| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $\omega = \dfrac{10\pi}{60}$ (rad s$^{-1}$) | B1 |
| | $F = mg\mu$ (N) | B1 |
| | $F = m \times 0.2\left(\dfrac{\pi}{6}\right)^2 = \dfrac{m\pi^2}{180}$ | M1A1ft |
| | $mg\mu \geq \dfrac{m\pi^2}{180}$ | dM1 |
| | $\mu_{\min} = \dfrac{\pi^2}{180g}, \quad (0.0056, \quad 0.00560)$ | A1 |
| | | **[6]** |
| **B1** | Correct angular speed in radians per second, seen anywhere | |
| **B1** | Correct inequality or equation for Friction, seen or used anywhere | |
| **M1** | Attempt the equation of motion along the radius. Must only contain friction and resultant force (give BOD unless clearly not friction). Allow with their $\omega$ or just $\omega$. | |
| **A1ft** | Correct equation. Follow through their $\omega$ | |
| **dM1** | Eliminate $F$ and solve to find $\mu$. Allow with an inequality or equation. Dependent on previous M1. | |
| **A1** | Correct answer, as shown or 2/3 sf decimal (0.00560). Must not be an inequality now. | |

Special Case: If $F \geq mg\mu$ or $F < mg\mu$ used, leading to $\mu = \dfrac{\pi^2}{180g}$ award max B1B0 M1A1 M1A0

| Question Number | Scheme | Marks |
|---|---|---|
| **2(a)** | $v = \frac{dx}{dt} = \frac{1}{(4x+3)}$ | |
| | $\frac{dt}{dx} = 4x+3$ | |
| | $t = \int (4x+3)\,dx, = \frac{1}{2}\times 4x^2 + 3x + c$ | |
| | OR $\int_0^2 dt = \int_0^X (4x+3)\,dx, = \left[\frac{1}{2}\times 4x^2 + 3x\right]_0^X$ | M1,dM1A1 |
| | $c = 0$ | |
| | $t = 2 = \frac{1}{2}\times 4x^2 + 3x \quad 2x^2 + 3x - 2 = 0$ | dM1 |
| | $x = \frac{1}{2} \; (x = -2)$ | A1cso (5) |
| **(b)** | $a = v\frac{dv}{dx} \qquad alt: \; a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ | M1 |
| | $= \frac{1}{4x+3}\times\frac{-4}{(4x+3)^2}$ | dM1A1 |
| | $\lvert F\rvert = \frac{1}{2}\times\frac{4}{(2+3)^3} = \frac{2}{125} = 0.016\,\text{N}$ | M1 A1cso (5) |
| | | **[10]** |
| **(a)** | | |
| **M1** | Rewrite as $\frac{dx}{dt}$ **and** separate variables to reach a form ready for integration | |
| **dM1** | Attempt the integration (at least one power going up). | |
| **A1** | Correct integration. Constant/limits not needed. | |
| **dM1** | Use $t = 2$ in their expression or substitute correct limits, and solve their 3 term quadratic to find $x$. If solving an incorrect quadratic, evidence of a correct method must be seen. Depends on the previous M mark. | |
| **A1cso** | Obtain $x = \frac{1}{2}$ (and reject -2 if seen) from completely correct work. Constant of integration must have been seen, although we do not need to see evidence of evaluation. | |
| **(b)** | | |
| **M1** | Use $a = v\frac{dv}{dx}$ | |
| **dM1** | Differentiate the given expression for $v$ and obtain an expression for $a$. We need to see a power of 2 (or a power of 3 if using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$). Depends on the previous M mark. | |
| **A1** | Correct expression, any form. | |
| **M1** | Use their acceleration in an equation of motion to obtain a value for $F$. Mass must be included and they must use their value of $x$. Independent, but must have found an expression for acceleration. | |
| **A1cso** | Correct magnitude of $F$. Correct solution only. Can be fraction or decimal. Must be positive. | |
| **3** (a) | $\angle PBA = 30°$ | B1 |

| Question Number | Scheme | Marks |
|---|---|---|
| | $R(\uparrow)\quad T\cos 30° + R\cos 60° = mg$ | M1 |
| | NL2 horizontally: $T\cos 60° + R\cos 30° = mr\omega^2,\ = ma\omega^2\cos 30°$ | M1A1,A1 |
| | $T = \dfrac{m\sqrt{3}}{2}\left(2g - a\omega^2\right)$ o.e. | dM1A1 (7) |
| **(b)** | $R = 2mg - \dfrac{3m}{2}\left(2g - a\omega^2\right) = \dfrac{3ma\omega^2}{2} - mg$ | M1A1 |
| | Use $R \geq 0$ | M1 |
| | $\omega \geq \sqrt{\dfrac{2g}{3a}}$ * | A1cso (4) |
| | | **[11]** |
| **(a)** | | |
| **B1** | Correct angle, seen explicitly, implied by a correct trig ratio, or used. | |
| **M1** | Attempt a vertical equation with 3 forces, $T$ and $R$ resolved. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces. | |
| **M1** | Equation of motion horizontally, two forces resolved and acceleration in either form. Attempt at radius not needed. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces. | |
| **A1** | Correct LHS | |
| **A1** | Correct acceleration with correct radius (which might be seen later in part (a)). | |
| **dM1** | Eliminate $R$ and solve to find expression for $T$. Depends on both previous M marks. Allow this mark even if they have not found an angle. | |
| **A1** | Correct expression for $T$ (any correct equivalent). | |
| **(b)** | | |
| **M1** | Attempt to obtain an expression in $R$. Independent of the M marks in (a), but must have come from 2 equations in $T$ and $R$. | |
| **A1** | Correct unsimplified expression in R | |
| **M1** | Use of the correct inequality for $R$ | |
| **A1cso*** | Obtain given result from fully correct working. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4(a)** | $mg\sin\alpha\times\left(\frac{3l}{2}+e\right)=\mu mg\cos\alpha\times\left(\frac{3l}{2}+e\right)+\frac{1}{2}\times\frac{2mg}{l}e^2$ | M1B1B1A1 |
| | $\frac{3}{5}\left(\frac{3l}{2}+e\right)=\frac{4\mu}{5}\left(\frac{3l}{2}+e\right)+\frac{e^2}{l}$ | |
| | $\mu=\frac{9l^2+6le-10e^2}{4l(3l+2e)}$ * | dM1A1cso (6) |
| **(b)** | $e=l\ \Rightarrow\ \mu=\frac{1}{4}$ or $0.25$ | B1 |
| | $F=\frac{1}{5}mg$ | B1ft |
| | Change in acceleration is due to change of direction of $F$ | |
| | $F_1=2mg-mg\sin\alpha+F_r\left(=\frac{8}{5}mg\right)$ and $F_2=2mg-mg\sin\alpha-F_r\left(=\frac{6}{5}mg\right)$ | M1 |
| | Mag of change in accel $=\frac{F_1-F_2}{m}=\frac{2g}{5}=3.92$ or $3.9\ (\mathrm{m\,s^{-2}})$ | M1A1 (5) |
| | | **[11]** |
| **(a) M1** | Attempt a work-energy equation with a GPE term, a single EPE term and the work done against friction. . (Allow $EPE=k\frac{\lambda x^2}{l}$) | |
| **B1** | Correct EPE at C. (Ignore any extra EPE terms for this mark) | |
| **B1** | Correct GPE | |
| **A1ft** | Correct equation. Follow through their EPE and GPE terms providing they are of the correct form | |
| **dM1** | At least one line of **correct** working to rearrange towards $\mu=$. They do not need to reach $\mu=$ for this mark. | |
| **A1cso*** | Given result obtained with no errors seen and at least one line of correct rearrangement. Must be exactly as printed on paper. | |
| **(b)** | | |
| **B1** | Correct numerical value for $\mu$ seen anywhere in (b). This might be implied by later working. | |
| **B1ft** | Correct value for $F$, seen anywhere in (b). Follow through their $\mu$ but must be dimensionally correct. $\mu$ | |
| **M1** | Attempt 2 equations of motion to find resultant force. (Use of $Change=2F$ ) would imply this mark. | |
| **M1** | Subtract and divide by $m$ to obtain the mag of the change in the acceleration. | |
| **A1** | Must be $\frac{2g}{5}$, or 3.9 or 3.92 (m s$^{-2}$) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **5(a)** | $3amg = \frac{1}{2}m \times 7ag - \frac{1}{2}mv^2$ | M1A2 |
| | $v^2 = ag \quad v = \sqrt{ag}$ | A1 (4) |
| **(b)** | $amg = \frac{1}{2}mw^2 - \frac{1}{2}m \times 7ag$ | M1 |
| | $w^2 = 9ag$ | |
| | $T_1 - mg = \frac{mw^2}{4a}$ | M1 |
| | $T_1 = \frac{13mg}{4}$ | A1 |
| | Speed immediately after impact $= \frac{1}{2}\sqrt{ag}$ | |
| | $4amg = \frac{1}{2}mV^2 - \frac{1}{2}m \times \frac{1}{4}ag$ | M1 |
| | $V^2 = \frac{33}{4}ag$ | |
| | $T_2 - mg = \frac{mV^2}{4a}$ | M1 |
| | $T_2 = \frac{49}{16}mg$ | A1 |
| | $T_1 : T_2 = \frac{13}{4} : \frac{49}{16} = 52 : 49$ | A1 (7) |
| | | [11] |

| | |
|---|---|
| **(a)** | |
| **M1** | Energy equation from projection to reaching the ceiling. Must have at least one GPE term and 2 KE terms |
| **A2** | Correct equation. -1 for each error. |
| **A1cso** | Correct expression for $v$ from fully correct work |
| **(b)** | |
| **M1** | Energy equation from the point of projection to $B$. Must have all required terms |
| **M1** | Form equation of motion at $B$ and eliminate $w^2$ to obtain an expression for $T_1$ Must have attempted a velocity at $B$. Condone $r = a$. |
| **A1** | Correct expression for $T_1$ |
| **M1** | Form energy equation from leaving the ceiling to reaching $B$. Must have attempted to use the coeff of restitution to find the initial speed for this equation. Condone $r = a$. |
| **M1** | Attempt an equation of motion at $B$ and eliminate $V^2$ to obtain an expression for $T_2$. Must have attempted a velocity at $B$. |
| **A1** | Correct expression for $T_2$ |
| **A1cao** | Correct ratio. Question asks for simplest form, so must be 52:49 (Condone $\frac{52}{49}$) |

| Question Number | Scheme | Marks |
|---|---|---|
| **6(a)** | $\frac{20(0.2-x)}{0.4}-\frac{20(0.2+x)}{0.4}=0.4\ddot{x}$ | M1A1 |
| | $-100x=0.4\ddot{x}$ | |
| | $\ddot{x}=-250x \quad \therefore \text{SHM}$ | dM1A1cso (4) |
| **(b)** | $\text{Period}=\frac{2\pi}{\sqrt{250}}$ oe | B1ft (1) |
| **(c)** | $v_{\max}=\frac{2}{0.4}=5\,\text{m s}^{-1}$ | B1 |
| | $a\omega=5 \quad a=\frac{5}{\sqrt{250}}=\frac{1}{\sqrt{10}} \quad (=0.3162...)\,\text{m}$ | M1A1ft (3) |
| **(d)** | $x=a\cos\omega t \quad 0.1=\frac{1}{\sqrt{10}}\cos\sqrt{250}t$ or $x=a\sin\omega t \quad 0.1=\frac{1}{\sqrt{10}}\sin\sqrt{250}t$ | M1A1ft |
| | $t=\frac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $t=\frac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$ | A1 |
| | Time for which $AP>0.5$ | |
| | $=\frac{2\pi}{\sqrt{250}}-2\frac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $=\frac{\pi}{\sqrt{250}}+2\frac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$ | dM1 |
| | $=0.2393...\text{s}$ | A1cso (5) [13] |
| **(a)** | | |
| **M1** | Attempt an equation of motion using a difference of 2 tensions obtained from Hooke's law and having different variable extensions. $\ddot{x}$ or $a$ allowed. Can be in algebraic form. | |
| **A1** | Correct equation. $\ddot{x}$ or $a$ allowed but if $a$ used the signs must indicate it is in the same direction as $\ddot{x}$ Can be in algebraic form. | |
| **dM1** | Rearrange their equation to the required form $\ddot{x}=-\omega^2 x$. Must be $\ddot{x}$. They cannot just lose terms to get to the required form. | |
| **A1ft** | Correct equation, can be numerical as shown or algebraic $\left(e.g.\quad \ddot{x}=-\frac{4\lambda}{ml}x\right)$, **and** state conclusion. If algebraic this must include stating that their "$\omega^2$" is positive. | |
| **(b)** | | |
| **B1ft** | Correct period (numerical) as shown or equivalent. Follow through their $\omega$ from $\ddot{x}$ or $a=\pm\omega^2 x$ (0.40 or better) | |
| **(c)B1** | Correct max speed, seen explicitly or used | |
| **M1** | Using $v_{\max}=a\omega$ to obtain a value for $a$ | |
| **A1ft** | Correct value, exact or decimal (0.32 or better) | |
| **(d)** | | |
| **M1** | Use $x=a\cos\omega t$ or $x=a\sin\omega t$ with $x=\pm 0.1$, their $\omega, a$. | |
| **A1ft** | Correct equation, follow through their $\omega, a$ | |
| **A1** | Correct expression for time from their choice of equation (if only decimal seen, award for 2sf or better 0.078997.. for cos,0.020349… for sin) Must have come from fully correct work in (a) | |
| **dM1** | Complete correct method to obtain the required time. Dependent on previous M mark. | |
| **A1cso** | Correct final answer. 2s.f. or better. Must have come from fully correct work in (a) and (d), although they might not have used $\ddot{x}$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)(i)** | $V=\pi\int_1^2\left(x^2+4\right)^2\,\mathrm{d}x=\pi\int_1^2\left(x^4+8x^2+16\right)\mathrm{d}x$ <br> $=\pi\left[\frac{1}{5}x^5+\frac{8}{3}x^3+16x\right]_1^2=\frac{613\pi}{15}\ (\text{cm}^3)$ * | M1A1 <br> A1cso |
| **(ii)** | $(\pi)\int_1^2 x\left(x^2+4\right)^2\,\mathrm{d}x=(\pi)\int_1^2\left(x^5+8x^3+16x\right)\mathrm{d}x$ <br> $=(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2$ $\quad alt\,(\pi)\left[\frac{\left(x^2+4\right)^3}{6}\right]_1^2$ | M1A1 |
| | $\bar{x}=\dfrac{(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2}{\frac{613}{15}(\pi)}=\dfrac{\frac{129}{2}}{\frac{613}{15}}=1.578...=1.58\ (\text{cm})$ | M1dM1A1 (8) |
| **(b)** | Mass $\quad \frac{613\pi}{15}M \quad 9\pi M \quad 45\pi M \quad \left(36\pi+\frac{613\pi}{15}\right)M=\frac{1153\pi}{15}M$ <br> Dist from $B$ $\quad 0.578 \quad 0.5 \quad 0.5 \quad \bar{y}$ <br> $\frac{613\pi}{15}\times0.578-9\pi\times0.5+45\pi\times0.5=\left(36\pi+\frac{613\pi}{15}\right)\bar{y}$ <br> $\bar{y}=\frac{1249}{2306}=0.5416...=0.54\ \ (\text{cm})$ | B1 <br> B1ft <br> M1A1ft <br> A1 (5) <br> **[13]** |

| | |
|---|---|
| **(a)(i)M1** | Attempt the squaring and integrating (at least one power going up). Allow w/o $\pi$ |
| **A1** | Correct integration allow w/o $\pi$ |
| **A1*cso** | Correct volume, with no errors seen. (Must include $\pi$ and no $V = \ldots$ w/o $\pi$ must have been seen.) |
| **(ii)M1** | Attempt $\int x\left(x^2+4\right)^2 dx$ . Must either expand or obtain $k\left(x^2+4\right)^3$. $\pi$ not needed. Limits not needed |
| **A1** | Correct algebraic integration, $\pi$ not needed. Limits not needed |
| **M1** | Substitute the (correct) limits in their integrated function. Independent, but must have been attempting $\int xy^2 dx$ |
| **M1** | Divide the two integrals (correct way up). Depends on the 1st and 2nd M marks. π and ρ in both or neither. |
| **A1** | Correct final result. Must be 3 sf. |
| | **(SC** Correct answer with no algebraic integration shown can score M0A0 M1 M0A0) |
| **(b)** | |
| **B1** | Correct masses seen explicitly or in an equation. |
| **B1ft** | Correct distances from $B$ (or any vertical axis). Follow through distance from (a). |
| **M1** | Form a moments equation, with lighter cylinder subtracted and the heavier one added |
| **A1ft** | Correct equation, follow through their distance from (a). |
| **A1** | Correct distance from $B$, 2 sf or better |

Alt (b) Find mass and CoM of $S_1$ first $\quad \text{Mass} = \dfrac{478\pi}{15}M \quad CoM = \dfrac{287}{478} \approx 0.6004$

Award B1B1 when all component masses and distances are seen. Complete method needed for M1.
Award first A1 for correct masses/distances initially used in forming both equations.

(Note: Use of 0.58 leads to $\bar{x} = 0.603$ (cm) for $S_1$. This gives a final answer 0.543. If they give 0.54, award full marks, as premature approximation does not affect final answer, but penalise 0.543)

**SC –** If the use $M$ and $5M$ for the masses, award max B0B1 M1A0A0