Question Number	Scheme	Marks
1(a)	$V=\pi\int_1^a y^2\,dx$	
	$V=(\pi)\int_1^a \frac{1}{x^2}\,dx=(\pi)\left[-\frac{1}{x}\right]_1^a$	M1A1
	$V=\pi\left(1-\frac{1}{a}\right)$*	A1*
		(3)
(b)	$(\pi)\int xy^2\,dx$	M1
	$(\pi)\int_1^a \frac{1}{x}\,dx=(\pi)[ln\,x]_1^a=(\pi)\,ln\,a$	dM1A1
	$(\pi)\left(1-\frac{1}{a}\right)\bar{x}=(\pi)\,ln\,a$	
	$\bar{x}=\frac{a\,ln\,a}{a-1}$	M1A1
		(5)
		[8]

Question Number	Scheme	Marks
2(a)	$F = \dfrac{k}{(x+R)^2}$	M1
	$x = 0, F = mg \rightarrow mg = \dfrac{k}{R^2}$	M1
	$k = mgR^2 \rightarrow F = \frac{mgR^2}{(x+R)^2}*$	A1*
		(3)
(b)		
	$mv\dfrac{dv}{dx} = -\dfrac{mgR^2}{(x+R)^2} \quad or \quad m\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -\dfrac{mgR^2}{(x+R)^2}$	M1
	$\dfrac{1}{2}v^2 = -\int \dfrac{gR^2}{(x+R)^2}dx$	dM1
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x+R}(+c)$	A1
	$x = R, v = U$	M1
	$\dfrac{U^2}{2} = \dfrac{gR^2}{2R} + c \rightarrow c = \dfrac{U^2 - gR}{2}$	A1
	$x = 0 \rightarrow \dfrac{1}{2}v^2 = gR + \dfrac{U^2 - gR}{2}$	
	$v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$	M1, A1
		(7)
		[10]
ALT1 (b)	$\dfrac{mv^2}{2} - \dfrac{mU^2}{2} = -m\int_R^0 \dfrac{gR^2}{(x+R)^2}\, dx$	M1
	$\dfrac{v^2}{2} - \dfrac{U^2}{2} = \left[\dfrac{gR^2}{x+R}\right]_R^0$	dM1 A1
	$\dfrac{v^2}{2} - \dfrac{U^2}{2} = \dfrac{gR^2}{R} - \dfrac{gR^2}{2R}$	M1 A1
	$v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$	M1, A1

Question Number	Scheme	Marks
ALT2 (b)	$mv\dfrac{dv}{dx} = -\dfrac{mgR^2}{(x+R)^2}$	M1
	$\int_U^V v dv = -\int_R^0 \dfrac{gR^2}{(x+R)^2}\, dx$	dM1
	$\left[\dfrac{v^2}{2}\right]_U^v = \left[\dfrac{gR^2}{x+R}\right]_R^0$	A1
	$\dfrac{v^2}{2} - \dfrac{U^2}{2} = \dfrac{gR^2}{R} - \dfrac{gR^2}{2R}$	M1 A1
	$v^2 = U^2 + gR \rightarrow v = \sqrt{U^2 + gR}$	M1, A1
		(7)

Question Number	Scheme	Marks
3(a)	$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$	B1
	$\omega = \pi$	B1
	$T_A \cos\theta - T_B \cos\theta = 600g$	M1A1
	$(T_A - T_B = 750g)$	
	$T_A \sin\theta + T_B \sin\theta = 600 \times \omega^2 \times (5\sin\theta)$	M1A1
	$(T_A + T_B = 3000\pi^2)$	
	Solve their two equations simultaneously	dM1
	$T_A = 1500\pi^2 + 375g = 18000(N)$, $18500(N)$, 18kN, 18.5kN	A1
	$T_B = 1500\pi^2 - 375g = 11000(N)$, $11100(N)$, 11kN, 11.1kN	A1
		(9)
(b)	If the length of the arms increased, then the radius of the circle would increase.	B1
	Therefore the total tension would increase.	dB1
		(2)
		[11]

Question Number	Scheme					Marks
4(a)		Top cone	inside cone	C	S	
	Mass ratio	(-) 1	(-) 1	8	6	B1
	y distance	$5a$	$3a$	$2a$	$\bar{y}$	B1
	My	$5a$	$3a$	$16a$	$6\bar{y}$	
	$8\times 2a - 1\times 5a - 1\times 3a = 6\bar{y}$					M1A1 **ft**
	$6\bar{y} = 8a \to \bar{y} = \frac{4}{3}a$					A1
						(5)
(b)	$\tan\alpha = \frac{3}{8}$ $(\alpha = 20.556 \ldots . or\ 69.44 \ldots .)$					B1
	$\tan\beta = \frac{\frac{3a}{2}}{4a - \frac{4a}{3}} = \frac{9}{16}$ $(\beta = 29.357 \ldots or\ 60.642 \ldots .)$					M1A1ft
	$\alpha + \beta = \ \theta = 50^o$ (or better 49.91379…)					A1
	$or\ 180 - 69.44 \ldots \ - \ 60.64 \ldots . = 50°$					**(4)**
						[9]
ALT (b)	$\cos\theta = \frac{AB^2 + BG^2 - AG^2}{2\ \times AB \times\ BG}$					B1
	$BG^2 = (1.5a)^2 + (4a - \bar{y})^2$ $(= \frac{337a^2}{36})$ $AG^2 = (3a)^2 + (\bar{y})^2$ $(= \frac{97a^2}{9})$ $\{AB^2 = (1.5a)^2 + (4a)^2$ $(= \frac{73a^2}{4})\}$					M1
	$\cos\theta = \ \ldots \ldots \ldots \ldots \ldots \ldots = 0.6439 \ldots \ldots.$					A1ft
	$\theta = 50^o$ (or better 49.91379…)					A1

Question Number	Scheme	Marks
5(a)	$\frac{2mge_1}{2a}$ or $\frac{6mg(4a-e_1)}{4a}$	B1
	$mg+\frac{2mge_1}{2a}=\frac{6mg(4a-e_1)}{4a}$	M1A1
	Solve to find either extension	dM1
	$e_1=2a$ and $e_2=4a-e_1=2a$*	A1*
		(5)
ALT (a)	$mg+\frac{2mge_1}{2a}=\frac{6mge_2}{4a}$, $e_1+e_2=4a$	M1A1
	Solve simultaneously to find either extension	dM1
	$e_1=2a$ and $e_2=4a-e_1=2a$*	A1*
(b)		
	$mg+\frac{2mg(2a-x)}{2a}-\frac{6mg(2a+x)}{4a}=m\ddot{x}$	M1A1A1
	$\ddot{x}=-\frac{5g}{2a}x$ ∴ SHM	A1
		(4)
(c)	$\omega^2=\frac{5g}{2a}$	B1ft
	$v^2=\frac{5g}{2a}\left(a^2-\left(\frac{a}{2}\right)^2\right)$	M1A1
	$v=\sqrt{\frac{15ga}{8}}=\frac{\sqrt{30ga}}{4}$	A1 cso
		(4)
ALT (c)	$\frac{2mga^2}{4a}$ or $\frac{6mg(3a)^2}{8a}$ or $\frac{2mg(\frac{3a}{2})^2}{4a}$ or $\frac{6mg(\frac{5a}{2})^2}{8a}$	B1
	$\frac{2mga^2}{4a}+\frac{6mg(3a)^2}{8a}=\frac{2mg(\frac{3a}{2})^2}{4a}+\frac{6mg(\frac{5a}{2})^2}{8a}+\frac{mga}{2}+\frac{mv^2}{2}$	M1A1
	$v=\sqrt{\frac{15ga}{8}}$	A1
		(4)
		[13]

(a)

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically, with two variable tensions and weight (M0 for setting both extensions as e)

A1 Correct equation.

dM1 Solve to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

(b)

M1 Vertical equation of motion with two different variable tensions, weight and $m\ddot{x}$ (allow ma)

A1 Equation with at most one error (allow ma for this mark, which does not count as an error).

A1 Fully correct equation. Must now be $m\ddot{x}$

A1 $\ddot{x} = -\frac{5g}{2a}x$ $\therefore$ SHM. **Must have concluding statement.**

(c)

B1ft Use of their ω^2

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Follow through their ω. Needs amplitude a and $x = \frac{1}{2}a$

A1 Correct equation. No follow through now.

A1 cso

ALT (a) using simultaneous equations

B1 Correct use of Hooke's law for either string. Must include an unknown extension.

M1 Resolve vertically with two tensions in e_1 and e_2 and weight AND give a second equation for $e_1 + e_2$

A1 Both equations correct.

dM1 Solves both equations simultaneously to find either extension.

A1* Correct extensions found for both strings, from fully correct working.

ALT (c)

B1 Use of correct EPE

M1 Complete method to find speed at $\frac{7}{2}a$ above A. Allow with $EPE = k\frac{\lambda x^2}{l}$. Must have all terms.

A1 Correct equation.

A1 Correct final answer

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 + mg(2a) = \frac{1}{2}m(3\sqrt{ag})^2 - mg(2a\cos 60^o)$	M1A1A1
	$(v^2 = 3ag)$	
	$T + mg = \dfrac{mv^2}{2a}$	M1A1
	$T = \dfrac{m(3ag)}{2a} - mg = \dfrac{mg}{2}$	dM1A1
	$T > 0$, therefore string remains taut and particle performs complete vertical circles.	A1
		(8)
(b)	From initial: $\frac{1}{2}mV^2 = \frac{1}{2}m(3\sqrt{ag})^2 + mg(2a - 2a\cos 60^o)$	M1A1
	Or from top: $\frac{1}{2}mV^2 = \frac{1}{2}m(3ag) + mg(4a)$	
	$(V^2 = 11ag)$	
	$T - mg = \dfrac{m(11ag)}{2a}$	M1A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1
		(5)
ALT		**[13]**
(a)	$\dfrac{1}{2}mv^2 + mg(2a\cos 60^o - 2a\cos\theta) = \dfrac{1}{2}m(3\sqrt{ag})^2 \rightarrow v^2 = ag(7 + 4\cos\theta)$	M1A1A1
	$T - mg\cos\theta = \dfrac{mv^2}{2a}$	M1A1
	$T - mg\cos\theta = \frac{mag}{2a}(7 + 4\cos\theta)$ AND $\theta = \pi$ or $\cos\theta \geq -1$	dM1
	$T + mg = \frac{mg}{2}(3) \rightarrow T = \frac{mg}{2}$	A1
	$T > 0$, string stays taut and particle completes vertical circles.	A1
(b)	$\theta = 2\pi \rightarrow T - mg\cos 2\pi = \frac{mg}{2}(7 + 4\cos 2\pi)$	M1, M1
	$T - mg = \dfrac{m(11ag)}{2a}$	A1, A1
	$T = \frac{13mg}{2} < 7mg$. Tension less than critical value, so particle completes vertical circles.	A1

Question Number	Scheme	Marks
7(a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7} \left(= \frac{3g}{14} = 2.1\right)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5g(x+2)\sin\theta + F_r(x+2) + \frac{3x^2}{2 \times 2}$	M1A1A1
	$64 = 14(x+2) + 3x^2$	
	$3x^2 + 14x - 36 = 0$	M1
	$x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
(b)	$T = \frac{3 \times 1.84}{2} (= 2.76)$	B1ft
	$0.5a = 2.76 + 1.4 - 2.1 \quad (= 2.06)$ Acceleration down slope, so particle does not remain at A.	M1A1
		(3)
		[11]

ALT (a)	$0.5u = 4 \rightarrow u = 8$	B1
	$F_{max} = \frac{\sqrt{5}}{5} \times 0.5g \times \frac{\sqrt{45}}{7} (= 2.1)$	B1
	$\frac{1}{2} \times 0.5 \times 8^2 = 0.5gd\sin\theta + F_r d + \frac{3(d-2)^2}{2 \times 2}$	M1A1A1
	$64 = 14d + 3(d-2)^2$	
	$3d^2 + 2d - 52 = 0$	M1
	$d = 3.8 \rightarrow x = 1.8(m) \quad (1.84m)$	M1A1
		(8)
ALT (b)	$T = \frac{3 \times 1.84}{2} (= 2.76)$	B1ft
	Upslope: $F_{max} (= 2.1)$, Downslope: $0.5g\sin\theta + T\ (= 4.16)$	M1
	$4.16 > 2.1$ so there is a resultant force down slope, so the particle does not remain at A	A1 **(3)**