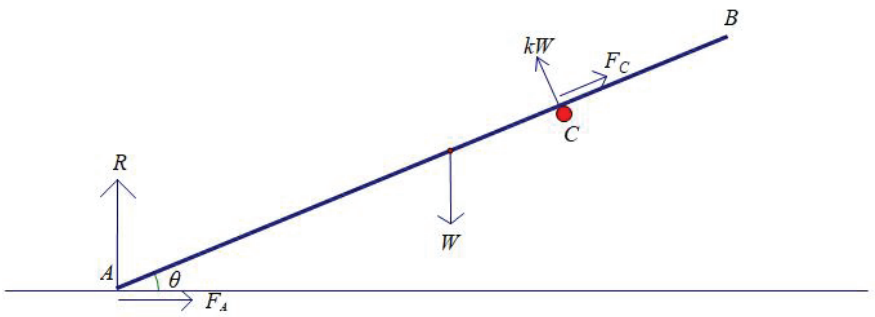
Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

1			Accept vectors given as column vectors throughout this question.
a	Differentiate **r** with respect to t	M1	Obtain $\left(ae^{2t}\right)\mathbf{i}+\left(12+be^{2t}\right)\mathbf{j}$ where $ab\neq 0$. If **i** or **j** missing and not recovered in (a) then M0.
	Obtain $\left(\mathbf{v}=\right)\left(8e^{2t}\right)\mathbf{i}+\left(12-2e^{2t}\right)\mathbf{j}$	A1	Correct only (seen or implied). Condone missing brackets and allow missing **i** or **j** only if recovered in (a).
	Substitute $t=3$ to obtain $\left(\mathbf{v}=\right)\left(8e^{6}\right)\mathbf{i}+\left(12-2e^{6}\right)\mathbf{j}\left(\text{m s}^{-1}\right)$	A1	Or exact equivalent. Condone missing brackets but must be a vector. ISW if go on to find the speed.
		[3]	
b	Differentiate **v** with respect to t	M1	Obtain $\left(ce^{2t}\right)\mathbf{i}+\left(de^{2t}\right)\mathbf{j}$ where $cd\neq 0$. Must be a vector.
	$\left(\mathbf{a}=\right)\left(16e^{2t}\right)\mathbf{i}-\left(4e^{2t}\right)\mathbf{j}\left(\text{m s}^{-2}\right)$	A1	Correct only.
	Equate the **i** and **j** components of their **v** and solve for T or e^{2T}	M1	$8e^{2T}=12-2e^{2T}\Rightarrow e^{2T}=\frac{6}{5}$ or equivalent $\left(T=\frac{1}{2}\ln\frac{6}{5}\right)$ Allow working in t
	$\left(\mathbf{a}=\right)19.2\mathbf{i}-4.8\mathbf{j}\left(\text{ms}^{-2}\right)$	A1	Or exact equivalent. Allow, for example: $\frac{96}{5}\mathbf{i}-\frac{24}{5}\mathbf{j}$, $16e^{\ln\frac{6}{5}}\mathbf{i}-4e^{\ln\frac{6}{5}}\mathbf{j}$ ISW once correct answer seen or if they go on to find the magnitude
		[4]	
		(7)	

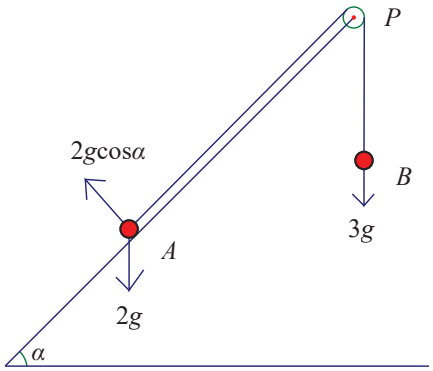
2a	Impulse momentum equation	M1	Dimensionally correct. Condone subtraction either way round but must be subtraction.
	$\pm\mathbf{I} = 5(a\mathbf{i}+\mathbf{j}) - 5(2\mathbf{i}+3\mathbf{j})$ $\left(= 5(a-2)\mathbf{i} - 10\mathbf{j}\right)$	A1	Correct unsimplified equation. Allow vector or scalar for **I** including e.g. $10\sqrt{2}$ or $10\sqrt{2}\mathbf{i}$ LHS may be implied by subsequent working.
	Correct use of Pythagoras' in an equation $2\sqrt{2} = \sqrt{(a-2)^2 + (-2)^2}$	M1	$25(a-2)^2 + 100 = 200$ or equivalent. Allow numerical slip but must be addition of components^2.
	$\Rightarrow a = 4$	A1	Obtain correct answer from correct working If $a = 0$ is stated, it must be rejected.
		[4]	
2b	$\theta° = \tan^{-1}\frac{3}{2} - \tan^{-1}\frac{1}{"4"}$ **or** $\tan^{-1}\frac{"4"}{1} - \tan^{-1}\frac{2}{3}$	M1	Correct use of trig to find the required angle using their a
	$\theta = 42.3$	A1	Accept ±42 or better (42.2736….) or 318 or better.
		[2]	
2b alt	$\cos\theta° = \frac{2\times"4"+3}{\sqrt{13}\sqrt{1^2+"4"^2}}$	M1	Correct use of scalar product or cosine rule with their "a"
	$\theta = 42.3$	A1	Accept 42 or better (42.2736….) or 318 or better
		[2]	
		(6)	

3a	M(A)	M1	Dimensionally consistent (i.e. allow without a) with the required terms and no others. Condone sin / cos confusion but LHS must have an attempt at resolving. M0 if kW is vertical.
	$3a\cos\theta \times W = 4a \times kW$	A1	Correct unsimplified equation. Allow N, R_C, etc. for kW
	$k = \frac{3}{5}$	A1	Exact value only
		[3]	
			
3b	Resolve horizontally	M1	Dimensionally correct. Contains all relevant terms and no others. Condone sign errors and sine / cosine confusion. F_A and F_C must be different. kW and R must be different.
	$F_A + F_C\cos\theta = kW\sin\theta$	A1	Correct unsimplified equation. Allow any alternative for kW but must not be same as R.
	Resolve vertically	M1	Dimensionally correct. Contains all relevant terms and no others. Condone sign errors and sine / cosine confusion. kW and R must be different.
	$R + kW\cos\theta + F_C\sin\theta = W$	A1	Correct unsimplified equation.

			Allow any alternative for kW but must not be same as R.
	$F_A = "\frac{3}{5}" \times W \times \frac{3}{5} - \frac{1}{6} \times "\frac{3}{5}" \times W \times \frac{4}{5} \left(= \frac{7}{25} W \right)$ $R = W - "\frac{3}{5}" \times W \times \frac{4}{5} - \frac{1}{6} \times "\frac{3}{5}" \times W \times \frac{3}{5} \left(= \frac{23}{50} W \right)$	DM1	Substitute for $\cos\theta$, sin θ and k in their equations and use $F = \mu R$ at A and at C Dependent on the two preceding M marks. Condone inexact working for this mark.
	$\Rightarrow \mu = \frac{14}{23}$	A1	From exact working only. Or exact equivalent e.g. $\frac{350}{575}$
		[6]	
		(9)	

N.B. Either of the vertical / horizontal equations could be replaced by any of the below.
If multiple equations given, mark the "best" two. On epen, mark the highest scoring first.

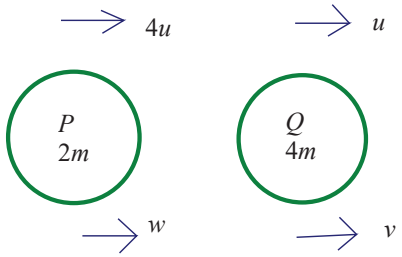
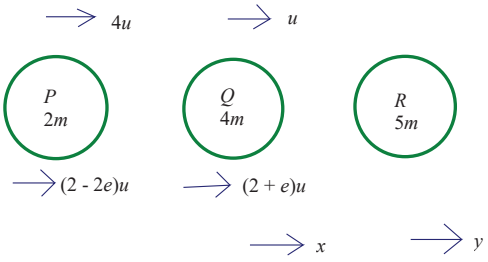
	Resolve parallel to beam	M1	M1: Dimensionally consistent. Contains all relevant terms and no others. Condone sign errors and sin/cos confusion. F_A and F_C must be different. kW and R must be different.
	$R\sin\theta + F_A\cos\theta + F_C = W\sin\theta$	A1	
	Resolve perpendicular to beam	M1	
	$R\cos\theta + kW = F_A\sin\theta + W\cos\theta$	A1	
	Moments about C	M1	
	$4a\cos\theta \times R = a\cos\theta \times W + 4a\sin\theta \times F_A$	A1	
	Moments about B	M1	A1: Correct unsimplified equation. Allow any alternative for kW but must not be same as R.
	$6a\cos\theta \times R + 2a \times kW = 3a\cos\theta \times W + 6a\sin\theta \times F_A$	A1	
	Moments about midpoint of AB	M1	
	$3a\cos\theta \times R = a \times kW + 3a\sin\theta \times F_A$	A1	

4a			
	$F_{\max} = \frac{1}{4} 2g\cos\alpha$	M1	Correct use of $F = \mu R$ Condone sine / cosine confusion
	$= 2.94(\text{N})$ or $2.9(\text{N})$ or $0.3g$	A1	2 sf or 3 sf only if exact multiple of g not seen. E.g. $\frac{147}{50}$ on its own is A0 but $\frac{3g}{10} = \frac{147}{50}$ is A1
		[2]	
4b	Work done against friction = $2.5 \times F_{\max} = 2.5 \times \frac{1}{4} \times 2g \times \frac{3}{5}$	B1ft	Correct substituted expression. May see within a work-energy equation. Condone sine / cosine confusion. Award for 7.4(J) or 7.35(J) or 2.5 × *their (a)*
	GPE lost = GPE lost by B – GPE gained by A	M1	Need both terms. May see within a work-energy equation. Dimensionally correct, Allow subtraction in either order (addition is M0). Condone sine / cosine confusion.
	$= 3g \times 2.5 - 2g \times 2.5\sin\alpha$ (=73.5 – 39.2 = 34.3)	A1	Correct unsimplified expression. Award for GPE gain of –34.3
	Work energy equation: KE gained = GPE lost – WD against friction (or equivalent)	M1	Must be using work-energy Need all terms and no extras (M0 if any GPE is double-counted or missing or if WD against friction is just a force). Condone sign errors.
	$\frac{1}{2} \times 2v^2 + \frac{1}{2} \times 3v^2 = \textit{their}\ 34.3 - \textit{their}\ 7.35$	A1ft	Ft their values for WD against friction and GPE lost.
	$v = 3.3\left(\text{m s}^{-1}\right)$ or $3.28\left(\text{m s}^{-1}\right)$	A1	2 sf or 3 sf only
	N.B. May see all parts contained in a single equation. Still assign marks as: B1ft (friction), M1A1 (GPE), M1A1ft (KE = WD), A1 (answer). It is possible to score e.g. B1M0A0M1A1A0.		

		[6]	
		(8)	

<table>
<tr><td>5a</td><td>Distance from $C = \dfrac{6a\sin\frac{1}{4}\pi}{3\times\frac{1}{4}\pi}$</td><td>M1</td><td>Correct use of the given formula.
Use of $\alpha = \dfrac{\pi}{2}$ is M0
Use of degrees is M0 but allow a “restart”</td></tr>
<tr><td></td><td>Distance from CD
$= \dfrac{6a\sin\frac{1}{4}\pi}{3\times\frac{1}{4}\pi}\times\cos\frac{1}{4}\pi = \dfrac{4a}{\pi}$ *</td><td>A1*</td><td>Obtain given answer from full and correct working. Might use Pythagoras or $\times\sin\frac{1}{4}\pi$; may see working marked on Figure 3. Must have some justification for multiplying by $\dfrac{\sqrt{2}}{2}$</td></tr>
<tr><td></td><td></td><td>[2]</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>5b</td><td><table>
<tr><td></td><td>Rectangle</td><td>Sector</td><td>Lamina</td></tr>
<tr><td>Area</td><td>$24a^2$</td><td>$\dfrac{9a^2}{4}\pi$</td><td>$a^2\left(24-\frac{9\pi}{4}\right)$</td></tr>
<tr><td>CoM from AF</td><td>$3a$</td><td>$6a-\dfrac{4a}{\pi}$</td><td>$(\,d\,)$</td></tr>
<tr><td>CoM from CE</td><td>$3a$</td><td>$\dfrac{4a}{\pi}$</td><td>$(\,6a-d\,)$</td></tr>
</table></td><td>B1</td><td>Correct area ratios for all three (allow with a or a^2 correctly cancelled)</td></tr>
<tr><td></td><td></td><td>B1</td><td>Correct distances for rectangle and sector. Allow from any parallel axis (e.g. CE).</td></tr>
<tr><td></td><td>Moments about AF or a parallel axis</td><td>M1</td><td>Dimensionally consistent. Condone sign errors.</td></tr>
<tr><td></td><td>E.g. $24a^2\times3a-\frac{9a^2}{4}\pi\times\left(6a-\frac{4a}{\pi}\right)=a^2\left(24-\frac{9\pi}{4}\right)d$
or $24\times3a-\frac{9}{4}\pi\times\left(\frac{4a}{\pi}\right)=\left(24-\frac{9\pi}{4}\right)\bar{x}$</td><td>A1</td><td>Correct unsimplified equation (allow $\bar{x}$ etc. instead of d). Condone invisible brackets for this mark.
$\bar{x}$ may be implied, E.g. by a moments expression divided by total area.</td></tr>
<tr><td></td><td>$d = \dfrac{24\times3a-\frac{9}{4}\pi\times\left(6a-\frac{4a}{\pi}\right)}{\left(24-\frac{9\pi}{4}\right)} = \dfrac{96\times3a-54\pi a+36a}{(96-9\pi)}$
$d = \dfrac{108-18\pi}{32-3\pi}a$ *</td><td>A1*</td><td>Obtain given answer from full and correct working (if brackets are missing from working penalise via this mark). Must see at least one line of working between moments equation and printed answer.
Need to include $d =$ … not just $\bar{x}$</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>5c</td><td>Moments about AC or a parallel axis</td><td>M1</td><td>Dimensionally consistent. Condone sign errors</td></tr>
</table>

	Eg. $24a^2 \times 2a - \frac{9\pi a^2}{4} \times \frac{4a}{\pi} = \bar{y}_{AC} \times a^2\left(24 - \frac{9\pi}{4}\right)$ or $24 \times 2a - \frac{9\pi}{4} \times \left(4a - \frac{4a}{\pi}\right) = \bar{y}_{FE} \times \left(24 - \frac{9\pi}{4}\right)$ $\left[\bar{y}_{AC} = \frac{156a}{96 - 9\pi} = \frac{52a}{32 - 3\pi}\right]$ or $\left[\bar{y}_{FE} = \frac{76 - 12\pi}{32 - 3\pi}a\right]$	A1	Correct unsimplified equation for their axis
	$\text{angle} = \tan^{-1}\left(\frac{d}{\bar{y}_{AC}}\right) = \tan^{-1}\frac{108 - 18\pi}{52}$	M1	Correct use of trig with d as given in (a) and denominator as their distance from AC to find the required angle. Use of $\bar{y} = 2a$ is M0.
	$= 44.7°$ or 0.78 radians	A1	Accept $45°$ or better. 44.696127… (0.780094…radians)
		[4]	
		(11)	

6a			
	Equation for CLM	M1	o.e. e.g equal and opposite impulse Need all terms. Dimensionally consistent. Condone sign errors
	$2m\times 4u+4m\times u=2m\times w+4m\times v$ $(6u=w+2v)$	A1	Correct unsimpliified equation
	Correct use of impact law	M1	Must be used the right way round and dimensionally correct. Condone sign errors
	$v-w=e(4u-u)\quad (v-w=3eu)$	A1	Correct unsimplified equation
	$\Rightarrow 3v=6u+3eu,\quad v=2u+eu$ *	A1*	Obtain given answer from full and correct working.
		[5]	
6b	$w=2u-2eu$ or $v=3u-\frac{w}{2}$	B1	Or equivalent. Must be seen in (b).
	$0<w<2u$	B1	Uses $e>0$ or $v>2u$ or $w<v$ to form inequality from correct working only. Must include lower inequality and both must be strict.
		[2]	
6c			
	Form equations for CLM and impact law.	M1	Both equations dimensionally correct and contain all relevant terms with initial speed of Q as given in part (a) in the correct direction.
	$4mu(2+e)=4mx+5my$ $y-x=e\times(2+e)u$	A1	Correct unsimplified equations
	Solve for x	M1	Solve for x in terms of u and e.
	$x=\frac{1}{9}(8u-6ue-5e^2u)$	A1	Or equivalent (could be negative) e.g. $x=\frac{1}{9}(2+e)(4-5e)u$

	E.g. $w-x\left(=(2-2e)u-\frac{1}{9}(2+e)(4-5e)u\right)$ $=\frac{u}{9}(10-12e+5e^2)$	M1	Find the difference between their x and their w to produce a three-term quadratic expression in e
	$5\left(e-\frac{6}{5}\right)^2+\frac{14}{5}>0$ So $w > x$ and there will be a second collision.	A1*	Correct conclusion with correct justification (e.g. $x < w$ so collision) Condone missing $\frac{1}{9}u$ for the justification. Might use discriminant and coefficient of e^2 as justification.
		[6]	
		(13)	

7a	Correct use of $P = Fv$	M1	Need to see value substituted correctly. E.g. $F = \frac{12000}{v}$ Condone incorrect number of zeros
	Equation of motion for the van and trailer	M1	Need all terms. Condone sine / cosine confusion. Condone sign errors. Allow with F or D. Dimensionally correct. M0 if acceleration included unless subsequently taken as zero. If equations of motion for car and trailer are separate, award only when combined.
	$\frac{12000}{v} - 600 - 1100g\sin\theta = 0$	A1	Correct unsimplified equation in v (may also have θ and g)
	Obtain 8.0 or 8.01	A1	2 sf or 3 sf only
		[4]	
7b	Need 2 equations. M1A1 for each equation. Use the mass from the ma term to determine which part of the system is being considered. If three equations given, mark the "best" two. On epen, mark the highest scoring first.		
	First equation e.g. Equation of motion for the van and the trailer	M1	Need all terms. Condone sine / cosine confusion. Condone sign errors. Dimensionally correct (E.g. 4000×2 for driving force is M0).
	$\frac{4000}{2} - 600 - 1100g\sin\theta = 1100a$ $\left(a = 0.456\,\text{m s}^{-2}\right)$	A1	Correct unsimplified equation
	Second equation e.g. Equation of motion for the van or the trailer	M1	Need all terms. Condone sine / cosine confusion. Condone sign error Dimensionally correct (E.g. 4000×2 for driving force is M0).
	$T - 300g\sin\theta - 200 = 300a$ Or $\frac{4000}{2} - 400 - 800g\sin\theta - T = 800a$	A1	Correct unsimplified equation
	Substitute for trig and solve for T	DM1	Dependent on the two preceding M marks
	$T = 582\left(\text{N}\right)$ or 580(N)	A1	2 sf or 3 sf only (over-accuracy or under-accuracy of correct answers should only be penalised once per complete question)
		[6]	
		(10)	

8a	Equation for horizontal motion	M1	Complete method to form an equation in t and θ only
	$49\cos\theta\times t=40$	A1	Correct unsimplified equation
	Equation for vertical motion	M1	Complete method to form an equation in t and θ only. Must be using height of 3.
	$3=49\sin\theta\times t-\frac{1}{2}gt^2$	A1	Correct unsimplified equation
	$t=\frac{40}{49\cos\theta}\Rightarrow$ $3=49\sin\theta.\frac{40}{49\cos\theta}-\frac{1}{2}\times9.8\times\frac{40^2\sec^2\theta}{49^2}$ $6=80\tan\theta-\frac{320}{49}\left(1+\tan^2\theta\right)$	DM1	Form an equation in $\tan\theta$ only using a correct trigonometric identity oe. Dependent on the first 2 M marks Any equivalent form.
	$320\tan^2\theta-3920\tan\theta+614=0$ $a=320, b=-3920, c=614$	A1	Or equivalent to 3 significant figures e.g. A1 for: $a=160, b=-1960, c=307$ or $a=7840, b=96000, c=15000$ but A0 for $a=7840, b=96040, c=15043$
		[6]	
8b	Min speed $\left(=49\cos30°\right)=42\left(\text{m s}^{-1}\right)$	B1	Or better $\left(\frac{49\sqrt{3}}{2}=42.435...\right)$. Must be seen as the answer to (b). Do not ISW if they go on to find a different speed but condone rounding errors after a correct answer is seen.
		[1]	
8c	Time to point above Y $\frac{40}{42.4...}=0.94(3)\left(\text{s}\right)$	M1	Complete method using *suvat*.
	Vertical speed $v=49\sin30°-gt$ $\left(=15.2....\left(\text{m s}^{-1}\right)\right)$	M1	Complete method using *suvat*. Use of s=3 (e.g. with $v^2=u^2+2as$) is M0.
	$v^2=\left(15.2...\right)^2+42.4^2$	DM1	Complete method to obtain the speed at required position (i.e. not top of pole). Dependent on second M mark (i.e. having a vertical speed at required point).
	$v=45\left(\text{m s}^{-1}\right)$ or $45.1\left(\text{m s}^{-1}\right)$	A1	2 sf or 3 sf only
		[4]	

8c alt	Use equation of the trajectory with $\theta = 30°$	M1	$y = 40\tan 30° - \frac{160}{49}\left(1+\tan^2 30°\right)$ $y \neq 3$ i.e. not using their answer to (a) with $\theta = 30°$
	Use $x = 40$ to obtain y	M1	$y = 18.7....$
	Form equation for conservation of energy	DM1	$\frac{1}{2}m\left(v^2 - 49\right) = mgy$ Dependent on second M mark i.e. having found a y.
	$v = 45\left(\text{m s}^{-1}\right)$ or $45.1\left(\text{m s}^{-1}\right)$	A1	2 sf or 3 sf only
		[4]	
		(11)	