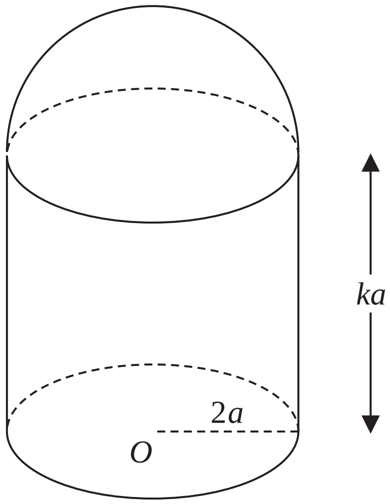

Figure 3

A thin uniform right hollow cylinder, of radius $2a$ and height ka, has a base but no top. A thin uniform hemispherical shell, also of radius $2a$, is made of the same material as the cylinder. The hemispherical shell is attached to the end of the cylinder forming a container C. The open circular rim of the cylinder coincides with the rim of the hemispherical shell. The centre of the base of C is O, as shown in Figure 3.

(a) Show that the distance from O to the centre of mass of C is

$$\frac{(k^2 + 4k + 4)}{2(k + 3)}a$$

(5)

The container is placed with its circular base on a plane which is inclined at 30° to the horizontal. The plane is sufficiently rough to prevent C from sliding. The container is on the point of toppling.

(b) Find the value of k.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. A particle P is moving along the x-axis. At time t seconds the displacement of P from the origin O is x metres, where $x = 4\cos\left(\frac{1}{5}\pi t\right)$

(a) Prove that P is moving with simple harmonic motion.

(3)

(b) Find the period of the motion.

(2)

(c) State the amplitude of the motion.

(1)

(d) Find, in terms of π, the maximum speed of P

(2)

The points A and B lie on the x-axis, on opposite sides of O, with $OA = 1.5\,\text{m}$ and $OB = 2.5\,\text{m}$.

(e) Find the time taken by P to move directly from A to B.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6. A particle P of mass 1.2 kg is attached to the midpoint of a light elastic string of natural length 0.5 m and modulus of elasticity λ newtons.

The fixed points A and B are 0.8 m apart on a horizontal ceiling. One end of the string is attached to A and the other end of the string is attached to B.

Initially P is held at rest at the midpoint M of the line AB and the tension in the string is 30 N.

(a) Show that $\lambda = 50$

(3)

The particle is now held at rest at the point C, where C is 0.3 m vertically below M. The particle is released from rest.

(b) Find the magnitude of the initial acceleration of P

(6)

(c) Find the speed of P at the instant immediately before it hits the ceiling.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 15 marks)

7.

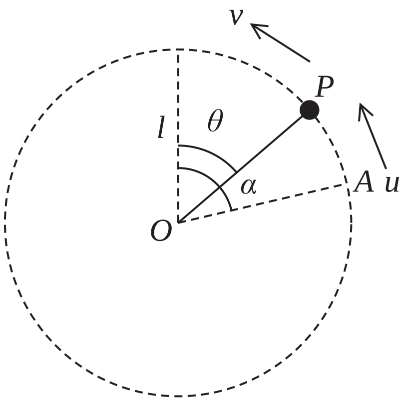

Figure 4

A particle P of mass m is attached to one end of a light rod of length l. The other end of the rod is attached to a fixed point O. The rod can rotate freely in a vertical plane about O. The particle is projected with speed u from a point A. The line OA makes an angle α with the upward vertical through O, where $\alpha < \frac{\pi}{2}$

When OP makes an angle θ with the upward vertical through O, the speed of P is v, as shown in Figure 4.

(a) Show that $v^2 = u^2 - 2gl(\cos\theta - \cos\alpha)$

(4)

Given that $\cos\alpha = \frac{2}{5}$ and that $u = \sqrt{3gl}$

(b) show that P moves in a complete vertical circle.

(4)

As the rod rotates, the least tension in the rod is T and the greatest tension is kT

(c) Find the exact value of k

(9)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

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- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. A particle P moves in a straight line with simple harmonic motion between two fixed points A and B. The particle performs 2 complete oscillations per second. The midpoint of AB is O and the midpoint of OA is C

The length of AB is 0.6 m.

(a) Find the maximum speed of P

(4)

(b) Find the time taken by P to move directly from O to C

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

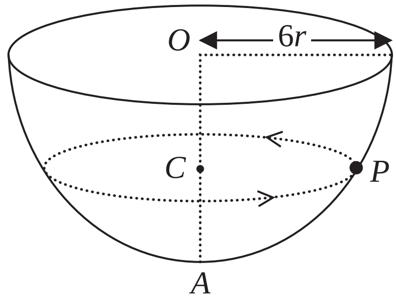

Figure 1

A hemispherical bowl of internal radius $6r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O. A particle P moves in a horizontal circle, with centre C, on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\dfrac{g}{4r}}$. The point C lies on OA, as shown in Figure 1.

Find, in terms of r, the distance OC

(9)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A particle P is moving along a straight line.

At time t seconds, P is a distance x metres from a fixed point O on the line and is moving away from O with speed $\frac{50}{2x+3}$ m s^{-1}

(a) Find the deceleration of P when $x = 12$

(5)

Given that $x = 4$ when $t = 1$

(b) find the value of t when $x = 12$

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4.

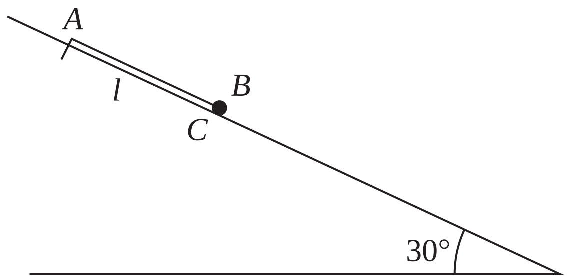

Figure 2

One end of a light elastic string, of natural length l and modulus of elasticity λ, is fixed to a point A on a smooth plane. The plane is inclined at 30° to the horizontal.

A small ball B of mass m is attached to the other end of the elastic string. Initially, B is held at rest at the point C on the plane with the elastic string lying along a line of greatest slope of the plane.

The point C is below A and $AC = l$, as shown in Figure 2.

The ball is released and comes to instantaneous rest at a point D on the plane.

The points A, C and D all lie along a line of greatest slope of the plane and $AD = \frac{5l}{4}$

The ball is modelled as a particle and air resistance is modelled as being negligible.

Using the model,

(a) show that $\lambda = 4mg$

(4)

(b) find, in terms of g and l, the greatest speed of B as it moves from C to D

(7)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

5. (a) Use algebraic integration to show that the centre of mass of a uniform solid hemisphere of radius r is at a distance $\frac{3}{8}r$ from the centre of its plane face.

[*You may assume that the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$]

(5)

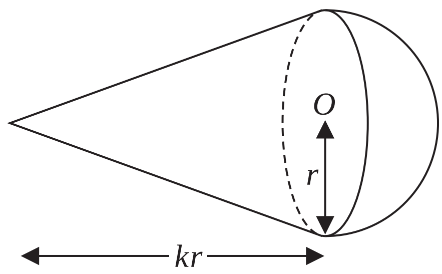

Figure 3

A uniform solid hemisphere of radius r is joined to a uniform solid right circular cone made of the **same material** to form a toy. The cone has base radius r and height kr. The centre of the base of the cone is O. The plane face of the cone coincides with the plane face of the hemisphere, as shown in Figure 3.

The toy can rest in equilibrium on a horizontal plane with any point of the curved surface of the hemisphere in contact with the plane.

(b) Find the exact value of k

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6.

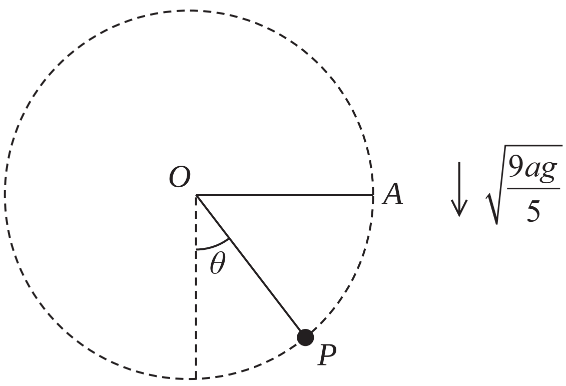

Figure 4

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where $OA = a$ and OA is horizontal, as shown in Figure 4.

The particle is projected vertically downwards with speed $\sqrt{\frac{9ag}{5}}$

When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is S.

(a) Show that $S = \frac{3}{5}mg(5\cos\theta + 3)$

(6)

At the instant when the string becomes slack, the speed of P is v

(b) Show that $v = \sqrt{\frac{3ag}{5}}$

(3)

(c) Find the maximum height of P above the horizontal level of O

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7.

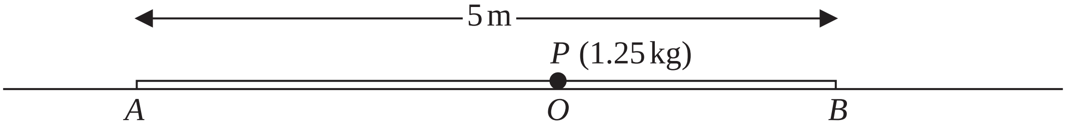

Figure 5

Figure 5 shows two fixed points, A and B, which are 5 m apart on a smooth horizontal floor.

A particle P of mass 1.25 kg is attached to one end of a light elastic string, of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to A

A second light elastic string, of natural length 1.2 m and modulus of elasticity λ newtons, has one end attached to P and the other end attached to B

Initially P rests in equilibrium at the point O, where $AO = 3$ m

(a) Show that $\lambda = 15$

(3)

The particle is now projected along the floor towards B

At time t seconds, P is a displacement x metres from O in the direction OB

(b) Show that, while both strings are taut, P moves with simple harmonic motion where $\ddot{x} = -18x$

(4)

The initial speed of P is $10\,\text{m s}^{-1}$

(c) Find the speed of P at the instant when the string PB becomes slack.

(3)

Both strings are taut for T seconds during one complete oscillation.

(d) Find the value of T

(6)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\text{ m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

1.

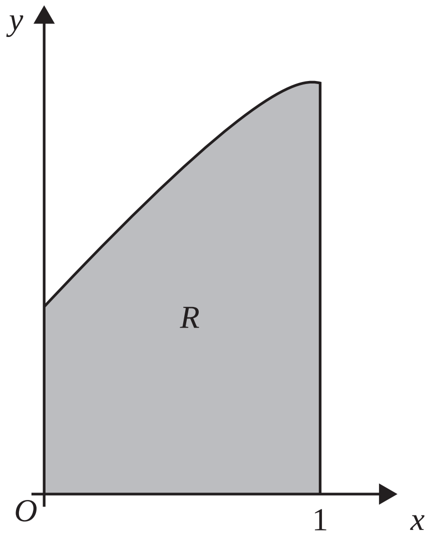

Figure 1

The shaded region R is bounded by the x-axis, the line with equation $x = 1$, the curve with equation $y = 1 + \sqrt{x}$ and the y-axis, as shown in Figure 1. The unit of length on both of the axes is 1 m.

The region R is rotated through 2π radians about the x-axis to form a solid of revolution which is used to model a uniform solid S.

Show, using the model and **algebraic integration**, that

(a) the volume of S is $\frac{17\pi}{6}$ m^3

(3)

(b) the centre of mass of S is $\frac{49}{85}$ m from O.

(5)

Question 1 continued

(Total for Question 1 is 8 marks)

2.

Figure 2

A light elastic string AB has natural length l and modulus of elasticity $2mg$.

The end A of the elastic string is attached to a fixed point. The other end B is attached to a particle of mass m. The particle is held in equilibrium, with the elastic string taut and horizontal, by a force of magnitude F. The line of action of the force and the elastic string lie in the same vertical plane. The direction of the force makes an angle α, where $\tan \alpha = \frac{3}{4}$, with the upward vertical, as shown in Figure 2.

Find, in terms of l, the length AB.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3.

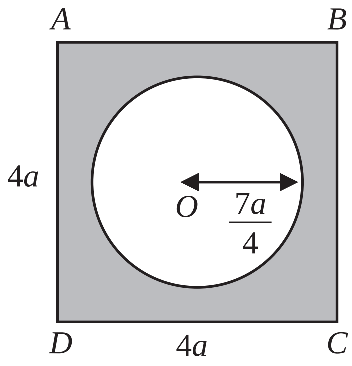

Figure 3

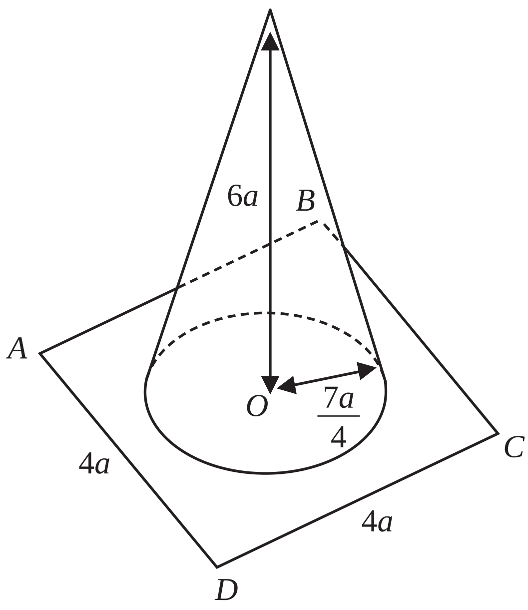

Figure 4

A square *ABCD* of side $4a$ is made from thin uniform cardboard. The centre of the square is *O*. A circle with centre *O* and radius $\frac{7a}{4}$ is then removed from the square to form a template *T*, shown shaded in Figure 3.

A right conical shell, with no base, has radius $\frac{7a}{4}$ and perpendicular height $6a$.

The shell is made of the same thin uniform cardboard as *T*.

The shell is attached to *T* so that the circumference of the end of the shell coincides with the circumference of the circle centre *O*, to form the hat *H*, shown in Figure 4.

[*The surface area of a right conical shell of radius r and slant height l is* πrl.]

(a) Show that the exact distance of the centre of mass of *H* from *O* is

$$\frac{175\pi a}{(63\pi + 128)}$$

(8)

A fixed rough plane is inclined to the horizontal at an angle α. The hat *H* is placed on the plane, with *ABCD* in contact with the plane, and *AB* parallel to a line of greatest slope of the plane. The plane is sufficiently rough to prevent the hat from sliding down the plane.

Given that the hat is on the point of toppling,

(b) find the exact value of $\tan\alpha$, giving your answer in simplest form.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages in your working.**

Solutions relying entirely on calculator technology are not acceptable.

A particle P is moving along the x-axis.

At time t seconds, where $0 \leqslant t \leqslant \frac{2}{3}$, P is x metres from the origin O and is moving with velocity $v\ \text{m s}^{-1}$ in the positive x direction where

$$v = (2x + 1)^{\frac{3}{2}}$$

When $t = 0$, P passes through O.

(a) Find the value of x when the acceleration of P is $243\ \text{m s}^{-2}$

(4)

(b) Find v in terms of t.

(6)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

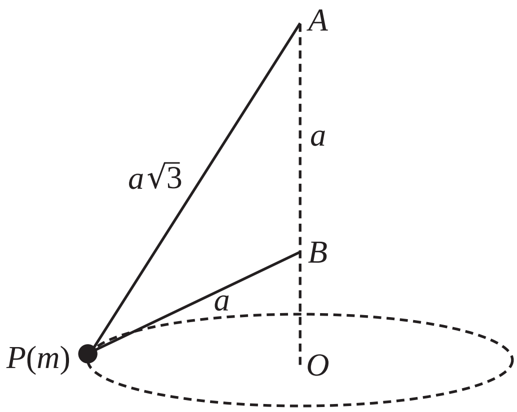

Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length $a\sqrt{3}$. The other end of the string is attached to a fixed point A. The particle P is also attached to one end of a second light inextensible string of length a. The other end of this string is attached to a fixed point B, where B is vertically below A, with $AB = a$.

The particle P moves in a horizontal circle with centre O, where O is vertically below B.

The particle P moves with constant angular speed ω, with both strings taut, as shown in Figure 5.

(a) Show that the upper string makes an angle of 30° with the downward vertical and the lower string makes an angle of 60° with the downward vertical.

(2)

(b) Show that the tension in the upper string is $\frac{1}{2}m\sqrt{3}(2g - a\omega^2)$.

(8)

(c) Show that $\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 14 marks)

6.

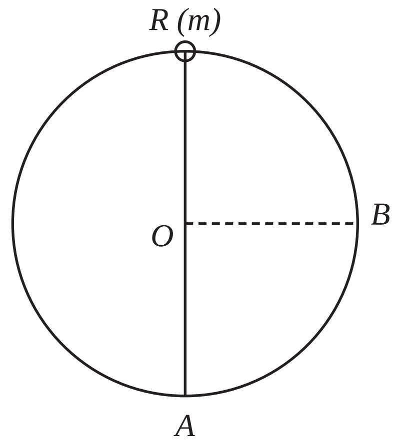

Figure 6

A small smooth ring R of mass m is threaded on to a smooth wire in the shape of a circle with centre O and radius l. The wire is fixed in a vertical plane. The ring R is attached to one end of a light elastic string of natural length l and modulus of elasticity mg. The other end of the elastic string is attached to A, the lowest point of the wire. The point B is on the wire and OB is horizontal.

The ring R is at rest at the highest point of the wire, as shown in Figure 6.

The ring R is slightly disturbed from rest and slides along the wire.

At the instant when R reaches the point B, the speed of R is v and the magnitude of the force exerted on R by the wire is N.

(a) Show that

$$v^2 = 2gl\sqrt{2}$$

(7)

(b) Show that

$$N = \frac{1}{2}mg\left(5\sqrt{2} - 2\right)$$

(7)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 14 marks)

7.

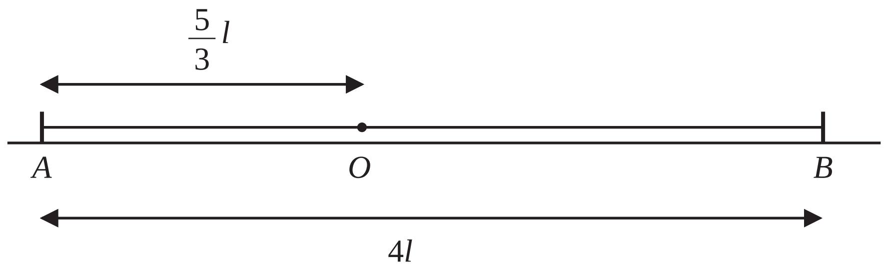

Figure 7

Two points A and B lie on a smooth horizontal table where $AB = 4l$.

A particle P of mass m is attached to one end of a light elastic spring of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to A. The particle P is also attached to one end of another light elastic spring of natural length l and modulus of elasticity mg. The other end of the spring is attached to B.

The particle P rests in equilibrium on the table at the point O, where $AO = \frac{5}{3}l$, as shown in Figure 7.

The particle P is moved a distance $\frac{1}{2}l$ along the table, from O towards A, and released from rest.

(a) Show that P moves with simple harmonic motion of period T, where

$$T = 2\pi\sqrt{\frac{l}{3g}}$$

(6)

(b) Find, in terms of l and g, the speed of P as it passes through O.

(1)

(c) Find, in terms of g, the maximum acceleration of P.

(1)

(d) Find the exact time, in terms of l and g, from the instant when P is released from rest to the instant when P is first moving with speed $\frac{3}{4}\sqrt{gl}$

(5)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 6 June 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information

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- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. **In this question you must show all stages in your working.**

Solutions relying on calculator technology are not acceptable.

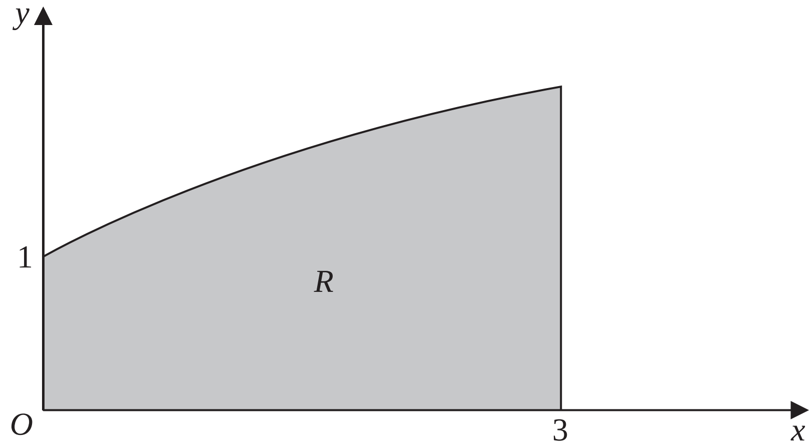

Figure 1

The finite region R, shown shaded in Figure 1, is bounded by the x-axis, the line with equation $x = 3$, the curve with equation $y = \sqrt{(x+1)}$ and the y-axis.
Find the ***y* coordinate** of the centre of mass of a uniform lamina in the shape of R.

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

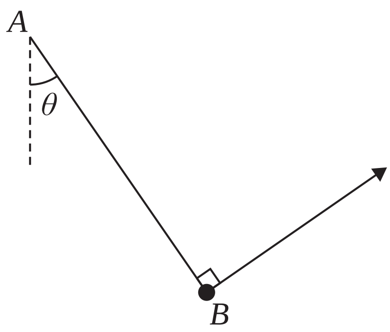

Figure 2

A light elastic string AB has modulus of elasticity $2mg$ and natural length ka, where k is a constant.
The end A of the elastic string is attached to a fixed point. The other end B is attached to a particle of mass m. The particle is held in equilibrium, with the elastic string taut, by a force that acts in a direction that is perpendicular to the string. The line of action of the force and the elastic string lie in the same vertical plane. The string makes an angle θ with the downward vertical at A, as shown in Figure 2.

Given that the length $AB = \frac{21}{10}a$ and $\tan\theta = \frac{3}{4}$, find the value of k.

(6)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 6 marks)

3. A uniform solid right circular cone C has base radius r, height H and vertex V.
A uniform solid S, shown in Figure 3, is formed by **removing** from C a uniform solid right circular cone of height h $(h < H)$ that has the same base and axis of symmetry as C.

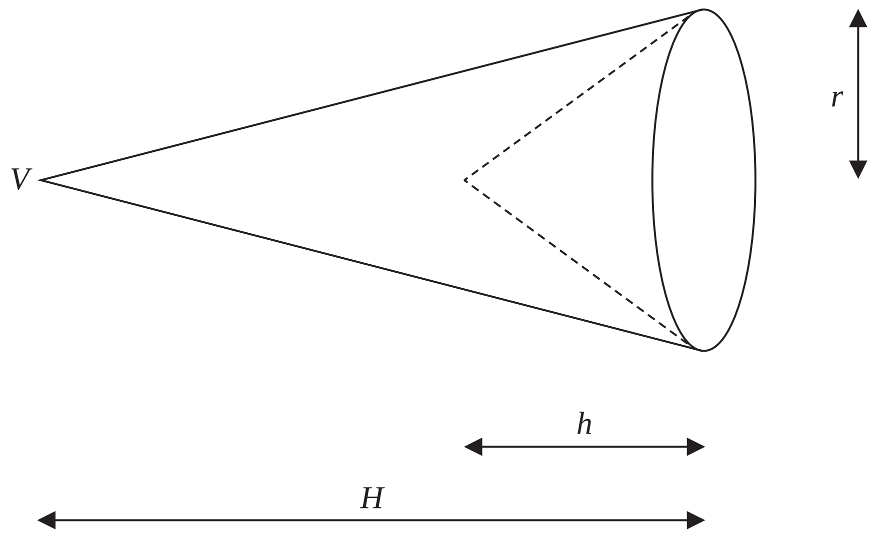

Figure 3

(a) Show that the distance of the centre of mass of S from V is

$$\frac{1}{4}(3H - h)$$

(5)

The solid S is suspended by two vertical light strings. The first string is attached to S at V and the second string is attached to S at a point on the circumference of the circular base of S.
The solid S hangs freely in equilibrium with its axis of symmetry horizontal.
The tension in the first string is T_1 and the tension in the second string is T_2

(b) Find $\frac{T_1}{T_2}$, giving your answer in terms of H and h, in its simplest form.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4.

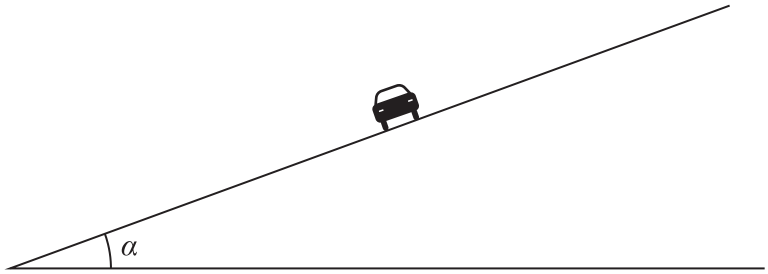

Figure 4

A car is travelling round a circular track. The track is **banked** at an angle α to the horizontal, as shown in Figure 4.

The car and driver are modelled as a particle.

The car moves round the track with constant speed in a horizontal circle of radius r.

When the car is moving with speed $\frac{1}{2}\sqrt{gr}$ round the circle, there is **no** sideways friction between the tyres of the car and the track.

(a) Show that $\tan\alpha = \frac{1}{4}$

(5)

The sideways friction between the tyres of the car and the track has coefficient of friction μ, where $\mu < 4$

The maximum speed at which the car can move round the circle without slipping sideways is V.

(b) Find V in terms of μ, r and g.

(7)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)

5. The centre of the Earth is the point O and the Earth is modelled as a fixed sphere of radius R.
At time $t = 0$, a particle P is projected vertically upwards with speed U from a point A on the surface of the Earth.

At time t seconds, where $t \geqslant 0$

- P is a distance x from O
- P is moving with speed v
- P has acceleration of magnitude $\dfrac{gR^2}{x^2}$ directed towards O

Air resistance is modelled as being negligible.

(a) Show that $v^2 = \dfrac{2gR^2}{x} + U^2 - 2gR$

(6)

Particle P is first moving with speed $\dfrac{1}{2}\sqrt{gR}$ at the point B.

(b) Given that $U = \sqrt{gR}$ find, in terms of R, the distance AB.

(3)

(c) Find, in terms of g and R, the smallest value of U that would ensure that P never comes to rest, explaining your reasoning.

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6.

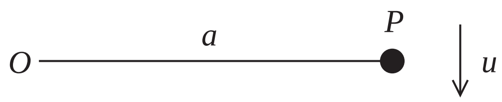

Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at rest with the string taut and horizontal and is then projected vertically downwards with speed u, as shown in Figure 5.

Air resistance is modelled as being negligible.

At the instant when the string has turned through an angle θ and the string is taut, the tension in the string is T.

(a) Show that $T = \dfrac{mu^2}{a} + 3mg\sin\theta$

(7)

Given that $u = 2\sqrt{\dfrac{3ag}{5}}$

(b) find, in terms of a and g, the speed of P at the instant when the string goes slack.

(4)

(c) Hence find, in terms of a, the maximum height of P above O in the subsequent motion.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length l. The other end of the string is attached to a fixed point on a ceiling. The particle P hangs in equilibrium at a distance D below the ceiling.

The particle P is now pulled vertically downwards until it is a distance $3l$ below the ceiling and released from rest.

Given that P comes to instantaneous rest just before it reaches the ceiling,

(a) show that $D = \frac{5l}{3}$

(6)

(b) Show that, while the elastic string is stretched, P moves with simple harmonic motion, with period $2\pi\sqrt{\frac{2l}{3g}}$

(6)

(c) Find, in terms of g and l, the exact time from the instant when P is released to the instant when the elastic string first goes slack.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 19 January 2024

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information

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- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

1. A spacecraft S of mass m moves in a straight line towards the centre, O, of a planet.

The planet is modelled as a fixed sphere of radius R.

The spacecraft S is modelled as a particle.
The gravitational force of the planet is the only force acting on S.

When S is a distance x $(x \geqslant R)$ from O

- the gravitational force is directed towards O and has magnitude $\dfrac{mgR^2}{2x^2}$
- the speed of S is v

(a) Show that

$$v^2 = \frac{gR^2}{x} + C$$

where C is a constant.

(3)

When $x = 3R$, $v = \sqrt{3gR}$

(b) Find, in terms of g and R, the speed of S as it hits the surface of the planet.

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2.

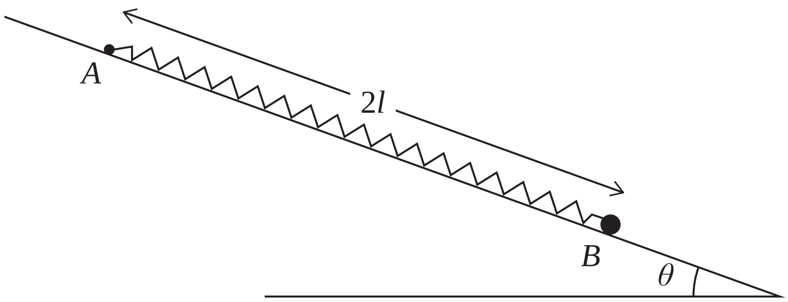

Figure 1

A light elastic **spring** has natural length l and modulus of elasticity λ
One end of the spring is attached to a point A on a smooth plane.

The plane is inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle P of mass m is attached to the other end of the spring.

Initially P is held at the point B on the plane, where AB is a line of greatest slope of the plane.

The point B is lower than A and $AB = 2l$, as shown in Figure 1.

The particle is released from rest at B and first comes to instantaneous rest at the point C on AB, where $AC = 0.7l$

(a) Use the principle of conservation of mechanical energy to show that

$$\lambda = \frac{100}{91}mg$$

(5)

(b) Find the acceleration of P when it is released from rest at B.

(4)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3.

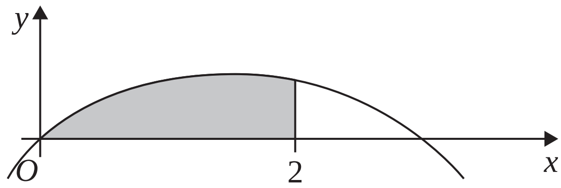

Figure 2

The shaded region in Figure 2 is bounded by the x-axis, the line with equation $x = 2$ and the curve with equation $y = \frac{1}{4}x(3 - x)$.

This region is rotated through 2π radians about the x-axis, to form a solid of revolution which is used to model a uniform solid S.

The volume of S is $\frac{2}{5}\pi$

(a) Use the model and algebraic integration to show that the x coordinate of the centre of mass of S is $\frac{31}{24}$

(5)

The solid S is placed with its circular face on a rough plane which is inclined at $\alpha°$ to the horizontal. The plane is sufficiently rough to prevent S from sliding.

The solid S is on the point of toppling.

(b) Find the value of α

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4.

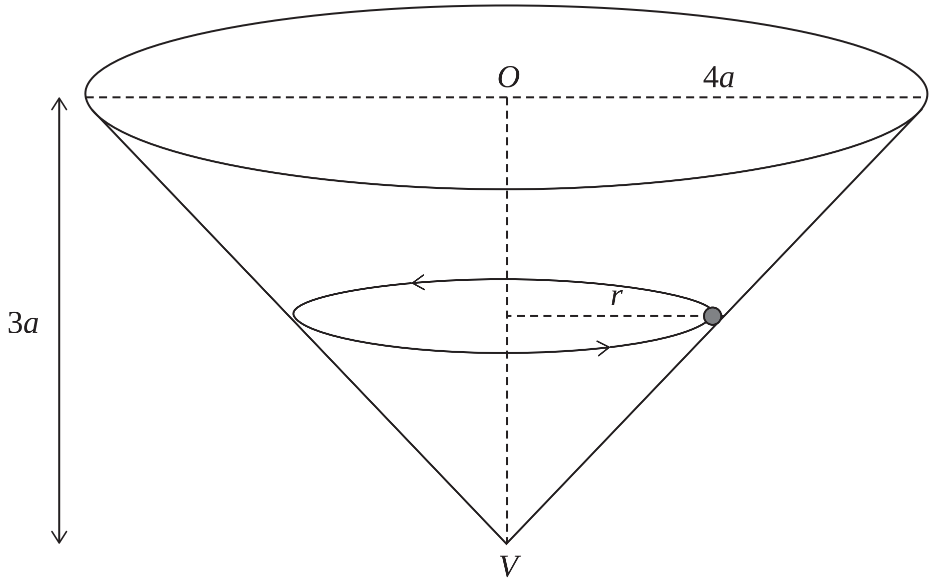

Figure 3

Figure 3 shows a thin hollow right circular cone fixed with its circular rim horizontal.

The centre of the circular rim is O. The vertex V of the cone is vertically below O.

The radius of the circular rim is $4a$ and $OV = 3a$.

A particle P of mass m moves in a horizontal circle of radius r $(0 < r < 4a)$ on the inner surface of the cone.

The coefficient of friction between P and the inner surface of the cone is $\frac{1}{4}$

The particle moves with a constant angular speed.

Show that the maximum possible angular speed is $\sqrt{\frac{16g}{13r}}$

(9)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. (a) Use algebraic integration to show that the centre of mass of a uniform semicircular disc of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from the diameter through O

[You may assume, without proof, that the area of a circle of radius r is πr^2]

(5)

A uniform lamina L is in the shape of a semicircle with centre B and diameter $AC = 8a$. The semicircle with diameter AB is removed from L and attached to the straight edge BC to form the template T, shown shaded in Figure 4.

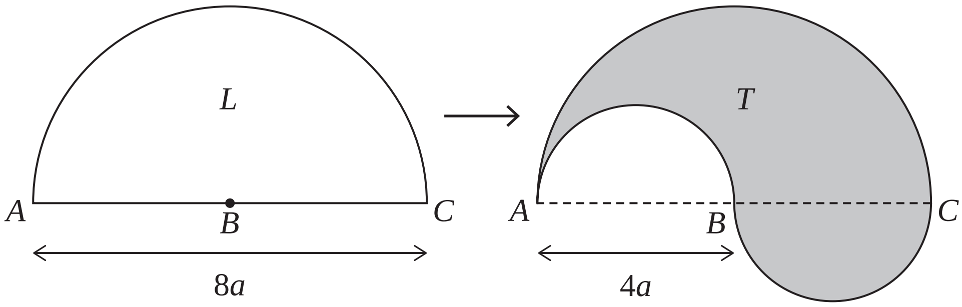

Figure 4

The distance of the centre of mass of T from AC is d.

(b) Show that $d = \frac{4a}{\pi}$

(5)

The template T is freely suspended from A and hangs in equilibrium with AC at an angle θ to the downward vertical.

(c) Find the exact value of $\tan\theta$

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 16 marks)

6. The fixed point A is **vertically above** the fixed point B, with $AB = 3l$

A light elastic string has natural length l and modulus of elasticity $4mg$
One end of the string is attached to A and the other end is attached to a particle P of mass m

A second light elastic string also has natural length l and modulus of elasticity $4mg$
One end of this string is attached to P and the other end is attached to B.

Initially P rests in equilibrium at the point E, where AEB is a **vertical** straight line.

(a) Show that $AE = \frac{13}{8}l$

(4)

The particle P is now held at the point that is a distance $2l$ vertically below A and released from rest.

At time t, the vertical displacement of P from E is x, where x is measured vertically downwards.

(b) Show that $\ddot{x} = -\frac{8g}{l}x$

(4)

(c) Find, in terms of g and l, the speed of P when it is $\frac{1}{8}l$ below E.

(3)

(d) Find the length of time, in each complete oscillation, for which P is more than $1.5l$ from A, giving your answer in terms of g and l

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 14 marks)

7.

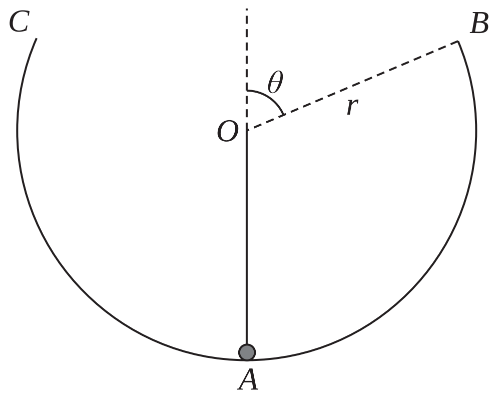

Figure 5

A thin smooth hollow spherical shell has centre O and radius r. Part of the shell is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point A is the lowest point of the bowl, as shown in Figure 5.

The point B is on the rim of the bowl, with OB at an angle θ to the upward vertical, where $\tan\theta = \frac{12}{5}$

A small ball is placed in the bowl at A. The ball is projected from A with horizontal speed u and moves in the vertical plane AOB. The ball stays in contact with the bowl until it reaches B.

At the instant when the ball reaches B, the speed of the ball is v.

By modelling the ball as a particle and ignoring air resistance,

(a) use the principle of conservation of mechanical energy to show that

$$v^2 = u^2 - \frac{36}{13}gr$$

(3)

(b) show that $u^2 \geqslant \frac{41}{13}gr$

(4)

The point C is such that BC is a diameter of the rim of the bowl.

Given that $u^2 = 4gr$

(c) use the model to show that, after leaving the inner surface of the bowl at B, the ball falls back into the bowl before reaching C.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 12 June 2024

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

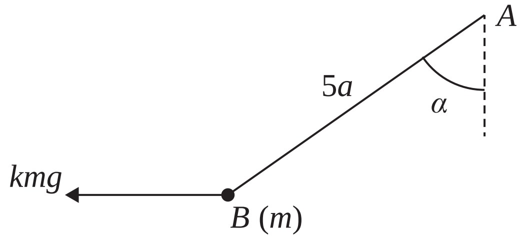

Figure 1

A light elastic string AB has natural length $4a$ and modulus of elasticity λ.
The end A is attached to a fixed point and the end B is attached to a particle of mass m.
The particle is held in equilibrium, with the string stretched, by a horizontal force of magnitude kmg.
The line of action of the horizontal force lies in the vertical plane containing the elastic string.

The string AB makes an angle α with the vertical, where $\tan\alpha = \frac{4}{3}$

With the particle in this position, $AB = 5a$, as shown in Figure 1.

(a) Show that $\lambda = \frac{20mg}{3}$

(4)

(b) Find the value of k.

(4)

Question 1 continued

(Total for Question 1 is 8 marks)

2.

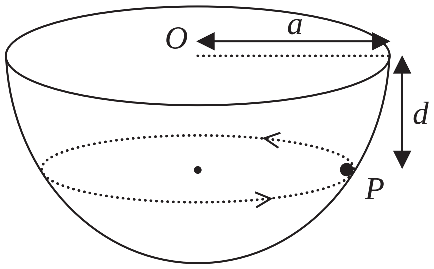

Figure 2

A thin hemispherical shell, with centre O and radius a, is fixed with its open end uppermost and horizontal.

A particle P of mass m moves in a horizontal circle on the smooth inner surface of the shell. The vertical distance of P below the level of O is d, as shown in Figure 2.

(a) Find, in terms of m, g, d and a, the magnitude of the force exerted on P by the inner surface of the hemisphere.

(3)

The particle moves with constant speed v.

(b) Find v in terms of g, a and d.

(5)

Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P is moving along the x-axis.

At time t seconds, where $t \geqslant 0$, the displacement of P from the origin O is x metres and P is moving with velocity $v\,\text{m}\,\text{s}^{-1}$ in the positive x direction.

The acceleration of P is $\dfrac{3\sqrt{x+1}}{4}\,\text{m}\,\text{s}^{-2}$ in the positive x direction.

When $t = 0$, $x = 15$ and $v = 8$

(a) Show that $v = (x+1)^{\frac{3}{4}}$

(4)

(b) Find t in terms of v.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. In a harbour, the water level rises and falls with the tides with simple harmonic motion.

On a particular day, the depths of water in the harbour at low and high tide are 4 m and 10 m respectively.

Low tide occurs at 12:00 and high tide occurs at 18:20

(a) Find, in $m h^{-1}$, the speed at which the water level is rising on this particular day at 13:35

(6)

A ship can only safely enter the harbour when the depth of water is at least 8.5 m.

(b) Find the earliest time after 12:00 on this particular day at which it is safe for the ship to enter the harbour, giving your answer to the nearest minute.

(4)

Question 4 continued

(Total for Question 4 is 10 marks)

5. A uniform right solid circular cone C has radius r and height $4r$.

(a) Show, using algebraic integration, that the distance of the centre of mass of C from its vertex is $3r$.

[You may assume that the volume of C is $\frac{4}{3}\pi r^3$]

(4)

A uniform solid S, shown below in Figure 3, is formed by **removing** from C a uniform solid right circular cylinder of height r and radius $\frac{1}{2}r$, where the centre of one end of the cylinder coincides with the centre of the plane face of C and the axis of the cylinder coincides with the axis of C.

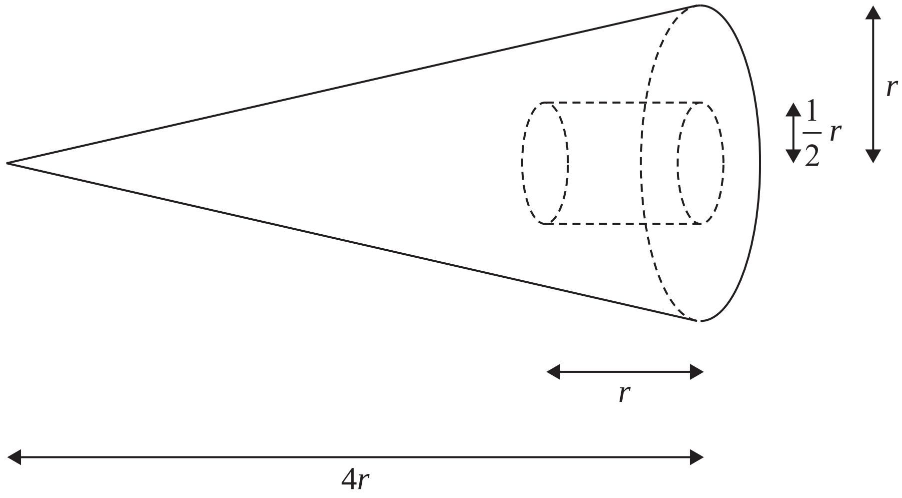

Figure 3

(b) Show that the distance of the centre of mass of S from the vertex of C is $\frac{75}{26}r$

(5)

A rough plane is inclined at an angle α to the horizontal.

The solid S rests in equilibrium with its plane face in contact with the inclined plane.

Given that S is on the point of toppling,

(c) find the exact value of $\tan\alpha$

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6.

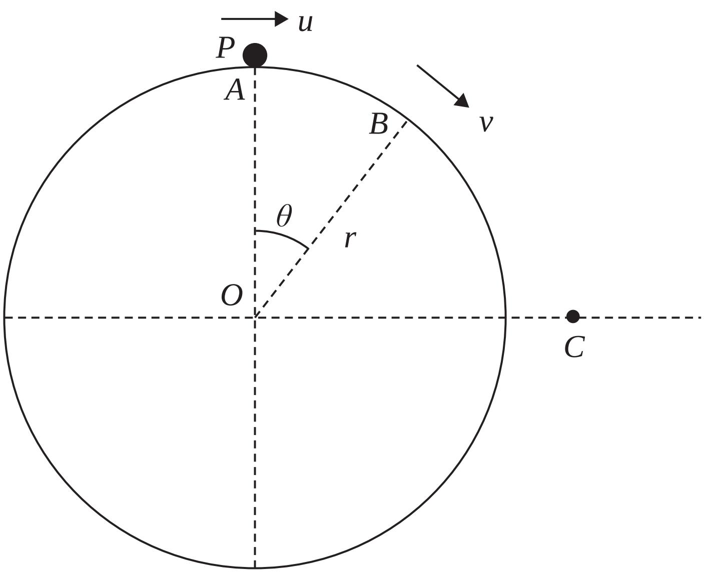

Figure 4

A fixed solid sphere has centre O and radius r.
A particle P of mass m is held at rest on the smooth surface of the sphere at A, the highest point of the sphere.
The particle P is then projected horizontally from A with speed u and moves on the surface of the sphere.
At the instant when P reaches the point B on the sphere, where angle $AOB = \theta$, P is moving with speed v, as shown in Figure 4.

At this instant, P loses contact with the surface of the sphere.

(a) Show that

$$\cos\theta = \frac{2gr + u^2}{3gr}$$

(7)

In the subsequent motion, the particle P crosses the horizontal through O at the point C, also shown in Figure 4.

At the instant P passes through C, P is moving at an angle α to the horizontal.

Given that $u^2 = \dfrac{2gr}{5}$

(b) find the exact value of $\tan\alpha$.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point A on a smooth horizontal table. The particle P is at rest at the point B on the table, where $AB = l$.

At time $t = 0$, P is projected along the table with speed U in the direction AB.

At time t

- the elastic string has not gone slack
- $BP = x$
- the speed of P is v

(a) Show that

$$v^2 = U^2 - \frac{2gx^2}{l}$$

(4)

(b) By differentiating this equation with respect to x, prove that, before the elastic string goes slack, P moves with simple harmonic motion with period $\pi\sqrt{\frac{2l}{g}}$

(5)

Given that $U = \sqrt{\frac{gl}{2}}$

(c) find, in terms of l and g, the exact total time, from the instant it is projected from B, that it takes P to travel a total distance of $\frac{3}{4}l$ along the table.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS